# Simulation of road traffic

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## Abstract

A cellular automaton model was constructed to model realistic traffic flow which contains characteristic features of non-deterministic traffic behaviour, such as non-linear backward motion of congestion and the bottlenecking of traffic flow. Bottleneck situations were further analysed by the implementation of traffic lights, which were regulated by introducing a concept called phase. Our model simulation produced fundamental diagrams of traffic flow which were consistent with real data results. The model implemented elements from a macroscopic and microscopic description of highway traffic containing a single lane. This aimed to understand the effects of various model parameters, such as the number of traffic lights and vehicular density, on traffic flow. For example, an exponential relationship between the number of traffic lights and the mean flow rate was demonstrated. Chaotic behaviour was generally observed for all simulations which essentially exhibited the non-deterministic nature of realistic traffic flow. This resulted in a substantial amplification to the variation of flow in the critical density region which was identified by the large standard deviations.

# 1. Introduction

The subject of traffic flow is a typical case study in the realm of complex systems. Although there is no clear consensus on the definition of a complex system, there are distinct features common to them all. In particular, complex systems display the property of emergence. This can be briefly described as the macroscopic effect which is caused by the underlying microscopic effects [1]. These emerging effects result from the mutual interactions between individual entities within the system as well as the rules that govern the behaviour of the individual entities. It is essential to note that without considering the mutual interactions, the study of any individual entity independently is insufficient. Traffic flow fits this description perfectly.

In traffic flow there are a wide variety of interactions that may be analysed. For example, interactions between vehicles, traffic lights, pedestrians, public transport, and the type of road that is being modelled, such as a roundabout or highway etc. Modelling and optimising traffic flow involves various disciplines of mathematics, physics, computer science and engineering which enables the development of efficient traffic infrastructure. For example, the use of chaos theory alongside nonlinear dynamics has been proven effective in obtaining more accurate and predictive models as was successfully seen in the case of the Yu-Wu highway in China [2]. A property of chaotic behaviour is the sensitivity to initial conditions [3]. Small perturbations to these initial conditions can produce significant variations in the mean flow. This report aims to observe these phenomena empirically.

The analysis of traffic flow is often examined and observed on three scales; macroscopic, mesoscopic and microscopic, which are well-defined scales in physics [4] . In particular, macroscopic effects observed in traffic, such as smooth and congested flow are analogous to laminar and turbulent flow, respectively, in the field of fluid dynamics. Congested flow occurs when the number of vehicles per unit distance reaches a threshold known as the critical density. The concept of critical density is explored in this report by observing key features, such as 'backward motion' of congestion, in systems with and without traffic lights. These concepts have been used in the automatic detection of traffic flow transition (from laminar to turbulent) and to develop better traffic infrastructure in Germany [5]. This model aims to investigate the flow of highway traffic in a single lane and the causes of bottleneck situations using a cellular automaton model.

# 2. The Model

As mentioned in the introduction, the analysis of highway traffic flow can often be examined and observed on three scales [4]. At the microscopic scale, each vehicle is considered independently, the dynamics of which, are governed by a set of rules. The cellular automaton extension to these rules may permit local interactions by considering vehicles ahead or behind. The mathematical formulation for a simple model generally involves an ordinary differential equation for each vehicle. At the macroscopic scale, relationships between holistic variables of the system are determined, the formulation of which generally relies upon a system of partial differential equations. The mesoscopic scale follows a mathematically similar approach to statistical mechanics. In the context of traffic flow analysis, the kinematics of a vehicle are probabilistic with relation to its position and velocity at a time $t$.

The procedure outlined in this report contains elements from the microscopic and macroscopic scale, which contains parallel aspects to [5]. To ensure a simplistic model, a mesoscopic model was not used, but it would allow and account for dynamic responses of external road conditions such as varying intensities of congestion.

Before constructing a computational model, considerations to the set of rules must be made to ensure realistic traffic flow. This was achieved by allowing random input which ensures non-deterministic behaviour. Therefore, we would predict a non-linear response that is constrained by the holistic variables of the system and independent of the initial conditions. Thus, an empirical analysis was considered to be a suitable approach to describe the behaviour.

