Fairness in Ranking: A Survey

MEIKE ZEHLIKE, Humboldt University of Berlin and Max Planck Institute for Software Systems, Germany

KE YANG, New York University, NY, USA

JULIA STOYANOVICH, New York University, NY, USA

In the past few years, there has been much work on incorporating fairness requirements into algorithmic rankers, with contributions coming from the data management, algorithms, information retrieval, and recommender systems communities. In this survey we give a systematic overview of this work, offering a broad perspective that connects formalizations and algorithmic approaches across subfields. An important contribution of our work is in developing a common narrative around the value frameworks that motivate specific fairness-enhancing interventions in ranking. This allows us to unify the presentation of mitigation objectives and of algorithmic techniques to help meet those objectives or identify trade-offs.

arXiv:2103.14000v1 [cs.IR] 25 Mar 2021

1 INTRODUCTION

The research community recognizes several important normative dimensions of information technology including privacy, transparency, and fairness. In this survey we focus on fairness — a broad and inherently interdisciplinary topic of which the social and philosophical foundations are still unresolved [20].

Research on fair machine learning has mainly focused on classification and prediction tasks [7, 20], while we focus on ranking. As is customary in fairness research, we assume that input data describes *individuals* — natural persons seeking education, employment, or financial opportunities, or being prioritized for access to goods and services. While some of the algorithmic techniques described here can be applied to entities other than people, we believe that the concept of fairness, along with the corresponding normative frameworks, applies predominantly to scenarios where data describes people. For consistency, we will refer to the set of individuals in the input to a ranking task as *candidates*.

We consider two types of ranking tasks: score-based and supervised learning. In score-based ranking, a given set of candidates is sorted on the score attribute, which may itself be computed on the fly, and returned in sorted order. In supervised learning, a preference-enriched training set of candidates is given, with preferences among them stated in the form of scores, preference pairs, or lists; this training set is used to train a model that predicts the ranking of unseen candidates. For both score-based and supervised learning tasks, we typically return the best-ranked k candidates, the top-k. Set selection is a special case of ranking that ignores the relative order among the top-k, returning them as a set.

To make this discussion concrete, we now present our running example from university admissions, a domain in which ranking and set selection are very natural and are broadly used.

candidate	A_1	A_2	X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	τ_1	τ_2	τ_3
b	male	White	4	5	5	{cs:0.9; art:0.2}	14	9	1	b	c	k
c	male	Asian	5	3	4	{math:0.9; cs:0.5}	12	9	1	c	b	l
d	female	White	5	4	2	{lit:0.8; math:0.8}	11	4	6	d	e	b
e	male	White	3	3	4	{math:0.8; econ:0.4}	10	7	6	e	f	d
f	female	Asian	3	2	3	{econ:0.9; math:0.5}	8	5	8	f	d	e
k	female	Black	2	2	3	{lit:0.9;art:0.8}	7	1	9	k	o	f
l	male	Black	1	1	4	{lit:0.5; math:0.7}	6	6	2	l	l	c
o	female	White	1	1	2	{econ:0.9; cs:0.8}	4	7	8	o	k	o

(a)   (b)   (c)   (d)

Fig. 1. (a) dataset C of college applicants, with demographic attributes A_1 (sex) and A_2 (race), numerical attributes X_1 (high school GPA), X_2 (verbal SAT), and X_3 (math SAT), and attribute X_4 (choice) that is a vector extracted from the applicants' essays; (b) is a ranking τ_1 on Y_1, computed as the sum of X_1, X_2, and X_3; (c) is a ranking on Y_2, predicted based on historical performance of STEM (cs, econ, math) majors; (d) is a ranking on Y_3, predicted based on historical performance of humanities (art, lit) majors. In all cases, the top-4 candidates will be interviewed in score order, and potentially admitted.

1.1 Running example: university admissions

Consider an admissions officer at a university who selects candidates from a large applicant pool. When making their decision, the officer pursues some or all of the goals listed below. Some of these goals may be legally mandated, while others may be based on the policies adopted by the university, and include admitting students who:

- are likely to succeed: complete the program with high marks and graduate on time;
- show strong interest in specific majors like computer science, art, or literature; and
- form a demographically diverse group in terms of their demographics, both overall and in each major.

Figure 1 shows a dataset C of applicants and illustrates the admissions process. Each applicant submits several quantitative scores, all transformed here to a discrete scale of 1 (worst) through 5 (best) for ease of exposition: X_1 is the high school GPA (grade point average), X_2 is the verbal portion of the SAT (Scholastic Assessment Test) score, and X_3 is the mathematics portion of the SAT score. Attribute X_4 (choice) is a weighted feature vector extracted from the applicant's essay, with weight ranging between 0 and 1, and with a higher value corresponding to stronger interest in a specific major. For example, candidate b is a White male with a high GPA (4 out of 5), perfect SAT verbal and SAT math scores (5 out of 5), a strong interest in studying computer science (feature weight 0.9), and some interest in studying art (weight 0.2).

The admissions officer uses a suite of tools to sift through the applications and identify promising candidates. Many of these tools are *rankers*, illustrated in Figure 2. A ranker takes a dataset of candidates, described by structured features, text, or both, as input and produces a permutation of these candidates, also called a *ranking*. The admissions officer will take the order in which the candidates appear in a ranking under advisement when deciding whom to consider more closely, interview, and admit.

These tools include score-based rankers (Sections 2.1) that compute the score of each candidate based on a formula that the admissions officer gives, and then return some number of highest-scoring applicants in ranked order. This *scoring formula* may, for example, specify the score as a linear combination of the applicant's high-school GPA and the
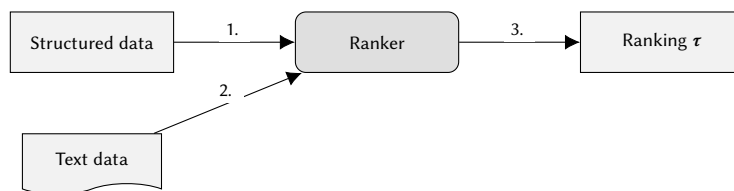
Fig. 2. Functional principle of rankers: (1. and 2.) Structured and text data that correspond to candidates serve as inputs to a ranker; (3.) The ranker outputs a ranking of the candidates τ.

two components of their SAT score, each carrying an equal weight. This is done in Figure 1(a), where a candidate's score is computed as $Y_1 = X_1 + X_2 + X_3$ and then ranking τ_1 in Figure 1(b) is produced.

Predictive analytics are also among the admissions officer's toolkit (Sections 2.2). For example, multiple ranking models may be trained, one per undergraduate major or set of majors, on features X_1, X_2, X_3, X_4 of the successful applicants from the past years, to predict applicant's standing upon graduation (based, e.g., on their GPA in the major). These ranking models are then used to predict a ranking of this year's applicants. In our example in Figure 1(a), feature Y_2 predicts performance in a STEM (Science, Technology, Engineering, Mathematics) major such as computer science (cs), economics (econ), or mathematics (math) and leads to ranking τ_2 in Figure 1(c), while feature Y_3 predicts performance in a humanities major such as literature (lit) or fine arts (art) and leads to ranking τ_3 in Figure 1(d).

The promising applicants identified in this way—with the help of either a score-based ranker or a predictive analytic—will then be considered more closely, *in ranked order*: invited for an interview and potentially admitted.

Let us recall that, in addition to incorporating quantitative scores and students' choice, an admissions officer also aims to admit a demographically diverse group of students to the university and to each major. Further, the admissions officer is increasingly aware that the data on which their decisions are based may be biased, in the sense that this data may carry results of historical discrimination or disadvantage, and that the computational tools at their disposal may be exacerbating or introducing new forms of bias, or even creating a kind of a self-fulfilling prophecy. (See discussion of the types of bias in Section 3.2.) For this reason, the officer may elect to incorporate one or several fairness objectives into the ranking process.

For example, they may assert, for legal or ethical reasons, that the proportion of the female applicants among those selected for further consideration should match their proportion in the input. Applying this requirement to ranking τ_1 in Figure 3 (in which we elaborate on the already familiar example in Figure 1) yields ranking τ_2 in Figure 3(c). Further, the admissions officer may assert that, because applicants are interviewed in ranked order, it is important to achieve proportional representation by sex in *every prefix* of the produced ranking, which yields ranking τ_3 in Figure 3(d). In this survey we give an overview of the technical work that would allow an admissions officer to compute ranked results under these and other fairness requirements.

1.2 Scope and contributions of the survey

In the past few years, there has been much work on incorporating fairness requirements into algorithmic rankers. Giving a systematic overview of this work is the primary goal of our survey.

Which specific fairness requirements an admissions officer will assert depends on the values they are operationalizing and, thus, on the mitigation objectives. An important goal of this survey is to create an explicit mapping between mitigation objectives, which we will characterize in Section 3.3, and existing technical methods for fairness in ranking,

candidate	A_1	A_2	X_1	X_2	X_3	Y
b	male	White	4	5	5	14
c	male	Asian	5	3	4	12
d	female	White	5	4	2	11
e	male	White	3	3	4	10
f	female	Asian	3	2	3	8
k	female	Black	2	2	3	7
l	male	Black	1	1	4	6
o	female	White	1	1	2	4

τ_1	τ_2	τ_3
b	b	b
c	c	d
d	d	c
e	f	f
f	e	e
k	k	k
l	l	l
o	o	o

(a)   (b)   (c)   (d)

Fig. 3. (a) dataset C of college applicants. Score Y is computed by a score-based ranker $f(X) = X_1 + X_2 + X_3$; (b) ranking τ_1 of Y in C; (c) ranking τ_2 with proportional representation by sex at the top-4; (d) ranking τ_3 with proportional representation by sex in every prefix of the top-4. The top-4 candidates will be interviewed in score order and potentially admitted.

discussed in Sections 5, 6, and 7. Without such a mapping, an admissions officer in our running example would have a difficult time selecting an appropriate fairness-enhancing intervention, and would not know which interventions are mutually comparable and which are not.

In our survey we will present a selection of approaches for fairness in ranking that were developed in several subfields of computer science, including data management, algorithms, information retrieval, and recommender systems. We are aware of several recent tutorials on fairness in ranking at RecSys 2019 [32] and VLDB 2020 [4], pointing to the need to systematize the work in this area and motivating our survey. Our goal is to offer a broad perspective, connecting work across subfields.

The primary focus of this survey is on associational fairness measures for ranking, although we do include one recently proposed causal framework.

The rest of this survey is organized as follows: We start with the preliminaries and fix notation in Section 2. We then go on to present classification frameworks along which we relate the technical methods surveyed in this paper in Section 3. In Sections 5, 6, and 7, we describe technical work on fairness in score-based ranking, supervised learning, and recommender systems, respectively. We discuss evaluation datasets and frameworks in Section 8, present important directions of future work in Section 9, and conclude in Section 10.

2 PRELIMINARIES AND NOTATION

In this section we will build on our running example to discuss score-based and supervised learning-based rankers more formally, and fix the necessary notation. We summarize notation in Table 1 and illustrate it throughout this section.

2.1 Score-based ranking

Formally, we are given a set C of candidates; each candidate is described by a set of features X and a score attribute Y. Additionally we are given a set of sensitive attributes A $\subseteq$ X, which are categorical, denoting membership of a candidate in demographic groups.[1] A sensitive attribute $A \in$ A may be binary, with one of the values (e.g., $A = 1$

---

[1]While some sensitive attributes like age or degree of disability may be drawn from a continuous domain, we are not aware of any approaches that represent them as such, and so will assume that sensitive attributes are categorical in the remainder of this survey.

Table 1. Summary of notation used throughout the survey.

C	A set of candidates to be ranked
a, b, c	Candidates in C
n	Number of candidates $|C|$
X	a set of features of the candidates in C
X_a	Features of candidate a
A	A set of sensitive features, $A \subseteq X$
$\mathcal{G}$	A group (subset) of candidates, $\mathcal{G} \subseteq C$
$\mathcal{G}_1$	A protected group (subset) of candidates, $\mathcal{G}_1 \subseteq C$
$\mathcal{U}$	A set of users that use the ranking system
Q	A set of queries
f	a ranker
$\hat{f}$	a ranker learned from training data
Y	the score feature and ground truth for supervised learning
$\hat{Y}$	the scores predicted by $\hat{f}$
Y_a	the score of candidate a
τ	Ranking: permutation of candidates from C
$\tau(i)$	The candidate at position i in τ
$v(i)$	the position bias of rank i
$U^k(\tau)$	Utility of the top-k candidates in τ
$U^k(\tau, \mathcal{G})$	Utility of the top-k candidates of group $\mathcal{G}$ in τ
$U(\tau, a)$	Utility of candidate a in τ
$D(a, b)$	Disparity in visibility between candidates a and b
$D(\mathcal{G}_1, \mathcal{G}_2)$	Disparity in visibility between groups $\mathcal{G}_1$ and $\mathcal{G}_2$

or A = female, as in Figure 1) denoting membership in a minority or historically disadvantaged group (often called "protected group") and with the other value (e.g., $A = 0$ or A = male) denoting membership in a majority or privileged group. Alternatively, a sensitive attribute may take on three or more values, for example, to represent ethnicity or (non-binary) gender identity of candidates.

A *ranking* τ is a permutation over the candidates in C. Letting $n = |C|$, we denote by $\tau = \langle \tau_1, \ldots, \tau_n \rangle$ a ranking that places candidate τ_i at rank i. We denote by $\tau(i)$ the candidate at rank i, and by $\tau^{-1}(a)$ the rank of candidate a in τ. We are often interested in a sub-ranking of τ containing its best-ranked k elements, for some integer $k \leq n$; this sub-ranking is called the top-k and is denoted $\tau_{1\ldots k}$. For example, Figure 3b presents $\tau = \langle b, c, d, e, f, k, l, o \rangle$, $\tau(3) = d$, $\tau^{-1}(l) = 7$, and the top-4 is $\tau_{1\ldots 4} = \langle b, c, d, e \rangle$.

Utility. Because score Y is assumed to encode a candidate's appropriateness, quality, or *utility*, a score-based ranking usually satisfies:

$$Y_{\tau(1)} \geq Y_{\tau(2)} \geq \ldots \geq Y_{\tau(n)} \tag{1}$$

We will find it convenient to denote by $U^k(\tau)$ the utility of $\tau_{1\ldots k}$. Different methods surveyed in this paper adopt different notions of utility, and we will make their formulations precise as appropriate. The simplest method is to treat $\tau_{1\ldots k}$ as a set (disregarding candidate positions), and to compute the utility of the set as the sum of scores of its

elements:

$$U^k(\boldsymbol{\tau}) = \sum_{i=1}^{k} Y_{\boldsymbol{\tau}(i)} \tag{2}$$

Another common method incorporates position-based discounts, following the observation that it is more important to present high-quality items at the top of the ranked list, since these items are more likely to attract the attention of the consumer of the ranking. For example, we may compute position-discounted utility of a ranking as:

$$U^k(\boldsymbol{\tau}) = \sum_{i=1}^{k} \frac{Y_{\boldsymbol{\tau}(i)}}{\log_2(i+1)} \tag{3}$$

For example, the utility at top-4 of $\boldsymbol{\tau}$ in Figure 3b is 30 based on Equation 2 and 19.6 based on Equation 3. Note that the base of the logarithm in the denominator of Equation 3 is empirically determined, and it can be set to some value $b > 1$ other than 2.

For these variants of utility and for others, it is often useful to quantify utility realized by candidates belonging to a particular demographic group $\mathcal{G} \subseteq C$, defined by an assignment of values to one or several sensitive attributes. For example, $\mathcal{G}$ may contain female candidates, or Asian female candidates. We can then compute to the utility of $\boldsymbol{\tau}_{1\ldots k}$ (per Equation 2) for group $\mathcal{G}$ as:

$$U^k(\boldsymbol{\tau}, \mathcal{G}) = \sum_{i=1}^{k} Y_{\boldsymbol{\tau}(i)} \times \mathbb{1}\left[\boldsymbol{\tau}(i) \in \mathcal{G}\right] \tag{4}$$

Here, $\mathbb{1}$ is an indicator variable that returns 1 when $\boldsymbol{\tau}(i) \in \mathcal{G}$ and 0 otherwise. Position-discounted utility (per Equation 3) for group $\mathcal{G}$ can be defined analogously. For the ranking in Figure 3b, $U^4(\boldsymbol{\tau}, sex = \texttt{male}) = 36$, $U^4(\boldsymbol{\tau}, sex = \texttt{male} \wedge race = \texttt{White}) = 24$, and $U^4(\boldsymbol{\tau}, sex = \texttt{male} \wedge race = \texttt{Black}) = 0$.

Fairness. To satisfy objectives other than utility, such as *fairness*, we may output a ranking $\hat{\boldsymbol{\tau}}$ that is not simply sorted based on the observed values of Y as in Equation 1. As is the case for classification and prediction, numerous fairness measures have been defined for rankings. These measures can be used both to assess the fairness of a ranking and to intervene on unfairness, for example, by serving as basis for constraints.

A prominent class of fairness measures corresponds to *proportional representation* in the top-k treated as a set, or in every prefix of the top-k. These measures are motivated by the need to mitigate different types of bias, based on assumptions about its origins and with a view of specific objectives (to be discussed in Section 3). For example, ranking $\boldsymbol{\tau}_2$ in Figure 3c re-ranks candidates to satisfy proportional representation by gender at the top-4 (treating it as a set), swapping candidates e and f. The ranking $\boldsymbol{\tau}_3$ in Figure 3d additionally reorders candidates c and d to achieve proportional representation by gender in every prefix of the top-4.

In addition to fairness measures, *diversity* measures have also been proposed in the literature [29]. In this survey we will focus on coverage-based diversity that is most closely related to fairness, and requires that members of multiple, possibly overlapping, groups, be sufficiently well-represented among the top-k, treated either as a set or as a ranked list. Diversity constraints may, for example, be stated to require that members of each ethnic group, each gender group, and of selected intersectional groups on ethnicity and gender, all be represented at the top-k in proportion to their prevalence in the input.

When candidates are re-ranked to meet objectives other than score-based utility, we may be interested to compute *Y-utility loss*, denoted $L_Y(\boldsymbol{\tau}, \hat{\boldsymbol{\tau}})$. We can use a variety of metrics that quantify the distance between ranked lists for
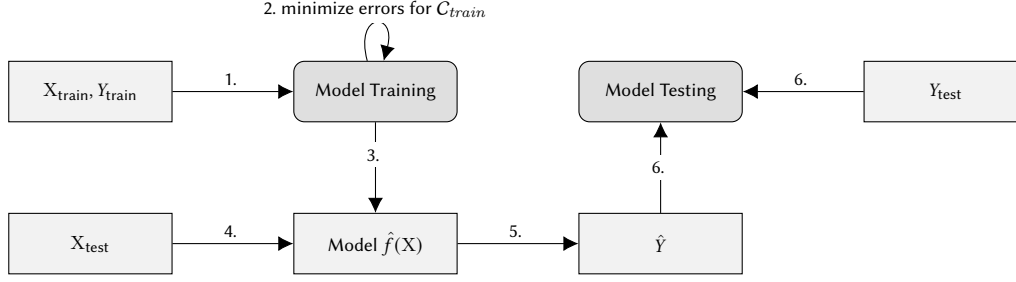
Fig. 4. Functional principle of supervised learning in ranking, commonly denoted learning-to-rank, or LtR. (1.) Training data C_{train}, consisting of tuples (X, Y), is given as input to an LtR algorithm that (2.) trains a ranking function $\hat{f}(X)$ (3.). This is done by minimizing the errors $\hat{f}(X)$ makes when predicting the scores $\hat{Y}$ for C_{train}. (4.) To test predictive accuracy of the model, the features X_{test} of C_{test} are given as input to $\hat{f}$ to (5.) predict scores $\hat{Y}$. (6.) Then, $\hat{Y}$ is compared to the ground truth Y_{test}.

this purpose, including, for example, the Kendall distance that counts the number of pairs that appear in the opposite relative order in τ and $\hat{\tau}$, or one in a family of generalized distances between rankings [48]. However, loss functions that compare rankings τ and $\hat{\tau}$ in their entirety are uncommon. Rather, Y-utility loss is usually specified over the top-k. The simplest formulation is:

$$L_Y^k(\boldsymbol{\tau}, \hat{\boldsymbol{\tau}}) = U^k(\boldsymbol{\tau}) - U^k(\hat{\boldsymbol{\tau}}) \tag{5}$$

Alternatively, we may normalize this quantity:

$$L_Y^k(\boldsymbol{\tau}, \hat{\boldsymbol{\tau}}) = 1 - \frac{U^k(\hat{\boldsymbol{\tau}})}{U^k(\boldsymbol{\tau})} \tag{6}$$

Further, we may be interested to quantify utility loss for a particular demographic group $\mathcal{G}$. In that case, we define the utility of τ and $\hat{\tau}$ for group $\mathcal{G}$, as was done in Equation 4, or analogously for other utility formulations. Interestingly, underrepresented groups may see a gain, rather than a loss, in Y-utility, because they may receive better representation at the top-k when a fairness objective is applied.

In our discussion so far we focused on score-based rankers, with an example in Figure 3. We now discuss ranking methods that use supervised learning and present concepts that are specific to such rankers.

2.2 Supervised learning to rank

In supervised learning to rank (LtR), we are given a set C of candidates; each candidate is described by a set X of features, including also sensitive features $A \subseteq X$. Each candidate $a \in C$ has an associated score attribute Y, which describes their quality with respect to a given task (e.g., college admissions). Every such association forms an instance of either the training dataset C_{train} or the test dataset C_{test}. Like score-based rankers, LtR rankers compute candidate scores and return a ranking τ with the highest-scoring candidates appearing closer to the top (per Eq. 1). The difference between score-based and LtR rankers is in how the score is obtained: in score-based ranking, a function is given to calculate the scores Y, while in supervised learning, the ranking function $\hat{f}$ is learned from a set of training examples and the score $\hat{Y}$ is estimated.[2]

---

[2]Note that the literature distinguishes point-wise, pair-wise and list-wise LtR methods and that Y has a slightly different meaning for each of them [51]. However, because the overall procedure remains the same, we will focus on point-wise LtR in the remainder of this section, and give technical details for pair-wise and list-wise methods in later sections, as appropriate.

candidate	A_1	A_2	X_1	X_2	X_3	X_4	Y_1	Y_2
b	male	White	4	5	5	{cs:0.9; art:0.2}	9	1
c	male	Asian	5	3	4	{math:0.9; cs:0.5}	9	1
d	female	White	5	4	2	{lit:0.8; math:0.8}	8	1
e	male	White	3	3	4	{math:0.8; econ:0.4}	7	6
f	female	Asian	3	2	3	{econ:0.9; math:0.5}	5	8
k	female	Black	2	2	3	{lit:0.9;art:0.8}	1	9
l	male	Black	1	1	4	{lit:0.5; math:0.7}	6	7
o	female	White	1	1	2	{econ:0.9; cs:0.8}	7	8

τ
d
l

(b)

$\hat{\tau}$
l
d

(c)

(a)

Fig. 5. (a) Dataset C of college applicants, with demographic attributes A_1 (sex) and A_2 (race), numerical attributes X_1 (high school GPA), X_2 (verbal SAT), and X_3 (math SAT), and attribute X_4 (choice) that is a vector extracted from the applicants' essays. Scores Y_1 and Y_2 are the respective ground truth scores for queries $Q_1, Q_2 \in Q$. We randomly distribute C into a training dataset $C_{train} = \{b, c, e, f, k, o\}$ and a test dataset $C_{test} = \{d, l\}$ (blue lines); (b) The ground truth ranking of C_{test} for query Q_1. The ranking model $\hat{f}$ should reproduce this ordering of candidates d and l when presented C_{test}; (c) A ranking predicted by a model with a bias against women. Note that by randomly choosing candidates d and l for C_{test} we accidentally injected a bias against women into our training data (all women are now ranked below men). A learning model is likely to pick up this bias and wrongly assign feature A_1 a high weight. (We remark that this is a very simple example on data bias for illustrative purposes).

