CSCI 599: Deep Learning and its Applications

Lecture 4

Fall 2017
Joseph J. Lim

Lots of help from **Shao-Hua Sun**, **Youngwoon Lee**, and **Te-Lin Wu**

Disclaimer

- This course is taught for the 1st time @ USC. This course is 599, and thus an **experimental** course.
- The syllabus, course policy, and grading details **may change** over the semester (**check website!**)
- If you prefer a well-structured course, this is **NOT** a course for you, and I encourage you to take the course next year. We really mean this.
- But, it will be **fun** and **challenging**!

Welcome to CSCI 599!

- 175 students
  - PhD: 61
  - Master's: 102
  - Undergraduate: 12
- About 20 different majors
  - CSCI: ~110
  - EE: ~30
  - Others: ~30

Communication

- Please use **Piazza** for any general communication including questions
  https://piazza.com/usc/fall2017/csci599
- Use e-mail ONLY when it is necessary. Seriously I don't know when…
  But, the staff e-mail address is: ~~deeplearning-staff-l@usc.edu~~
- Any non-necessary e-mail will be ignored.

Communication

- Please do NOT
  - e-mail us individually (**we will not reply**)
  - come to our office without appointment

Assignment 1

- Assignment 1 will be released next week (not this week)
- DUE October 18th, 2017 (week 8)

Important Dates

Subject to change!

- **Midterm: week 7**
- Assignment 1: **week 8** (changed from week 6)
- Assignment 2: week 11
- Project
  - **Team formation: week 4**
  - Project proposal: week 8
    - **Project meeting with TA #1: between week 4 - week 8**
    - Project meeting with Instructor #1: week 8 (M-W)
  - Project mid-report: week 12
    - Project meeting with TA #2: between week 8 - week 12
    - Project meeting with Instructor #2: week 11 (M-W)
  - Project report + Final presentation: week 15 (5-9:30pm) **4.5 hours**
    - Project meeting with TA #3: between week 12 - week 15

Project Evaluation

- Creativity and difficulty of the problem setup
- Novelty of the approach
- Thoroughness of the experiments
- Quality of student's presentation, report, and meetings with TA/instructor

Extra credit for creating your own project (OK to discuss and get help from TAs)

Team Formation

- Submit your team information now
- It is OK to change the team until the proposal day.
- Link: https://goo.gl/forms/i7XnI8y08qDEoeE73

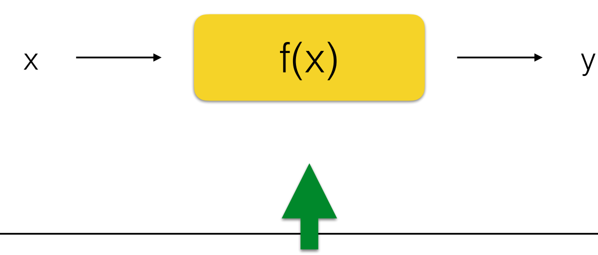
Today's agenda

- Part 1
  - Loss function & Optimization
- Part 2
  - Neural Networks
- Part 3
  - Project Discussion

Today's agenda

- Part 1
  - Loss function & Optimization
- Part 2
  - Neural Networks
- Part 3
  - Project Discussion

Recap:Our course



(1) How do we learn this function (using deep learning)?
(2) How to formulate a problem into this

Loss function & Optimization

x → f(x) → y

Loss function & Optimization

x → f(x; W) → y

Showing W (to learn) explicitly
Parametric function

Loss function & Optimization

x → f(x; W) → y

Showing W (to learn) explicitly
Parametric function

- **Loss function (L)** measures how well learned W can map X to Y (compared to f*).

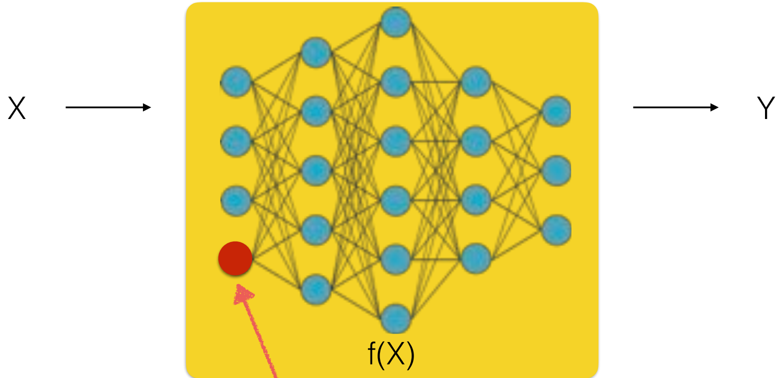
Loss function & Optimization

x → f(x; W) → y

Showing W (to learn) explicitly
Parametric function

- **Loss function (L)** measures how well learned W can map X to Y (compared to f*).
- **Optimization** finds the best W given a loss function L (i.e. finding W that minimizes L).

Recap: Linear Classification



Apparently, this is an unit used in deep neural networks too.

Recap: Linear Classification

$$\underbrace{f(x;\ W)}_{5\text{x}1} = \underbrace{W}_{5\text{x}7500}\underbrace{x}_{7500\text{x}1} + \underbrace{b}_{5\text{x}1}$$

X → f(X; W) → Y

50x50x3 numbers
(7500 numbers)

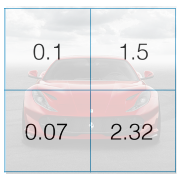
Weights
(that we want to learn)

One-hot class vector
(e.g. <0,0,0,1,0>)

Y: output
X: input
W, b: **learned weight**

Modified from CS 231N @ Stanford

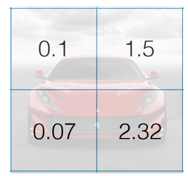
Recap: Linear Classification



$$\underbrace{\begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 0.1 \\ 1.5 \\ 0.07 \\ 2.32 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} 0.631 \\ 0.969 \\ 0.71 \end{bmatrix}}_{f(x;\ W,b)} \begin{array}{l} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{array}$$

Modified from CS 231N @ Stanford

Recap: Linear Classification



$$\underbrace{\begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 0.1 \\ 1.5 \\ 0.07 \\ 2.32 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} 0.631 \\ \boxed{0.969} \\ 0.71 \end{bmatrix}}_{f(x;\ W,b)} \begin{array}{l} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{array}$$

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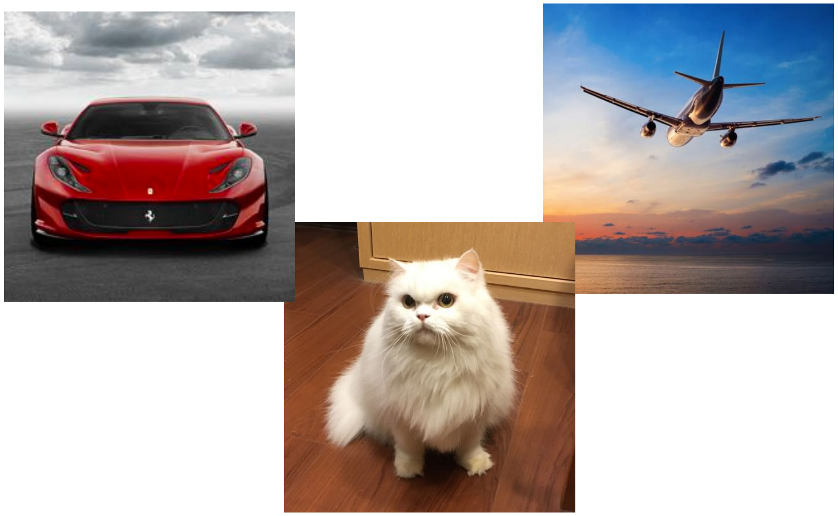
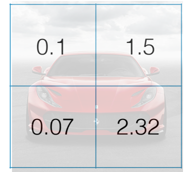
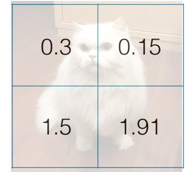
Linear Classification



Linear Classification





0.1	-0.15	0	0.3
-0.5	0.2	0.1	0.35
-0.45	0.4	0.1	0.15

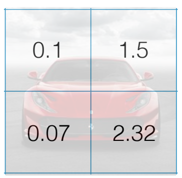
W

0.15
-0.1
-0.2

b

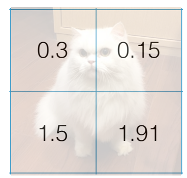
f(x; W,b)