The procedure to construct a computational highway model was adapted from [5] and began by defining a one-dimensional array of $N$ cells. A cell may be empty or occupied by a single vehicle and the speed of the vehicle is represented by an integer value $v$ between 0 and 5. A single step in time (timestep) represents a time increase of one second. The suitability of this definition is explored at the end of Section 4. Each timestep evolves the array configuration by performing a set of operations that obey a sequence of rules. All rules are applied simultaneously to all vehicles. Additionally, to ensure initial parameters such as vehicle density, are constant, a circular boundary condition was imposed. This condition implies vehicles exiting the last cell re-enter the array at the first cell.

The vehicle density $\rho_v$ and traffic light density $\rho_t$ was defined as the number of vehicles and traffic lights per unit cell of the array, respectively. The time-averaged flow $q$ was determined by calculating the number of vehicles crossing a given cell per unit time,

$$q = \frac{1}{T}\sum_{t=0}^{T} n_i\,(t), \tag{1}$$

where $T$ is the total time and $n_i(t)$ equals the number of vehicles crossing the $i^{th}$ cell at time $t$.

## 2.1 The rules

The sequence of rules performed after each timestep are as follows:

1) Vehicle advancement: Each vehicle progresses $v$ cells.
2) Acceleration: If the number of cells to the next vehicle is larger than $v + 1$, the speed increases by 1 ($v \rightarrow v + 1$), given that $v < 5$.

3) Deceleration: If acceleration does not occur, a vehicle at the $i^{th}$ cell will react to the next vehicle at the $j^{th}$ cell by reducing its speed to $v \rightarrow j - i - 1$.
4) Randomisation: Each vehicle has a given probability $p$ of reducing its speed by one ($v \rightarrow v - 1$). In our setup the probability $p$ was set to 0.3.

In a system containing traffic lights, an additional rule was imposed:

5) Stopping: A vehicle at the $i^{th}$ cell which sees a red traffic light at the $j^{th}$ will reduce its speed to $v \rightarrow j - i - 1$.

The application of these rules are in accordance with the principles of cellular automata. Rule 4 is necessary in simulating realistic traffic flow and ensuring non-deterministic behaviour. Furthermore, it was observed that the exclusion of this rule results in the congestion shifting one cell backwards after each timestep (linear backward motion). The initial conditions were set by randomly assorting vehicles in the highway array (ensuring there were no contradictions to the rules). This allows us to quantify the chaotic nature of the model. The highway is illustrated in Figure 1 below.

Space (highway)

Time

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.........2.......1.1.2..01.01.1.2......2.........5.......4.......1.......
...........3......0.0..00.00.0.0..2......3............2......3....1......
..............3...0.0..01.01.1.1....2.......4...........2.......0..2.....
.................01.1..0.00.0.0.1.....2.........4.........3.....1....3...
.................0.0.1.1.01.1.1..2......3...........5........1...2......4
...5.............1.1..0.00.0.0.1...3.......4.............2....1....2.....
........5.........0.1.1.01.0.0..2.....4........5...........1...2.....3...
.............4....0..0.00.00.1....3.......4.........5.......2....3......4
...5.............01..1.01.01..2......4........5..........1....2.....2....
........4........0.1..00.00.1...3........4.........4......2.....3.....3..
4...........4....1..1.01.01..2.....4.........5.........1....3......1.....
....4...........0.1..00.00.0...3.......5..........4.....2......2....2....
........5.......1..1.01.01.1......3.........5.........1...3......2....3..
4............1...1..00.00.0.2........4...........4.....1.....2.....2.....
....5.........2...0.01.01.0...3..........5...........0..2......2.....3...
.........4......1.1.0.00.01......4.............5......0....2......2......4
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Figure 1: Illustration of a highway simulation modelled in Python 3 at a moderate vehicle density, $\rho_v = 0.2$. Empty cells are represented by dots and vehicles are represented by an integer denoting its speed $v$. Each vehicle traverses $v$ dots after a single timestep. Each new line displays the traffic configuration after one timestep.

## 2.2 Traffic light profile and phase

There are numerous factors which determine the signal time intervals in realistic scenarios, such as road conditions, intersection layout, time of day etc., which results in unpredictable behaviour in a traffic light network. To produce meaningful results, a simplified model was constructed to regulate the traffic light network.