Figure 4 describes the LtR process. We are given two datasets C_{train} and C_{test}.[3] We use C_{train} to train an LtR model, learning a ranking function $\hat{f}(X)$ that minimizes the prediction errors on Y_{train}. This is usually done by minimizing the sum of the individual errors $\hat{f}$ makes between the ground truth Y and its prediction $\hat{Y}$ for C_{train}. To evaluate the performance of the model $\hat{f}$, we apply it to C_{test}, and then compare ground truth scores and predictions. If model testing succeeds, meaning that the ranker's predictions are deemed sufficiently accurate, then $\hat{f}$ is deployed: a new set of candidates is ranked by predicting their scores $\hat{Y} = \hat{f}(X)$, and ranking the candidates according to these predictions.

As an example, consider Figure 5 that revisits our college admissions example from Figure 1 in a supervised learning setting. We are given six features, as previously described, and two ground truth scores Y_1 and Y_2 for each candidate.

First, training data is prepared as input: We divide the data into a training set $C_{train} = \{b, c, e, f, k, o\}$ and a test set $C_{test} = \{d, l\}$ (blue lines). Then a model is trained and tested using any available LtR method, such as RankNet [15] or ListNet [17]. Figure 5b depicts the ground truth ranking of C_{test} based on score Y_1.

Prediction accuracy. In traditional supervised learning, the term *utility* is often used to refer to prediction accuracy of $\hat{f}$. A common measure of prediction accuracy in LtR is the Normalized Discounted Cumulative Gain (NDCG) [42], which compares a ranking generated by model $\hat{f}$ to a ground-truth ranking (sometimes called the "ideal" ranking). NDCG measures the usefulness, or *gain*, of the candidates based both on their scores and on their positions in the ranking. NDCG incorporates position-based discounts, capturing the intuition that it is more important to retrieve high-quality (according to score) candidates at higher ranks, and is hence closely related to Equation 3. NDCG of a predicted ranking is computed relative to the gain of the ground-truth ranking, IDCG, and thus NDCG measures the extent to which the model is able to reproduce the ground-truth ranking from C_{train} in its predictions $\hat{Y}$. We are usually interested in NDCG at the top-k (denoted NDCG^k), and so normalize the position-discounted gain of the top-k in the predicted ranking by

---

[3]To follow machine learning best practices, we may also produce a separate validation dataset, used to tune model hyperparameters. We leave this out from our discussion for brevity.

the position-discounted gain of the top-k in the ideal ranking (denoted IDCG^k), per Equation 7.

$$\text{NDCG}^k = \frac{1}{\text{IDCG}^k} \cdot \sum_{i=1}^{k} \frac{\hat{Y}\boldsymbol{\tau}(i)}{log_2(i+1)} \tag{7}$$

An important application of LtR are information retrieval systems, where users issue search queries, and expect the system to find relevant information and rank the results by decreasing relevance to their queries. Consider again our example in Figure 5, and suppose that we are additionally given a set Q of queries, each associated with C via a score.

In our example, two queries are given, Q_1 = "What are the most promising candidates to admit to a STEM major?" associated with score Y_1, and Q_2 = "What are the most promising candidates to admit to a humanities or arts major?" associated with Y_2. The training and test sets are formed by assigning the candidate features X and their respective scores Y per query: $C_{train} = \left\{(X_{train}, Y_Q)\right\}_{Q \in Q}$ and $C_{test} = \left\{(X_{test}, Y_Q)\right\}_{Q \in Q}$. With these sets as input, we can use the LtR procedure shown in Figure 4 to train a single model.

To evaluate model performance, its accuracy measures need to be extended to handle multiple queries. Commonly used measures are NDCG (Eq. 7) averaged over all queries and Mean Average Precision (MAP). MAP [55] consists of several parts: first, precision at position k ($P@k$) is calculated as the proportion of query-relevant candidates in the top-k positions of the predicted ranking $\hat{\boldsymbol{\tau}}$. This proportion is computed for all positions in $\hat{\boldsymbol{\tau}}$, and then averaged by the number of relevant candidates for a given query to compute average precision (AP). Finally, MAP is calculated as the mean of AP values across all queries. MAP enables a performance comparison between models irrespective of the number of queries that were given at training time.

Fairness. As is the case in score-based ranking, discussed in Section 2.1, LtR methods may incorporate fairness objectives in addition to utility. Fairness interventions in LtR are warranted because the procedure described in Figure 4 is prone to pick up and amplify different types of bias (see Section 3.2 for details). For example, let us return to Figure 5 and note that, by randomly selecting $C_{train} = \{b, c, e, f, k, o\}$, in which all men are ranked above all women, we may have accidentally injected a strong gender bias into the learned model. This, in turn, may result in an estimated ranking in Figure 5c that places the male candidate l above the female candidate d, although their ground truth scores would place them in the opposite relative order.

This may lead to biased future predictions for rankings that systematically disadvantage women (Fig. 5c).[4]

As a remedy, two main lines of work on measuring fairness in rankings, and enacting fairness-enhancing interventions, have emerged over the past several years: probability-based and exposure-based. Both interpret fairness as a requirement to provide a predefined share of visibility for one or more protected groups throughout a ranking.

Probability-based fairness is defined by means of statistical significance tests that ask how likely it is that a given ranking was created by a fair process, such as by tossing a coin to decide whether to put a protected-group or a privileged-group candidate at position i [88, 90].

Exposure-based fairness is defined by quantifying the expected attention received by a candidate, or a group of candidates, typically by comparing their average *position bias* [43] to that of other candidates or groups.

$$\text{Exposure}\,(\boldsymbol{\tau}(i)) = \mathbb{E}_{\boldsymbol{\tau} \sim \pi}\,[\text{v}(\boldsymbol{\tau}(i))] \tag{8}$$

Here, $\pi : rnk(C) \rightarrow [0, 1]$ is the probability mass function over the ranking space, and position bias $\text{v}(\boldsymbol{\tau}(i))$ refers to the observation that users of a ranking system tend to prefer candidates at higher positions, and that their attention

---

[4]We denote that this is a very simplified example for technical bias which we use for illustrative purposes.

decreases either geometrically or logarithmically with increasing rank [6, 43]. Logarithmic position-based discounting when computing exposure is in-line with position-based discounting of utility for score-based rankers (Eq. 3) and with the NDCG measure for supervised LtR (Eq. 7).

The algorithmic fairness community is familiar with the distinction between individual fairness, a requirement that individuals who are similar with respect to a task are treated similarly by the algorithmic process, and group fairness, a requirement that outcomes of the algorithmic process be in some sense equalized across groups. Probability-based fairness definitions are designed to express group fairness goals. In contrast, and depending on the specific formalization, exposure-based fairness can serve the goals of either individual fairness or group fairness. Individual unfairness in exposure can be expressed as the discrepancy $D(.)$ in exposure between two candidates a and b:

$$D(a, b) = |\text{Exposure}(a) - \text{Exposure}(b)| \tag{9}$$

Group unfairness can be expressed as the discrepancy $D(.)$ in the average exposure between two groups $\mathcal{G}_1$ and $\mathcal{G}_2$:

$$D(\mathcal{G}_1, \mathcal{G}_2) = \left| \frac{1}{|\mathcal{G}_1|} \sum_{a \in \mathcal{G}_1} \text{Exposure}(a) - \frac{1}{|\mathcal{G}_2|} \sum_{b \in \mathcal{G}_2} \text{Exposure}(b) \right| \tag{10}$$

Note that consensus on a definition of exposure has not yet been found and, while many measures feature position bias in some way, they disagree on its importance. An additional distinctive characteristic of fairness definitions is that some of them consider a notion of a candidate's merit when measuring disparities in exposure, while others explicitly leave it out. Most of the former understand merit as the utility score Y at face value. However, as we will discuss in Section 3.3, the understanding of merit depends on worldviews and on one's conception of equal opportunity. In Sections 6 and 7 we will present different interpretations of merit and exposure that have been used by LtR methods in information retrieval and recommender systems.

3 FOUR CLASSIFICATION FRAMEWORKS FOR FAIRNESS-ENHANCING INTERVENTIONS

Operationally, algorithmic approaches surveyed in this paper differ in how they represent candidates (e.g., whether they support one or multiple sensitive attribute, and whether these are binary), in the type of bias they aim to surface and mitigate, in what fairness measure(s) they adopt, in how they navigate the trade-offs between fairness and utility during mitigation, and, for supervised learning methods, at what stage of the pipeline a mitigation is applied. Conceptually, these operational choices correspond to normative statements about the types of bias being observed and mitigated, and the objectives of the mitigation. In this section we give four classification frameworks that allow us to relate the technical choices with the normative judgments they encode, and to identify the commonalities and the differences between the many algorithmic approaches.

3.1 Group structure

Recall that fairness of a method is stated with respect to a set of categorical sensitive attributes (or features). Individuals who have the same value of a particular sensitive attribute, such as gender or race, are called a *group*. In this survey, we consider several orthogonal dimensions of group structure, based on the handling of sensitive attributes.

Cardinality of sensitive attributes. Some methods consider only *binary* sensitive attributes (e.g., binary gender, majority or minority ethnic group), while other methods handle higher-cardinality (*multinary*) domains of values for sensitive attributes. If a multinary domain is supported, methods differ in whether they consider one of the values to be

protected (corresponding to a designated group that has been experiencing discrimination), or if they treat all values of the sensitive attribute as potentially being subject to discrimination.

Number of sensitive attributes. Some methods are designed to handle a *single sensitive attribute* at a time (e.g., they handle gender or race, but not both), while other methods handle *multiple sensitive attributes* simultaneously (e.g., they handle both gender and race).

Handling of multiple sensitive attributes. Methods that support multiple sensitive attributes differ in whether they handle these *independently* (e.g., by asserting fairness constraints w.r.t. the treatment of both women and Blacks) or *in combination* (e.g., by requiring fairness w.r.t. Black women). Note that any method that supports a single multinary attribute can be used to represent multiple sensitive attributes with the help of a computed high-cardinality sensitive attribute. For example, a computed sensitive attribute *gender-race-disability* can represent the Cartesian product $\{male, female, non\text{-}binary\} \times \{White, Black, Asian\} \times \{disabled, non\text{-}disabled\}$. We may be tempted to say that such methods take the point of view of intersectional discrimination [25, 54]. However, as we will discuss in Section 3.3, detecting and mitigating intersectional discrimination is more nuanced, and so it is in general not true that if a method takes a Cartesian product of sensitive attribute values then handles intersectional discrimination, and if a method treats sensitive attributes independently then it does not.

3.2 Type of bias

We study ranking systems with respect to the types of bias that they attempt to mitigate, namely, pre-existing bias, technical bias, and emergent bias, as defined by [37].

Pre-existing bias. This type of bias includes all biases that exist independently of an algorithm itself and has its origins in society. For an example of pre-existing bias in rankings, consider the Scholastic Assessment Test (SAT). College applicants in the US are commonly ranked on their SAT score, often in combination with other features. It has been documented that the mean score of the math section of the SAT differs across racial groups, as does the shape of the score distribution. According to a Brookings report that analyzed 2015 SAT test results, "The mean score on the math section of the SAT for all test-takers is 511 out of 800, the average scores for blacks (428) and Latinos (457) are significantly below those of whites (534) and Asians (598). The scores of black and Latino students are clustered towards the bottom of the distribution, while white scores are relatively normally distributed, and Asians are clustered at the top" [63]. This disparity is often attributed to racial and class inequalities encountered early in life, and presenting persistent obstacles to upward mobility and opportunity.

Technical bias. This type of bias arises from technical constraints or considerations, such as the screen size or a ranking's inherent position bias — the geometric drop in visibility for items at lower ranks compared to those at higher ranks. Position bias arises because in Western cultures we read from top to bottom, and from left to right, and so items appearing in the top-left corner of the screen attract more attention [6]. A practical implication of position bias in rankings that do not admit ties is that, even if two items are equally suitable for a searcher, only one of them can be placed above the other in a ranking, suggesting to the searcher that it is better and should be prioritized.

Note that, as all rankings carry an inherent position bias, any method that produces rankings with equalized candidate visibility implicitly addresses this technical bias. However, we will only assign a method to technical bias mitigation, if the paper is explicitly concerned with it, such as [11].
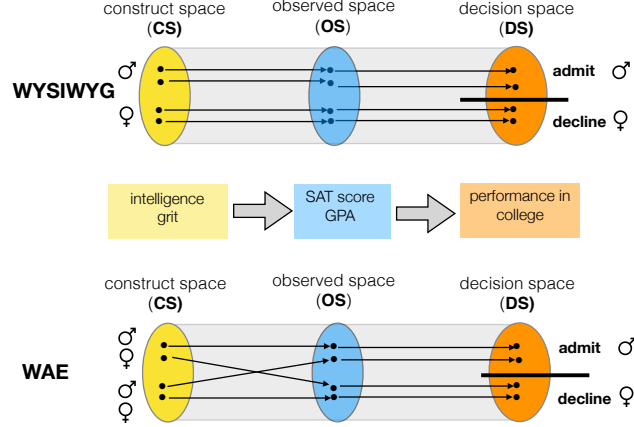
Fig. 6. An illustration of the worldviews from Frieder et al. [36]: "What you see is what you get" (WYSYWIG) vs. "We are all equal" (WAE). WYSYWIG assumes that the mapping from the construct space (*CS*) to the observed space (*OS*) shows very low distortion. In contrast, WAE assumes that the mapping from *CS* to *OS* distorts the structure of the groups in *CS*, leading to structural bias.

Emergent bias. This type of bias arises in a context of use and may be present if a system was designed with different users in mind or when societal concepts shift over time. In the context of ranking and recommendation it arises most notably because searchers tend to trust the systems to indeed show them the most suitable items at the top positions [61], which in turn shapes a searcher's idea of a satisfactory answer for their search. These feedback loops can create a "the-winner-takes-it-all" situation in which consumers increasingly prefer one majority product over everything else.

3.3 Mitigation objectives

Worldviews. Friedler et al. [36] reflect on the impossibility of a purely objective interpretation of algorithmic fairness (in the sense of a lack of bias): "In order to make fairness mathematically precise, we tease out the difference between beliefs and mechanisms to make clear what aspects of this debate are opinions and which choices and policies logically follow from those beliefs." They model the decision pipeline of a task as a sequence of mappings between three metric spaces: construct space (*CS*), observed space (*OS*), and decision space (*DS*), and define worldviews (belief systems) as assumptions about the properties of these mappings.

The spaces and the mappings between them are illustrated in Figure 6 for the college admissions example. Individuals are represented by points. *CS* represents the "true" unobservable properties of an individual (e.g., intelligence and grit), while *OS* represents the properties that we can measure (e.g., SAT score as a proxy for intelligence, high school GPA as a proxy for grit) and serves as the feature space of an algorithmic ranker. An observation process $g(p) = \hat{p}$ maps from an individual $p \in CS$ to an entity $\hat{p} \in OS$. An example of such a process is an SAT test. The decision space *DS* maps from *OS* to a metric space of decisions, which for rankings represent the degree of relevance of an entity $\hat{p}$ by placing it at a particular position in the ranking.

Note that the mappings between the spaces are prone to distortions, of which those that map from *CS* to either *OS* or *DS* are by definition unobservable. Because the properties of these mapping cannot be independently verified, a belief system has to be postulated. Friedler et al. [36] describe two extreme cases: WYSIWYG ("what you see is what you get") and WAE ("we are all equal"). The former assumes that *CS* and *OS* are essentially the same and any distortion between the two is at most ϵ. The latter assumes that any differences between the utility distributions of different groups are

due to an erroneous or biased observation process g. In our college admissions example this would mean that any differences in the GPA or IQ distributions across different groups are solely caused by biased school systems and IQ tests. It is also assumed that g shows different biases across groups, to which the authors refer as *group skew*.

The authors further define different terms from the Fairness, Accountability, Transparency, and Ethics (FATE) literature in terms of the underlying group skew. Their *fairness* definition is inspired by Dwork et al. [30] and says that items that are close in construct space shall also be close in decision space, which is widely known as individual fairness. Group fairness, however, is defined indirectly through the terms *direct discrimination* and *non-discrimination*. Direct discrimination is absent if the group skew of a mapping between OS and DS is less than ϵ. Non-discrimination is present, if the group skew of a mapping between CS and DS is less than ϵ. Note that the last definition requires a choice of world view beforehand in order to be evaluated. If WYSIWYG is chosen, group fairness is given as soon as there is no direct discrimination, because $CS \approx OS$.

We will classify the investigated algorithms in terms of which worldview they choose and which of the three terms (fairness, direct discrimination, non-discrimination) they aim to optimize.

When categorizing surveyed methods with respect to worldview, we consider whether their fairness objective aims for equality of outcome or equality of treatment. If the goal of a method is to achieve equality of outcome, and if it is asserted that OS is not trustworthy because of biased or erroneous distortion g between CS and OS, then we consider this method to fall under the WAE worldview. If, on the other hand, the goal is to achieve equality of treatment and it is asserted that the mapping between CS and OS shows low distortion, then the method falls under the WYSIWYG worldview.

Equality of Opportunity. Heidari et al. [40] show an application of equality of opportunity (EOP) frameworks to algorithmic fairness. EOP emphasizes the importance of personal qualifications, and seeks to minimize the impact of circumstances and arbitrary factors on individual outcomes. "At a high level, in these models an individual's outcome/position is assumed to be affected by two main factors: his/her circumstance c and effort e. Circumstance c is meant to capture all factors that are deemed irrelevant, or for which the individual should not be held morally accountable; for instance c could specify the socio-economic status they were born into. Effort e captures all accountability factors—those that can morally justify inequality." [40]

Several conceptions of EOP have been proposed, differing in what features they consider to be relevant (or morally acceptable to use) and which they deemed irrelevant. One of these is the *libertarian* view that focuses on an individual's freedoms and liberties. According to this view, any information about an individual that was legally obtained can be used to make a decision. Under this view there is no attempt to equalize access to opportunity, and so it is not, properly speaking, an EOP framework [46], despite being denoted as such by Heidari et al. [40]. If a fairness definition assumes that all individuals are comparable in all dimensions as long as there is no gross violations of their privacy rights during the comparison, then we map this definition to the libertarian view.

Formal EOP considers a competition to be fair when candidates are evaluated on the basis of their relevant qualifications, and the most qualified candidate wins. This view rejects any qualifications that are irrelevant, such as hereditary privileges or social status, but it makes no attempt to correct for arbitrary privileges and disadvantages leading up to the competition that can lead to disparities between people's qualifications at the time of competition. Formal EOP is typically understood in the algorithmic fairness literature as fairness-through-blindness — disallowing the direct impact from sensitive attributes (e.g., gender and race) on the outcome but allowing them to impact the outcome through proxies.

Limiting formal EOP to fairness through blindness has been challenged in recent work by Khan et al. [46], who argue for a broader interpretation: "For example, think of the SAT as a predictor of college success: when students can afford to do a lot of test prep, scores are an inflated reflection of students' college potential. When students don't have access to test prep, the SAT underestimates students' college potential. The SAT systematically overestimates more privileged students, while systematically underestimating less privileged students. The test's validity as a predictor of college potential varies across groups. That's also a violation of formal EOP. After all, in the college admissions contest, applicants should only be judged by 'college-relevant' qualifications–but this test's accuracy as a yardstick for college potential varies with students' irrelevant privilege'." Despite this recent development, we will only map fairness-through-blindness definitions to formal EOP in this survey.

Substantive EOP views, including Rawlsian and luck-egalitarian, assume that individuals are only comparable in the dimension of their *intrinsic* effort that is not affected directly or indirectly by their sensitive attributes (e.g., gender and race). In other words, the comparison across individuals shall be conditioned on their effort to assign opportunities. While Rawlsian EOP assumes that this effort itself is not affected by sensitive attributes, luck-egalitarian EOP states that the effort can also be affected by sensitive attributes. We map a fairness definition to Rawlsian EOP when its implied comparison across individuals is conditioned on a dimension of effort that is independent of sensitive attributes. And we map a fairness definition to luck-egalitarian EOP when its implied comparison across individuals is conditioned on their circumstances (i.e., conditioned on effort, and effort is conditioned on circumstances).

We will classify surveyed approaches with respect to the EOP framework based on how their fairness definition compares individuals according to some qualifications (e.g., test scores, credit amount, and times of being arrested). However, such a mapping is elusive if a paper does not clearly state its underlying concept of an individual's effort. Note also that we explicitly map the *fairness definitions*, not the overall approaches. This is because many methods *combine* a fairness and a utility objective into a single optimization problem and, by doing so, lose a clear association with a particular framework. As a result, many of the methods we survey fall between the WAE and WYSIWYG worldviews, and do not cleanly map to a single EO category. Some are even designed to allow a continuous shift between the frameworks, by providing a tuning parameter [92].

Worldviews vs. Equality of Opportunity. There is an ambiguity between the worldviews [36] and the EOP [40] classification frameworks: It is not clear at what point in time the features from construct space are to be considered and therefore it is not clear whether WAE relates to Rawlsian EOP or luck-egalitarian EOP. If we have to interpret effort as an absolute term, then construct space features (e.g., a student's grit) have to be considered by the time a decision is made. In this case WAE relates to Rawlsian EOP. If, instead, we interpret effort as a relative term and therefore allow for the fact, that it might be shaped by our circumstances (e.g., a student's grit may flourish better when they grow up in households where education is deemed important), we should consider features in *CS* much earlier, probably at the time of birth. In this case WAE relates to luck-egalitarian EOP. The definition of construct space and WAE in Friedler et al. [36] suggest the former case, however the authors do not explicitly state this and also discuss the issue of when to consider construct space features, but without stating any assumption for their own work.

We show in our method analysis that different methods are not free of this ambiguity and we will work out which algorithm assumes WAE as Rawlsian EO and which assumes WAE as luck-egalitarian EO, whenever possible. We believe that more research and discussion is needed to resolve the ambiguity of the WAE worldview.

Intersectional discrimination. Intersectional Discrimination [25, 54] states that individuals who belong to several protected groups simultaneously (e.g., Black women) experience stronger discrimination compared to individuals
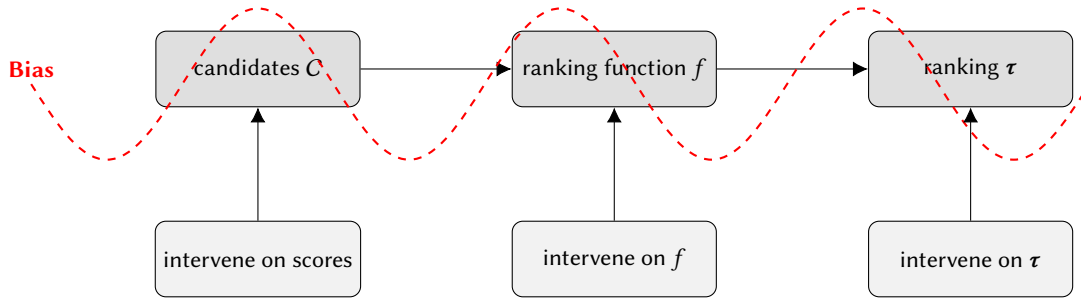
Fig. 7. Bias mitigation in score-based ranking: intervening on the score distribution of the candidates in C, on the ranking function f (e.g.,, by adjusting attribute weights), or on the ranked outcome (e.g., by enforcing proportional representation constraints).

who belong to a single protected group (e.g., White women or Black men), and that this disadvantage compounds more than additively. This effect has been demonstrated by numerous case studies, and by theoretical and empirical work [23, 28, 58, 70]. The most immediate interpretation for ranking is that, if fairness is taken to mean proportional representation among the top-k, then it is possible to achieve proportionality for each gender subgroup (e.g., men and women) and for each racial subgroup (e.g., Black and White), while still having inadequate representation for a subgroup defined by the intersection of both attributes (e.g., Black women).

