Day:

Date: 27-09-2023

Assignment # 01

Name :

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Reg. NO:

FA20-BSM-018

subject :

Graph Theory

Batch :

Five

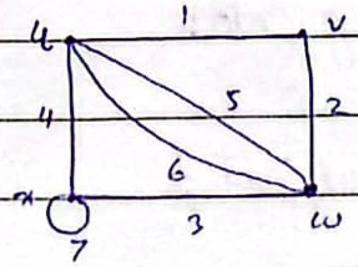
Submit to:

Sir Haider Ali

Day: ______ Date: ______

Question : 2.1

consider the following graph G shown on the right which statement holds true for G.



(a) vertices v and x are adjacent

no it is false.

(b) edge 6 is incident with w

yes, it is true.

(c) x is incident with edge 4

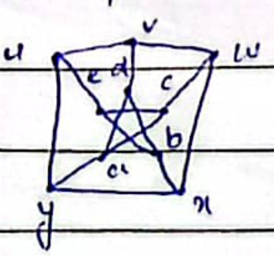
yes it is true.

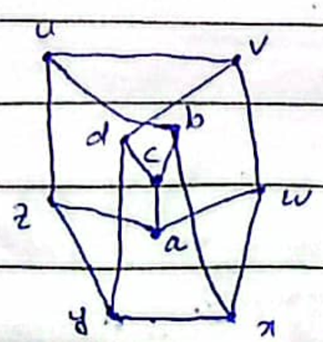
(d) w and edge 6, 5 from subgraph of G.

no it is not true.

Question : 2.2

By suitably labelling the vertices show that the following graphs are isomorphic.





Day: ______ Date: ______

Solution:

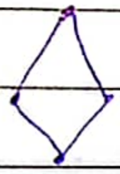
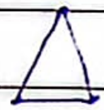
no of vertices = 10

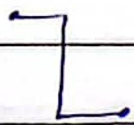
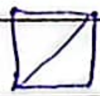
no of edges = 15

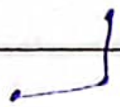
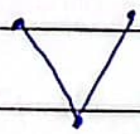
In both graphs no of vertices and edges are same, also degree of all vertices is 3. and conveying the same information. So, they are isomorphic.

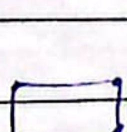
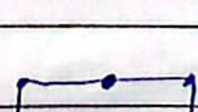
Question: 2.3

Draw eleven unlabelled simple graphs.

(i)  (ii)  (iii) 

(iv)  (v)  (vi) 

(vii)  (viii)  (ix) 

(x)  (xi) 

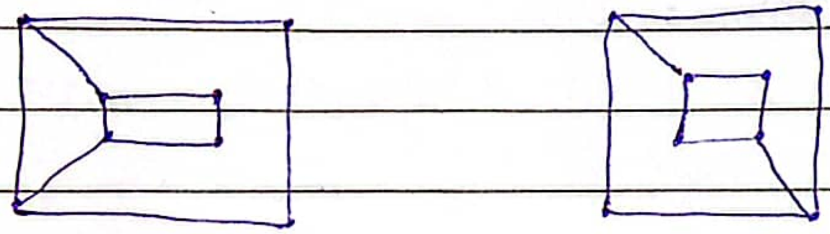
Day: ______ Date: ______

Question: 2.4

(a) if two graphs have the same degree sequence must they be isomorphic?

Solution:

Having the same degree sequence is not sufficient for two graphs to be isomorphic for example:



These graphs having same degree but not isomorphic.

(b) if two graphs are isomorphic must they have same degree sequence?

Solution:

Yes it is necessary if two graph are isomorphic, they must have same degree sequence.

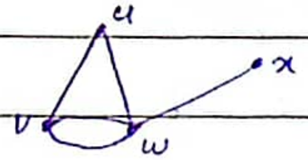
Day: Date:

Question : 2.5

Let G be graph with degree sequence {1, 2, 3, 4} write down the no of edges and construct such graph. Are there any simple graph with degree sequence {1, 2, 3, 4}.

Solution:

The graph would be the following shape.



The no of vertices = 4

The no of edges = 5

so there is no simple graph with degree sequence {1, 2, 3, 4}.

Question : 2.6

if G is a simple graph with at least two vertices then G has two or more vertices of same degree.