Linear Classification



$$\underbrace{\begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 0.1 \\ 1.5 \\ 0.07 \\ 2.32 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} 0.631 \\ \boxed{0.969} \\ 0.71 \end{bmatrix}}_{f(x;\, W,b)} \begin{array}{l} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{array}$$

Linear Classification

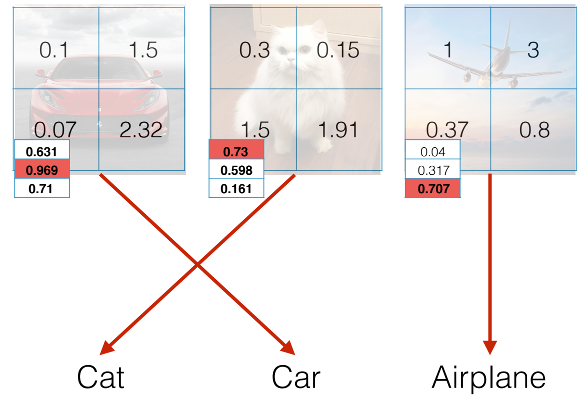


$$\underbrace{\begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 0.3 \\ 0.15 \\ 1.5 \\ 1.91 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} \boxed{0.73} \\ 0.598 \\ 0.161 \end{bmatrix}}_{f(x;\, W,b)} \begin{array}{l} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{array}$$

Linear Classification

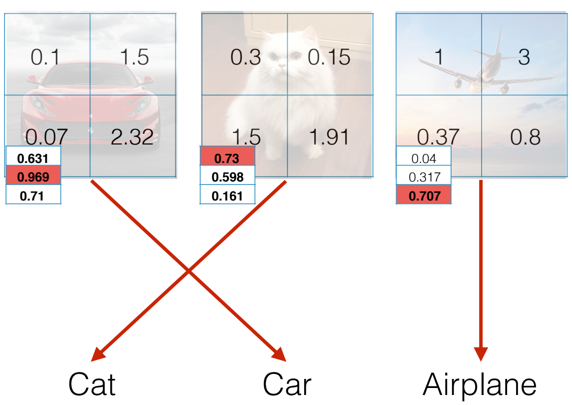
$$
\underset{W}{\begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}}
\underset{x}{\begin{bmatrix} 1 \\ 3 \\ 0.37 \\ 0.8 \end{bmatrix}}
+ \underset{b}{\begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}}
= \underset{f(x;\, W,b)}{\begin{bmatrix} 0.04 \\ 0.317 \\ \boxed{0.707} \end{bmatrix}}
\begin{matrix} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{matrix}
$$

Linear Classification



$$
W = \begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}
\qquad
b = \begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}
$$

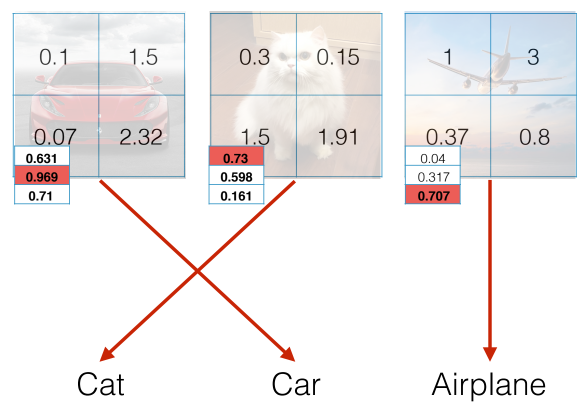
Linear Classification



$$
W = \begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}
\qquad
b = \begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}
$$

Looks perfect! Doesn't it?

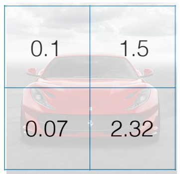
Linear Classification

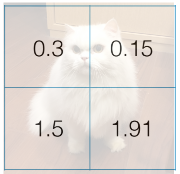


$$
W = \begin{bmatrix} 0.1 & -0.15 & 0 & 0.3 \\ -0.5 & 0.2 & 0.1 & 0.35 \\ -0.45 & 0.4 & 0.1 & 0.15 \end{bmatrix}
\qquad
b = \begin{bmatrix} 0.15 \\ -0.1 \\ -0.2 \end{bmatrix}
$$

But… What if we have different W or b?

Linear Classification

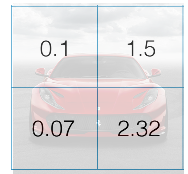




$$W = \begin{bmatrix} -0.25 & 0.1 & 0.05 & 0.175 \\ 0.05 & -0.075 & 0 & 0.15 \\ -0.225 & 0.2 & 0.05 & 0.075 \end{bmatrix} \qquad b = \begin{bmatrix} 0.35 \\ 0.1 \\ 0 \end{bmatrix}$$

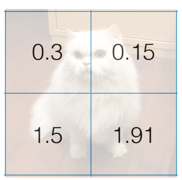
f(x; W,b)

Linear Classification



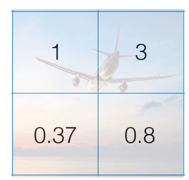
$$\underbrace{\begin{bmatrix} -0.25 & 0.1 & 0.05 & 0.175 \\ 0.05 & -0.075 & 0 & 0.15 \\ -0.225 & 0.2 & 0.05 & 0.075 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 0.1 \\ 1.5 \\ 0.07 \\ 2.32 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0.35 \\ 0.1 \\ 0 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} \boxed{0.884} \\ 0.34 \\ 0.455 \end{bmatrix}}_{f(x;\, W,b)} \begin{array}{l} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{array}$$

Linear Classification



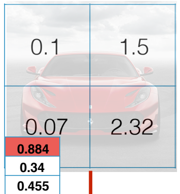
$$\underbrace{\begin{bmatrix} -0.25 & 0.1 & 0.05 & 0.175 \\ 0.05 & -0.075 & 0 & 0.15 \\ -0.225 & 0.2 & 0.05 & 0.075 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 0.3 \\ 0.15 \\ 1.5 \\ 1.91 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0.35 \\ 0.1 \\ 0 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} \boxed{0.699} \\ 0.39 \\ 0.181 \end{bmatrix}}_{f(x;\, W,b)} \begin{array}{l} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{array}$$

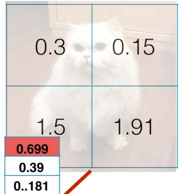
Linear Classification

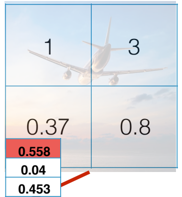


$$\underbrace{\begin{bmatrix} -0.25 & 0.1 & 0.05 & 0.175 \\ 0.05 & -0.075 & 0 & 0.15 \\ -0.225 & 0.2 & 0.05 & 0.075 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 1 \\ 3 \\ 0.37 \\ 0.8 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0.35 \\ 0.1 \\ 0 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} \boxed{0.558} \\ 0.04 \\ 0.453 \end{bmatrix}}_{f(x;\, W,b)} \begin{array}{l} \text{Cat} \\ \text{Car} \\ \text{Airplane} \end{array}$$

Linear Classification







-0.25	0.1	0.05	0.175
0.05	-0.075	0	0.15
-0.225	0.2	0.05	0.075

W

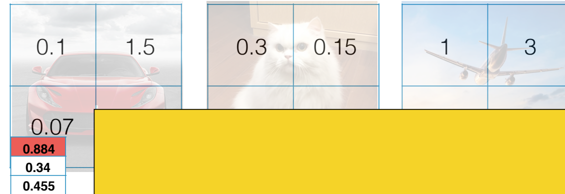
0.35
0.1
0

b

Cat Car Airplane

Oops…

Linear Classification



-0.25	0.1	0.05	0.175
0.05	-0.075	0	0.15
5	0.075		

W matters!