Intersectional concerns also arise in more subtle ways. For example, Yang et al. [86] observed that when representation constraints are stated on individual attributes, like race and gender, and when the goal is to maximize score-based utility subject to these constraints, then a particular kind of unfairness can arise, namely, utility loss can be particularly severe in historically disadvantaged intersectional groups. When discussing specific technical methods, we will speak to whether they consider intersectional discrimination and, if so, which specific concerns they aim to address.

3.4 Mitigation method

Score-based and supervised learning based rankers use different types of bias mitigation methods.

In score-based ranking, we categorize mitigation methods into those that intervene on the score distribution, or on the scoring function, or on the ranked outcome, as illustrated in Figure 7. Methods that *intervene on the score distribution* aim to mitigate disparities in candidate scores, either before these candidates are processed by an algorithmic ranker or during ranking. Methods that *intervene on the ranking function* identify a function that is similar to the input function but that produces a ranked outcome that meets the specified fairness criteria. Methods that *intervene on the ranked outcome* impose constraints to require a specific level of diversity or representation among the top-k as a set, or in every prefix of the top-k.

In supervised learning, we categorize mitigation methods into pre-processing, in-processing, and post-processing, Illustrated in Figure 8. *Pre-processing* methods seek to mitigate discriminatory bias in training data, and have the advantage of early intervention on pre-existing bias. *In-processing* methods aim to learn a bias-free model. Finally, *post-processing* methods re-rank candidates in the output subject to given fairness constraints [38].

The advantage of post-processing methods in supervised learning is that they provide a guaranteed share of visibility for protected groups. In contrast, in-processing methods only consider fairness at training time and make no guarantees about fairness of the test set. However, post-processing methods may be subject to legal challenges because of due process concerns that may make it illegal to intervene at the decision stage (e.g., Ricci v. DeStefano [78]). Thus, like all
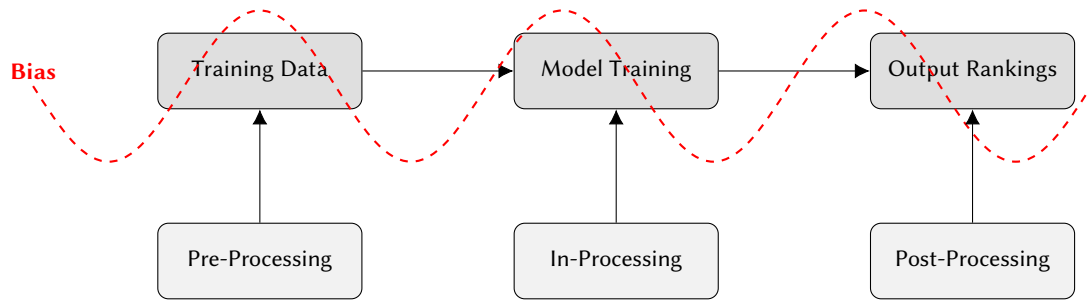
Fig. 8. Bias mitigation in different stages of a supervised learning method: pre-processing, in-processing, and post-processing.

technical choices, the choice of whether to use a pre-, in-, or post-processing fairness-enhancing intervention is not purely technical, but must also consider the social and legal context of use of the algorithmic ranker.

4 DATASETS

Before diving into a description of the fair ranking methods, we present the experimental datasets used by them. We summarize the datasets in Table 2, where we highlight the following aspects: size (usually the number of candidates), sensitive attributes, scoring attributes, and the surveyed papers that use this dataset in their evaluation. We then briefly describe each dataset, and refer the reader to the description of each method in Sections 5-7 for details about that dataset's use. All datasets are publicly available under the referenced links unless otherwise indicated.

The papers surveyed here rarely substantiate their choice of an experimental dataset, other than that by the fact that the dataset was available, and that items in it have scores on which to rank. Both of these reasons can be seen as purely technical (or even syntactic) rather than conceptual. Unfortunately little explicit attention has been paid to explaining whether a particular dataset was collected with a ranking task in mind, and *why* it is deemed appropriate for the specific fairness definition, that is, whether and to what extent the task for which the dataset was collected or can plausibly be used exhibits unfairness of the kind that the proposed fairness definition is designed to address. We see this as an important limitation of empirical studies in fairness in ranking and, more generally, in algorithmic fairness research, and posit that the use of a dataset must be explicitly justified.

AirBnB [1]. This dataset consists of 10,201 house listings from three major cities: Hong Kong (4,529 items), Boston (3,944 items), and Geneva (1,728 items). The gender of the hosts is used as the sensitive attribute, and the ranking score is computed as the ratio of the rating and the price.

COMPAS (Correctional Offender Management Profiling for Alternative Sanctions) [62]. This dataset is derived based on a recidivism risk assessment tool called COMPAS. The dataset contains the COMPAS scores from the Broward County Sheriff's Office in Florida in 2013 and 2014, and the profile of each person's criminal history collected by ProPublica [2]. In total there are 7,214 data points, with sensitive attributes gender and race.

CS department rankings [26]. This dataset contains information about 51 computer science departments in the U.S.. The methods in [86, 87] use the number of publications as the ranking score. Two categorical attributes are treated

Dataset	Size	Sensitive attrs	Score	Used in
AirBnB [1]	10,201 houses	gender of host	rating, price	[11, 49]
COMPAS [62]	7,214 people	gender, race	risk scores	[3, 88, 90]
CS departments [26]	51 departments	department size, geographic region	number of publications in different CS areas	[86]
DOT [60]	1.3 million flights	airline name	departure delay, arrival delay, taxi-in time	[3]
Engineering students [76]	5 queries, 650 students per query	gender, high school type	academic performance after first year	[91]
Forbes richest U.S. [35]	400 people	gender	net worth	[75]
German credit [64]	1,000 people	gender, age	credit amount, duration	[72, 83, 88, 90]
IIT-JEE [79]	384,977 students	birth category, gender, disability status	test scores	[18]
LSAC [69]	21,792 students	gender, race	LSAT scores	[92]
MEPS [59]	15,675 people	gender, race, age	number of trips requiring medical care	[86]
NASA astronauts [57]	357 astronauts	major in college	flight hours	[75]
Pantheon [22]	11,341 people	occupation	popularity of Wikipedia page	[75]
SAT [66]	1.6M students	gender	SAT score	[90]
StackExchange [74]	253,000 queries 6M documents	domains	document relevance	[11]
SSORC [67]	8,975,360 papers	gender of authors (inferred)	number of citations	[18]
W3C experts [81]	60 queries, 200 experts per query	gender	probability of being an expert	[91]
XING [84]	40 candidates	gender	years of experience, education	[49, 90]
Yahoo LTR [85]	26,927 queries 638,794 documents	N/A	relevance	[72]
Yow news [95]	unknown	source of news	relevance	[71]

Table 2. Experimental datasets used in the surveyed papers.

as sensitive: department size (with values "large" and "small") and geographic area (with values "North East", "West", "Middle West", "South Center", and "South Atlantic").

DOT (Department of Transportation) [60]. This dataset consists of about 1.3 million records of flights conducted by 14 U.S. airlines in the first three months of 2016. The dataset was collected by Asudeh et al. [3] from the flight on-time database that is published by the U.S. Department of Transportation. Three scoring attributes are used in [3]: departure delay, arrival delay, and taxi-in time. The name of the airline conducting the flight is treated as the sensitive attribute.

Engineering students [76]. This dataset contains the results of a Chilean university admissions test from applicants to a large engineering school in five consecutive years. The task in [91] is to predict the students' academic performance after the first year based on their admissions test results and school grades. The sensitive attributes are gender and whether the applicants graduated from a private or public high school. This dataset is only accessible upon request, see referenced link for details.

Forbes richest Americans [35]. This dataset consists of 400 individuals from the 2016 Forbes US Richest list [5], ranked by their net worth. Gender is the sensitive attribute, with 27 female vs. 373 male individuals in the dataset.

German credit [64]. This dataset, hosted by the UCI machinle learning repository [52], contains financial information of 1,000 individuals, and is associated with a binary classification task that predicts whether an individual's credit is good or bad. The sensitive attributes are gender and age, where age is categorized into younger or older based on a threshold (25 or 35 years old is variably used as the threshold). Attributes credit amount and duration (how long an individual has had a line of credit) have been used as scoring attributes in fair ranking papers.

IIT-JEE (The Joint Entrance Exam of Indian Institutes of Technology) [79]. This dataset consists of scores of 384,977 students in the Mathematics, Physics, and Chemistry sections of IIT-JEE 2009, along with their gender, birth category (see [8]), disability status, and zip code. The students are scored on a scale from −35 to +160 points in all three sections, with an average total score of +28.36, a maximum score of +424, and a minimum score of −86.

LSAC [69]. This dataset consists of a U.S. national longitudinal bar exam passage data gathered from the class that started law school in Fall 1991. Data is provided by the students, their law schools, and state boards of bar examiners over a 5-year period [82]. The dataset consists of 21,791 students, with the sensitive attributes sex and race. Rankings are produced based on LSAT scores.

MEPS (Medical Expenditure Panel Survey) [59]. This dataset consists of 15,675 people and their information regarding the amount of health expenditures [21, 24]. The sensitive attributes are gender, race, and age of each individual, where age is categorized into younger or older based on a threshold (35 years old) in [86, 87]. The ranking score is based on utilization, defined by the IBM AI Fairness 360 toolkit [9] as the total number of trips requiring medical care. Utilization is computed as the sum of the number of office-based visits, the number of outpatient visits, the number of ER visits, the number of inpatient nights, and the number of home health visits.

NASA astronauts [57]. This dataset consists of 357 astronauts with their demographic information. The method in [75] ranks this dataset by the number of space flight hours, and assigns individuals to categories based on their undergraduate major, treating is a the sensitive attribute. A total of 83 majors are represented in the dataset, the 9 most frequent are assigned to their individual categories — Physics (35), Aerospace Engineering (33), Mechanical Engineering (30), etc, and the remaining 141 individuals are combined into the category "Other", resulting in 10 groups.

SAT [66]. This dataset contains about 1.6 million data points, in which the score column corresponds to an individual's results in the US Scholastic Assessment Test (SAT) in 2014 [80]. The sensitive attribute is gender.

SSORC [67]. The Semantic Scholar Open Research Corpus contains the meta-data of 46,947,044 published research papers in computer science, neuroscience, and biomedicine from 1936 to 2019 on Semantic Scholar. The meta-data for

---

[5]https://www.forbes.com/forbes-400/list/

each paper includes the list of authors of the paper, the year of publication, the list of papers citing it, and the journal of publication, along with other details. The sensitive attribute is the gender of the authors, collected by Celis et al. [18]. The ranking score is the number of citations of each paper.

StackExchange [74]. This dataset contains a query log and a document collection using the data from the Stack Exchange Q&A community (dump as of 13-06-2016) [12]. It consists of about 6 million posts inside the type "Question" or "Answer" in 142 diverse subforums (e.g., Astronomy, Security, Christianity, Politics, Parenting, and Travel). The questions are translated into about 253,000 queries, and the respective answers serve as the documents for the queries. The sensitive attribute is the query domain.

W3C experts [81]. The task behind this dataset corresponds to a search of experts for a given topic based on a corpus of e-mails written by possible candidates. The sensitive attribute is the gender of the expert. The experimental setup in [91] investigates situations in which bias is unrelated to relevance: expertise has been judged correctly, but ties have been broken in favor to the privileged group (i.e., all male experts are followed by all female experts, followed by all male non-experts, followed finally by all female non-experts).

Xing [84]. This dataset was collected by Zehlike et al. [90] from a German online job market website[6]. The authors collected the top-40 profiles returned for 54 queries, and computed an ad-hoc score based on educational features, job experience and profile popularity. The sensitive attribute is gender, which was inferred based on the first name associated with the profile and the profile picture, when available. Items are ranked based on an ad-hoc score.

Yahoo! LTR [85]. This dataset consists of 19,944 training queries and 6,983 test set queries. Each query has a variable sized candidate set of documents that needs to be ranked. There are 473,134 training and 165,660 test documents. The query-document pairs are represented by a 700-dimensional feature vector. For supervision, each query-document pair is assigned an integer relevance judgments from 0 (bad) to 4 (perfect). The dataset is used to evaluate the effectiveness of Learning to Rank methods in [72], thus no sensitive attribute is specified.

Yow news [95]. This dataset contains explicit and implicit feedback from a set of users for news articles in the "people" topic produced by Yow [96]. The ranking score is the explicitly given relevance field. The source of news is treated as the sensitive attribute.

5 SCORE-BASED RANKING

In this section we present several methods for fairness in score-based ranking. Table 3 summarizes the methods presented in this section according to the frameworks of Section 3.

Recall that, in score-based ranking we categorize mitigation methods into those that intervene on the ranking process, on the score distribution, or on the scoring function. In Section 5.1, we describe methods that *intervene on the ranked outcome* by ensuring proportional representation across groups. Next, in Section 5.2, we discuss several methods that formulate *fairness* and *coverage-based diversity* constraints by specifying bounds on the number of candidates from groups of interest to be present in prefixes of a ranked list. These methods also intervene on the ranked outcome. Then, in Section 5.3, we describe methods that *intervene on the score distributions*. Finally, in Section 5.4, we present a method that treats the fairness objective as a black-box and proposes a geometric interpretation of score-based ranking to reach the objective by *intervening on the ranking function.*

---

[6]https://www.xing.com

5.1 Intervening on the Ranked Outcome: Rank-aware Proportional Representation

To the best of our knowledge, Yang and Stoyanovich [88] were the first formalize rank-aware fairness, under the assumption that the scores based on which the ranking is produced encode pre-existing bias.

Consider a ranking in which candidates are assigned to one of two groups, $\mathcal{G}_1$ or $\mathcal{G}_2$, according to a single binary sensitive attribute (e.g.,, binary gender), and with one of these groups, $\mathcal{G}_1$, corresponding to the protected group (e.g., the female gender). The fairness measures proposed in this paper are based on the following intuition: Because it is more beneficial for an item to be ranked higher, it is also more important to achieve proportional representation at higher ranks. The idea, then, is to take several well-known proportional representation measures and to make them *rank-aware*, by placing them within a framework that applies position-based discounts.

Fairness definition and problem formalization. Recall from Section 2 that position-based discounting is commonly used to quantify utility (Eq. 3) or prediction accuracy (Eq. 7) in a ranking. In a similar vein, the use of position-based discounting in Yang and Stoyanovich [88] is a natural way to make set-wise proportional representation requirements rank-aware. Specifically, the idea is to consider a series of prefixes of a ranking τ, for $k = 10, 20, \ldots$, to treat each top-k prefix $\tau_{1\ldots k}$ as a set, to compute *statistical parity* at top-k, and to compare that value to the proportion of the protected group in the entire ranking. (Naturally, perfect statistical parity is achieved when $k = n$.) The values computed at each cut-off point are summed up with a position-based discount. Based on this idea, the authors propose three fairness measures that differ in the specific interpretation of statistical parity: normalized discounted difference (rND), ratio (rRD), and KL-divergence (rKL).

Normalized discounted difference (rND) (Equation 11) computes the difference between the proportions of the protected group $\mathcal{G}_1$ at the top-k and in the over-all population. Normalizer Z is computed as the highest possible value of rND for the given number of items n and protected group size $|\mathcal{G}_1|$.

$$\text{rND}(\tau) = \frac{1}{Z} \sum_{k=10,20,\ldots}^{n} \frac{1}{\log_2 k} \left(\frac{|\tau_{1\ldots k} \cap \mathcal{G}_1|}{k} - \frac{|\mathcal{G}_1|}{n} \right) \tag{11}$$

Method	Group structure	Bias	Worldview	EOP	Intersectional
Rank-aware proportional representation [88]	one binary sensitive attr.	pre-existing	WAE	luck-egalitarian	no
Constrained ranking maximization [19]	multiple sensitive attrs.; multinary; handled independently	pre-existing	WAE	Rawlsian	no
Balancing utility loss across groups [86]	multiple sensitive attrs., multinary, handled independently	pre-existing & technical	WAE	luck-egalitarian	yes
Diverse k-choice secretary [75]	one multinary sensitive attr.	pre-existing	WAE	luck-egalitarian	no
Selection with implicit bias [47]		*forthcoming*			
Intervening on rankings with implicit bias [18]		*forthcoming*			
Causal intersectionality for fair ranking [87]		*forthcoming*			
Designing fair scoring functions [3]		*forthcoming*			

Table 3. Classification of score-based ranking methods according to the frameworks in Section 3.

Normalized discounted ratio (rRD) is defined analogously, as follows:

$$\text{rRD}(\boldsymbol{\tau}) = \frac{1}{Z} \sum_{k=10,20,\ldots}^{n} \frac{1}{\log_2 k} \left(\frac{|\boldsymbol{\tau}_{1\ldots k} \cap \mathcal{G}_1|}{|\boldsymbol{\tau}_{1\ldots k} \cap \mathcal{G}_2|} - \frac{|\mathcal{G}_1|}{|\mathcal{G}_2|} \right) \tag{12}$$

When either the numerator or the denominator of a term in Eq. 12 is 0, the value of the term is set to 0.

Finally, normalized discounted KL-divergence (rKL) uses Kullback-Leibler (KL) divergence to quantify the expectation of the logarithmic difference between two discrete probability distributions, P_k that quantifies the proportions in which groups are represented at the top-k:

$$P_k = \left(\frac{|\boldsymbol{\tau}_{1\ldots k} \cap \mathcal{G}_1|}{k}, \frac{|\boldsymbol{\tau}_{1\ldots k} \cap \mathcal{G}_2|}{k}, \right) \tag{13}$$

and Q that quantifies the proportions in which groups are represented in the over-all ranking:

$$Q = \left(\frac{|\mathcal{G}_1|}{n}, \frac{|\mathcal{G}_2|}{n}, \right) \tag{14}$$

KL-divergence between P_k and Q, denoted $D_{KL}(P_k||Q)$, is computed at every cut-off point k, and position-based discounting is applied as the values are compounded, with normalizer Z defined analogously as for rND:

$$\text{rKL}(\boldsymbol{\tau}) = \frac{1}{Z} \sum_{k=10,20,\ldots}^{n} \frac{1}{\log_2 k} D_{KL}(P_k||Q) \tag{15}$$

Note that, unlike rND and rRD, which are limited to a binary sensitive attribute, rKL can handle a multinary sensitive attribute and so is more flexible.

Experiments and observations. The authors evaluate the empirical behavior of the proposed fairness measures using real and synthetic datasets. Real datasets used are COMPAS [62] and German Credit [64], see Section 4 for details. Synthetic datasets are generated using an intuitive data generation procedure described below. This procedure was later used in the work of Zehlike et al. [90] and Wu et al. [83], and is of independent interest.

Recall that $\mathcal{G}_1$ represents the protected group and $\mathcal{G}_2$ represents the privileged group, and suppose for simplicity that each group constituted one half of the candidates C. An example is given in Figure 9a, in which C contains 8 candidates, 4 female ($\mathcal{G}_1$) and 4 male ($\mathcal{G}_2$). The data generation procedure, presented in Algorithm 1, takes two inputs: a ranking $\boldsymbol{\tau}$ of C and a "fairness probability" p, and it produces a ranking $\tilde{\boldsymbol{\tau}}$. The input ranking $\boldsymbol{\tau}$ is assumed to be generated by the vendor according to their usual process (e.g., based on candidate scores, as in Figure 9b). Algorithm 1 splits up $\boldsymbol{\tau}$ into two rankings: $\boldsymbol{\tau}_1$ of candidates in $\mathcal{G}_1$ and $\boldsymbol{\tau}_2$ of candidates in $\mathcal{G}_2$. It then repeatedly considers pairs of candidates at the top of the lists, $\boldsymbol{\tau}_1(1)$ and $\boldsymbol{\tau}_2(1)$, and decides which of these should be ranked above the other, selecting $\boldsymbol{\tau}_1(1)$ with probability p and $\boldsymbol{\tau}_2(1)$ with probability $1 - p$, and appending the selected candidate to $\tilde{(\boldsymbol{\tau})}$.

The parameter p specifies the relative preference between candidates in $\mathcal{G}_1$ and in $\mathcal{G}_2$. When $p = 0.5$, groups are mixed in approximately equal proportion for as long as there are items in both groups. This is illustrated in Figure 9c for the sensitive attribute A_1 (gender) and in Figure 9e for the sensitive attribute A_2 (race). When $p > 0.5$, members of the protected group $\mathcal{G}_1$ are preferred, and when $p < 0.5$ members of the privileged group $\mathcal{G}_2$ are preferred. In extreme cases, when $p = 0$, all (or most) members of $\mathcal{G}_2$ will be placed before any members of $\mathcal{G}_1$, as shown in Figure 9d for the sensitive attribute A_1 (gender). Note that candidates within a group always remain in the same relative order in $\tilde{\boldsymbol{\tau}}$ as in $\boldsymbol{\tau}$ (that is, there is *no reordering within a group*), but there may be *reordering between groups*.

Algorithm 1: FairGen

Require: Ranking τ, fairness probability p.

{Initialize the output ranking $\tilde{\tau}$.}

1: $\tilde{\tau} \leftarrow \emptyset$

2: $\tau_1 = \tau \cap \mathcal{G}_1$

3: $\tau_2 = \tau \cap \mathcal{G}_2$

4: **while** $(\tau_1 \neq \emptyset) \wedge (\tau_2 \neq \emptyset)$ **do**

5:    $r = random([0, 1])$

   {Append the next selected item to $\tilde{\tau}$}

6:    **if** $r < p$ **then**

7:       $\tilde{\tau} \leftarrow pop(\tau_1)$

8:    **else**

9:       $\tilde{\tau} \leftarrow pop(\tau_2)$

10:    **end if**

11: **end while**

   {If any items remain in τ_1 or τ_2, append them to $\tilde{\tau}$}

12: $\tilde{\tau} \leftarrow \tau_1$

13: $\tilde{\tau} \leftarrow \tau_2$

14: **return** $\tilde{\tau}$

---

candidate	A_1	A_2	Y	τ	$\tilde{\tau}$	$\tilde{\tau}$	$\tilde{\tau}$
b	male	White	9	b	b	b	b
c	male	Black	8	c	d	c	c
d	female	White	7	d	c	e	d
e	male	White	6	e	f	l	o
f	female	White	5	f	e	d	e
k	female	White	4	k	k	f	f
l	male	White	3	l	l	k	k
o	female	Black	2	o	o	o	l
	(a)			(b)	(c)	(d)	(e)

Fig. 9. **(a)** A set of applicants for college admissions C, with two binary sensitive attributes: A_1 (gender), with protected group $\mathcal{G}_F = \{d, f, k, o\}$ and privileged group $\mathcal{G}_M = \{b, c, e, l\}$; and A_2 (race), with protected group $\mathcal{G}_B = \{c, o\}$ and privileged group $\mathcal{G}_W = \{b, d, e, f, k, l\}$. Protected values of A_1 and A_2 are shown in orange, and privileged values—in blue. **(b)** Ranking τ sorts the applicants in descending order of their score Y, as shown in Figure 9b, with male candidates appearing in higher proportion at the top ranks. **(c)** Ranking $\tilde{\tau}$ for A_1 mixes candidates in approximately equal proportion by gender, with $p = 0.5$ in Algorithm 1, and is expected to achieve statistical parity for this attribute, since gender groups are represented in equal proportion in C. **(d)** Ranking $\tilde{\tau}$ for A_1, with $p = 0$ in Algorithm 1, places all, or most, male applicants about the female applicants. **(e)** Ranking $\tilde{\tau}$ for A_2 (race), with $p = 0.5$ in Algorithm 1, is expected to achieve equal representation by race at top ranks, but not statistical parity, since C is not balanced by race.