The procedure was to define a ‘light profile’ as a one-dimensional array with a fixed length of 24 elements which contains various proportions of ‘R’s’ (red lights) and ‘G’s’ (green lights), such as

$$L_{12} = [\text{RRRRRRRRRRRRGGGGGGGGGGGG}]. \tag{2}$$

A single light profile is applied to all traffic lights on the highway. Each traffic light begins at a specified position in the light profile and shifts one position to the right every timestep. For example, applying $L_{12}$ to a given traffic light will begin at the $0^{th}$ element $(R)$ and will remain red for 12 timesteps before turning green $(G)$. Light profiles contain circular boundary conditions, therefore, after 12 more timesteps, the traffic light will return to the $0^{th}$ element and turn red. Thus, each traffic light repeats the above procedure every 24 seconds.

For a road containing multiple traffic lights, it is possible to define a concept called phase which relates the initial positions in the light profile for each traffic light. The phase ϕ is normalised between 0 and 1. A phase of $\phi = 0$ implies that all traffic lights begin at the same initial position in the light profile and a phase of $\phi = 1$ implies that all traffic lights' initial position are equally spaced in the light profile. An intermediate phase value will be between these two extremes. In general, by defining the largest spacing between two adjacent lights in the light profile as $l = 24/N\rho_t$ (where $N$ is the number of cells in the highway array and $\rho_t$ is the traffic light density), the initial position $x_k$ of the $k^{th}$ traffic light in the light profile array is given by

$$x_k = kl\phi; \quad \text{where } k = 0, 1, 2, \ldots, K-1, \tag{3}$$

and $K$ is the total number of traffic lights. An illustration of the highway model with traffic lights implemented is shown in Figure 2 below.

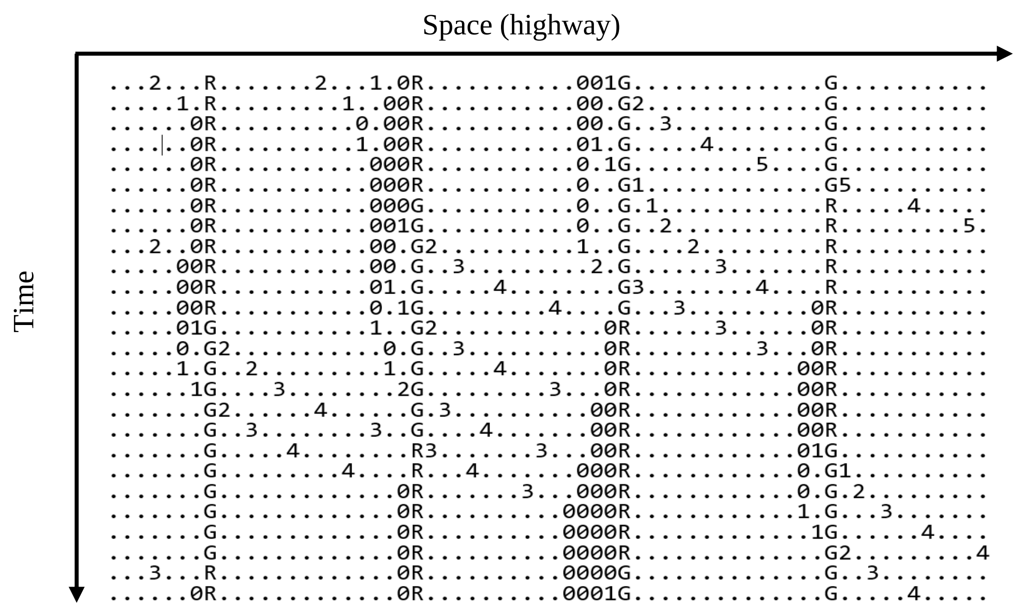


Figure 2: Illustration of a highway simulation modelled in Python 3 at a low vehicle density and traffic light density, $\rho_v = 0.12$ and $\rho_t = 4/60$ for $N = 60$ cells. The light profile applied is $L_{12}$ (Equation 2) with a phase $\phi = 1$.