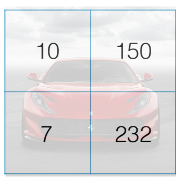
Cat Car Airplane

0

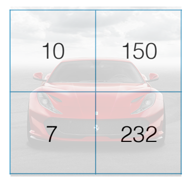
b

Oops…

Loss functions



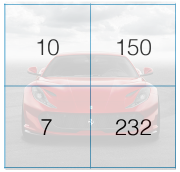
Loss functions



Cat	0
Car	1
Airplane	0

Ground truth

Loss functions



W1

0.1	-0.15	0	0.3
-0.5	0.2	0.1	0.35
-0.45	0.4	0.1	0.15

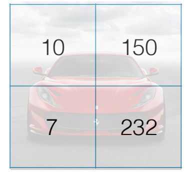
W2

-0.25	0.1	0.05	0.175
0.05	-0.075	0	0.15
-0.225	0.2	0.05	0.075

Cat	0
Car	1
Airplane	0

Ground truth

Loss functions



W1

0.1	-0.15	0	0.3
-0.5	0.2	0.1	0.35
-0.45	0.4	0.1	0.15

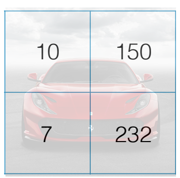
W2

-0.25	0.1	0.05	0.175
0.05	-0.075	0	0.15
-0.225	0.2	0.05	0.075

	Ground truth	W1	W2
Cat	0	0.631	0.884
Car	1	0.969	0.34
Airplane	0	0.71	0.455

Ground truth

Loss functions



W1

0.1	-0.15	0	0.3
-0.5	0.2	0.1	0.35
-0.45	0.4	0.1	0.15

W2

-0.25	0.1	0.05	0.175
0.05	-0.075	0	0.15
-0.225	0.2	0.05	0.075

	Ground truth	W1	W2
Cat	0	0.631	0.884
Car	1	0.969	0.34
Airplane	0	0.71	0.455

Ground truth

Which one is better?

Loss functions

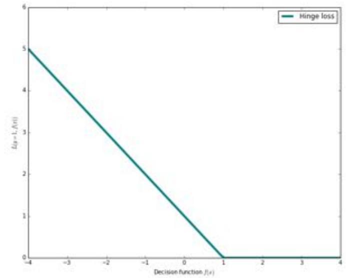
x → f(x; W) → y

Showing W (to learn) explicitly
Parametric function

L measures how well learned W can map X to Y.

- Hinge Loss
- L_1, L_2 Loss
- Cross-Entropy

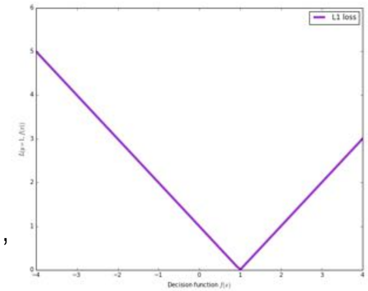
Hinge Loss



- Given f(x; W,b) & examples (x_i, y_i), minimize:

$$Loss = \frac{1}{N}\sum_{i=1}^{N}\sum_{j \neq y_i} max(0, 1 + f(x_i; W, b)_j - f(x_i; W, b)_{y_i})$$

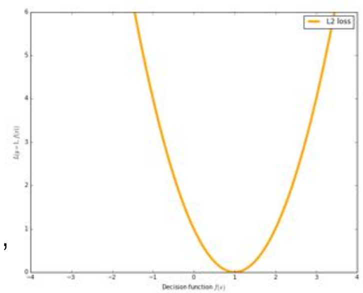
L_1 Loss



- Given f(x; W,b) & examples (x_i, y_i), minimize:

$$Loss = \frac{1}{N}\sum_{i=1}^{N} \| y_i - f(x_i; W, b)\|_1$$

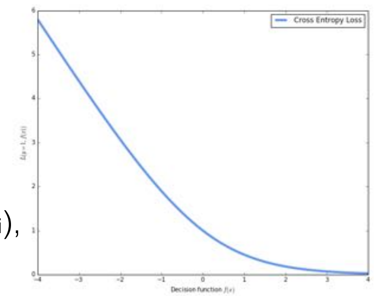
L_2 Loss



- Given f(x; W,b) & examples (x_i, y_i), minimize:

$$Loss = \frac{1}{N}\sum_{i=1}^{N} \| y_i - f(x_i; W, b)\|_2^2$$

Cross-Entropy Loss

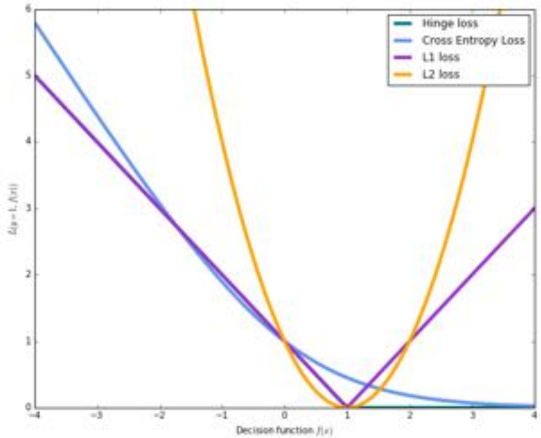


- Given f(x; W,b) & examples (x_i, y_i), minimize:

$$Loss = -\frac{1}{N}\sum_{i=1}^{N} \hat{y_i}\, log(P(y_i = j \mid x_i))$$

$$P(y_i = j \mid x_i) = \frac{e^{f(x_i;W,b)}}{\sum_{j=1}^{C} e^{f(x_j;W,b)}}$$

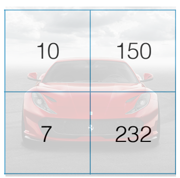
Loss functions



Each loss function penalizes differently.

Does a loss function matter?

Does a loss function matter?



W1

W_{11}	W_{11}	W_{11}	W_{11}
W_{11}	W_{11}	W_{11}	W_{11}
W_{11}	W_{11}	W_{11}	W_{11}

W2

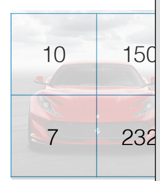
W_{21}	W_{21}	W_{21}	W_{21}
W_{21}	W_{21}	W_{21}	W_{21}
W_{21}	W_{21}	W_{21}	W_{21}

	Ground truth	W1	W2
Cat	0	0	0.07
Car	1	1.2	0.9
Airplane	0	0	0.05

Ground truth

Which one is better?

Does a loss function matter?



	f(x; W1)	f(x; W2)
L_1-loss	0.2	0.22

W2

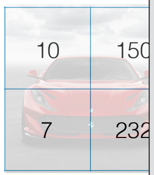
W_{21}	W_{21}
W_{21}	W_{21}
W_{21}	W_{21}

	Ground truth	W1	W2
Cat	0	0	0.07
Car	1	1.2	0.9
Airplane	0	0	0.05

Ground truth

Which one is better?

Does a loss function matter?



	f(x; W1)	f(x; W2)
L1-loss	0.2	0.22
L2-loss	0.04	0.0174

W2

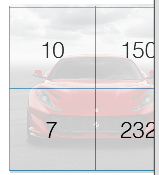
W_{21}	W_{21}
W_{21}	W_{21}
W_{21}	W_{21}

Cat	0	0	0.07
Car	1	1.2	0.9
Airplane	0	0	0.05

Ground truth

Which one is better?

Does a loss function matter?



	f(x; W1)	f(x; W2)
L1-loss	0.2	0.22
L2-loss	0.04	0.0174

Loss function matters!

W2

W_{21}	W_{21}
W_{21}	W_{21}
W_{21}	W_{21}

Cat	0	0	0.07
Car	1	1.2	0.9
Airplane	0	0	0.05

Ground truth

Which one is better?

Regularization

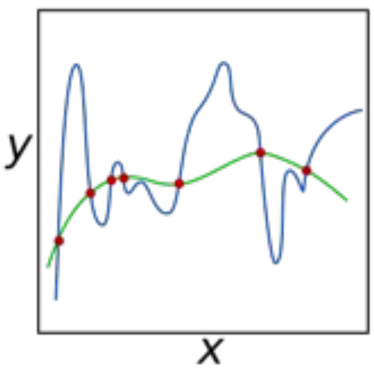
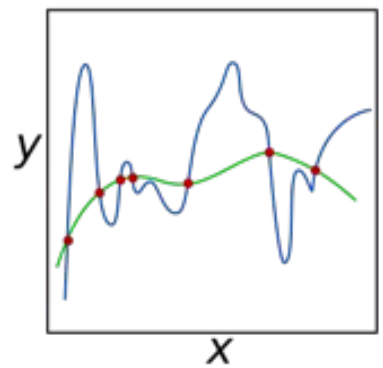


Image from Wikipedia

Regularization



Ockham's razor:
"Among competing hypotheses, the simplest is the best."
William of Ockham, 1285 - 1347

Image from Wikipedia

Regularization

Data loss | Regularization

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} \| y_i - f(x_i; W, b)\|_2^2}_{\text{Data loss}} + \underbrace{\lambda R(W)}_{\text{Regularization}}$$

Ockham's razor:
"Among competing hypotheses, the simplest is the best."
William of Ockham, 1285 - 1347

Regularization

Data loss | Regularization

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} \| y_i - f(x_i; W, b)\|_2^2 + \lambda R(W)$$

fitting to the data

Ockham's razor:
"Among competing hypotheses, the simplest is the best."
William of Ockham, 1285 - 1347

Regularization

Data loss | Regularization

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} \| y_i - f(x_i; W, b)\|_2^2 + \lambda R(W)$$

fitting to the data | preferring a simple model

Ockham's razor:
"Among competing hypotheses, the simplest is the best."
William of Ockham, 1285 - 1347

Optimization

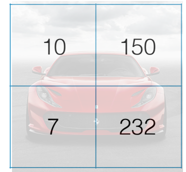
Find W that **minimize a loss function** (L).