The proposed fairness measures —normalized discounted difference (rND), ratio (rRD), and KL-divergence (rKL)— are evaluated on rankings produced by Algorithm 1 with a range of values for p and with different relative proportions of $\mathcal{G}_1$ and $\mathcal{G}_2$ in C. The authors conclude that rKL is the most promising measure both because it is smooth and because it naturally generalized to multinary sensitive attributes.

This paper also proposes a bias mitigation methodology, inspired by Zemel et al. [94], that integrates fairness objectives into an optimization framework that balance fairness against utility, with an experimental evaluation on the German credit dataset [64] (see details in Sec. 4).

Insights. The fairness definitions of this paper aim to address pre-existing bias, per classification in Section 3.2.

Fairness is interpreted as equality of outcomes, suggesting an underlying assumption of WAE, per classification in Section 3.3. Assuming the existence of indirect discrimination in candidate scores (i.e., that the observation process between construct space CS and observable space OS is biased), the paper aims to ensure a similar representation of groups in the ranked outcomes.

The approach is designed around a relative view of effort: candidates are ranked according to score within a demographic group, and a ranked outcome is considered fair if the groups are mixed in equal proportion when the input is balanced, as in Figure 9a by A_1 (gender), or, more generally, when statistical parity is achieved at high ranks. This clearly links to luck-egalitarian EOP, per classification in Section 3.3.

5.2   Intervening on the Ranked Outcome: Diversity Constraints

In Section 5.1 we discussed how fairness measures that are based on (set-wise) proportional representation can be made rank-aware. The methods described in this section start with the observation that if the total number of candidates in C, and the number of candidates in each demographic group of interest, is available as input (i.e., that these quantities are known a priori or can be estimated), then any measure that aims to equalize or bound the difference in proportions can be equivalently re-formulated with the help of counts. Specifically, proportional representation constraints and coverage-based diversity constraint [29] for *set selection tasks* can be expressed by specifying a lower-bound $L_k^{\mathcal{G}}$ and an upper-bound $U_k^{\mathcal{G}}$ on the representation of group $\mathcal{G} \subseteq C$ among the top-k set of a ranking. Such constraints can be formulated for one or several demographic groups of interest, and also for their intersections, and a score-based ranker can then optimize utility under such constraints. Generalizing beyond set selection, constraints $L_p^{\mathcal{G}}$ and $U_p^{\mathcal{G}}$ can be specified over every prefix of the top-k of a ranked list, with $p \in [k]$, or, more practically, at some specific cut-off points within the top-k.

Similarly to the methods of Section 5.1, the methods described in this section are designed to enforce fairness and diversity in the sense of representation. In contrast Section 5.1, these methods are designed to handle multiple sensitive attributes simultaneously—individually or in combination.

5.2.1   Celis et al. [19]. **Fairness definition and problem formalization.** The authors formulate the *constrained ranking maximization problem*: Consider a set of n candidates C, and the integer $k \ll n$, along with 1) the utility of placing a candidate in a particular position in the ranking, 2) the collection of sensitive attributes (e.g., gender, race, or disability status) that map candidates to groups $\boldsymbol{\mathcal{G}}$, and 3) a collection of lower-bound constraints $L_p^{\mathcal{G}}$ and upper-bound constraints $U_p^{\mathcal{G}}$ that, for each prefix $p \in [k]$ and for each group $\mathcal{G} \in \boldsymbol{\mathcal{G}}$, bound the number of candidates from that group that are allowed to appear in the top-p positions of the ranking. The goal is to output a ranking that maximizes overall utility with respect to the original utility metric, while respecting the constraints. Note that this problem formulation has the flexibility to explicitly associate a utility with an assignment of candidate $a \in C$ to rank position $j \in [k]$, and may already incorporate position-based discounting (per Equation. 3). However, for consistency and ease of exposition, we will assume that utility score Y is fixed per candidate.

An example of the constrained ranking maximization problem is given in Figure 10, where the goal is to select $k = 4$ candidates, with at least two of each gender ($L_4^{\mathsf{M}} = 2$, $L_4^{\mathsf{F}} = 2$) and at least one of each race ($L_4^{\mathsf{W}} = 1$, $L_4^{\mathsf{B}} = 1$, $L_4^{\mathsf{A}} = 1$)

among the top-k, and with no further constraints on the prefixes of the top-k. (For convenience, we are referring to each groups by the first letter of the attribute value that defines it, such as M for male and A for Asian). Figure 10b shows τ_1, a ranking of the top-4 candidates selected based on utility Y: two of them are male and two are female, and all are White. Applying diversity constraints on gender and race yields τ_2, a ranking of the top-4 in Figure 10c, selecting the top-scoring White male candidates a and b, and two lower-scoring female candidates, g and k. Computing total utility as the sum of scores of selected candidates (for simplicity), we observe that $U(\tau_1) = 68$ and $U(\tau_2) = 53$ in this example.

Note that the example in Figure 10 is deliberately constructed to highlight disparities in scores due to pre-existing bias on gender and race: all male candidates are ranked above all female candidates of a given race, and all Whites are ranked above all Black, who are in turn ranked above all Asians. For this reason, imposing diversity constraints leads to a substantial drop in score-utility of τ_2 in Figure 10c.

candidate	A_1	A_2	Y
a	male	White	19
b	male	White	18
c	female	White	16
d	female	White	15
e	male	Black	11
f	male	Black	11
g	female	Black	10
h	female	Black	9
i	male	Asian	7
j	male	Asian	7
k	female	Asian	6
l	female	Asian	3

(a)

τ_1
a
b
c
d

(b)

τ_2
a
b
g
k

(c)

τ_3
a
c
e
k

(d)

Fig. 10. **(a)** A set of applicants for college admissions C, with two sensitive attributes: A_1 (gender), with groups $\mathcal{G}_M = \{a, b, e, f, i, j\}$ and $\mathcal{G}_F = \{c, d, g, h, k, l\}$, and A_2 (race), with groups $\mathcal{G}_W = \{a, b, c, d\}$, $\mathcal{G}_B = \{e, f, g, h\}$, and $\mathcal{G}_A = \{i, j, k, l\}$. **(b)** Top-4 ranking τ_1 selects the highest-scoring candidates according to Y; all selected candidates are White, two of them are female and two are male. The utility of τ_1, computed as the sum of scores, is 68. **(c)** Top-4 ranking τ_2 selects highest-scoring candidates subject to constraints to select at least two candidates of each gender, $L_4^M = 2$, $L_4^F = 2$, and at least one candidate of each race, $L_4^W = 1$, $L_4^B = 1$, $L_4^A = 1$. The utility of τ_1, computed as the sum of scores, is 53. **(d)** Top-4 ranking τ_3, subject to the same diversity constrains as τ_2, but additionally balancing utility loss within each group. This ranking has utility 52, and it returns the highest-scoring male, female, White, and Black candidates.

In the example we constructed, diversity constraints are satisfiable. However, as was shown by Celis et al. [19], the constrained ranking maximization problem can be seen to generalize various NP-hard problems such as independent set, hypergraph matching and set packing, and so is hard in the general case. It turns out that even checking if there is a complete feasible ranking is NP-hard. The authors show that a special case of the problem, in which each candidate is assigned to (at most) one group, and so the assignment induces a partitioning on C, can be solved in polynomial time. In this case, diversity constraints can only be specified with respect to a single sensitive attribute, which may be binary or multinary, and so can represent multiple sensitive attributes in combination (see discussion on group structure in Section 3.1).

Recall that the problem formulation allows to associate a utility with an assignment of candidate $a \in C$ to rank position $j \in [k]$. While the nature of these assignments can in principle be arbitrary, many reasonable utility metrics, including NDCG, Bradley-Terry [14] or Spearman's rho [73], are non-increasing with increasing rank position, and with decreasing utility score Y, which is intuitively interpreted to mean that, if $Y_a \geq Y_b$ then placing a above b cannot decrease the utility of the overall ranking. Such metrics are said to be monotone and to satisfy the Monge property. For this family of utility metrics, the authors propose an exact dynamic programming algorithm that solves the constrained ranking optimization problem in time polynomial in the number of candidates m and size of the selected set k, and exponential in the number of possible assignments of candidates to groups (typically the product of domain cardinalities of the sensitive attributes, $card(A_1) \times card(A_2) = 6$ in our example in Figure 10). The authors also propose approximation algorithms that allow violations of diversity constraints, and study the quality of these approximations.

Insights. The focus of this work is on the formal properties of the constrained ranking maximization problem, including its hardness and approximability under different assumptions about the sensitive attributes, the divsersity constrains, and the properties of the utility metric. The paper does not include an experimental evaluation.

The paper states that "[...] left unchecked, the output of ranking algorithms can result in decreased diversity in the type of content presented, promote stereotypes, and polarize opinions." The goal of imposing diversity constrains is to counteract pre-existing bias. Further, since these constraints enforce equality of outcomes, the method relates to the WAE worldview.

When the candidates are partitioned by a single sensitive attribute (either binary or multinary), the method will select the highest-scoring members of each group to satisfy diversity constraints, suggesting that it operates under a luck-egalitarian EOP view. However, when candidates are associated with two or more sensitive attributes, as is the case in the example in Figure 10, a single utility distribution is assumed, in the sense that a higher-scoring candidate will be preferred to a lower-scoring one irrespective of their group membership, whenever constraints permit. This was illustrated in Figure 10c, where the highest-scoring White and male candidates were selected among the top-k, but the highest-scoring female, Black, and Asian candidates were skipped. Based on this observation, we map this method to Rawlsian EOP. We will elaborate on this specific concern in the next sub-section.

5.2.2 Yang et al. [86]. **Fairness definition and problem formalization.** The authors further investigate the constrained ranking maximization problem with two or more sensitive attributes, and observe that members of multiple historically disadvantaged groups may still be treated unfairly by this process. For an intuition, consider again Figure 10, and recall that the goal is to select the top-4 candidates, with at least two of each gender ($L_4^M = 2$, $L_4^F = 2$) and at least one of each race ($L_4^W = 1$, $L_4^B = 1$, $L_4^A = 1$). Maximizing utility subject to these constraints being met yields, ranking τ_2 in Figure 10c that selects the best (according to score) White and male candidates a and b, but it does not select the best Black, Asian, or female candidates.

Our example was deliberately constructed to highlight the following: if some population groups have systematically lower scores, then it costs less to skip their best-scoring members in the name of diversity. This runs contrary to the nature of the diversity objective, which is to equalize access to opportunity. This also represents unfairness, under the luck-egalitarian view. To see why, suppose that scores represent effort (e.g., how hard someone studied to do well on a test), and that we consider it important to reward effort. We may then take a relative view of effort, and assert that scores are more informative *within* a group than *across groups*. Taken together, this means that the best-scoring individuals from historically disadvantaged groups should have a chance to be selected among the top-k. Figure 10d represents a

ranked outcome that gets closer to this objective; it presents τ_3, a top-4 ranking that contains the highest-scoring male, female, White and Black candidates.

Yang et al. [86] formalize this intuition by stating that, when multiple sensitive attributes (e.g., gender and race) are considered simultaneously, it is crucial to consider the utility loss that is incurred within each group, and to balance that loss across groups. The authors propose two measures to quantify in-group utility, IGF-Ratio and IGF-Aggregated, both taking on values from the range (0, 1], with 1 corresponding to perfect utility within a group (no loss), and with high loss corresponding to values close to 0. Both IGF-Ratio and IGF-Aggregated can be computed over the top-k as a set, or in rank-aware manner, by considering every prefix of length $p \in [k]$. In what follows, we will illustrate one of these measures, IGF-Ratio, taking the set interpretation for simplicity.

IGF-Ratio, quantifies the utility within a group (e.g., female or Black) by computing the ratio of the utility score of the highest-scoring skipped candidate from that group and the lowest-scoring selected candidate. Consider again ranking τ_2 in Figure 10c. We compute IGF-Ratio$(\tau_2, \mathcal{G}_M)$ = IGF-Ratio$(\tau_2, \mathcal{G}_W)$ = 1, since the highest-scoring male and White candidates were selected. For the female, Black, and Asian groups, we compute IGF-Ratio$(\tau_2, \mathcal{G}_F) = \frac{Y_g}{Y_c} = \frac{10}{16}$; IGF-Ratio$(\tau_2, \mathcal{G}_B) = \frac{Y_g}{Y_e} = \frac{10}{11}$; and IGF-Ratio$(\tau_2, \mathcal{G}_A) = \frac{Y_k}{Y_i} = \frac{6}{7}$.

IGF-Aggregated is based on similar intuition as IGF-Ratio, but rather than comparing the utility due to a pair of items for each group — one selected and one skipped — it compares the sum of scores of all items from a group up to a particular position with the sum of scores of all selected items from that group (up to the same position).

The authors go on to use IGF-Ratio and IGF-Aggregated to state that loss in these measures should be balanced across groups. They implement this requirement as an additional set of constraints, and formalize the induced optimization problem that (1) meets diversity constraints for each group, (2) balances utility loss across groups, and (3) maximizes over-all utility subject to (1) and (2), as integer linear programs.

Experiments and observations The authors conduct experiments on two real datasets, CS departments [26] and MEPS [59] (see details in Section 4). They use these datasets to quantify the feasible trade-offs between, diversity, overall utility, and utility loss across groups. Further, they show that utility loss can be balanced effectively, and that the over-all utility cost of such interventions is low.

Insights Similarly to papers surveyed earlier in this section, the work of Yang et al. [86] aims to address pre-existing bias by equalizing outcomes, and so relates to the WAE worldview. Further, because of an explicit focus on ensuring that the best-qualified candidates from each group have an opportunity to be selected, or to appear at higher ranks, this work takes a relative view of effort and so is firmly in the luck-egalitarian EOP camp.

The main insight on which this work is based is that membership in multiple sensitive groups can lead to unfair treatment, and that the effects can be particularly pronounced for individuals who are multiply marginalized and who may, for example, be denied opportunity along the dimensions of both race and gender. This insight is surfacing an important dimension of intersectional discrimination in algorithmic rankers and is, to the best of our knowledge, the first approach in this area to have observed and proposed ways to counteract intersectional effects.

5.2.3   Stoyanovich et al. [75]. **Fairness definition and problem formalization** The final method we discuss in this section aims to incorporate diversity constraints of the kind that were used by Celis et al. [19] and Yang et al. [86] into online set selection. This setting models a sequence of job or college admissions interviews: candidates arrive one-by-one and their qualification score Y is revealed at the time of the interview. Candidates are assumed to arrive in random order according to score, and their total number n is known or can be estimated. The decision maker must hire

or to reject the candidate being considered as soon as their score Y is revealed, before advancing to the next candidate in the sequence.

The classic version of this problem, known as the Secretary problem [31, 53], aims to select a single candidate with the highest score Y. It was shown by Lindley [53] and by Dynkin [31] that the optimal hiring strategy is to interview $s = \lfloor \frac{n}{e} \rfloor$ candidates without making any offers (this is called the "warm-up period"), and make an offer to the first candidate whose score is better than the best score of the of the first s candidates (or accept the last candidate if no better candidate is seen). This strategy yields the highest-scoring candidate with probability $\frac{1}{e}$, and is said to have "competitive ratio" e. Further, this is the best such strategy for the secretary problem (i.e., with the highest competitive ratio) [34]. This problem has been extended by Babaioff et al. [5] to select k candidates, maximizing the expected sum of their scores. Stoyanovich et al. [75] postulate the diverse k-choice secretary problem that enriches the k-choice secretary problem of Babaioff et al. [5] with diversity constraints.

candidate	a	e	**b**	f	c	**d**
Y	7	4	8	5	9	3

(a)

candidate	a	e	**b**	**f**	c	d
Y	7	4	8	5	9	3

(b)

Fig. 11. An instance of the diverse k-choice secretary problem. A set of $n = 6$ college applicants C, are arriving for in-person interviews. The order of interviews is from left to right, with a arriving first, followed by e, etc. A candidate's score Y is revealed when they are interviewed. C is partitioned into two groups based on (binary) gender, $G_M = \{a, b, c\}$ with $n_M = 3$ candidates, and $G_F = \{d, e, f\}$ with $n_F = 3$ candidates. The goal is to select $k = 2$ candidates, with one of every gender ($L_k^M = 1$, $L_k^F = 1$), and to maximize the expected sum of Y-scores subject to these diversity constraints. **(a)** using a common warm-up period yields candidates b and d, selecting the 2nd best male candidate but the lowest-scoring female candidate **(b)** separating warm-up per-group yields candidates b (as before) and f, the top-scoring female candidate.

The diverse k-choice secretary problem is formalized as follows: In addition to a qualification score Y, each candidate is associated with one of $i \geq 2$ groups G based on the value of a single multinary sensitive attribute (e.g., gender, race, or disability status). Both the total number of candidates n, and the number of candidates in each group $n_1 \ldots n_i$, are known ahead of time or can be estimated. The goal of the decision maker is to select k candidates, maximizing the expected sum of their scores, subject to diversity constraints, stated in the form of per-group lower-bounds L_k^G and upper-bounds L_k^G. Figure 11 gives an example: a set of $n = 6$ college applicants, of whom $n_M = 3$ are male and $n_F = 3$ are female, are being interviewed in the order shown in Figure 11, left-to-right. The admissions officer wishes to select $k = 2$ applicants, with one of each gender, specified by the lower-bound constraints $L_k^M = 1$ and $L_k^F = 1$.

The key idea in Stoyanovich et al. [75] is that, if score distributions are expected to differ between the groups, then separate warm-up periods should be conducted for each group to better estimate the scores of that group's desirable candidates. As illustrated in Figure 11a, measuring the higher-scoring male candidates and the lower-scoring female candidates against the same (higher-scoring) standard will allow high-scoring male candidates to be selected. However, the female candidates selected in this way are those that happen to be at the end of the interview queue: they were chosen "at the last minute" to satisfy the diversity constraint. This is problematic for the reasons we outlined when discussing Yang et al. [86] earlier in this section — it withholds opportunity from the relatively better-qualified candidates of a historically disadvantaged group, and it can build bad precedent if the lesser-qualified candidates from that group are selected but do not perform well on the task. Figure 11b shows the result of a selection in which warm-up was conducted separately per group, yielding a higher-scoring female candidate.

The authors propose additional techniques to handle cases where the sum of the per-group lower bound is less than k, leaving the freedom to select high-scoring candidates from any group. Finally, they consider the case where a constant-size waiting list of candidates is allowed, showing that it can lead to higher-utility outcomes.

Experiments and observations The experimental evaluation of the proposed algorithms for variants of the diverse k-choice secretary problems is conducted using three real datasets: Forbes richest Americans [35], NASA astronauts [57], and Pantheon [22] (see details in Section 4). Additional results on synthetic datasets are provided, to simulate differences in score distributions between groups. The evaluation on real datasets shows that the algorithms can select candidates that meet the desired diversity constraints while paying a very small cost in terms of the utility loss. The evaluation on synthetic datasets shows that if a difference in the observed scores is expected between groups, then these groups must be treated separately during processing. Otherwise, a solution may be derived that meets diversity constraints, but that results in lower utility for the disadvantaged groups.

Insights This work focuses on pre-existing bias that exhibits itself through differences in expected scores between groups of candidates. Diversity constraints, and the mechanism used to enact them, aims to equalize outcomes across groups, and so this method clearly links to the WAE worldview.

The core idea in this work is that effort, as represented by scores, should be seen as relative: scores are estimated per group, and individuals from a particular group are evaluated against that group's score threshold. This places the method firmly within the luck-egalitarian EOP camp.

5.3 Intervening on the Score Distribution

This section is forthcoming. It will cover methods by Kleinberg and Raghavan [47], Celis et al. [18], and Yang et al. [87] that model implicit bias in score distributions, and intervene on the distributions to mitigate that bias.

5.4 Intervening on the Ranking Function

This section is forthcoming. It will cover methods that design fair scoring functions by Asudeh et al. [3].

6 SUPERVISED LEARNING

In this section, we present several methods for fairness in learning-to-rank and information retrieval. We will continue to use the notation that we introduced in Section 2.2 wherever appropriate to illustrate the commonalities of the fields.

Recall from Section 3.2 that we speak of three different bias categories: pre-existing, technical and emergent bias. The following section presents different approaches to mitigate these biases and enhance fairer ranking results through various strategies and definitions of fairness. Their categorization into pre-, in- and post-processing approaches describes whether they mitigate bias, before, during, or after model training (recall Figure 8 at page 16). Table 4 summarizes the classification of the methods we present in the following into the frameworks described in Section 3.

6.1 Pre-Processing Methods: Learning Fair Training Data

Pre-processing approaches are usually concerned with biases in the training data which they try to mitigate. Those biases can be of all three types: pre-existing biases appear in any data collection procedure in various ways. It is the way we interrogate, the decision which information we collect and which not, etc. Technical bias makes its way into data as rounding errors, different number and category encoding or the strategy choice how to handle missing values. Emergent bias arises when data is used in a different way than intended during collection.

Method	Mitigation Point	Group structure	Bias	Worldview	EO Framework
iFair [49]	pre-proc.	multinary; intersectional	pre-existing	WAE	Rawlsian
DELTR [91]	in-proc.	binary; intersectional	pre-existing	WAE	luck-egalitarian
Fair-PG-Rank [72]	in-proc	binary; intersectional	technical	WYSIWYG	formal
Pairwise Ranking Fairness [10]	in-proc.	binary; intersectional	?	?	?
FA*IR [90]	post-proc.	binary; intersectional	pre-existing	continuous	formal – luck-egal.
CFAθ [92]	post-proc.	multinary; intersectional	pre-existing	continuous	formal – Rawlsian.
Fairness of Exposure [71]	post-proc.	binary; intersectional	pre-existing & technical	WYSIWYG & WAE	formal & luck-egal.
Equity of Attention [11]	post-proc.	multinary; intersectional	technical & emergent	WYSIWYG	formal

Table 4. Summary of method classification. "Pre-, in-" and "post-proc" refer to whether a method can be classified as pre-, in- or post-processing. "Binary" vs. "multinary" tells whether the method can handle two or more protected groups per one attribute at a time (e.g. young/old is a binary manifestation of the attribute "age", while kid/teen/adult/old is a multinary manifestation of said attribute). "Intersectional" expresses whether and how a method can handle more than one protected attribute at a time. Intersectional means that groups are formed by a combination of attribute values (e.g. black females). See Section 3 for reference.

General advantages of pre-processing methods are:

- Pre-processing methods consider fairness as first concern in the machine learning pipeline.
- Most in- and post-processing methods rely on the availability of group labels during or after training, respectively. Pre-processing approaches instead commonly operate on a distance measure between individuals which allows to be agnostic to group membership. It is sufficient to define who should be similar to whom, based on the features that are available.
- Additionally it is possible to control for certain types of fairness across groups, even if only sparse information about group membership is available [50].

General disadvantages are:

- Machine learning methods that rely on a separate feature engineering step are not applicable because the features identified by domain experts may be rendered meaningless, if fair representations are learned from the raw data.
- Current methods only operationalize individual fairness and treat group fairness as a special case of it.