# 4. Results and discussion

Figure 3 shows the relationship between the vehicle density and time-averaged flow (known as the fundamental diagram of traffic flow) using the highway model with circular boundary conditions and no traffic lights. For each density value, the simulation ran 100 times to determine the time-averaged mean flow rate $\bar{q}$ (represented by dots) and the standard deviation $\sigma$ (indicated by error bars). Each simulation ran for $10^4$ timesteps. This procedure of determining uncertainties was repeated throughout the rest of this section.

The critical density $\rho_c$ was identified near $\rho_c = 0.08$ which corresponds to the peak flow rate $q_c$. For densities greater than $\rho_c$, a clear linear relationship can be observed. This is supported by the fact that the best fit line in this region has a reduced $\chi^2$ value of 2.29 and is illustrated on Figure 3. The validity of the model is supported by the similarities between these results and those obtained from real data (Figure 4).

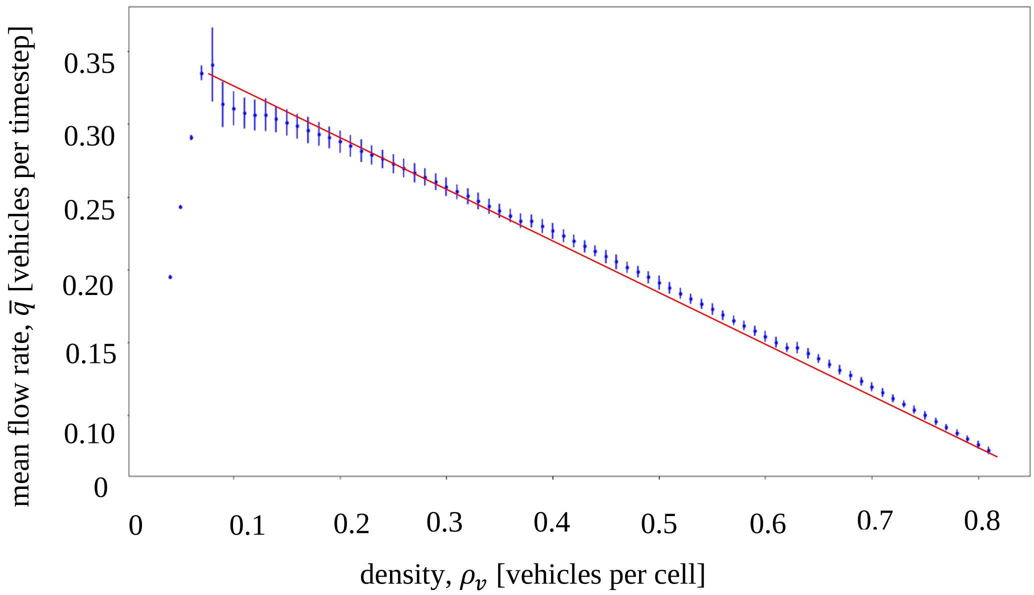


Figure 3: Diagram illustrating the fundamental relation of traffic flow for $N = 96$ cells. Each simulation ran for $10^4$ timesteps. Uncertainties corresponding to $1\sigma$ have been annotated on the diagram. Some uncertainties are too small to be visible on this scale. The reduced $\chi^2$ was calculated as 2.29 for a linear best fit line for data points after the critical density data point (after the first 5 data points). The best fit parameters were determined as $m = -0.350 \pm 0.001$ and $c = 0.363 \pm 0.001$.

Although the initial conditions vary with each run (whilst constant $\rho_v$ is maintained), each data point in our model still exhibits random fluctuations. This demonstrates the random, non-deterministic behaviour of our model, which supports the notion that the model is representative of a realistic scenario. A significant uncertainty was observed at the peak flow rate which implies that the sensitivity to initial conditions is amplified in these regions. Furthermore, Figure 1 illustrates the traffic flow (where $\rho_v = 0.2$), qualitatively, which exhibits non-linear backward motion of congestion, a characteristic of realistic traffic flow.

Moreover, it can be observed from Figure 3 that the standard deviation $\sigma$ decreases as vehicle density increases in the region beyond the critical density. This may suggest that the random fluctuations become more determinable at higher densities. This is supported by a similar effect on the spread of data points in the real data (Figure 4).