Proof:

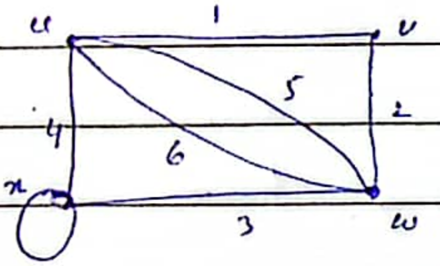
suppose the graph G has $n > 1$

Day:______ Date:______

vertices then $0 \le deg(v) \le n-1$ for any vertex v But a vertex of degree 0 is not adjacent to any other vertex, while a vertex of degree $n-1$ is adjacent to every other vertex cannot have both a vertex of degree 0 and $n-1$. it follows that $0 \le deg(v) \le n-1$ for any vertex v. In either case, there are $n-1$ possible values for collection or n integers so two or integers must be equal.

Question: 2.7

For the graph shown on the right, wright down.



(a) a walk of length 7 between u and w.

uvuxuvvw

(b) all the cycles of length 1, 2, 3, 4

length 1: The loop xx

length 2: The multiple edge uwu

length 3: The triangle uwxu

Day: Date:

length 4: the quadrilaterials uvwxu

(c) a path of max length

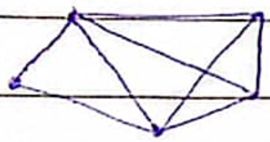
uvwx

Question: 2.8

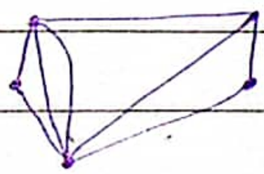
Draw four connected graphs G_1, G_2, G_3 and G_4 with 5 vertices and 8 edges satisfying.

Ans:

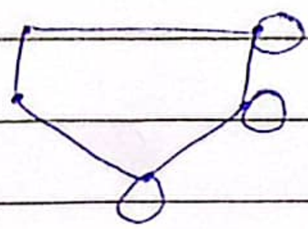
- G_1 is simple graph.



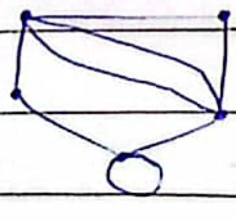
- G_2 is non simple graph with no loops:



- G_3 is non simple graph with no multiple edges



- G_4 is graph with both loops and multiple edges.

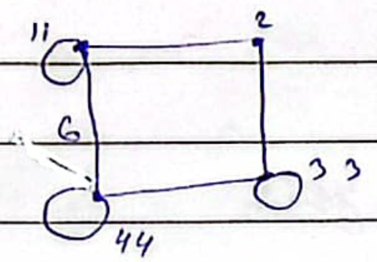


Day: ____________ Date: ____________

Question : 2.9

Draw a simple connected graph

(a) with degree sequence {1, 1, 2, 3, 3, 4, 4, 6}



~~Question : 2.10~~

determine no of edges in the following graphs.

(a) C_{10}

10

(b) $K_{9,10}$

$9 \cdot 10 = 90$

Question : 2.11

Determine the number of edges in the following graphs

(a) K_{10}

$10(10-1)/2 = \frac{10(9)}{2} = 90/2 = 45$

(b) Q_5

$5 \times 2^{5-1} = 5 \times 2^4 = 16 \times 5 = 80$

Day: ______ Date: ______

(c) Q_5

$$5 \times 2^{5-1} = 2^4 \times 5 = 16 \times 5 = 80$$

(d) The dodecahedron:

24

Question : 2.13

The girth of graph G is the length of shortest cycle and circumference is length of longest cycle. find both for

(a) peterson graph:

girth = 4

circumference = 9

Question : 2.14

(a) The 9 web graph Q_4:

girth = 4

circumference = 9

Question : 2.15

prove that if every cycle of a graph has an even number of edges then graph is bipartite?

Ans: if G is bipartite with vertex sets

Day: Date:

v_1 and v_2. every step along a walk takes you either from v_1 to v_2 or v_2 to v_1, to end up where you started therefore even steps takes but in this situation

suppose every cycle of G is even and v_0 be any vertex. for each vertex v_1 the same component shortest path from v_1 to v_0

Do same for G component it would even cycle and thus G is bipartite.