6.1.1 iFair [49]. **Fairness Definition.** This work operates on an individual fairness objective to learn fair representations of training data points. It is based on the fairness definition by Dwork et al. [30], which states that similar individuals should be treated similarly. The goal is to transform an input record X_a (the feature vector for candidate a) into fairer data representations $\widetilde{X}_a$ using a mapping ϕ, such that two individuals a and b, who are indistinguishable on their non-sensitive attributes $X \setminus A$ (marked by X^*) should also be nearly indistinguishable in their fair representations $\phi(X_a)$ (where sensitive attributes are included):

$$\left| d\left(\phi(X_a), \phi(X_b)\right) - d(X_a^*, X_b^*) \right| \leq \epsilon$$

Note that this definition assumes that a similarity measure d is available that can correctly (and free of bias) capture the differences between two individuals. As the paper uses the family of Minkowsky p-metrics, let us assume we choose the Minkowsky metric with $p = 1$, i.e., the Manhattan distance as d. Reconsidering our college admission example, in Figure 12 we see that candidates b and c, as well as d and e have a distance of 0 in their non-protected features: $d(X_1^*, X_2^*) = d(X_3^*, X_4^*) = 0$. When comparing candidates across groups we see that the female group shows a Manhattan distance of 1 to the male group: $d(X_1^*, X_3^*) = d(X_2^*, X_3^*) = d(X_1^*, X_4^*) = d(X_2^*, X_4^*) = 1$ The algorithm, named "iFair", would create a new feature set $\phi(X)$ that preserves those distances *and* includes sensitive attributes in a way that they do not correlate with non-protected attributes anymore. In our example A_1 correlates with X_2, which may be picked up by a ranking model. To avoid that iFair would assign non-correlating values to A_1, e.g., by switching the sex of candidates b and d.

Insights. Though not clearly stated the wording suggests that attributes are measured in observable space OS and the definition seeks to reduce technical bias. Depending on the distance metric choice of d, the method would potentially be capable of learning representations that ignore *all* information about group membership, even if it is encoded partly in the non-protected features. In this case it would assign low weights to those non-protected features that indirectly encode protected ones. However, the choice of Minkowsky metrics, where distances are measured in terms of absolute numbers, suggests that the authors assume construct space $CS \sim OS$ and hence a leaning to a WYSIWYG worldview and to formal equality of opportunity.

The authors do not address the question if or why Minkowsky metrics are not prone to reproduce biases from training data, that may arise from biased observation processes. They tackle this problem in a follow-up work [50], where the distance metric is replaced by a *fairness graph* that captures pairwise similarities of individuals. In the graph a node represents an individual a and an edge between two nodes indicates that these individuals are to be considered similar. This approach has two advantages: it allows a comparison of an individual's non-protected attributes across different domains (e.g., the h-index of a successful researcher in programming languages is typically lower than those of a successful researcher in machine learning), and it allows for sparse similarity judgments, as individuals can be grouped into clusters based on their in-group relevance scores (e.g., the top-10% in the female group).

We do not further present Lahoti et al. [50] here, because the paper focuses on classification tasks in its experimental section. Pre-processing methods claim to be application agnostic, however Lahoti et al. [49] is the only work by the publication of this thesis that has been proven to work for ranking tasks.

Algorithm. The problem is formalized as a probabilistic clustering problem: given K clusters of similar individuals (with $K < n$) each is represented by a prototype vector v_k. A candidate record X_a is assigned to one of the v_k based on

candidate	A_1 (sex)	A_2 (race)	X_1 (GPA)	X_2 (SAT)
b	male	White	4	5
c	male	Asian	5	4
d	female	Black	5	3
e	female	White	4	4

Fig. 12. Data set of college applicants. Score Y is the again the ground truth. If we assume the Manhattan distance as distance measure d, the method tries to find a fair feature representation to ensure that a learning to rank method considers candidates only based their non-sensitive features (here X_1 and X_2). In this case candidate b and c, as well as e and c should have the same fair feature representation $\widetilde{X}$ where A_1 and A_2 are not relevant for the ranking decision.

a record-specific probability distribution P_a that reflects the distances of the record from each of the prototype vectors and thus forms the fair representation:

$$\phi(X_a) = \widetilde{X}_a = \sum_{k=1..K} P_{ak} \cdot v_k$$

This is used to formalize a utility objective to ensure a low reconstruction loss and a fairness objective that demands that ϕ should preserve pair-wise distances on non-protected attributes between data records:

$$L_{\text{util}}(X, \widetilde{X}) = \sum_{a \in C} ||X_a - \widetilde{X}_a||_2$$

$$L_{\text{fair}}(X, \widetilde{X}) = \sum_{a,b \in C} \left(d(\widetilde{X}_a, \widetilde{X}_b) - d(X_a^*, X_b^*) \right)^2$$

The two objectives are combined into an objective function that the algorithm optimizes by use of gradient descent:

$$L = \lambda \cdot L_{\text{util}}(X, \widetilde{X}) + \mu \cdot L_{\text{fair}}(X, \widetilde{X})$$

The algorithm supports multiple groups and it is not necessary to specify the protected groups a-priori.

Experiments. Experiments are performed on five real-world and a synthetic data set. From the real-world data sets, two are used for learning-to-rank tasks, namely the XING data set [84] and the AirBnB data set [1] (the other ones are used for classification tasks and hence we do not cover them). The synthetic experiments show that representations learned by iFair remain nearly the same for all configurations, irrespective of changes in group membership. This means that changing the value of the protected attribute does not influence the learned representation, i.e. when training a model on these representations it will not allow any conclusions about the protected attribute.

The results show that applying learning algorithms on representations learned by iFair leads to more consistent decisions w.r.t. the distribution of items across a ranking than when applying the same algorithm to the original data. This means that two items with similar non-protected features receive similar visibility in the resulting ranking.

6.2 In-Processing Methods: Learning a Fair Model

In-processing fair ranking methods extend the objective function of a learning-to-rank algorithm by a fairness term. Thus the algorithm's optimization problem consists of an accuracy objective *and* a fairness objective and the method learns to find the best balance between these two. General advantages are:

(i) In-processing methods yield better trade-offs between accuracy and fairness than post-processing ones, because finding this balance is the heart of their learning objective.

(ii) They are capable of handling different types of underlying biases without knowing which particular type is present (see Section 6.2.1).

General disadvantages are:

(i) The impact of the fairness objective on the resulting ranking is less apparent than it is in case of post-processing algorithms.

6.2.1 DELTR [91]. **Fairness Definition.** This method perceives unfairness as disparities in exposure, i.e. the average visibility a group receives (see Section 2.2 and Eq. 8 on page 9). The exposure of a document is defined as its probability $P_{\hat{Y}Q}(X_a)$ to appear in the top position of a ranking for query Q:

$$\text{Exposure}\left(X_a | P_{\hat{Y}Q}\right) = P_{\hat{Y}Q}(X_a) \cdot v_1 \tag{16}$$

where v_1 is the *position bias* of position 1, indicating its relative importance for users of a ranking system [42]. The exposure of group $\mathcal{G}$ is hence the average probability of its members to appear in the top position:

$$\text{Exposure}(\mathcal{G}|P_{\hat{Y}Q}) = \frac{1}{|\mathcal{G}|} \sum_{X_a \in \mathcal{G}} \text{Exposure}\left(X_a|P_{\hat{Y}Q}\right) \tag{17}$$

Insights. As the definition optimizes for equality of exposure (exposure is the outcome), rather than equity, this means that the (potentially biased) qualification of a candidate is not taken into account, which suggests that its underlying assumption is a WAE worldview. We denote however, that because exposure is defined through the probability to appear in the top position, it indirectly contains a measurement of document relevance. We further denote that by setting γ to 0, the WYSIWYG worldview can also be adopted.

The method is concerned with pre-existing biases that lead to a biased observation process and in turn disparate exposure distributions. Given that, the fairness definition belongs to the framework of substantive EO, yet the authors do not explicitly state their beliefs about score distributions in CS, and so they do not explicitly map their approach to either Rawlsian or luck-egalitarian EO framework. However, as the method is agnostic to "true" score distributions and optimizes for equal exposure in decision space DS irrespective of whether score distributions in CS are different for demographic groups, we argue that this meets the conditions for luck-egalitarian EO and place it into that category.

Algorithm. The algorithm incorporates its unfairness measure into the objective function of a list-wise learning algorithm, namely ListNet [17], such that the algorithm optimizes simultaneously for an accuracy metric L and an unfairness metric for two groups $D\left(\hat{Y}\right)$:

$$L_{\text{DELTR}}\left(Y, \hat{Y}\right) = L\left(Y, \hat{Y}\right) + \gamma D\left(\hat{Y}\right) \tag{18}$$

with

$$D\left(\hat{Y}\right) = \max\left(0, \text{Exposure}\left(\mathcal{G}_0|P_{\hat{Y}Q}\right) - \text{Exposure}\left(\mathcal{G}_1|P_{\hat{Y}Q}\right)\right)^2$$

where $\mathcal{G}_0$ denotes the non-protected group and $\mathcal{G}_1$ the protected one. The squared hinge loss ensures an asymmetric metric, i.e. unfairness is only detected, if the protected group receives less exposure than the non-protected group, but not vice versa. The optimization problem is solved using gradient descent.

Experiments. To illustrate how the method works, let us back back to our running example in Figure 5 on page 8: the algorithm gets as input the sensitive feature that forms a protected group, in this case A_1. As the training data C_{train} shows disparities in exposure for females (they are all ranked below males) a standard learning to rank algorithm is likely to pick up A_1 as a predominant criterion for its model and assign a high weight to it. DELTR instead will learn to ignore A_1 as a decision criterion because the unfairness metric penalizes ranking predictions that show high disparities in exposure for the different groups of A_1. This way DELTR can also compensate for systematic errors in the data or in the relevance measures (e.g. if the SAT test design favors male applicants, hence females systematically achieve lower scores).

The experiments are performed on three different datasets each of them containing different types of biases. The proposed methods handles all types without prior knowledge to what particular bias is present:

- **W3C experts:** [81] The experimental setup investigates on situations in which bias is unrelated to relevance: expertise has been judged correctly, but ties have been broken in favor to the non-protected group. In this case, including the sensitive feature during training yields very bad results in terms of disparate exposure and relevance, because all experts from the protected group are ranked to the bottom of the list.

- **Engineering students:** [76] The task is to predict the students' academic performance after the first year based on their admission test results and school grades. Here the same admission test score relates to different levels of academic performance across groups, i.e. a score of 500 from a protected candidate relates to better academic performance than a score of 500 from a non-protected candidate. This experiment investigates on situations in which bias is coming from different score distributions among groups and shows that trading-off accuracy for fairer rankings is not a necessary condition. In this case including the sensitive feature during training yields *better* results in terms of exposure and relevance.
- **Law School Admission Council:** [69] These experiments show that disparities in exposure due to differences in academic performances can be reduced by use of the approach at a cost of accuracy. However the trade-off is usually lower for in-processing methods than for post-processing ones, because the best balance between accuracy and fairness is found automatically and not manually.

6.2.2 Fair-PG-Rank [72]. **Fairness Definition.** The approach by Singh and Joachims [72] addresses technical bias i.e., bias that is created by the ranking system itself, namely through position bias that gives candidates beyond the first few positions significantly less visibility than those in the first positions. It also operates on a notion of document exposure, however this time exposure is not defined as the probability of a document to appear at the top position but through expected attention, which the authors define as equivalent to the expected position bias (as we defined in Equation 8 in Section 2.2 on page 9). Similar to Biega et al. [11] (Section 6.3.4) they operate on a merit-based constraint: each candidate $\tau(i) = a$ in ranking τ should receive exposure proportional to their utility $U(\tau, a)$:

$$U(\tau, a) \geq U(\tau, b) \rightarrow \frac{\text{Exposure}(a)}{U(\tau, a)} \leq \frac{\text{Exposure}(b)}{U(\tau, b)} \tag{19}$$

The work further proposes a definition of individual fairness per query Q that measures the disparities in visibility $\text{v}(.)$ for two candidates a, b in τ:

$$D(a, b|Q) = \frac{1}{|H^Q|} \sum_{(a,b) \in H^Q} \max\left[0, \frac{\text{v}(a)}{U(\tau, a)} - \frac{\text{v}(b)}{U(\tau, b)}\right] \tag{20}$$

with $H^Q = \{(a, b) \text{ s.t. } U(\tau, a) \geq U(\tau, b)\}$. The authors also propose a definition of group fairness for two groups, in which the individual document visibility from the above equation is replaced with a notion of group visibility:

$$D(\mathcal{G}_0, \mathcal{G}_1|Q) = \max\left[0, \frac{\text{v}(\mathcal{G}_0)}{U(\tau, \mathcal{G}_0)} - \frac{\text{v}(\mathcal{G}_1)}{U(\tau, \mathcal{G}_1)}\right]$$

with $\text{v}(\mathcal{G}) = \frac{1}{|\mathcal{G}|} \sum_{a \in \mathcal{G}} \text{v}(a)$ being the average exposure of group $\mathcal{G}$.

Algorithm. Using the fairness definitions the authors extend the ListNet [17] ranking function to incorporate their disparity measures. The algorithm Fair-PG-Rank is learning an optimal ranking τ^* via empirical risk minimization using the following learning objective:

$$\tau^*_\delta = \arg\max_{\tau} \frac{1}{N} \sum_{Q=1}^{N} [L(\tau)] - \lambda \frac{1}{N} \sum_{Q=1}^{N} [D(.|Q)] \tag{21}$$

with L being a loss function that measures the utility of τ for the user. The optimization is done using gradient descent.

Insights. As already mentioned the method is concerned explicitly with the inherent technical bias of a ranking that arises from showing result documents in a one-dimensional list. It assumes a WYSIWYG world in which a document's

merit in *OS* truly reflects its merit in *CS*, because no means are taken to reduce any errors that might have been introduced by mapping function *g*. It therefore operates on a formal EO framework.

Compared to the method described in the previous section (6.2.1) this approach is concerned with equity of exposure rather than equality. This means that documents shall only receive as much exposure as they "deserve" based on their relevance measure. Coming to our college admission example (Figure 5, page 8): we see that in the training data (white background lines) all females have worse scores than males and appear below in the ranking. As stated before, a standard LTR algorithm is therefore likely to treat A_1 as an important decision criterion and assign a high weight to it. Because Fair-PG-Rank optimizes for equity of exposure this approach ensures that documents receive equal exposure among those that "deserve" it based on their relevance measure. As such Fair-PG-Rank ensures that a model is not discriminating based on a sensitive attribute while assuming that the relevance data is correct and does not contain discriminating patterns.

Experiments. Experiments are done on a synthetic dataset, the German credit dataset [64] using the group fairness definition in the learning objective and the Yahoo! LTR dataset [85], using the individual fairness definition in the learning objective. The synthetic dataset has two features for each document, where Feature 2 is corrupted for the minority group G_1. The results show that with increasing values of λ, the weight of the corrupted feature is decreasing. With the real world datasets the authors show that the method works in a real world setting, but do not discuss its implications. The authors compare their method to [91], but as their measure of disparate exposure is vastly different and the bias they are focusing on is not the same, it is not clear if meaningful conclusions should be drawn from such comparisons. Further research is needed to understand how methods with opposing worldviews, conflicting understandings of equality of opportunity and different addressed biases can be compared in a meaningful way.

6.2.3 Pairwise Fairness for Rankings [10]. **Fairness Definition.** This method is the first one to introduce a pairwise fairness metric for ranking predictions. The authors setup their method as a component of a cascading recommender system, however it only considers the final ranking of items. We therefore present the method here rather than in Section 7, where we talk about fairness in recommendation systems.

The framework is formalized as follows: each query consists of user features U_i for user i and context features C. Each ranking candidate a is described by a feature vector X_a. Then a ranker $\hat{f}$ is trained to predict user engagement, which relates to clicks $\hat{y}$ *and* interaction after a click $\hat{z}$ (such as purchase, ratings, etc.). which are then mapped to a scalar value to rank items: $\hat{f}(X) = \hat{Y}$.

The focus of the fairness definition is on the risk for groups of ranked candidates to be under-recommended, where the group definition is binary, i.e. $A_a \in 0, 1$ for candidate a. For this, the authors first define pairwise accuracy, which describes the probability that a clicked candidate is ranked above another relevant un-clicked candidate, for the same query:

$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b\right)$$

Pairwise fairness asks if the pairwise accuracy is the same across the two groups:

$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = 0\right) = P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = 1\right)$$

To account for user engagement z the definition is extended to compare only those candidates with each other that receive the same amount of engagement $\tilde{z}$:

$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = 0, z_a = \tilde{z}\right) =$$
$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = 1, z_a = \tilde{z}\right) \forall \tilde{z}$$

The authors further extend the definition to also consider group exposure in rankings, because two rankings could have the same pairwise accuracy across groups while systematically putting candidates of one group to lower ranks of the list. To account for this, they split the definition into *intra-group* pairwise fairness:

$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = A_b = 0, z_a = \tilde{z}\right) =$$
$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = A_b = 1, z_a = \tilde{z}\right) \forall \tilde{z}$$

and *inter-group* pairwise fairness:

$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = 0, A_b = 1, z_a = \tilde{z}\right) =$$
$$P\left(\hat{f}(X_a) > \hat{f}(X_b) \mid Y_a > Y_b, A_a = 1, A_b = 0, z_a = \tilde{z}\right) \forall \tilde{z}$$

Intra-group fairness indicates whether across candidates from the same group, those that are more likely to be clicked on are ranked above those less likely, while inter-group fairness describes whether mistakes of the ranker are at the cost of one particular group.

Insights. The framework as is is not clearly classifiable into WAE or WYSIWYG, because the authors talk about click probability and user engagement without further specifying what these two are composed off. Particularly they do not tell, whether a measure of candidate merit is part of the click probability or not. Click through rate (CTR) is usually defined to contain some measure of relevance [65], in that case Y would contain a concept of merit (that is measured in OS) and hence the framework would correspond to WYSIWYG. The authors briefly mention that they assume the final ranking model $\hat{f}$ only operates on relevant documents, which supports the assumption that the underlying worldview of this framework is WYSIWYG. We face the same uncertainty when thinking about which EO framework this work corresponds to. Without knowledge of the actual underlying estimation of click probability and user engagement, and without a statement on an individual's effort, it is not clear to which EO framework or worldview a mathematical fairness definition belongs to. In fact, even though the authors state that they adopted a definition of "equal opportunity", depending on the exact definition of click probability and user engagement, their pairwise fairness definition could be mapped to any of the four equal opportunity definitions from section 3. An identification of the addressed bias is also not possible without the CTR definition.

Experiments. The experiments study the performance of the ranker with respect to a sensitive subgroup of candidates in a synthetic dataset, comparing the performance of this subgroup to the rest of the data, denoted by "not subgroup." The subgroup represents approximately 0.2% of all items. The authors compare two versions of their approach: a model without any pairwise fairness constraint and one with an inter-group fairness constraint. Performance measures (pairwise accuracy) are aggregated across user engagement levels and averaged. Then the metrics for the subgroup and not-subgroup are divided, such that a ratio of 1 means perfect fairness, a value below 1 means that the average pairwise accuracy for the subgroup is lower than for the other and a value above 1 means that it is higher. Engagement z is grouped into four levels. The overall pairwise fairness evaluation shows that the system under-ranks candidates
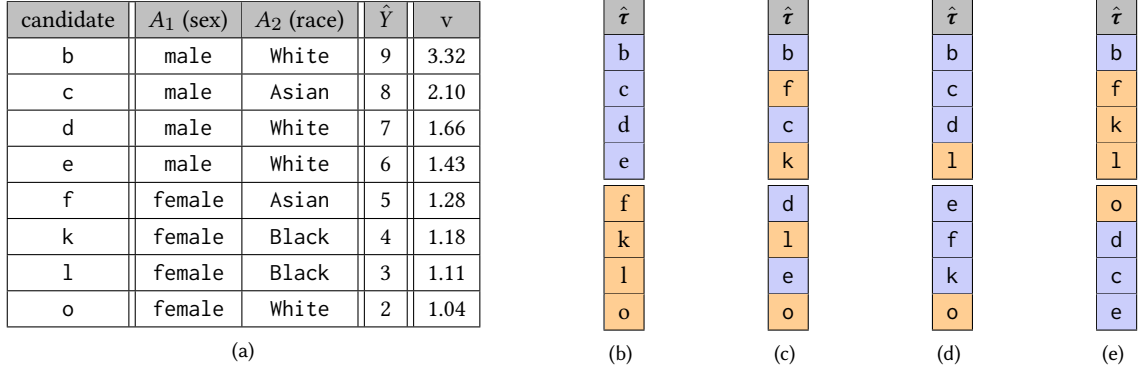
candidate	A_1 (sex)	A_2 (race)	$\hat{Y}$	v
b	male	White	9	3.32
c	male	Asian	8	2.10
d	male	White	7	1.66
e	male	White	6	1.43
f	female	Asian	5	1.28
k	female	Black	4	1.18
l	female	Black	3	1.11
o	female	White	2	1.04

(a)

$\hat{\tau}$
b
c
d
e
f
k
l
o

(b)

$\hat{\tau}$
b
f
c
k
d
l
e
o

(c)

$\hat{\tau}$
b
c
d
l
e
f
k
o

(d)

$\hat{\tau}$
b
f
k
l
o
d
c
e

(e)

Fig. 13. (a) dataset C of college applicants. Score $\hat{Y}$ is predicted by a learning to rank model. v is the position bias of positions $1 - 8$; (b) predicted ranking $\hat{\tau}$ based on $\hat{Y}$. Again, the top-4 candidates will be admitted. Note that under this scenario females are not admitted; (c) a ranking that is fair according to the ranked group fairness condition of the FA*IR algorithm defined by [90] with $p = 0.7$ and $\alpha = 0.1$. Also the outcome of the algorithm CFAθ with $\theta = 1$; (d) a ranking produced by CFAθ with $\theta = 1$ if the dataset was changed to candidate f and k being male. Note that CFAθ cannot achieve a ranking where the now only two female candidates l and o are both admitted; (e) a ranking with equal exposure across groups produced by the method of Singh and Joachims [71].

from the subgroup when the subsequent level of engagement is low, but interestingly slightly over-ranks candidates from the subgroup when the subsequent level of engagement is high.

The intra-group pairwise fairness evaluation shows that across all levels of engagement the model has more difficulty selecting the clicked candidate when comparing subgroup candidates than when comparing non-subgroup candidates. This is partly because the subgroup is small and less diverse.

The inter-group fairness evaluation shows that across all levels of engagement the subgroup candidates are significantly under-ranked relative to the non-subgroup candidates. Further, the results show that the pairwise accuracy for non-subgroup candidates in inter-group pairs is notably higher than in intra-group pairs, suggesting that subgroup candidates, even when of interest to the user, are ranked under non-subgroup candidates. When optimizing for inter-group fairness, these disparities are mitigated and candidates from the subgroup receive more exposure (which they measure as the probability that candidate a is ranked above item b).

6.3 Post-Processing Methods: Re-Ordering Ranked Items

Post-processing algorithms assume that a ranking model has already been trained. A predicted ranking is handed to the algorithm, which re-orders items to improve fairness. Most algorithms operate on a notion of group membership meaning that certain groups are called protected, while one group is non-protected. As such post-processing methods are often similar in their fairness concepts as the score-based fair ranking methods. We will use our running example (reusing Figure 3) to illustrate the differences and similarities between the presented methods in this section, but we omit the non-sensitive features X_1, X_2 and X_3 which are irrelevant at this point.

General advantages of post-processing methods are:

(i) The effect the algorithms have on a ranked output is easy to visualize and understand, because the original ranking before the application of a fairness method can be compared to its result in terms of how items are re-ordered and (in some cases) in terms of a loss in a ranking metric such as NDCG.

(ii) Many of them provide a guaranteed share of visibility for the protected group in the ranking.