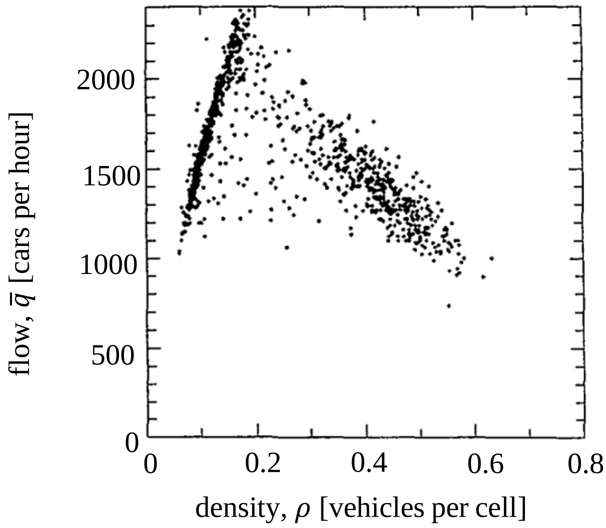


Figure 4: Real data showing the relationship between flow and density on a highway (containing no traffic lights) [5].

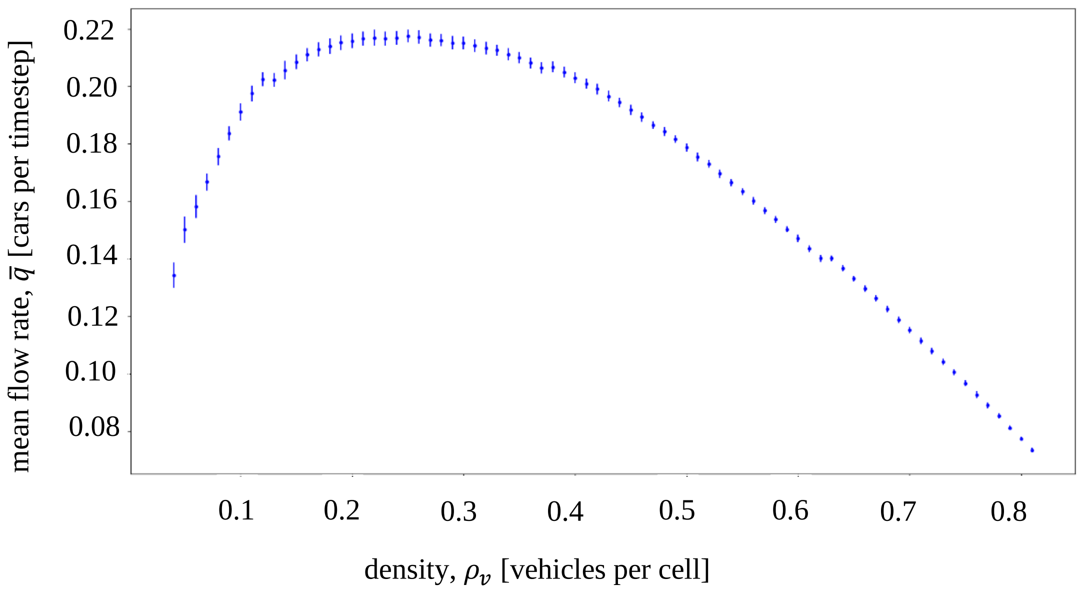


Figure 5: Diagram illustrating the fundamental relation of traffic flow for $N = 96$ cells with six traffic lights implemented. Each simulation ran for 2000 timesteps. Uncertainties corresponding to $1\sigma$ have been determined by applying the same procedure described at the start of Section 4 and have been annotated on the diagram. Some uncertainties are too small to be visible on this scale.

Figure 5 shows the fundamental diagram of traffic flow with six traffic lights implemented which contains similarities and differences to Figure 3. As density increases, both figures show a steep increase in flow to their respective peak values followed by a gradual decline. However, the peak flow rate $q_c$ is lower due to the implementation of traffic lights as expected. Figure 5 shows a transition from a steep increase to a state of approximately constant flow for a range of vehicular densities which defines a bottleneck situation [6].

The time-averaged flow for a highway model with circular boundaries and traffic lights can be analysed in relation to phase (as described in Section 2.2) and its ability to induce a bottleneck shown in Figure 6. By applying $L_{12}$ (Equation 2) to six traffic lights, the optimal phase range that maximises flow can be identified. $L_{12}$ was considered to be a suitable profile as traffic lights generally remain green and red for similar intervals of time.