Optimization

Find W that **minimize a loss function** (L).

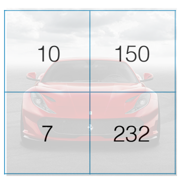
Note that we are finding W based on L, **NOT** f!

Optimization



x

Optimization

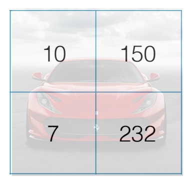


x

0
1
0

y* (ideal output)

Optimization



x

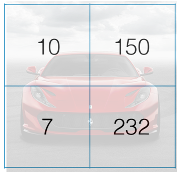
?	?	?	?
?	?	?	?
?	?	?	?

W

0
1
0

y* (ideal output)

Optimization



x

?	?	?	?
?	?	?	?
?	?	?	?

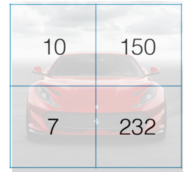
W

0
1
0

y* (ideal output)

f(x; W)

Optimization



x

?	?	?	?
?	?	?	?
?	?	?	?

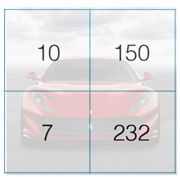
W

0
1
0

y* (ideal output)

f(x; W) → y*

Optimization



x

?	?	?	?
?	?	?	?
?	?	?	?

W

0
1
0

y* (ideal output)

f(x; W) → y*

L(W; f,x,y*)

Optimization

How do we find W minimizing L?

* Random search
* Analytic solution
* Numerical approach (gradient descent)

ut)

f(x; W) → y*

L(W; f,x,y*)

Optimization

How do we find W minimizing L?

* Random search
* Analytic solution
* Numerical approach (gradient descent)

ut)

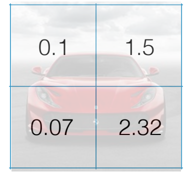
f(x; W) → y*

L(W; f,x,y*)

As an example, let's pick a simple loss function.

$L(W) = \| f(x; W) - y^* \|_2$

1. Random Search



x

?	?	?	?
?	?	?	?
?	?	?	?

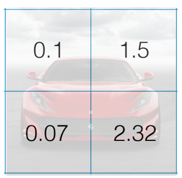
W

0
1
0

y* (ideal output)

$L(W) = \| f(x; W) - y^* \|_2$

1. Random Search



x

0.05	0.15	-0.1	0.2
-0.1	0.2	0.25	0.05
0.4	-0.15	0.25	0.15

W

$L(W) = \| f(x; W) - y^* \|_2$

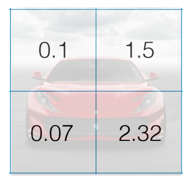
0.987
0.223
0.38

f(x; W)

0
1
0

y* (ideal output)

1. Random Search



x

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

$L(W) = \| f(x; W) - y^* \|_2$

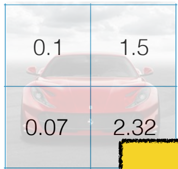
0.416
-0.118
1.167

f(x; W)

0
1
0

y* (ideal output)

1. Random Search



x

?	?	?	?
?	?	?	?

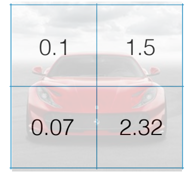
0
1
0

al output)

L(W) = |

Too slow

2. Analytic Solution



x

?	?	?	?
?	?	?	?
?	?	?	?

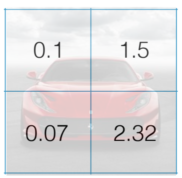
W

0
1
0

y* (ideal output)

L(W) = || f(x; W) - y* ||$_2$

2. Analytic Solution



x

?	?	?	?
?	?	?	?
?	?	?	?

W

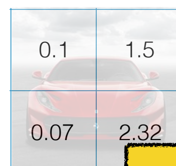
0
1
0

y* (ideal output)

L(W) = || f(x; W) - y* ||$_2$

$$\min_W L(W) = (X^T X)^{-1} X^T y^*$$

2. Analytic Solution



x

?	?	?	?
?	?	?	?

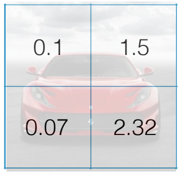
0
1
0

al output)

L(W) = |

Very hard to find.

3. Numerical Solution (gradient descent)



x

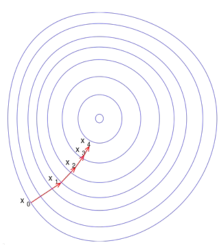
?	?	?	?
?	?	?	?
?	?	?	?

W

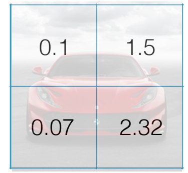
0
1
0

y* (ideal output)

L(W) = || f(x; W) - y* ||$_2$



3. Numerical Solution (gradient descent)



x

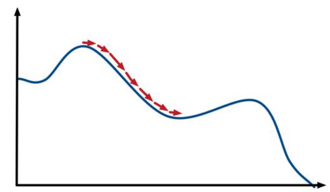
?	?	?	?
?	?	?	?
?	?	?	?

W

0
1
0

y* (ideal output)

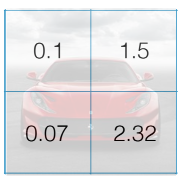
L(W) = || f(x; W) - y* ||$_2$



The derivative of a function in one-dimension

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Numerical Solution (gradient descent)



x

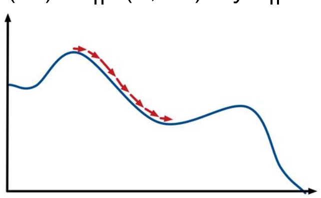
?	?	?	?
?	?	?	?
?	?	?	?

W

0
1
0

y* (ideal output)

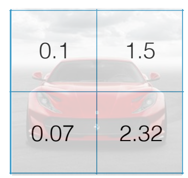
L(W) = || f(x; W) - y* ||$_2$



The **derivative** of a function in one-dimension

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Numerical Solution (gradient descent)



x

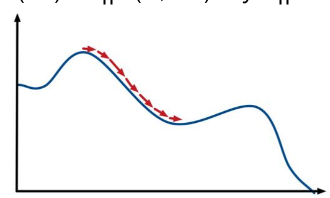
?	?	?	?
?	?	?	?
?	?	?	?

W

0
1
0

y* (ideal output)

L(W) = || f(x; W) - y* ||$_2$



The **derivative** of a function in one-dimension

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The **gradient** is the vector of partial derivatives in a higher dimension

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

?	?	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

?	?	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

?	?	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

0.416
-0.118
1.167

f(x; W+h)

0
1
0

y* (ideal output)

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

?	?	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

0.416
-0.118
1.167

f(x; W+h)

L(W+h) = 1.6693

0
1
0

y* (ideal output)

3. Numerical Solution (gradient descent)

W

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

-0.1+ 0.001	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

gradient (dW)

?	?	?	?
?	?	?	?
?	?	?	?

L(W) = 1.6688 L(W+h) = 1.6693

3. Numerical Solution (gradient descent)

W

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

-0.1+ 0.001	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

gradient (dW)

?	?	?	?
?	?	?	?
?	?	?	?

L(W) = 1.6688 L(W+h) = 1.6693

Gradient =

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Numerical Solution (gradient descent)

W

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

-0.1+ 0.001	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

gradient (dW)

?	?	?	?
?	?	?	?
?	?	?	?

L(W) = 1.6688 L(W+h) = 1.6693

Gradient = (1.6693-1.6688)/0.001 = 0.5

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Numerical Solution (gradient descent)

W

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

-0.1+ 0.001	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

gradient (dW)

0.5	?	?	?
?	?	?	?
?	?	?	?