General disadvantages are:

(i) The post-processing idea inherently suggests that fairness comes at the expense of accuracy, because the scores of a previously trained ranking model are taken as "the true anchor point".

(ii) Algorithms with a fixed fairness constraint [71, 90, 92] may cause a huge loss in performance w.r.t. the unaware ranking. This may be the case, if the score distribution of the protected group is much lower than for the non-protected group.

*6.3.1 FA*IR [90].* **Fairness Definition.** The fairness definition is based on the assumption that rankings are fair when the decisions on candidate placements are drawn from a Bernoulli distribution (coin tosses). The algorithm assures that the number of protected candidates does not fall far below a required minimum percentage p at any point in the ranking, by formulating this fairness as a statistical significance test to test whether a ranking was likely to have been produced by a Bernoulli process.

FA*IR operates in a binary group setting and is concerned with disparate impact, as it does not take any notion of merit into account for its re-ranking strategy. It accepts a ranking prefix of length k as fairly representing the protected group, if the number τ_G of protected items is not significantly i.e. α lower than the minimum target proportion p: $F(\tau_G k, p) > \alpha$ with F being the binomial cumulative distribution function. If this condition holds for each ranking prefix $k = 1, \ldots, n$, the entire ranking is accepted.

Insights. The existence of a minimum proportion for a protected group suggests a WAE worldview and a main concern with pre-existing bias, that has to be accounted for using said minimum proportion. However, the WYSIWYG worldview can also be adopted, if p is set to a low value, which permits a gradual transition from WYSIWYG to WAE. This however is a mere side effect of the method and not to be seen as a desired goal of the algorithm (as opposed to [92]). Critically, if p is chosen too high, the framework will rank a lot more protected items to the highest positions, than non-protected ones. This would be justified only under libertarian EO, or more importantly if one assumes that the protected group is actually a lot more relevant (they score very high in construct space), but the measurements in observable space are extremely biased or the scoring model produces inverted predictions for the protected group in decision space (this translates into low values in either OS or DS). While such cases do exist it is questionable, if one should actually attempt to correct such a flawed ranking, rather than disregard it entirely.

The framework assumes that differences between groups in the relevance distribution are an artifact of different circumstances for each group and therefore relates to substantive EO. As we stated in Section 3, WAE can relate to both of the described substantive EO frameworks and it is mostly the algorithmic definition itself, that makes it belonging to the one or the other. For FA*IR however, both mappings are possible, because the algorithm is agnostic to the source of relevance differences and merely implements an affirmative action policy. We therefore conclude that the preliminaries for luck-egalitarian EO are met.

As stated above the method ensures a given minimum proportion of a protected group in every prefix of a post-processed ranking. As an example consider Figure 13 where a model has predicted relevance scores $\hat{Y}$ based on which the ranking in Figure 13b is produced. This ranking is fair according to the FA*IR algorithm if the input is $p < 0.5, \alpha = 0.1$. If $p \geq 0.5$ the candidates have to be reordered. Figure 13c shows a ranking produced by FA*IR with $p = 0.7, \alpha = 0.1$.

Algorithm. For performance improvements the algorithm calculates a table that contains the minimum number of protected candidates at each position in the ranking given a minimum proportion p (see Table 5). This is done by

computing $F^{-1}(\alpha; k, p)$, the percent point function of the binomial CDF. Then the two groups are ranked separately by decreasing scores and merged into one ranking according to the table. Whenever the ordering by score would violate the minimum number requirement, a protected item is put at the respective position.

Experiments. The experimental evaluation is done on the three real datasets: COMPAS [62], German credit [64], and SAT [66]. The experimental evaluation shows the effects of the methods on performance in terms of NDCG and maximum rank loss. All scenarios under consideration relate to questions of distributive fairness, where certain benefits (scholarship, visibility, pardon) are to be distributed fairly across the two groups.

6.3.2 Continuous Fairness with Optimal Transport [92]. **Fairness Definition and Algorithm.** This work defines a mathematical framework CFAθ to continuously interpolate between the worldviews WYSIWYG and WAE. The authors argue that legally WYSIWYG maps with individual fairness, while WAE maps with current anti-discrimination law, that defines group fairness in terms of *statistical parity of outcome*. The question of what a fair distribution of outcome is depends on the estimated extend of indirect discrimination and pre-existing bias in the scoring model. Interestingly, this means that any fairness definitions departing from group fairness as statistical parity and individual fairness as meritocratic scores do not yet have any legal meaning. Furthermore the authors state that current ruling on anti-discrimination cases only involves actual unfair decisions and not on "softer" disadvantages such as reduced visibility in a ranking.

The WYSIWYG worldview corresponds to the meritocratic ideal and hence to the formal EO framework, while WAE corresponds to Rawlsian EO. This is because an individual's measurable effort (here, their raw score) is seen to be drawn from different distributions μ_k per group in OS, while in CS, there exists only one ν, meaning that all groups have essentially the same distribution of true effort.

The framework can handle multiple groups that are defined as a partition over all individuals: $X = \bigcup_{k \in \{0,1\}^N} g^{-1}(k)$, where $g : X \to \{0, 1\}^N$ is a mapping that returns 1 if an individual carries a certain trait from the set of N features. The k-th group is therefore $\mathcal{G}_k := g^{-1}(k)$. The authors explicitly include all features in the group definition instead of only taking certain attributes that are legally protected into account. This broad definition has the advantage that it can handle non-protected features that serve as proxies for protected ones.

The framework assumes a potentially biased scoring function S is given, that maps from space of individual traits X to a n-dimensional vector $S : X \to \mathbb{R}^n$ and that each group's score forms a probability distribution μ_k. The combined score distribution is called μ and corresponds to a metric from OS, which as usual is prone to pre-existing bias and other systematic errors with different group skews. The authors then define a score distribution $\nu_k = \mu_k \circ T_k^{-1}$ to be the fair score representation of group k obtained by an optimal transport map $T_k : \mathbb{R}^n \to \mathbb{R}^n$. Depending on the worldview, ν is defined differently: In WYSIWYG $\mu = \nu_k$ for all k groups, while in WAE any differences between the μ_k

p \ k	1	2	3	4	5	6	7	8	9	10	11	12
0.1	0	0	0	0	0	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	1	1	1	1	1	2
0.4	0	0	0	0	1	1	1	1	2	2	2	3
0.5	0	0	0	1	1	1	2	2	3	3	3	4
0.6	0	0	1	1	2	2	3	3	4	4	5	5
0.7	0	1	1	2	2	3	3	4	5	5	6	6

Table 5. Example values of the minimum number of protected items that must appear in the top k positions to pass the ranked group fairness test with $\alpha = 0.1$. Table from [90]

are solely product of bias and there exists a single v that is equal for all groups. In the former case the optimal transport matrix is the identity matrix. In the latter, the WAE fair representation distribution v that satisfies statistical parity is defined as the barycenter in Wasserstein space of the μ_k and a T_k has to be found for each group to transform μ_k into v while minimizing violations against decision maker utility and individual fairness. The framework further defines a displacement interpolation with a fairness parameter $\theta \in [0, 1]$, which allows to transform μ_k into any distribution $\mu_k^\theta = \mu_k \circ (T^\theta)^{-1}$ between the WYSIWYG (or individual fairness) policy μ and the WAE (or group fairness) policy v. This means that a high θ corresponds to more emphasis on group fairness, while a low one to more individual fairness and $\mu^0 = \mu$ and $\mu^1 = v$. A notable advantage of this approach is that it does not rely on the existence of a distance metric between individuals, in contrast to many other methods [30, 49]. This is important because it is not clear how such a distance metric, if actually available in OS, would be less prone to biases and errors than a normal optimization metric.

Insights. As CFAθ moves the group distributions of predicted scores closer to each other, this means that when setting $\theta = 1$ the algorithm achieves statistical parity for each group throughout the ranking *but not more* in contrast to FA*IR which can push candidates from the protected group even higher. Such a setting would result in the same ranking as in Figure 13c given that there are 50% of males and females in the dataset. However if there where only 25% females in the dataset (e.g. candidate f and k are male), then the algorithm will produce a ranking as shown in Figure 13d when given $\theta = 1$ as input. Note that with such a dataset the method cannot return a ranking in which the now only two female candidates are ranked among the admitted (i.e. into the top-4).

Experiments. The experimental evaluation is done on a synthetic dataset with 100,000 data points, a score feature and two sensitive features. Group membership for this experiment is defined by all combinations of the values of the sensitive features. Rankings are produced based on the score column and the performance of a fair ranking with different θ is measured in terms of NDCG. The fairness of a ranking is measured as the share of each group at each n positions, with n ranging from 10 to 1000.

A second experiment is conducted on the LSAC dataset [69]. The experiments confirm the general properties of post-processing methods: it is clearly visible how groups are distributed more evenly across all positions with increasing values of θ. However depending on the differences between the μ_k, a processed ranking based on the fair representation v can show significant declines in performance measures w.r.t. the raw score ranking.

6.3.3 *Fairness of Exposure [71].* **Fairness Definition.** The fairness objective of this work is set as a linear combination $a^T P_{a,i}^\tau v = h$ with a being a vector to encode group membership, $P_{a,i}^\tau$ as the probability that $\hat{f}$ places candidate a at rank i in τ and v reflecting the importance of a position in a ranking. This equation is solved under three different group fairness constraints based on a definition of exposure that a candidate a receives under P^τ:

$$\text{Exposure}(X_a | P^\tau) = \sum_{i=1}^{k} P_{a,i}^\tau v(i)$$

with $v(i)$ being the position bias of position i in ranking τ. The average exposure of a group $\mathcal{G}$ is defined as follows:

$$\text{Exposure}(\mathcal{G} | P^\tau) = \frac{1}{|\mathcal{G}|} \sum_{X_a \in \mathcal{G}} \text{Exposure}(X_a | P^\tau)$$

The goal is to distribute exposure fairly between two groups $\mathcal{G}_0$ and $\mathcal{G}_1$ by use of the following three definitions:

(1) **Demographic Parity:** states that the average exposure of groups shall be equal $\text{Exposure}(\mathcal{G}_0 | P^\tau) = \text{Exposure}(\mathcal{G}_1 | P^\tau)$.

(2) **Disparate Treatment:** requires equity of exposure across groups, i.e. the average exposure in relation to their average utility should be equal across groups:

$$\frac{\text{Exposure}(\mathcal{G}_0|\text{P}^\tau)}{U(\tau, \mathcal{G}_0)} = \frac{\text{Exposure}(\mathcal{G}_1|\text{P}^\tau)}{U(\tau, \mathcal{G}_1)} \tag{22}$$

(3) **Disparate Impact:** is measured in terms of disparate click through rate [65] per group, which shall be equal across groups given the groups average utility:

$$\text{CTR}(\mathcal{G}|\text{P}^\tau) = \frac{1}{|\mathcal{G}|} \sum_{a \in \mathcal{G}} \sum_{i=1}^{k} \text{P}^\tau_{a,i} Y_a \text{v}(i)$$

with Y_a being the relevance of candidate a.

$$\frac{\text{CTR}(\mathcal{G}_0|\text{P}^\tau)}{U(\tau, \mathcal{G}_0)} = \frac{\text{CTR}(\mathcal{G}_1|\text{P}^\tau)}{U(\tau, \mathcal{G}_1)} \tag{23}$$

The definition of demographic parity addresses the problem of pre-existing bias and corresponds to the WAE framework, as it tries to balance visibility across groups independently of their performance in OS (note however that a group's exposure is depending on P^τ and may hence indirectly depend on a utility measure, if P^τ is calculated based on document utility). The definition assumes that $CS \not\sim OS$ and therefore that an individuals true effort is different from the measured effort, which the demographic parity definition accounts for. As the authors do not specify their belief about CS it is not clear whether this demographic parity definition corresponds to Rawlsian or luck-egalitarian EO. However, as visibility is equalized without any assumptions on an individual's true effort, we would argue that the conditions for a luck-egalitarian framework are met.

The definition of disparate treatment explicitly addresses the technical bias of a ranking, also known as position bias, by ensuring that all documents of the same utility receive equal visibility. This corresponds to the formal EO framework. The method also corresponds to the WYSIWYG worldview because document utility is measured through features from observable space without taking into account that a biased observation process may exist, hence $CS \sim OS$. Note that it does not necessarily correspond to individual fairness, because utility and exposure are averaged across individuals of a group. This can lead to a downgrading of high scoring individuals in otherwise badly performing groups. Note that if this artefact alone was considered for the mapping to an equal opportunity framework, the best choice would actually be libertarian EO, because items are not assigned to positions based on their own utility anymore, but at least partly based on their group membership.

Insights. The definition of disparate impact is misleading in two ways, because it does not comply with the *legal* definition of disparate impact, which is described solely in terms of the deviation from statistical parity. First the click through rate contains a notion of document relevance and second it is set relative to the average group utility, which actually includes aspects of libertarian EO rather than substantive, as mentioned before. Statistical parity in contrast does not consider any relevance measure whatsoever, precisely because it assumes that these very measurements are subject to pre-existing biases and a biased mapping from CS to OS. As the given definition mostly corresponds to formal EO and a WYSIWYG worldview, it would be more appropriate to label it as a different version of disparate treatment that is concerned with click through rate instead of exposure.

Algorithm. The algorithmic framework is implemented as an ILP that maximizes ranking utility given one of the above constraints translated into a scalar h (contrary to Biega et al. [11], who constraint quality and optimize for disparate

treatment):

$$
\begin{aligned}
\arg\max_{\mathrm{P}} \quad & Y^T \mathrm{P} \mathrm{v} \\
\text{subject to} \quad & \mathbb{1}^T \mathrm{P} = \mathbb{1}^T, \\
& \mathrm{P}\mathbb{1} = \mathbb{1}, \\
& 0 \leq \mathrm{P}_{a,i} \leq 1, \\
& \mathrm{a}^T \mathrm{P} \mathrm{v} = h
\end{aligned}
\tag{24}
$$

Depending on the respective definition of fairness from this paper the outcome rankings will look quite differently. Let us assume that the model is absolutely sure about where to put each candidate such that P becomes the unit matrix and such the exposure of a group is calculated sum of each group members position bias in the ranking. This gives us a group exposure of 2.13 for the male group and 1.15 for the female group for the ranking in Figure 13b. Let us consider the demographic parity objective: Here the exposure should be the same for both groups. As the position bias v is a constant value, the algorithm will modify the scores $\hat{Y}$ until the parity objective is met. A possible solution is shown in Figure 13e with a group exposure of 1.66 for the male group and 1.62 for the female group. (We want to denote at this point that in college admissions the position bias does not play too much of a role and that a more appropriate example would be e.g. a search scenario. We applied the running example to make the outcome differences and similarities across methods visible). The other two objectives work in an similar manner except that there objective functions take a measure of ranking utility into account.

Experiments. The experiments are framed within three different scenarios of unfairness: biased allocation of opportunity (in job candidates rankings), misrepresentation of real-world distributions (biased Google image search for CEO), fairness as freedom of speech (equality of voice within new media channels like YouTube or Twitter). For these, the authors create a synthetic dataset with 100,000 entries and a binary protected attribute. Furthermore they use real dataset YOW news recommendation dataset [95].

6.3.4 Equity of Attention [11]. **Fairness Definition.** This work is one of the few from the FAccT community to address fairness as an issue that arises from the inherent technical bias of a ranking itself. As stated before, each ranking position carries an inherent position bias that increases with the position number, meaning that even if all items had the same relevance, those at the top would receive a lot more attention than others at lower ranks. The authors frame this discrepancy as a problem of distributive individual fairness where they want to achieve equity of attention on the level of individuals: the attention an item gets from users should be proportional to their relevance to the query. Assuming that relevance decreases linearly and attention decreases geometrically, there is a necessary discrepancy between the attention loss that subjects receive and their relevance decrease.

This work proposes a post-processing algorithm that optimizes equity of user attention with a constrained overall relevance loss (in contrast to 6.3.3 which constraints fairness and optimizes for relevance). As a single ranking can not be fair according to their definition due to position bias, the authors suggest an approach in which unfairness is additive. Their fairness definition ensures that *over time* the differences in attention that well suited candidates a and b receive when ranked in m rankings $\tau_{1 \ldots m}$ is equal:

$$
\frac{\sum_{i=1}^{m} att(\tau_i, a)}{\sum_{i=1}^{m} U(\tau_i, a)} = \frac{\sum_{i=1}^{m} att(\tau_i, b)}{\sum_{i=1}^{m} U(\tau_i, b)}
\tag{25}
$$

Hence unfairness is measured as the accumulated difference between the attention of items and their relevance:

$$\text{unfairness}(\boldsymbol{\tau}_1, \ldots, \boldsymbol{\tau}_m) = \sum_{a=1}^{n} \left| \sum_{i=1}^{m} att(\boldsymbol{\tau}_i, a) - \sum_{i=1}^{m} U(\boldsymbol{\tau}_i, a) \right| \tag{26}$$

Insights. The authors relate their work to the notion of individual fairness by [30] and treat group fairness (which they refer to as equality of attention) as a special case, in which the utility distributions are uniform across all rankings: $U(\boldsymbol{\tau}, a) = U(\boldsymbol{\tau}, b), \forall a, b$. It therefore relates to the formal EO framework and a WYSIWYG worldview. Note that this understanding of group fairness can not account for biases and errors in the data that are manifesting differently across groups, meaning that the error for the protected group might be high, while the data for the non-protected group is correct. Their definition of group fairness corresponds therefore explicitly with a WYSIWYG worldview and contrasts with most of the other definitions in the literature. Additionally the method is capable of addressing emergent bias, which may result from online learning algorithms that learn through user feedback and click data.

Algorithm. The fairness definition is implemented as a constrained optimization problem (which is then translated into an Integer Linear Program) in which unfairness is minimized subject to constraints on the maximum NDCG-loss in an online manner. This means that the algorithms allows unfairness minimization over time as subjects can enter and leave the system at any point. The algorithm reorders a given ranking such that unfairness is minimized given the cumulative attention and relevance distribution seen so far:

$$\begin{aligned}
\text{minimize} \quad & \sum_a \left| \sum_{i=1}^{l-1} att(\boldsymbol{\tau}_i, a) + att(\boldsymbol{\tau}_l, a) - \left(\sum_{i=1}^{l-1} U(\boldsymbol{\tau}_i, a) + U(\boldsymbol{\tau}_l, a) \right) \right| \\
\text{subject to} \quad & \text{NDCG@k}(\boldsymbol{\tau}_l, \boldsymbol{\tau}_{l*}) \geq \theta
\end{aligned} \tag{27}$$

with $\boldsymbol{\tau}_l$ being the current and $\boldsymbol{\tau}_{l*}$ the reordered ranking, and θ is the quality constraint, that is increased to ensure more fairness in the rankings.

As this method measures the attention each item received over time given their relevance, we have to consider several rankings to illustrate its effect. As attention is defined through position bias too, just as in [71], let us come back to Figure 13. It is likely that the first few rankings will look like in Figure 13b. However at some point, the received relative attention of e.g. candidates f w.r.t. candidate e will be too low, given that their score decreases linearly while the position bias decreases geometrically. If this happens, candidates e and f will be swapped for the next ranking to be produced by the model.

Experiments. The experiments are run on two real datasets AirBnB [1] and StackExchange [74] and a synthetic dataset, each with two different models for attention gains by position (geometric attention decrease, and only the first position gets all attention)

- **Synthetic Data:** Experiments are set up with three different relevance score distributions (uniform, linear, exponential) and the aforementioned attention models. In all cases the algorithm shows periodic behavior under lab conditions, meaning that every x rounds, it brings unfairness to 0. Furthermore fairness is not a steady state but will grow with each new ranking for each individual item.
- **AirBnB:** For these experiments rankings were created from AirBnB apartment listings in Hongkong (4529 items), Boston (3944 items) and Geneva (1728 items) under two scenarios – first using always the same query, second to varying queries. The results verify that the difference between the distributions for attention and relevance is generally huge in real world datasets and the question of whether unfairness can be brought to zero is highly
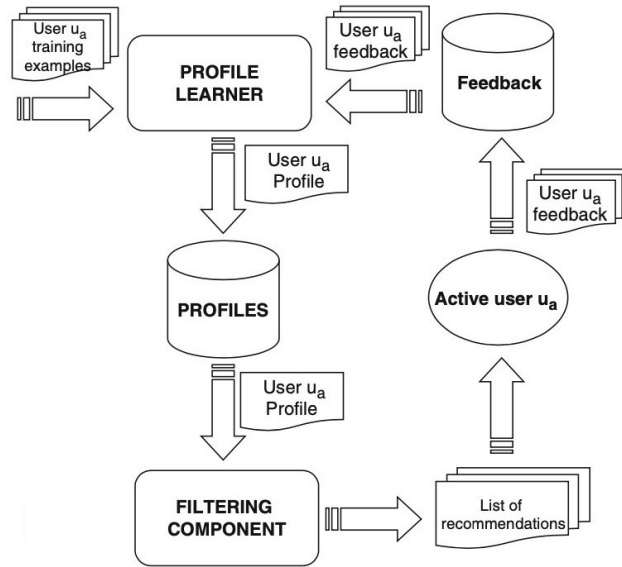
Fig. 14. Principle components of a recommender system. A profile learner determines user profiles from training examples (upper right) which are stored in a profile database. When the user issues a query, the filtering component processes the general search result, and returns a list of user specific recommendations. The interaction of the user are collected as feedback and stored in a feedback database. This data is used in turn to update the model of the profile learner. Image taken from [27].

dependant on the dataset at hand. In all cases, only if $\theta = 0$ (i.e. no quality ensuring constraint) unfairness is not growing over time.

- **StackExchange:** The experiments show that individual subjects appear in relatively few result rankings, causing an extended fairness amortization time.

7 FAIRNESS IN OTHER DOMAINS: RECOMMENDATION AND MATCHING

Today many web applications with search functionality also implement a recommendation system that provides personalized search results to the user with items dedicated precisely to their interest. The main objective of recommender systems is to facilitate transactions between multiple stakeholders in which personalization plays a key role and therefore fairness issues for more than one group of participants have to be considered. RecSys tasks can be grouped into three different types: finding good items that meet a user's interest, optimizing the utility of users, and predicting ratings of items for a user [45]. They often consist of multiple models, must balance multiple goals and are difficult to evaluate due to extreme and skewed sparsity [10]. Examples for such systems are user recommendations in web shops or on streaming platforms. Figure 14 shows the functional principle of a recommender system. A profile learner determines user profiles from training examples (upper right) which are stored in a profile database. When the user issues a query, the filtering component processes the general search result (and additional items to be displayed), and returns a list of user specific recommendations. The interaction of the user with the result are collected as feedback and stored in a feedback database. This data is used in turn to update the model of the profile learner.

Burke [16] identifies three different stakeholders in recommender systems, namely the consumers, producers and the platform itself. They further identify three different types of recommender systems, distinguished by the respective stakeholders that are considered for fairness. When considering fairness for the recommended items, they speak of producer-fairness, while fairness for the users of the system is considered under the term consumer-fairness. In a system that satisfies consumer fairness the disparate impact of recommendation on protected classes of consumers has to be taken into account, while the fairness of outcomes is not considered for producers. Producer fairness regards the producer side of the system, but not the consumer side, e.g. to ensure market diversity and avoid monopoly domination. At the same time the producers are a passive system component that do not seek out recommendation opportunities, but instead wait for users to request recommendations from the platform. The third fairness constraint simultaneously takes producers and consumers into account and has to be applied when protected groups exist among both stakeholders.

In our paper's language consumers would be called users $\mathcal{U}$ und producers would be called candidates C.