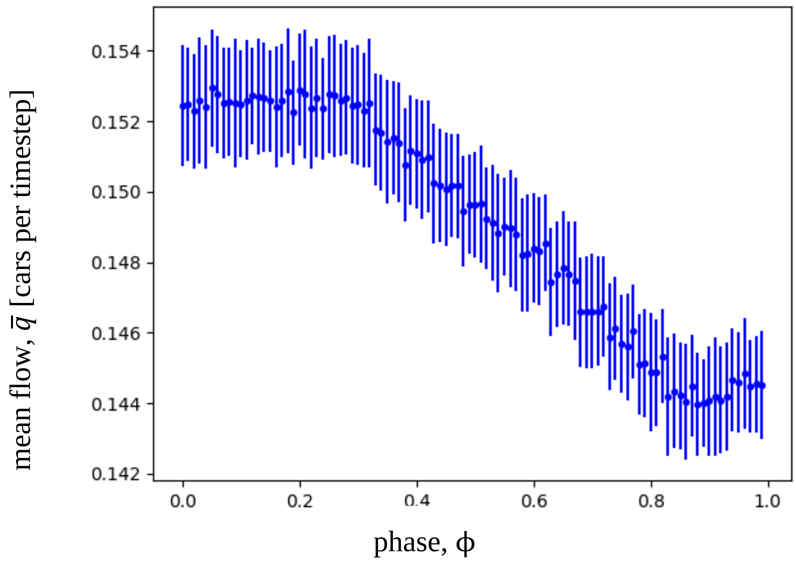


Figure 6: A graph of the phase-flow relation on a highway with circular boundary conditions for $N = 96$ cells, $\rho_t = 6/96$, $\rho_v = 0.15$ using light profile $L_{12}$. Uncertainties corresponding to $1\sigma$ have been determined by applying the same procedure described at the start of Section 4 and have been annotated on the diagram.

It is essential to note that phase-flow relation shown in Figure 6 is exclusive to $L_{12}$ and different profiles may have entirely different relations. Additionally, by comparing the scales in Figure 3 and Figure 5, it can be seen that the effect of varying phase is approximately an order of magnitude less than varying the density. The phase-flow relation appears to be continuous and contains no outstanding troughs which may suggest that initiating bottleneck situations are independent on the phase. Therefore, the situation in its bottleneck condition will be maintained for all phase values. However, a multivariate analysis treatment would be essential to substantiate this notion.

The relationship between the number of traffic lights and the mean flow rate was explored and resembled an exponential curve, qualitatively. This was validated by taking the natural logarithm of the flow values to obtain a linear form and determine a linear $\chi^2$ fit (Figure 7). The reduced $\chi^2$ value was determined as 0.81 which implies that the relationship between the number of traffic lights and the mean flow rate was exponential. Moreover, applying a linear fit to the mean flow rate directly produced a reduced $\chi^2$ value of 5.70 which further supports this notion.