L(W) = 1.6688 L(W+h) = 1.6693

Gradient = (1.6693-1.6688)/0.001 = 0.5

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	**0.2+ 0.001**	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

0.5	**?**	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	**0.2+ 0.001**	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

0.5	**?**	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

0.418
-0.118
1.167

f(x; W+h)

0
1
0

y* (ideal output)

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	**0.2+ 0.001**	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

0.5	**?**	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

0.418
-0.118
1.167

f(x; W+h)

L(W+h) = 1.6697

0
1
0

y* (ideal output)

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	**0.2+ 0.001**	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

0.5	**?**	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688

L(W+h) = 1.6697

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	**0.2+ 0.001**	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

0.5	**?**	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688 L(W+h) = 1.6697

Gradient = (1.6697-1.6688)/0.001 = 0.9

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Numerical Solution (gradient descent)

-0.1	0.2	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W

-0.1+ 0.001	**0.2+ 0.001**	0.15	0.05
0.25	0.05	0.2	-0.1
0.25	0.15	-0.15	0.4

W+h

0.5	**0.9**	?	?
?	?	?	?
?	?	?	?

gradient (dW)

L(W) = 1.6688 L(W+h) = 1.6697

Gradient = (1.6697-1.6688)/0.001 = 0.9

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. Numerical Solution (gradient descent)

Various optimization strategy for stability, accuracy, or speed

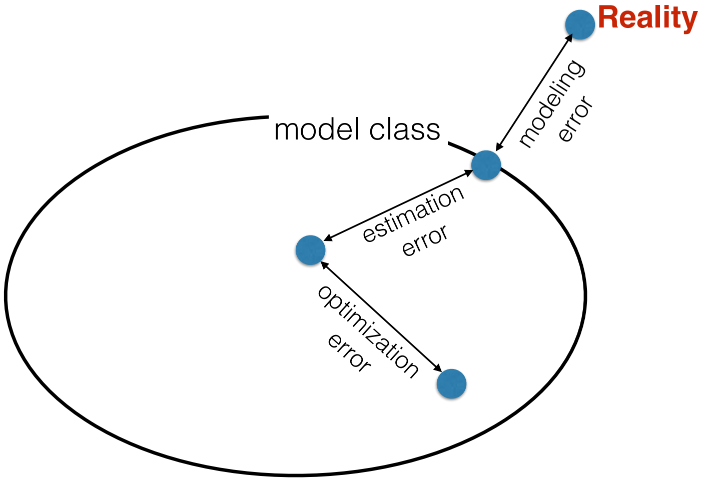
- Gradient Descent
- Stochastic gradient descent
- Adam optimization
- Batch and Minibatch algorithms

3. Numerical Solution (gradient descent)

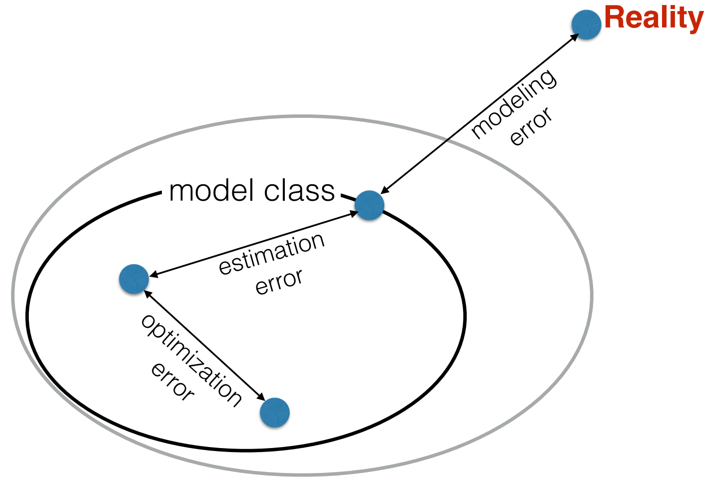
Issues

1. Local minima
2. Exploding gradient
3. Inexact gradient

Where do errors come from?



Where do errors come from?



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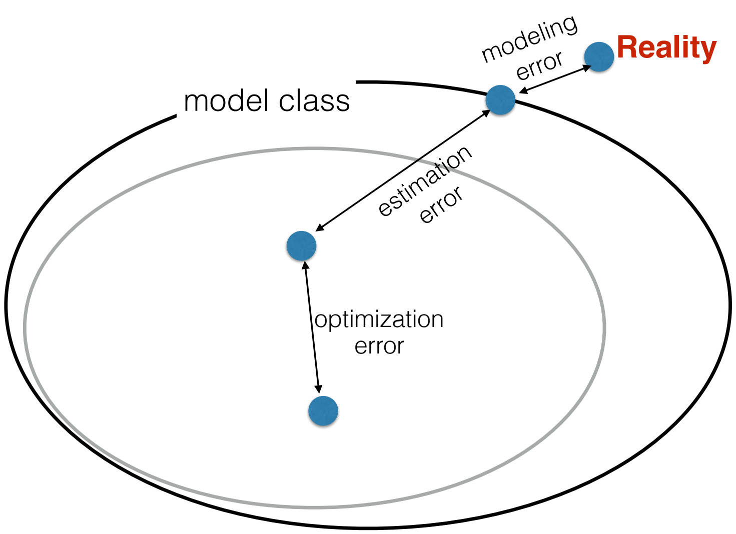


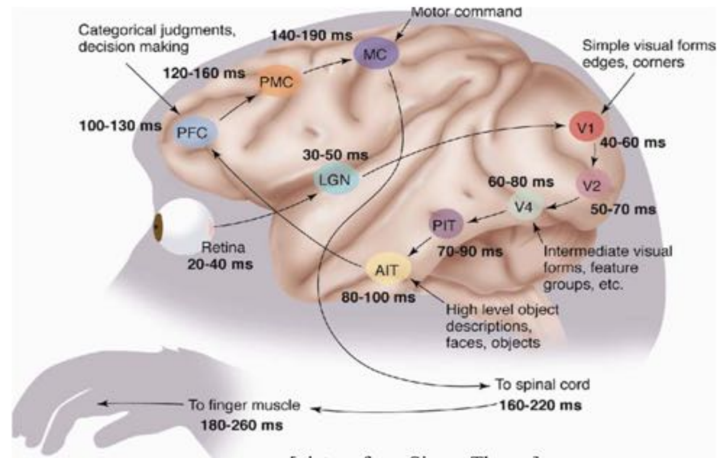
Break Time

See you in 15 mins!

Today's agenda

- Part 1
  - Loss function & Optimization
- Part 2
  - Neural Networks
- Part 3
  - Project Discussion

Deep Learning is motivated by human brain



[picture from Simon Thorpe]

Slide credit: Marc'Aurelio Ranzato, Yann LeCun

Neuron

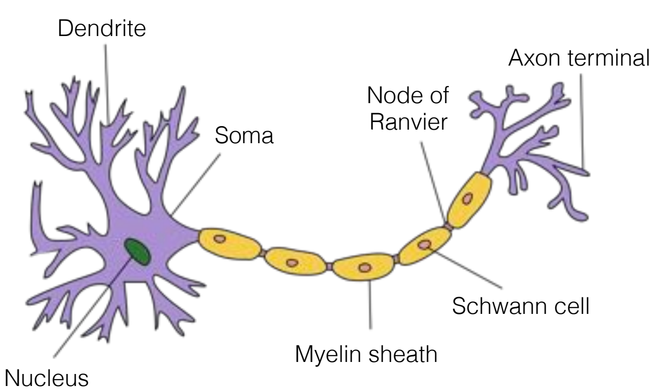


Image from Wikipedia

Neuron

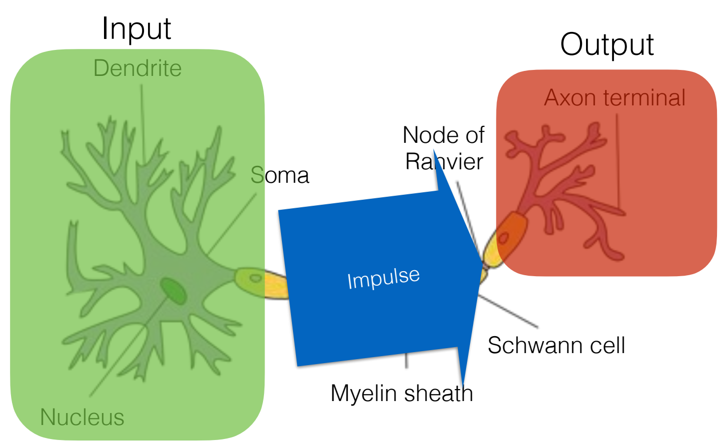
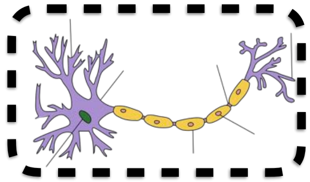