Recommender systems often use rankings to present the most suitable candidates to their users, and the question of the fairness for all stakeholders has been raised from different perspectives. We therefore want to give a short abstract on the topic of fairness for recommender systems, and point the curious reader to a recent extensive tutorial by Ekstrand et al. [33] for additional information. In this section we will use the terms users and consumers, as well as candidates and producers interchangeably to demonstrate that these concepts are analogous to each other, and to illustrate the similarities between the previously described methods, and those that follow in this section.

All methods presented in this section can be considered in-processing approaches.

7.1 Recommendation Independence

Fairness Definition. This work in [45] introduces a concept of fairness as *recommendation independence*: an unconditional statistical independence between a recommendation outcome and a specified piece of information, i.e. predictions of ratings should not be based on some previously specified feature. It is formalized as a regularization based approach that can deal with the encoding of sensitive information in the first and second moment of distributions, which means that independence shall be given in terms of of mean *and* standard deviation of the distribution of predicted user ratings. A binary sensitive feature will be specified by user or manager.

Users are represented by a random variable U, X represents items, A sensitive features and Y ground truth ratings. The i-th instance of the training dataset $\mathcal{T}_{\text{train}}$ is a 4-tuple (U_i, X_i, A_i, Y_i). The rating predictions $\hat{Y}$ are calculated using a modified loss function in which an independence term shall be maximized, meaning that the higher $\text{ind}(\hat{Y}, A)$, the less statistically dependent are $\hat{Y}$ and A:

$$\sum_{(U_i, X_i, A_i, Y_i) \in \mathcal{T}_{\text{train}}} \text{loss}(Y, \hat{Y}(U_i, X_i, A_i)) - \eta \, \text{ind}(\hat{Y}, A) + \text{reg}(\Theta)$$

The method can therefore be seen as an in-processing approach.

The paper gives three ways in which independence can be measured, all aiming to produce a rating model in which the distributions of predicted ratings are indistinguishable from a sensitive feature:

(1) **Mean Matching:** the means of two normal distributions shall match

$$-\left(\frac{\mathbb{S}^{(0)}}{N^{(0)}} - \frac{\mathbb{S}^{(1)}}{N^{(1)}} \right)$$

where $\mathbb{S}^{(\mathcal{G})}$ is the sum of predicted ratings per group and $N^{(\mathcal{G})}$ is the number of training items per group.

44

(2) **Distribution Matching:** The similarity between two distributions $Pr[\hat{Y}|A=0]$ and $Pr[\hat{Y}|A=1]$ is measured in terms of the negative Bhattacharyya-distance:

$$\frac{1}{2} \ln \left(\frac{2\sqrt{\mathbb{V}^{(0)}\mathbb{V}^{(1)}}}{\mathbb{V}^{(0)} + \mathbb{V}^{(1)}} \right) - \frac{\left(\frac{\mathbb{S}^{(0)}}{N^{(0)}} - \frac{\mathbb{S}^{(1)}}{N^{(1)}} \right)^2}{4(\mathbb{V}^{(0)} + \mathbb{V}^{(1)})}$$

where $\mathbb{V}^{(\mathcal{G})}$ is the variance of the training items per group.

(3) **Mutual information:** is defined as a degree of statistical independence through a differential entropy function for normal distributions:

$$-I(\hat{Y}; A) = -(H(\hat{Y}) - H(\hat{Y}|A))$$

where $H(\hat{Y}) = \frac{1}{2} \ln 2\pi e \mathbb{V}$ and $H(\hat{Y}|A) = \frac{1}{2} \ln 2\pi e \mathbb{V}^{(s)}$.

Insights. It is not explicitly stated, which worldview they believe in, however the goal of equalizing rating distributions can be associated with the WAE worldview, as the fairness definitions do not contain a measure of merit. Furthermore the fairness definition is agnostic to the underlying ground truth distributions and would work under the assumption that true effort can be shaped by circumstances. It therefore includes the luck-egalitarian idea of EO. Note however, that this analysis only applies if the parameter η is set high enough. As in section 6.2.1, the parameter shifts the framework from WAE and luck-egalitarian EO to WYSIWYG and formal EO when it takes lower values. The definition implicitly addresses the problem of pre-existing biases. Depending on the choice of the protected feature (e.g. popularity), it may also be capable to address emergent bias.

Algorithm. The algorithm is implemented using probabilistic matrix factorization to predict ratings:

$$\hat{Y}(U, X, A) = \mu^{(\mathcal{G})} + b_U^{(\mathcal{G})} + c_X^{(\mathcal{G})} + p_U^{(\mathcal{G})\top} q_X^{(\mathcal{G})}$$

where μ, b_U and c_X are global, per-user and per-item bias parameters respectively and p_U and q_X are K-dimensional parameter vectors, which represent the cross effects between users and items. Said parameters are learned by minimizing the loss function, where the loss is expressed as a L_2-norm:

$$\sum_{(U_i, X_i, A_i, Y_i) \in \mathcal{T}_{\text{train}}} (Y_i - \hat{Y}(U_i, X_i, A_i))^2 - \eta \operatorname{ind}(Y, A) + \operatorname{reg}(\Theta)$$

All independence measures are differentiable and can hence be optimized efficiently using conjugate gradient methods.

Experiments. The experiments are done on three datasets with different sensitive features:

(1) The ML1M **Movielens dataset** [39] contains 1M movie items. Two sensitive features where chosen for two separate settings: first whether the movie's release year was before 1990, and second the user's gender. The first setting investigates on fairness on the side of the producer, whereas the second relates to a fairness concern w.r.t. the users of the system.

(2) The **Flixster dataset** [41] is also a dataset on movie recommendations and contains almost 10M entries. The popularity of an item was chosen as the protected feature: movies that received the most user ratings (top 1%) belong to one group, while all movies that received less ratings form the other group.

(3) The **Sushi dataset** [44] contains around 50,000 data points of users and their preferences when ordering Sushi from 25 different restaurants. Three different choices of sensitive features where investigated: age (whether a person was a teen or not), gender (a user was male or female) and seafood (whether or not a type of sushi was seafood).

The performance evaluation uses the mean absolute error, while the independence (i.e. fairness) evaluation uses the Kolmogorov-Smirnoff test. The latter evaluates the area between two empirical cumulative distributions and shall be close to zero for high fairness. The experiments compare all three independence measures to each other. In all cases independence can be achieved, though at a small cost of accuracy.

7.2 Familiarity Bias and Superstar Economics

Fairness Definition. This work in [56] addresses the problem of disparities in exposure of items to users due to the combination of two factors: the pre-existing familiarity of the user with certain items (someone who likes action movies will likely know Tom Cruise) and the current recommendation strategies of two-sided markets ("superstar economics"). As recommendation systems optimize for relevance this leads to a lock-in of popular and relevant suppliers and thus causes suppliers at the tail end of the exposure distribution struggle to attract consumers. This may lead to a dissatisfaction with the marketplace.

The paper aims to understand the interplay of relevance, fairness and satisfaction and investigates on consumer relevance and supplier fairness on a music streaming platform. It introduces a notion of multinomial group fairness, which requires that the content shown to users be spread well across the wide long-tailed popularity spectrum, rather than focusing on a small subset of popular artists. From the popularity distribution of all artists, K bins of equal size are created and artists are grouped into these bins depending on their popularity:

$$\Psi(s) = \sum_{i=1}^{K} \sqrt{|t_j|_{\forall t_j \in P_i \cap T(s)}}$$

where P_i is the set of artists that belong to popularity bin i, s is the recommended set and $T(s)$ is the collection of artists in the set s with t_j being the j-th track in s. This definition rewards sets that are *diverse* in terms of the different artist popularity bins represented and, as per the current definition, *fair* to different popularity bins of suppliers. There is more benefit to selecting an artist from a popularity bin not yet having one of its artist already chosen. As soon as an artist is selected from a bin, other artists from the same bin start having diminishing gain owing to the square root function.

Insights. The work does not explicitly express the authors' beliefs about the mapping g from construct space to observable space. However the fairness definition aims to equalize the exposure of different artist independent of their popularity and can therefore be associated with a WAE worldview. It can also be associated to substantive EO, most likely luck-egalitarian EO which would acknowledge different distributions of popularity in construct space. However, it is not clear how an artist's popularity relates to their true underlying effort at all, as it is mainly driven by the *users* of the system, rather than by the artists themselves. The method is addressing a combination of pre-existing and technical bias, where the popularity bias can be seen as pre-existing and its reinforcement by the recommender system as technical bias. It may also address emergent bias w.r.t. popularity shifts in the future.

The paper then presents three different policies with $\phi(.)$ being a relevance and $\psi(.)$ a fairness measure:

(1) A weighted combination of relevance and fairness:

$$s_u^* = \arg\max_{s \in S_u}(1 - \beta)\phi(u, s) + \beta\psi(s)$$

where S_u is the collection of all sets pertinent to the user u.

(2) A probabilistic combination of relevance and fairness, where the weighting factor $\beta \in [0, 1]$ decides on whether to recommend a set based on relevance or fairness:

$$s_u^* = \begin{cases} \arg\max_{s \in S_u} \psi(s), & \text{if } p < \beta \\ \arg\max_{s \in S_u} \phi(u, s), & \text{otherwise} \end{cases}$$

where $p \in [0, 1]$ is a randomly generated number.

(3) A guaranteed relevance term to ensure that the minimum relevance is β:

$$s_u^* = \arg\max_{s \in S_u} \psi(s) \;\; s.t. \;\; \phi(u, s) \geq \beta$$

They further investigate on different affinities of users to fairness, i.e. some users only want to listen to one particular artist, where others are more flexible. This affinity is measured as the difference between the satisfactions of a user when recommended relevant content vs. when recommended fair content. The paper states that a fairness recommendation policy should be adaptive to this affinity ξ_u and therefore redefine the second fairness policy:

$$s_u^* = \begin{cases} \arg\max_{s \in S_u} \psi(s), & \text{if } \xi_u \geq 0 \\ \arg\max_{s \in S_u} \phi(u, s), & \text{otherwise} \end{cases}$$

Another version redefines the first policy to use the z-scored affinity, denoted as $\hat{\xi}_u$:

$$s_u^* = \arg\max_{s \in S_u} (1 - \hat{\xi}_u)\phi(u, s) + \hat{\xi}_u \psi(s)$$

As all definitions are modifying the objective function of the learning algorithm the method can be classified as in-processing approach.

Algorithm. The algorithm is implemented as a combinatorial contextual bandit problem with the following consecutive interactions between the customers and the recommendation system:

(1) The system observes context X drawn from a distribution of contexts $P(X)$.
(2) Based on X the system chooses the sets s to recommend to the user.
(3) A reward function $Y \in [0, 1]$ is drawn from distribution $P(Y|X, s)$ that expresses user satisfaction.
(4) The system maximizes Y under the different fairness policies.

As estimating user satisfaction is not easy, because it relies on user feedback which is hard to estimate offline, the recommender system is modeled as a stochastic policy π that specifies a conditional distribution $\pi(a|X)$. The value $V(\pi)$ of a policy is the expected reward, i.e. the user's satisfaction, if an action a is chosen under that policy. This value is to be estimated for a new policy π^* given logged training data, using the inverse propensity score (IPS) estimator, which is provably unbiased:

$$V_{\text{offline}} = \sum_{\forall (X, a, Y_a, p_a)} \frac{Y_a \mathbb{1}(\pi^*(X) = a)}{p_a}$$

with p_a being the propensity scores for an action a, that was randomly chosen from the space of all possible actions under context X, and $\mathbb{1}(\pi^*(x) = a)$ being the set indicator function that evaluates to 1 if the action selected by the target policy matches the logged training data. With this a metric of user satisfaction can be computed as:

$$Y_{\pi^*(x)} = \mathbb{E}_a \left[\frac{Y_a \mathbb{1}(\pi^*(X) = a)}{p_a} \right]$$

Experiments. The experimental part contains a trade-off analysis between user relevance of sets and fairness of sets, which shows that only very few highly relevant sets also achieve high scores of fairness. This means that a recommendation system that solely optimizes for user relevance will not automatically lead to fair and diverse sets for the suppliers, and in turn that two-sided marketplaces have to optimize for users and suppliers to satisfy both. The evaluation is done on a proprietary dataset from Spotify with 400K users, 49K artists and 5K sets (i.e. playlists). When maximizing relevance $\phi(u, s)$ only, the results show the highest user satisfaction, whereas when optimizing for fairness only, the user satisfaction drops by 35%. When optimizing for both, user satisfaction increases with an increase of the weight that is given to the relevance objective. However satisfaction does not drop significantly up to a fairness weight of 20%, such that policies could easily increase fairness up to that point without trading user satisfaction significantly. The guaranteed relevance policy yields the best satisfaction results in absolute numbers and shows a linear trend of user satisfaction with increasing weights for relevance. However for the same levels of β this policy achieves the least average fairness scores compared to the other two policies. This means that the usage of the interpolating fairness policy and the probabilistic fairness policy is preferable in situations where less fairness shall be traded for higher relevance values. When evaluating the adaptive policies, the results highlight that personalizing the recommendation policy and adapting based on user level affinity is better than globally balancing relevance and fairness Interestingly the adaptive policies lead to a relatively high fairness mean compared to the global policies while at the same time increasing the overall user satisfaction. This is another example where the general assumption, that a trade-off between fairness and relevance is a necessary evil, is disproved.

7.3 Fairness in Two-sided Matchings

Fairness Definition. This work in [77] gives a case study of a two-sided matching platform, namely a ride-hailing platform such as Uber. It discusses two-sided fairness of providers and consumers in a platform performing repeated matchings of providers and consumers over time. Here, the riders are consumers and the drivers are producers. Fairness is seen in terms of fair distribution of provider income given a driver's active time in the system. It is explicitly considered over time, because a single matching does not have a significant long-term effect on the life of the people that are matched and income equity can be amortized over a longer period, such as a week or a month. Note that the paper speaks explicitly about customers and drivers, but in a broader sense those can be seen as users $\mathcal{U}$ of the system and candidates C to be matched to them. We will therefore adopt the notation from before to the framework.

Customer utility $U_{\mathcal{U}}$ is described as a customer $\mathcal{U}_b$'s waiting time, which is approximated using the negative distance d of a driver C_a to them:

$$U_{\mathcal{U}}(b, a) = -d(\mathcal{U}_b, C_a)$$

Driver utility is described as the income a driver receives from transporting a customer, which is approximated using the distance from the customer's pick-up location to their destination, reduced by the distance the driver has to travel in order to arrive at the respective pick-up location:

$$U_C(a, b) = d(\mathcal{U}_b, \text{dest}(\mathcal{U}_b^t)) - d(\mathcal{U}_b, C_a)$$

The paper then defines two fairness concepts. First the authors introduce **parity fairness**: over time the sum of received utility shall be (almost) equal for all drivers in C:

$$\sum_a \sum_b |U_{C_a}^T - U_{C_b}^T| < \epsilon$$

with $U_{C_a}^T$ being the total utility driver a received until matching round T. Second the authors define **proportional fairness**: over time, the sum of received utility normalized by their active driving time shall be equal for all drivers

$$\sum_a \sum_b \left| \frac{U_{C_a}^T}{\Lambda_{C_a}^T} - \frac{U_{C_b}^T}{\Lambda_{C_b}^T} \right| < \epsilon$$

where $\Lambda_D^T(j)$ is the total amount of time a driver has been active on the platform until T.

Insights. The work does not talk about any protected groups and instead tries to equalize the (hourly) wage of all drivers in the system. Also the drivers' efforts are not considered other than in the sense of their active time and such it does not make sense to assign the method to a particular equality of opportunity framework. The assumption behind this work is that driving skills are essentially the same and therefore all drivers should be paid equally. It is an extreme case of the WAE worldview in which not only do all groups have the same qualification distribution, but the absolute qualification values are the same for each individual. The biases addressed are of a technical and emergent nature. The emergent bias would otherwise result in the sense that the drivers' locations are shaped by the platform and may be concentrated on certain hot spots while some locations remain less frequented.

Algorithm. The method defines a two-sided optimization objective to minimize the difference of the utilities of drivers (customers) as compared to the maximum utility gained by any driver (customer) up until the previous matching round:

$$\sum_a \sum_b \lambda \cdot \left| \max_{a'} U_{C_{a'}}^{T-1} - \left(U_{C_a}^{T-1} + M_{a,b}^T \cdot U_C^T(a,b) \right) \right|$$
$$+ (1-\lambda) \cdot \left| \max_{b'} U_{\mathcal{U}_{b'}}^{T-1} - \left(U_{\mathcal{U}_b}^{T-1} + M_{a,b}^T \cdot U_{\mathcal{U}}^T(b,a) \right) \right|$$

where $M_{a,b}^T$ is 1 if driver C_a is matched to customer $\mathcal{U}_b$ in round T and 0 otherwise and λ is a hyper-parameter to continuously interpolate between producer and consumer fairness. This objective is translated into an integer linear program.

Experiments. For the experiments income inequality is measured using the generalized entropy index. They are performed on a dataset from a ride hailing platform in an Asian city consisting of 1462 registered drivers. As passengers are not registered, every request is handled using a unique job id, with 231,268 jobs in total. The analysis of the dataset shows that supply exceeds demand by an order of magnitude and driver income is hence a scarce resource. Then different matching strategies are compared to each other w.r.t. their effects on driver and customer utility:

- **Nearest driver first** is a simple objective for low customer waiting times. It yields the highest *average* driver utility, but has the largest discrepancies in terms of driver income (i.e. a high GEI).
- **Worst-off driver first** yields almost equal driver income, but lowest customer utility. It also shows undesired effects, such as lowering hourly wages for drivers that have long active periods, because whenever a new driver joins the system, they have the lowest possible income so far, namely zero.
- The **two-sided optimization** achieves not only better results in terms of income equality for drivers but also in terms of customer waiting time, because drivers happen to be placed in better positions for following requests. It is not clear if this is a data artifact or a general property of the algorithm though. This confirms again, as for rankings in [91] and for two-sided platforms in [56], that a trade-off between fairness considerations and customer utility is not necessarily to be made, but that optimization for fairness can, in some cases increase system utility.

8   FRAMEWORKS AND BENCHMARKS

In this section, we survey several frameworks and benchmarks that focus on fairness in rankings. For each framework, we highlight the following aspects: type of rankers, datasets, system structure (i.e., how the framework works), integration of fairness, and accessibility (i.e., how the framework can be used).

8.1   Fair Search

FairSearch [93] is an open source API to provide fair search results, which is designed as stand-alone libraries (supported in Python and Java) and plugins of Elasticsearch (supported in Java). Users can run FairSearch together with their own datasets once these have been formatted as required.

FairSearch implements two algorithms to guarantee fair ranking as search results: an in-processing technique named as DELTR (see details in Sec. 6.2.1) and a post-processing one called FA*IR (details in Sec. 6.3.1) For the support of DELTR, FairSearch provides an off-line wrapper to train a fair ranking model using DELTR that is later uploaded and stored in Elasticsearch as plugins. For FA*IR, FairSearch applies FA*IR algorithm to rerank the search results provided by Elasticsearch and presents it as final search results to users. The implementation of FairSearch can be found in https://github.com/fair-search.

8.2   Fair TREC

FairTrec [13] is a project to coordinate research around the idea of fairness in search results, launched by National Institute of Standards and Technology (NIST) and organized by researchers from Microsoft and Boise State University in 2019. It is a part of Text REtrieval Conferences (TREC), which focuses on research in information retrieval and related applications by providing a large test collection, uniform scoring procedures, and a forum for organizations interested in comparing their results.

FairTrec2019 uses a set of open data from a specific search tool: the Semantic Scholar search engine, which provides 471 GB data files and can be downloaded from http://api.semanticscholar.org/corpus. Its goal is to provide fair exposure to different groups of authors defined by various factors such as demographics and stature. Their fairness definition belongs to the group fairness category (see details in Sec. 3.1). The participants are asked to perform a re-rank task on the provided data and their results are evaluated by fair exposure of groups of authors and relevance of documents, both are computed by the provided measurements. Note that the definition of groups is not provided by the organizers and will be a part of evaluation to test the robustness of the results for various group definitions.

8.3   Ranking Facts

Ranking Facts [89] is a Web-based application that generates a "nutritional label" for rankings. The nutritional label is made up of a collection of visual widgets that implement research results on fairness, stability, and transparency for rankings, and that communicate details of the ranking methodology, or of the output, to the end user. Ranking Facts supports rule-based rankers that are specified by users as input and can be used for any tabular datasets with numerical and categorical features. To use Ranking Facts, users are asked to upload a tabular dataset as in CSV format, then specify a rule-based ranking function and some sensitive features considering issues of fairness and diversity. For example, for a dataset of financial information, gender can be a feature to consider fair exposure of women and men in a generated ranking. Race can be a feature to consider equally distribution of different racial groups in a generated ranking. Ranking Facts generates the results according to the provided ranking function and the sensitive features.

Ranking Facts implements three algorithms to decide whether a generated ranking is fair to a disadvantaged group according to the user-specified sensitive feature. They are proportional representation, pairwise comparison, and FA*IR. An example of using Ranking Facts is illustrated in the following. The tool can be accessed at http://demo.dataresponsibly.com/rankingfacts. The code is available at https://github.com/DataResponsibly/RankingFacts.

8.4 Mirror Data Generator

Mirror Data Generator is an open source tool that generates synthetic data, mirroring the different issues in the real data collection. It has been used to produce rankings with intersectional concerns [87]. Mirror Data Generator takes a dependency graph as input. The dependency graph is a weighted DAG that describes the columns in the generated data as its nodes and the dependent relations among the columns as its weighted edges. The weight of an edge in a dependency graph represents how much the child node (the one on the right of the edge) depends on the parent node (the one on the left of the edge).

Mirror Data Generator can generate "dirty" synthetic data that includes the columns with missing values and erroneous values. The generation of these columns are operated as an additional step to remove or modify the values accordingly. A "dirty" column can be embedded in a dependency graph to depend on other columns and even on itself, which captures the special patterns of the missing values (e.g., not missing randomly). The code is available at https://github.com/DataResponsibly/MirrorDataGenerator.

9   DISCUSSION AND RECOMMENDATIONS

In the previous sections, we discussed the principal functioning of fair ranking methods, and made explicit the technical choices they make and the value frameworks they encode. In this section we discuss our insights and draw a set of recommendations for the evaluation of fair ranking methods.

Our recommendations are aimed at data science researchers and practitioners. With these recommendations we aim to establish best practices for the development, evaluation, and deployment of fair ranking algorithms, and to avoid potentially harmful uninformed transfer of methods from one application domain to another.

Recommendation 1: Make Values and Context of Use Explicit. Different application scenarios require different value frameworks. The classification frameworks we presented in this paper are meaningful if the application scenario is concerned with aspects of *distributive justice.* However, even if a situation requires distributive justice to be taken into account, the goods or benefits to be distributed play a key role in determining which framework should be applied.

For example, college admissions (educational opportunity) may require a different interpretation of fairness than hiring or user rating prediction in online shops (economic opportunity). To avoid algorithmic solutionism, users of fair rankers must first clearly articulate their own moral goals for a ranking task, and choose a fairness-enhancing method that is consistent with their goals. To facilitate this choice, the fairness concepts behind a fair rankers must be made explicit by their creators.

Values are rarely made explicit in fair ranking papers. A reader looking to adopt a method to their application context will often use the experiments section of a paper to decide whether the method will "work" for them. We often see experimental sections in which all available datasets, corresponding to vastly different contexts of use—from recidivism risk prediction, to credit scoring, to college admissions, to matching platforms like AirBnB—are used to show performance of a method, but without an explanation as to why the dataset was selected, other than that it was available and items in it have scores on which to rank. For example, the COMPAS dataset [2] is often used in

experiments for papers that propose methods for distributive justice [88, 90], yet the dataset was collected for a legal decision making task and so this use is out of scope. We caution against this practice and recommend that, when designing their experiments, the authors should carefully substantiate the appropriateness of using a proposed method on a particular dataset in the context of a specific ranking task. This substantiation should be made by mapping the method and the task to a value framework.