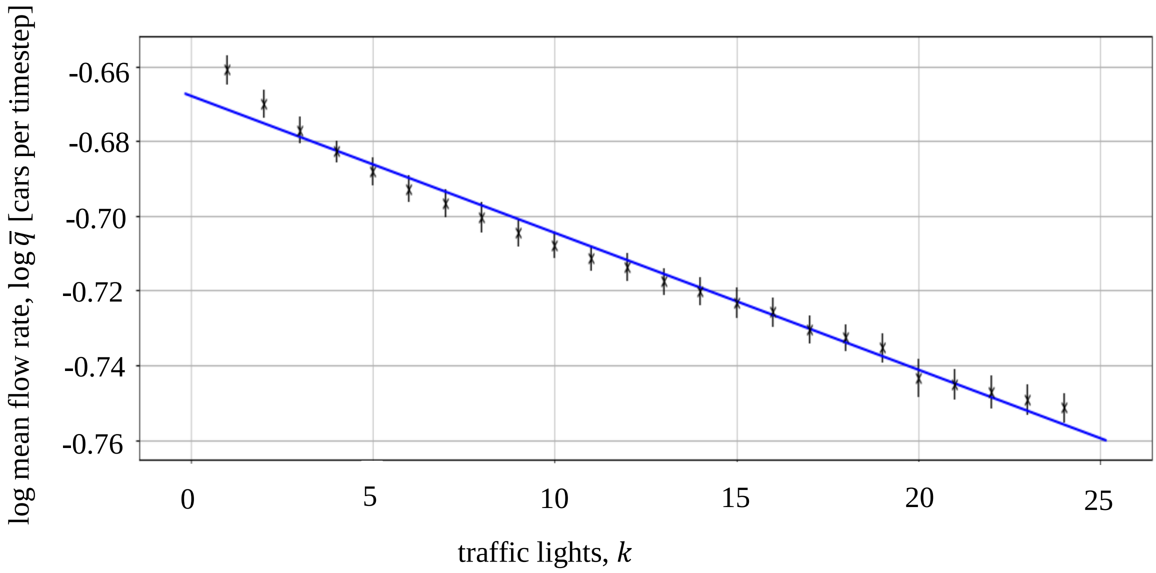


Figure 7: A graph of the relationship between the number of traffic lights and the log of the mean flow rate. The model parameters were $\rho_v = 0.4$, $\rho_t = 6/96$, $N = 9$ and $K = 24$. Uncertainties corresponding to $1\sigma$ have been determined by applying the same procedure described at the start of Section 4 and have been propagated accordingly. The reduced $\chi^2$ was calculated as 0.81 for a linear best fit line for data points The best fit parameters were determined as $m = -0.0037 \pm 0.0001$ and $c = -0.670 \pm 0.001$.

The effect on the flow rate due to the intervals of time for which a traffic light displays a given colour was also explored. Our procedure was to vary the number of consecutive timesteps $j$ for which a traffic light remains red or green in a light profile $L$ (with fixed length of 24 elements) while keeping the proportions of R's and G's constant. For example, $L_j$ for $j = 3$ has a profile

$$L_3 = [\mathrm{RRRGGGRRRGGGRRRGGGRRRGGG}]. \quad (4)$$

The relationship between $L_j$ (for $j = 1,2,3,4,6$ and 12) and the flow rate is shown in Figure 8. The density values explored were divided into two categories depending on the gradient of the curve in Figure 5. Vehicle densities $\rho_v = 0.2$ and $\rho_v = 0.4$ were defined as a steady bottleneck due to their relatively flat gradients whilst $\rho_v = 0.6$ and $\rho_v = 0.8$ were defined as a congested bottleneck due to their steep negative gradients.

Figure 8(a) suggests that consistent behaviour can be observed for steady bottleneck flow. However, for congested bottleneck flow shown in 8(b), there is an apparent deviation from this behaviour. In general, the flow rate is less affected by $j$ in congested scenarios. This may be considered intuitive when compared to realistic scenarios as traffic lights are less effective at higher vehicle densities. This is supported by the comparable behaviour between Figure 3 and Figure 5 in high density regions.

Furthermore, the peak at $L_2$ may indicate the optimal number of timesteps a light stays green or red to maximise flow.

Quantitative comparisons between realistic traffic and the model can be made with respect to the units of model parameters [5]. During maximum congestion, each vehicle occupies approximately 7.5 m, which represents the length of a single cell in the road array. The maximum speed of $v = 5$ in our model should correspond to the national speed limit in the UK [7] (approximately 110 kph). Therefore, the length of a timestep can be approximated as 1.23 seconds. This would imply an optimal time interval for a traffic light for a simple highway with a single lane would be 2.46 seconds. This underestimation is likely due to the limitations of a cellular automaton model confined to a small range of arbitrarily set integers as well as a non-rigorous approach in defining these model parameters. This is further suggested as the model implies the maximum braking distance is approximately 38 m whilst real data predicts a breaking distance of approximately 96 m assuming regular road conditions [8].

Other feasible approaches to define the model parameters include scaling the model to align with the fundamental diagram corresponding to real data as explored in [5].