Image from Wikipedia

Neuron



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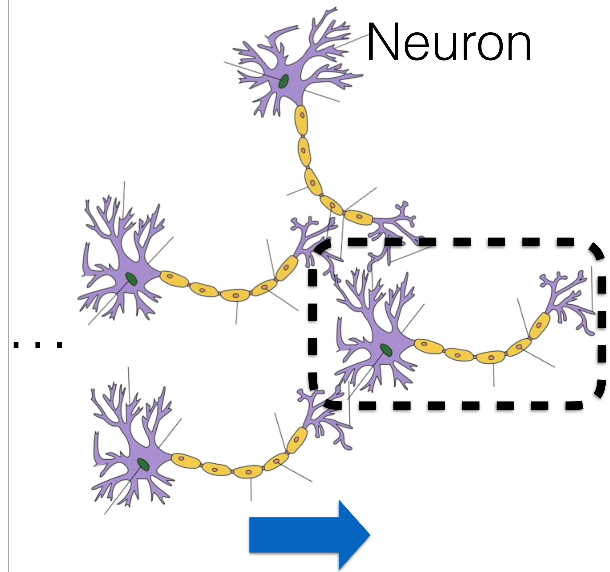
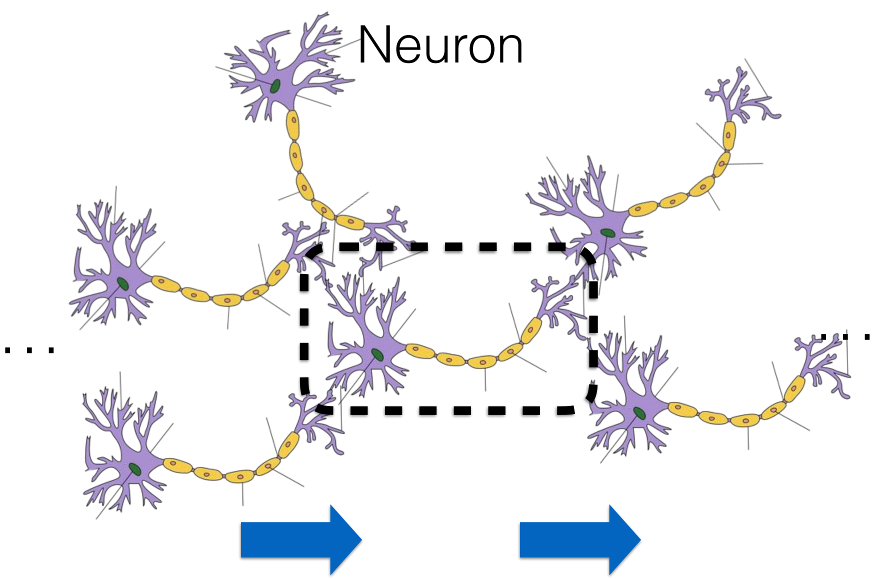
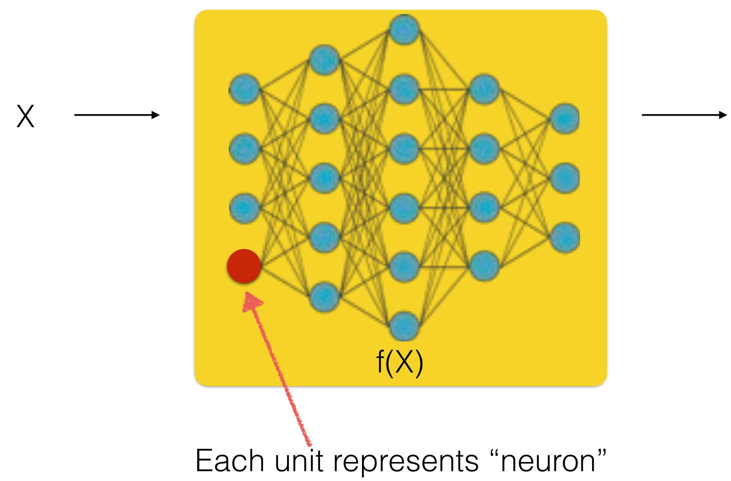


Neuron



Neural Networks



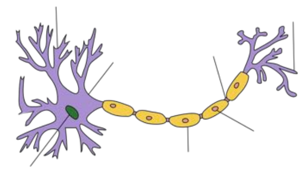
What is Neural Networks?

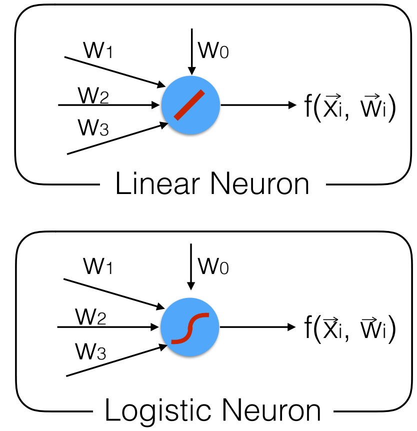
In simple terms,

a set of **neurons (atomic functions)**

connected in a **non-linear** way

“Neuron”

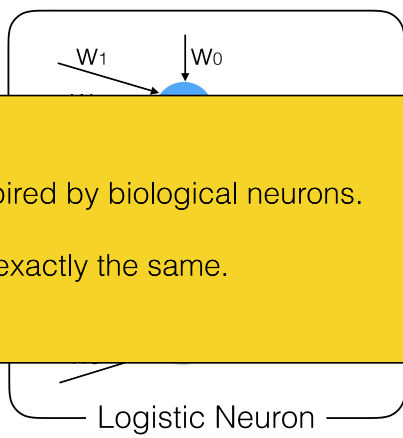




More neurons!

Modified from the HKUST slide

“Neuron”



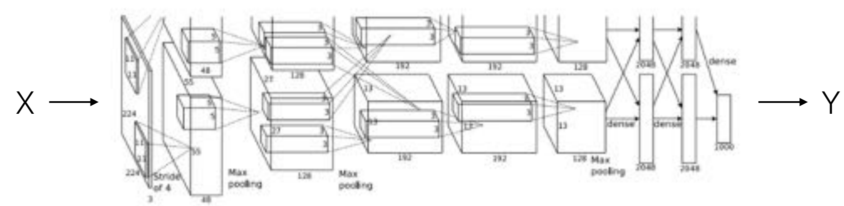
Artificial Neurons are inspired by biological neurons.

They are **NOT** exactly the same.

More neurons!

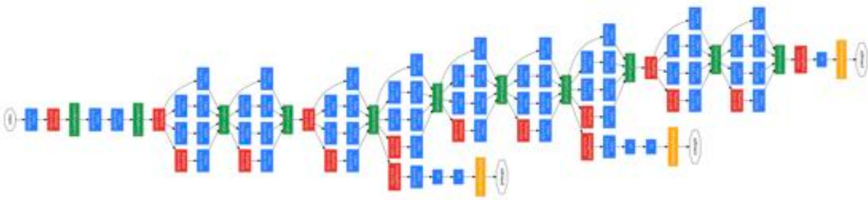
Modified from the HKUST slide

(Convolutional) Neural Networks



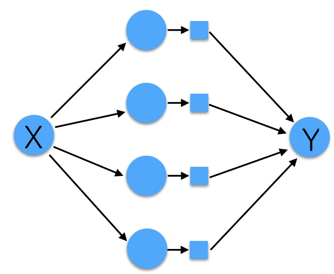
A Krizhevsky, et. al. ImageNet Classification with Deep Convolutional Neural Networks. NIPS 2012.

Deeper Neural Networks

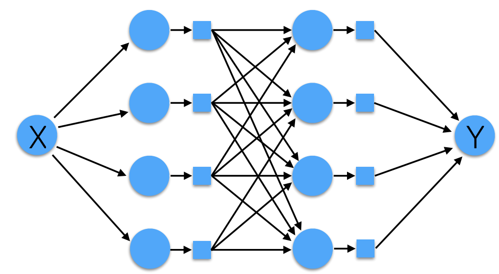


Szegedy et. al., Going deeper with convolutions, CVPR 2015

Why "Deep Learning"?