Recommendation 2: Surface Normative Consequences of Technical Choices. Algorithmic rankers are complex technical objects, and many implicit and explicit choices go into their design. In this paper we discussed an important technical dimension of ranker design, namely, the representation of group structure: how many sensitive attributes a ranker handles, and whether these are binary or multinary. This technical choice in turn impacts what type of discrimination a fair ranker can help address (e.g., on one or on several sensitive attributes), and whether it can address intersectional concerns and, if so, what specific concerns are in scope (e.g., representation constraints on intersectional groups, differences in score distributions, or imbalanced loss in utility). We deliberately discussed intersectional discrimination under the heading of mitigation objectives, rather than presenting it as a purely technical choice, and we recommend that designers of fair rankers explicitly discuss both their technical choices, and what consequences they have for applicability.

Another important technical dimension is where in the processing pipeline bias mitigation is applied (recall Figure 8 on page 16). Pre-processing methods have the advantage of early intervention on pre-existing bias. The advantage of in-processing methods is that they do not allow a biased model to be trained. The advantage of post-processing methods is that they provide a guaranteed share of visibility for protected groups. However, post-processing methods are be subject to legal debates in the US because of due process concerns that may make it illegal to intervene at the decision stage (e.g., Ricci v. DeStefano [78]). In the EU, post-processing methods can be used if other methods fail to comply with EU anti-discrimination law. We recommend that the designers of fair rankers substantiate the appropriateness of their technical choice based on the context of use, on for which their method was designed, as well as on the region of use to avoid legal pitfalls.

Recommendation 3: Draw Meaningful Comparisons. Additional research is needed to understand how methods with opposing worldviews, conflicting understandings of equality of opportunity, and different addressed biases can be compared in a meaningful way. For example, in their experiments the authors of Fair-PG-Rank [72] compare its results to those of DELTR [91], but as their measure of disparate exposure is vastly different and the bias they are focusing on is not the same, it is not clear what conclusion to draw from such comparisons. Making the values and the context of use explicit will go a long way towards helping design meaningful experimental comparisons between methods, rather than mechanically comparing apples to oranges.

10   CONCLUSION

In this paper we gave an extensive overview of the state-of-the-art literature of fair ranking in the domains of score-based and supervised learning-based ranking. We introduced important dimensions along which to classify fair ranking methods, mapping their assumptions and design choices to the normative values they incorporate. We outlined the technical details of all presented methods, presented differences and commonalities, and categorized each technical choice by its implicit values within the four normative dimensions. We discussed implications of normative choices and gave recommendations for researchers on how to make such choices in their work explicit.

Most fair ranking methods are concerned with the concept of *distributive justice*, as they aim to fairly distribute the visibility in a ranking among the candidates. Our focus on distributive justice allowed for the mapping between the worldviews and the equality of opportunity concepts in the framework we proposed. However, this mapping is only meaningful in a distributive context and most likely cannot be transferred to a different setting. In the future we hope to see work that relates to other concepts of justice, such as *procedural justice*, which is concerned with the fairness and transparency of a decision making *process*, and is therefore particularly important in legal decision making. It will be interesting to study whether fairness-enhancing methods designed for concerns of distributive justice can be transferred to the context of procedural justice in a meaningful way.

Another interesting direction to classify fair ranking methods within distributive justice contexts is to understand the properties of ranking scores with respect to different indexes of advantage. Commonly, the score models a candidate's *potential utility* to the user of the ranking, which stems from welfarism/utilitarianism and, as such, incorporates an idea of satisfaction and preference [68]. In contrast, Rawls judges the goodness of a distribution in terms of so-called *primary goods*. It is important to explore the implications of these different conceptions of advantage, and, crucially, understand whether they can be combined in a common fairness objective.

REFERENCES

[1] AirBnB. [n.d.]. AirBnB. https://insideairbnb.com

[2] Julia Angwin, Jeff Larson, Surya Mattu, and Lauren Kirchner. 2016. Machine bias. ProPublica. *See https://www. propublica. org/article/machine-bias-risk-assessments-in-criminal-sentencing* (2016).

[3] Abolfazl Asudeh, HV Jagadish, Julia Stoyanovich, and Gautam Das. 2019. Designing fair ranking schemes. In *Proceedings of the 2019 International Conference on Management of Data*. ACM, 1259–1276.

[4] Abolfazl Asudeh and H. V. Jagadish. 2020. Fairly Evaluating and Scoring Items in a Data Set. *Proc. VLDB Endow.* 13, 12 (2020), 3445–3448. https://doi.org/10.14778/3415478.3415566

[5] Moshe Babaioff, Nicole Immorlica, David Kempe, and Robert Kleinberg. 2008. Online auctions and generalized secretary problems. *SIGecom Exchanges* 7, 2 (2008). https://doi.org/10.1145/1399589.1399596

[6] Ricardo Baeza-Yates. 2018. Bias on the web. *Commun. ACM* 61, 6 (2018), 54–61. https://doi.org/10.1145/3209581

[7] Solon Barocas, Moritz Hardt, and Arvind Narayanan. 2017. Fairness in machine learning. *Nips tutorial* 1 (2017), 2.

[8] Surender Baswana, P. P. Chakrabarti, Yashodhan Kanoria, Utkarsh Patange, and Sharat Chandran. 2019. Joint Seat Allocation 2018: An algorithmic perspective. *CoRR* abs/1904.06698 (2019). arXiv:1904.06698 http://arxiv.org/abs/1904.06698

[9] Rachel K. E. Bellamy, Kuntal Dey, Michael Hind, Samuel C. Hoffman, Stephanie Houde, Kalapriya Kannan, Pranay Lohia, Jacquelyn Martino, Sameep Mehta, Aleksandra Mojsilovic, Seema Nagar, Karthikeyan Natesan Ramamurthy, John T. Richards, Diptikalyan Saha, Prasanna Sattigeri, Moninder Singh, Kush R. Varshney, and Yunfeng Zhang. 2018. AI Fairness 360: An Extensible Toolkit for Detecting, Understanding, and Mitigating Unwanted Algorithmic Bias. *CoRR* abs/1810.01943 (2018). arXiv:1810.01943 http://arxiv.org/abs/1810.01943

[10] Alex Beutel, Jilin Chen, Tulsee Doshi, Hai Qian, Li Wei, Yi Wu, Lukasz Heldt, Zhe Zhao, Lichan Hong, Ed H. Chi, and Cristos Goodrow. 2019. Fairness in Recommendation Ranking Through Pairwise Comparisons. In *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining* (Anchorage, AK, USA) *(KDD '19)*. ACM, New York, NY, USA, 2212–2220. https://doi.org/10.1145/3292500.3330745

[11] Asia J Biega, Krishna P Gummadi, and Gerhard Weikum. 2018. Equity of attention: Amortizing individual fairness in rankings. In *The 41st International ACM SIGIR Conference on Research & Development in Information Retrieval*. ACM, 405–414.

[12] Asia J Biega, Rishiraj Saha Roy, and Gerhard Weikum. 2017. Privacy through solidarity: A user-utility-preserving framework to counter profiling. In *Proceedings of the 40th International ACM SIGIR Conference on Research and Development in Information Retrieval*. 675–684.

[13] Malte Bonart. [n.d.]. Fair ranking in academic search. ([n. d.]).

[14] Ralph Allan Bradley and Milton E Terry. 1952. Rank analysis of incomplete block designs: I. The method of paired comparisons. *Biometrika* 39, 3/4 (1952), 324–345.

[15] Christopher JC Burges. 2010. From ranknet to lambdarank to lambdamart: An overview. *Learning* 11, 23-581 (2010), 81.

[16] Robin Burke. 2017. Multisided fairness for recommendation. *arXiv preprint arXiv:1707.00093* (2017).

[17] Zhe Cao, Tao Qin, Tie-Yan Liu, Ming-Feng Tsai, and Hang Li. 2007. Learning to rank: from pairwise approach to listwise approach. In *Proceedings of the 24th international conference on Machine learning*. ACM, 129–136.

[18] L Elisa Celis, Anay Mehrotra, and Nisheeth K Vishnoi. 2020. Interventions for ranking in the presence of implicit bias. In *Proceedings of the 2020 Conference on Fairness, Accountability, and Transparency*. 369–380.

[19] L Elisa Celis, Damian Straszak, and Nisheeth K Vishnoi. 2018. Ranking with Fairness Constraints. In *45th International Colloquium on Automata, Languages, and Programming (ICALP 2018)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

[20] Alexandra Chouldechova and Aaron Roth. 2020. A snapshot of the frontiers of fairness in machine learning. *Commun. ACM* 63, 5 (2020), 82–89. https://doi.org/10.1145/3376898

[21] Joel W Cohen, Steven B Cohen, and Jessica S Banthin. 2009. The medical expenditure panel survey: a national information resource to support healthcare cost research and inform policy and practice. *Medical care* (2009), S44–S50.

[22] MIT Collective Learning Group. [n.d.]. Pantheon. https://github.com/DataResponsibly/Datasets

[23] Patricia Hill Collins. 2002. *Black feminist thought: Knowledge, consciousness, and the politics of empowerment.* routledge.

[24] Amanda Coston, Karthikeyan Natesan Ramamurthy, Dennis Wei, Kush R Varshney, Skyler Speakman, Zairah Mustahsan, and Supriyo Chakraborty. 2019. Fair Transfer Learning with Missing Protected Attributes. In *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society, AIES.* ACM.

[25] Kimberle Crenshaw. 1990. Mapping the margins: Intersectionality, identity politics, and violence against women of color. *Stan. L. Rev.* 43 (1990), 1241.

[26] CS Rankings. [n.d.]. CSRankings: Computer Science Rankings. https://csrankings.org

[27] Marco De Gemmis, Pasquale Lops, Cataldo Musto, Fedelucio Narducci, and Giovanni Semeraro. 2015. Semantics-aware content-based recommender systems. In *Recommender systems handbook.* Springer, 119–159.

[28] Catherine D'Ignazio and Lauren F Klein. 2020. *Data feminism.* MIT Press.

[29] Marina Drosou, HV Jagadish, Evaggelia Pitoura, and Julia Stoyanovich. 2017. Diversity in Big Data: A review. *Big Data* 5, 2 (2017), 73–84.

[30] Cynthia Dwork, Moritz Hardt, Toniann Pitassi, Omer Reingold, and Richard Zemel. 2012. Fairness through awareness. In *Proceedings of the 3rd innovations in theoretical computer science conference.* ACM, 214–226.

[31] E.B. Dynkin. 1963. The optimum choice of the instant for stopping a Markov process. *Sov. Math. Dokl.* 4 (1963).

[32] Michael D. Ekstrand, Robin Burke, and Fernando Diaz. 2019. Fairness and discrimination in recommendation and retrieval. In *Proceedings of the 13th ACM Conference on Recommender Systems, RecSys 2019, Copenhagen, Denmark, September 16-20, 2019*, Toine Bogers, Alan Said, Peter Brusilovsky, and Domonkos Tikk (Eds.). ACM, 576–577. https://doi.org/10.1145/3298689.3346964

[33] Michael D. Ekstrand, Robin Burke, and Fernando Diaz. 2019. Fairness and Discrimination in Retrieval and Recommendation. In *Proceedings of the 42nd International ACM SIGIR Conference on Research and Development in Information Retrieval* (Paris, France) *(SIGIR'19).* Association for Computing Machinery, New York, NY, USA, 1403–1404. https://doi.org/10.1145/3331184.3331380

[34] Thomas S. Ferguson. 1989. Who Solved the Secretary Problem? *Statist. Sci.* 4, 3 (08 1989), 282–289. https://doi.org/10.1214/ss/1177012493

[35] Forbes. [n.d.]. Forbes Richest Americans. https://github.com/DataResponsibly/Datasets

[36] Sorelle A Friedler, Carlos Scheidegger, and Suresh Venkatasubramanian. 2016. On the (im) possibility of fairness. *arXiv preprint arXiv:1609.07236* (2016).

[37] Batya Friedman and Helen Nissenbaum. 1996. Bias in Computer Systems. *ACM Trans. Inf. Syst.* 14, 3 (1996), 330–347. https://doi.org/10.1145/230538.230561

[38] Sara Hajian, Francesco Bonchi, and Carlos Castillo. 2016. Algorithmic bias: From discrimination discovery to fairness-aware data mining. In *Proceedings of the 22nd ACM SIGKDD international conference on knowledge discovery and data mining.* ACM, 2125–2126.

[39] F Maxwell Harper and Joseph A Konstan. 2015. The movielens datasets: History and context. *Acm transactions on interactive intelligent systems (tiis)* 5, 4 (2015), 1–19.

[40] Hoda Heidari, Michele Loi, Krishna P Gummadi, and Andreas Krause. 2019. A moral framework for understanding fair ml through economic models of equality of opportunity. In *Proceedings of the Conference on Fairness, Accountability, and Transparency.* ACM, 181–190.

[41] Mohsen Jamali and Martin Ester. 2010. A matrix factorization technique with trust propagation for recommendation in social networks. In *Proceedings of the fourth ACM conference on Recommender systems.* 135–142.

[42] Kalervo Järvelin and Jaana Kekäläinen. 2002. Cumulated gain-based evaluation of IR techniques. *ACM Transactions on Information Systems (TOIS)* 20, 4 (2002), 422–446.

[43] Thorsten Joachims, Laura Granka, Bing Pan, Helene Hembrooke, and Geri Gay. 2017. Accurately interpreting clickthrough data as implicit feedback. In *ACM SIGIR Forum*, Vol. 51. Acm New York, NY, USA, 4–11.

[44] Toshihiro Kamishima. 2003. Nantonac collaborative filtering: recommendation based on order responses. In *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining.* 583–588.

[45] Toshihiro Kamishima, Shotaro Akaho, Hideki Asoh, and Jun Sakuma. 2018. Recommendation independence. In *Conference on Fairness, Accountability and Transparency.* 187–201.

[46] Falaah Arif Khan, Eleni Manis, and Julia Stoyanovich. 2021. Translation Tutorial: Fairness and Friends. In *Proceedings of the ACM Conference on Fairness, Accountability, and Transparency.*

[47] Jon Kleinberg and Manish Raghavan. 2018. Selection Problems in the Presence of Implicit Bias. In *9th Innovations in Theoretical Computer Science Conference (ITCS 2018) (Leibniz International Proceedings in Informatics (LIPIcs), Vol. 94)*, Anna R. Karlin (Ed.). Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, Dagstuhl, Germany, 33:1–33:17. https://doi.org/10.4230/LIPIcs.ITCS.2018.33

[48] Ravi Kumar and Sergei Vassilvitskii. 2010. Generalized distances between rankings. In *Proceedings of the 19th International Conference on World Wide Web, WWW 2010, Raleigh, North Carolina, USA, April 26-30, 2010*, Michael Rappa, Paul Jones, Juliana Freire, and Soumen Chakrabarti (Eds.).

ACM, 571–580. https://doi.org/10.1145/1772690.1772749

[49] Preethi Lahoti, Krishna P Gummadi, and Gerhard Weikum. 2019. ifair: Learning individually fair data representations for algorithmic decision making. In *2019 IEEE 35th International Conference on Data Engineering (ICDE)*. IEEE, 1334–1345.

[50] Preethi Lahoti, Krishna P Gummadi, and Gerhard Weikum. 2019. Operationalizing individual fairness with pairwise fair representations. *arXiv preprint arXiv:1907.01439* (2019).

[51] Hang Li. 2014. *Learning to Rank for Information Retrieval and Natural Language Processing, Second Edition.* Morgan & Claypool Publishers. https://doi.org/10.2200/S00607ED2V01Y201410HLT026

[52] M. Lichman. 2013. UCI Machine Learning Repository.

[53] D. V. Lindley. 1961. Dynamic Programming and Decision Theory. *Journal of the Royal Statistical Society* 10, 1 (March 1961), 39–51.

[54] Timo Makkonen. 2002. Multiple, compound and intersectional discrimination: Bringing the experiences of the most marginalized to the fore. *Institute for Human Rights, Åbo Akademi University* (2002).

[55] Christopher D Manning, Prabhakar Raghavan, and Hinrich Schütze. 2008. Evaluation in information retrieval. *Introduction to information retrieval* 1 (2008), 188–210.

[56] Rishabh Mehrotra, James McInerney, Hugues Bouchard, Mounia Lalmas, and Fernando Diaz. 2018. Towards a fair marketplace: Counterfactual evaluation of the trade-off between relevance, fairness & satisfaction in recommendation systems. In *Proceedings of the 27th acm international conference on information and knowledge management*. 2243–2251.

[57] NASA. [n.d.]. Astronauts. https://github.com/DataResponsibly/Datasets

[58] Safiya Umoja Noble. 2018. *Algorithms of oppression: How search engines reinforce racism.* nyu Press.

[59] Department of Health and Human Services. [n.d.]. Medical Expenditure Panel Survey. https://meps.ahrq.gov/mepsweb/

[60] Bureau of Transportation Statistics. [n.d.]. National Summary of U.S. Flights. https://www.transtats.bts.gov

[61] Bing Pan, Helene Hembrooke, Thorsten Joachims, Lori Lorigo, Geri Gay, and Laura Granka. 2007. In Google we trust: Users' decisions on rank, position, and relevance. *Journal of computer-mediated communication* 12, 3 (2007), 801–823.

[62] ProPublica. [n.d.]. Correctional Offender Management Profiling for Alternative Sanctions. https://github.com/propublica/compas-analysis

[63] Richard V. Reeves and Dimitrios Halikias. 2017. Race gaps in SAT scores highlight inequality and hinder upward mobility. (2017). https://www.brookings.edu/research/race-gaps-in-sat-scores-highlight-inequality-and-hinder-upward-mobility

[64] UCI Machine Learning Repository. [n.d.]. German Credit. https://archive.ics.uci.edu/ml/machine-learning-databases/statlog/german/

[65] Matthew Richardson, Ewa Dominowska, and Robert Ragno. 2007. Predicting clicks: estimating the click-through rate for new ads. In *Proceedings of the 16th international conference on World Wide Web*. 521–530.

[66] SAT. [n.d.]. SAT. https://www.qsleap.com/sat/resources/sat-2014-percentiles

[67] Semantic Scholar. [n.d.]. Semantic Scholar Open Research Corpus. https://api.semanticscholar.org/corpus/

[68] Amartya Sen. 1980. Equality of what? *The Tanner lecture on human values* 1 (1980), 197–220.

[69] Law School Admission Council Research Report Series. [n.d.]. LSAC national longitudinal bar passage study. https://github.com/MilkaLichtblau/DELTR-Experiments/tree/master/data/LawStudents

[70] Stephanie A Shields. 2008. Gender: An intersectionality perspective. *Sex roles* 59, 5-6 (2008), 301–311.

[71] Ashudeep Singh and Thorsten Joachims. 2018. Fairness of exposure in rankings. In *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*. ACM, 2219–2228.

[72] Ashudeep Singh and Thorsten Joachims. 2019. Policy Learning for Fairness in Ranking. *arXiv preprint arXiv:1902.04056* (2019).

[73] Charles Spearman. 1961. The proof and measurement of association between two things. (1961).

[74] StackExchange. [n.d.]. StackExchange. https://stackexchange.com/

[75] Julia Stoyanovich, Ke Yang, and H. V. Jagadish. 2018. Online Set Selection with Fairness and Diversity Constraints. In *Proceedings of the 21th International Conference on Extending Database Technology, EDBT 2018, Vienna, Austria, March 26-29, 2018*. 241–252. https://doi.org/10.5441/002/edbt.2018.22

[76] Engineering students. [n.d.]. Engineering students. https://github.com/MilkaLichtblau/DELTR-Experiments/tree/master/data/EngineeringStudents

[77] Tom Sühr, Asia J Biega, Meike Zehlike, Krishna P Gummadi, and Abhijnan Chakraborty. 2019. Two-sided fairness for repeated matchings in two-sided markets: A case study of a ride-hailing platform. In *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*. ACM, 3082–3092.

[78] Supreme Court of the United States. 2009. Ricci v. DeStefano (Nos. 07-1428 and 08-328), 530 F. 3d 87, reversed and remanded. https://www.law.cornell.edu/supct/html/07-1428.ZO.html.

[79] Indian Institute Of Technology. [n.d.]. IIT-JEE. https://indiankanoon.org/doc/1955304/

[80] The College Board. 2014. SAT Percentile Ranks.

[81] TREC. [n.d.]. W3C Experts. https://github.com/MilkaLichtblau/DELTR-Experiments/tree/master/data/TREC

[82] Linda F Wightman and Henry Ramsey. 1998. *LSAC national longitudinal bar passage study.* Law School Admission Council.

[83] Yongkai Wu, Lu Zhang, and Xintao Wu. 2018. On Discrimination Discovery and Removal in Ranked Data using Causal Graph. In *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, KDD 2018, London, UK, August 19-23, 2018*. 2536–2544. https://doi.org/10.1145/3219819.3220087

[84] Xing. [n.d.]. XING. https://github.com/MilkaLichtblau/xing_dataset

[85] Yahoo. [n.d.]. The Yahoo Webscope Program. https://webscope.sandbox.yahoo.com/

[86] Ke Yang, Vasilis Gkatzelis, and Julia Stoyanovich. 2019. Balanced Ranking with Diversity Constraints. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019, Macao, China, August 10-16, 2019*. 6035–6042. https://doi.org/10.24963/ijcai.2019/836

[87] Ke Yang, Joshua R. Loftus, and Julia Stoyanovich. 2020. Causal intersectionality for fair ranking. arXiv:2006.08688 [cs.LG]

[88] Ke Yang and Julia Stoyanovich. 2017. Measuring fairness in ranked outputs. In *Proceedings of the 29th International Conference on Scientific and Statistical Database Management*. ACM, 22.

[89] Ke Yang, Julia Stoyanovich, Abolfazl Asudeh, Bill Howe, HV Jagadish, and Gerome Miklau. 2018. A nutritional label for rankings. In *Proceedings of the 2018 International Conference on Management of Data*. ACM, 1773–1776.

[90] Meike Zehlike, Francesco Bonchi, Carlos Castillo, Sara Hajian, Mohamed Megahed, and Ricardo Baeza-Yates. 2017. Fa* ir: A fair top-k ranking algorithm. In *Proceedings of the 2017 ACM on Conference on Information and Knowledge Management*. ACM, 1569–1578.

[91] Meike Zehlike and Carlos Castillo. 2018. Reducing disparate exposure in ranking: A learning to rank approach. *arXiv preprint arXiv:1805.08716* (2018).

[92] Meike Zehlike, Philipp Hacker, and Emil Wiedemann. 2017. Matching code and law: achieving algorithmic fairness with optimal transport. *Data Mining and Knowledge Discovery* (2017), 1–38.

[93] Meike Zehlike, Tom Sühr, Carlos Castillo, and Ivan Kitanovski. 2020. FairSearch: A Tool For Fairness in Ranked Search Results. In *Companion Proceedings of the Web Conference 2020*. 172–175.

[94] Richard S. Zemel, Yu Wu, Kevin Swersky, Toniann Pitassi, and Cynthia Dwork. 2013. Learning Fair Representations. In *International Conference on Machine Learning*. http://jmlr.org/proceedings/papers/v28/zemel13.html

[95] Yi Zhang. [n.d.]. Yow News Recommendation. https://www.younow.com

[96] Yi Zhang. 2005. *Bayesian Graphical Model for Adaptive Information Filtering*. Ph.D. Dissertation. Carnegie Mellon University.