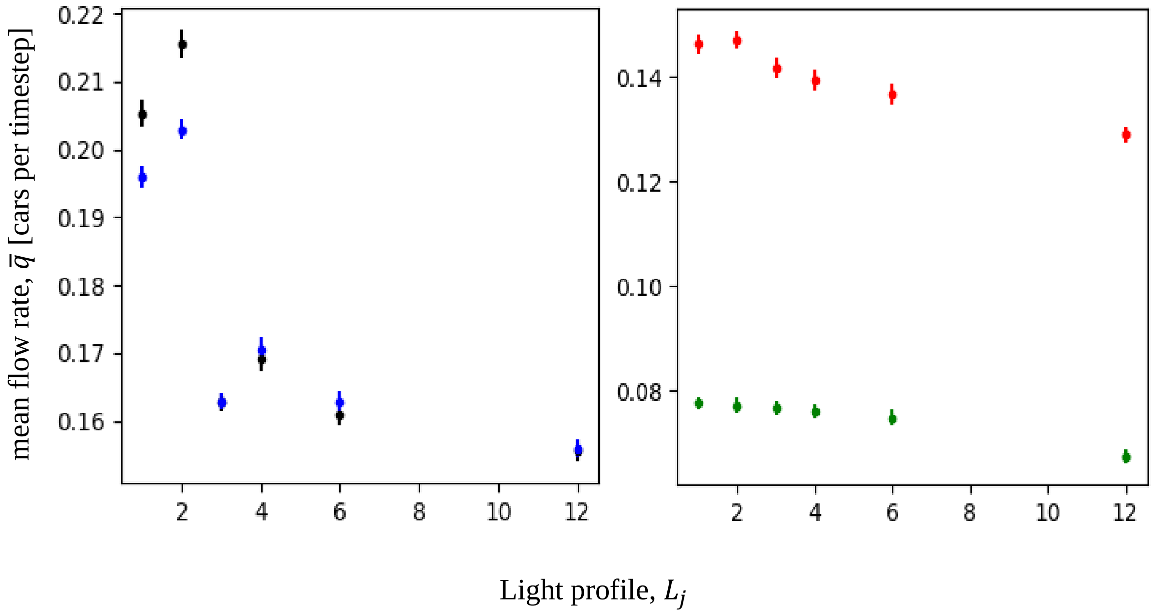


Figure 8: The relationship between mean flow rate and light profiles with various values of $j$. (a) displays the plot for laminar flow [$\rho_v = 0.2$ (black), 0.4 (blue)] and (b) displays the plot for congested flow [$\rho_v = 0.6$ (red), 0.8 (green)]. The model parameters were $\rho_v = 0.4$, $\rho_t = 6/96$, $N = 9$ and $K = 24$. Uncertainties corresponding to $1\sigma$ have been determined by applying the same procedure described at the start of Section 4 and have been annotated.

# 5. Conclusion

In conclusion, the model constructed shows characteristic features of realistic traffic flow, such as non-linear backward motion of congestion and the bottlenecking of traffic flow. Both of these features were observed qualitatively in the model simulation. The concept of phase was introduced and the effect of its variation was shown to be approximately an order of magnitude less than varying the vehicular density. Moreover, the phase-flow relation indicated that initiating bottleneck situations are independent of the phase. However, a multivariate analysis treatment would be essential to confirm this view. The simplicity of our model may exaggerate the effect of particular variables, such as phase, which would likely be dominated by external road conditions in realistic scenarios. Although chaotic behaviour was generally observed for all simulations, there was a substantial amplification to the variation in flow in the critical density region, which was identified by the large standard deviations.

Numerous limitations were identified with our computational model. As the speeds set in our cellular automaton model were confined to a small range of integers, underestimations and inconsistencies were encountered when simulating a realistic scenario. Translating these model parameters to represent a realistic scenario would involve rigorous methods. These methods include scaling the model to align with the fundamental diagram corresponding to real data [5]. Additionally, the random input was set arbitrarily to a single probability value whilst realistic scenarios would exhibit dynamic variation to account for non-deterministic road conditions. A mesoscopic model would facilitate these dynamic responses which was explored in [4].

Our model required macroscopic variables, such as mean flow rate, to be determined over large time intervals. It also lacked adaptive methods to forecast and control changing traffic flow conditions. Our model could be refined by incorporating elements from short-term predictive models such as the ‘Volterra’ model which was used to forecast short-term traffic flow and was effective in the prediction of chaotic traffic flow in highway scenarios [2]. These models are becoming increasingly viable as computational methods advance.

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