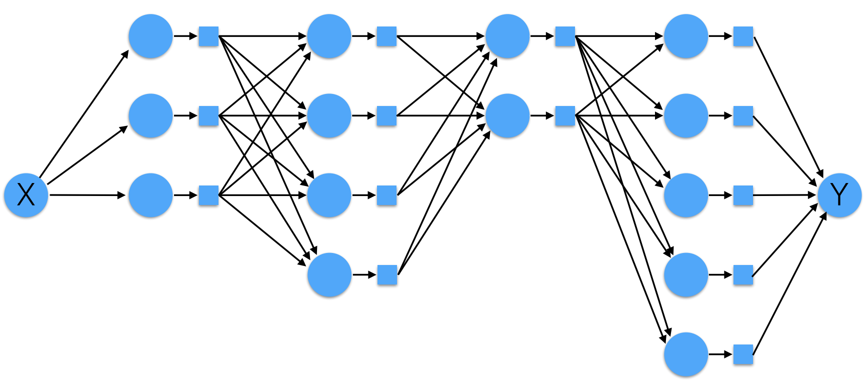
Shallow network



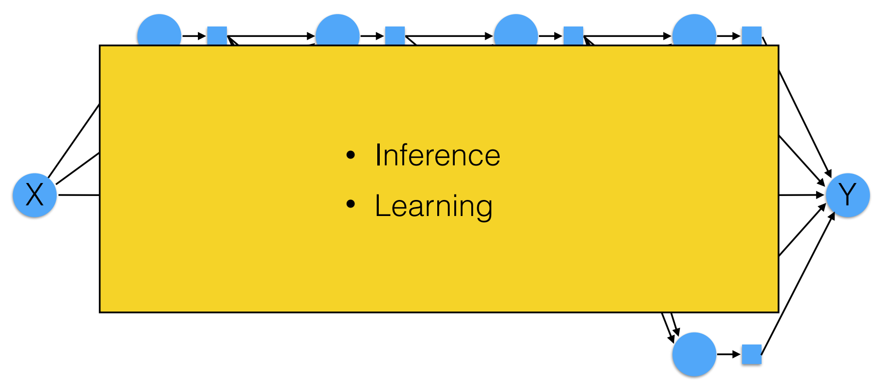
Deep network

The algorithm for training a **deep** network

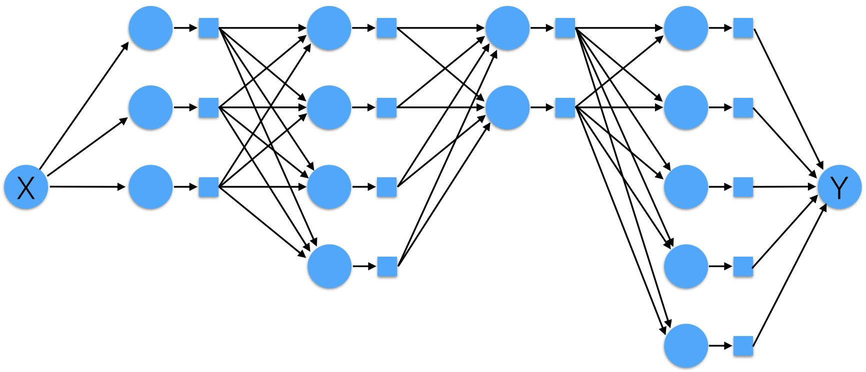
Neural Networks Example



Neural Networks Example



Forward propagation



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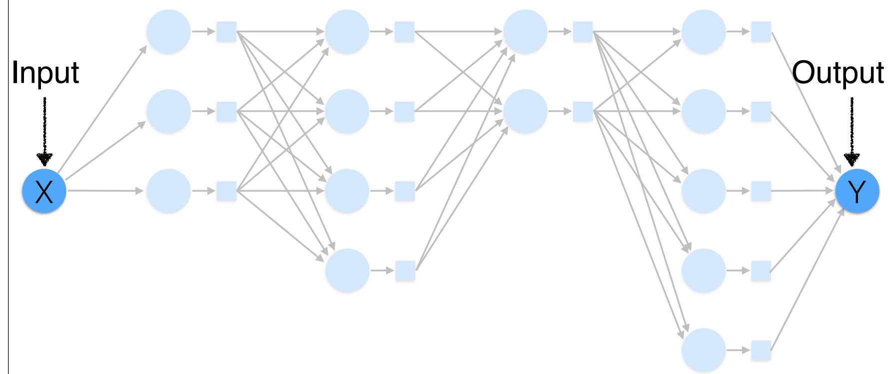

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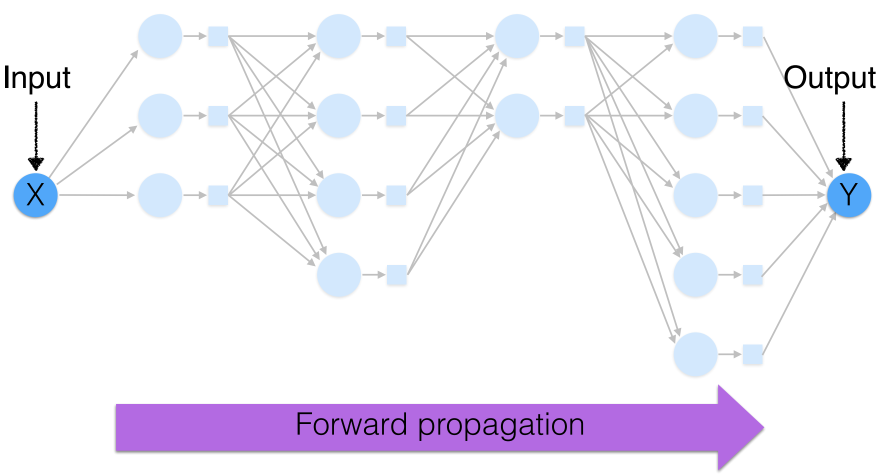

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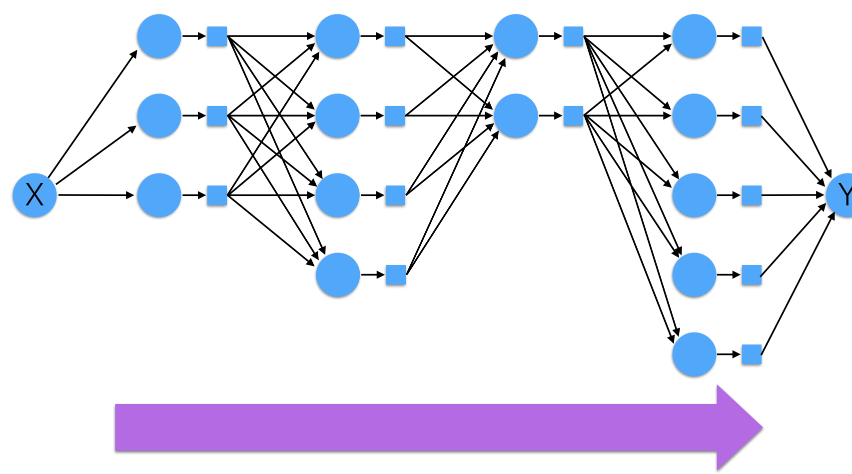

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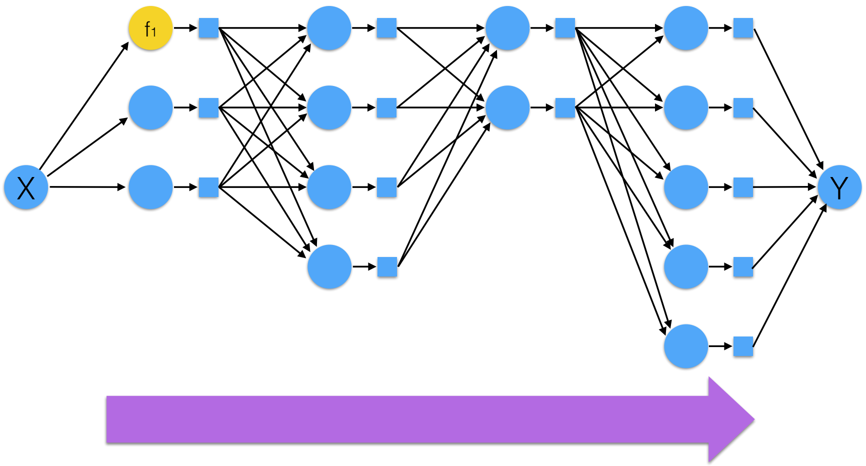

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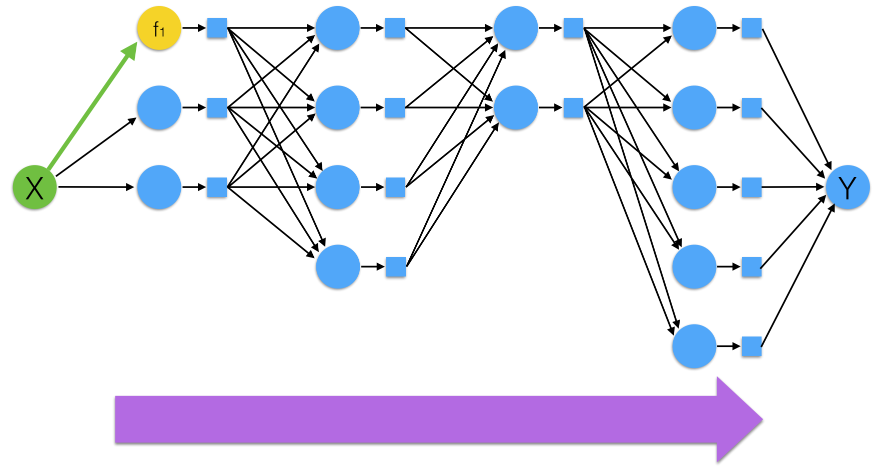

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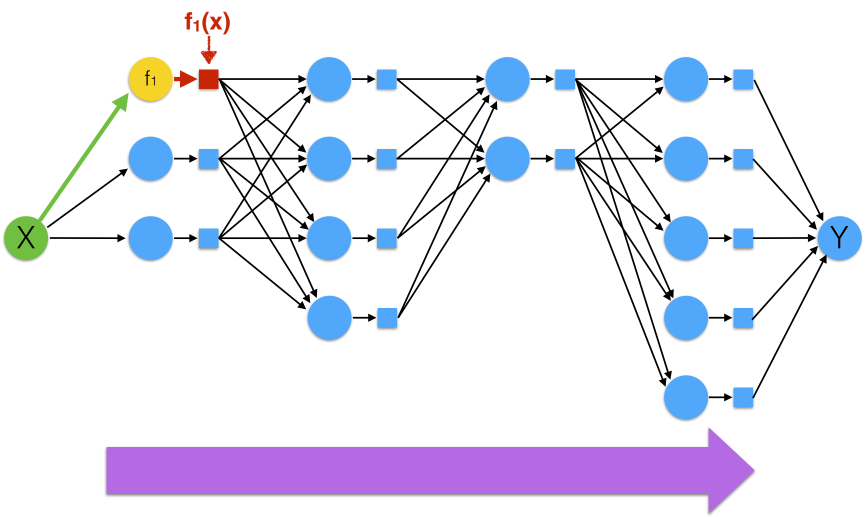

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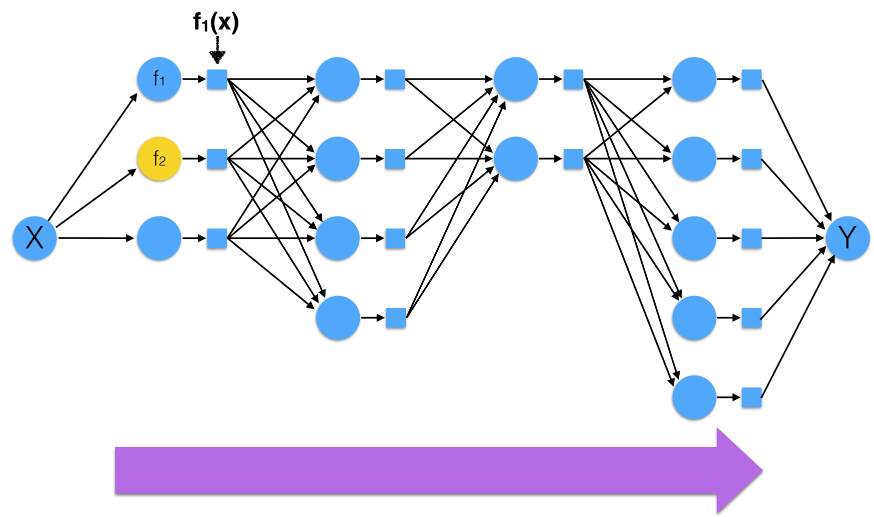

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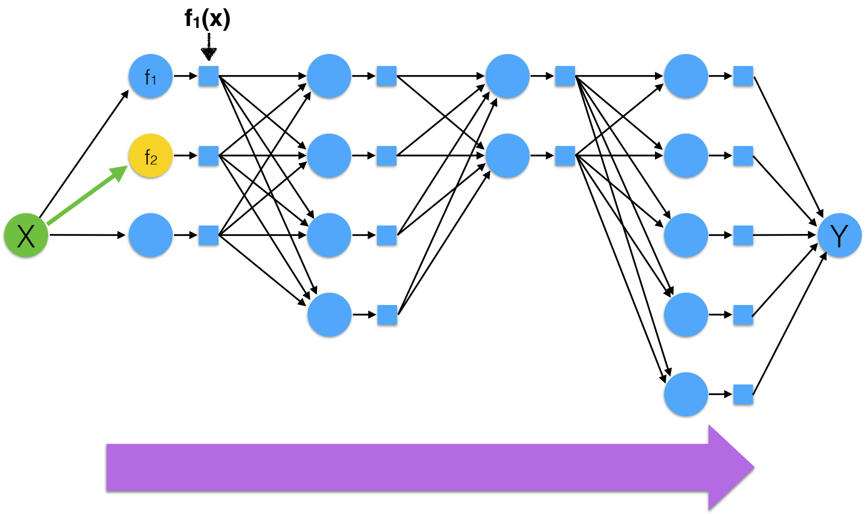

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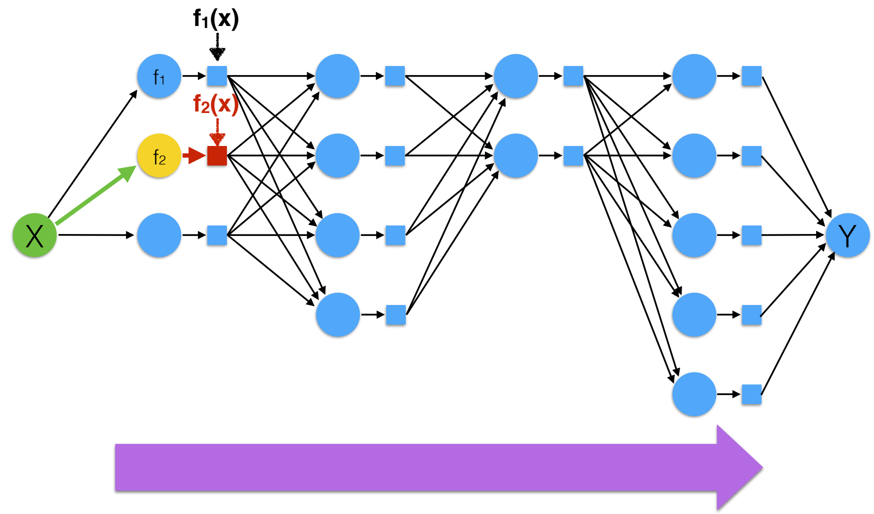

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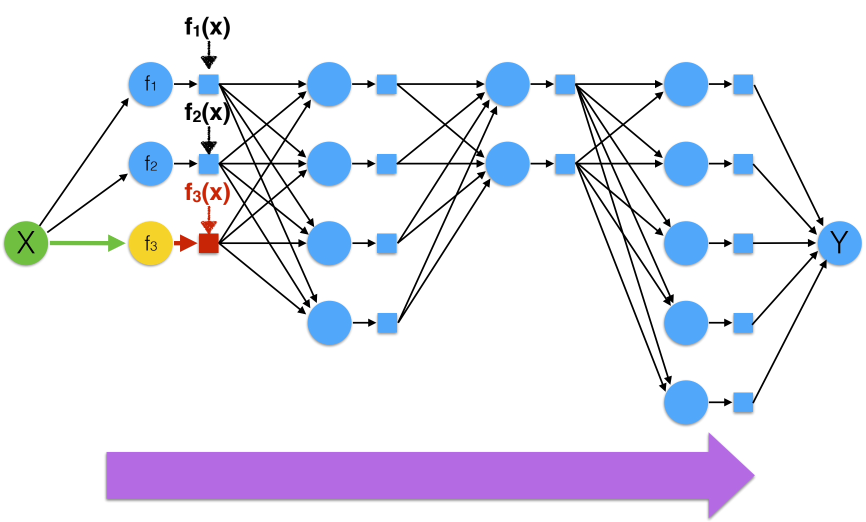
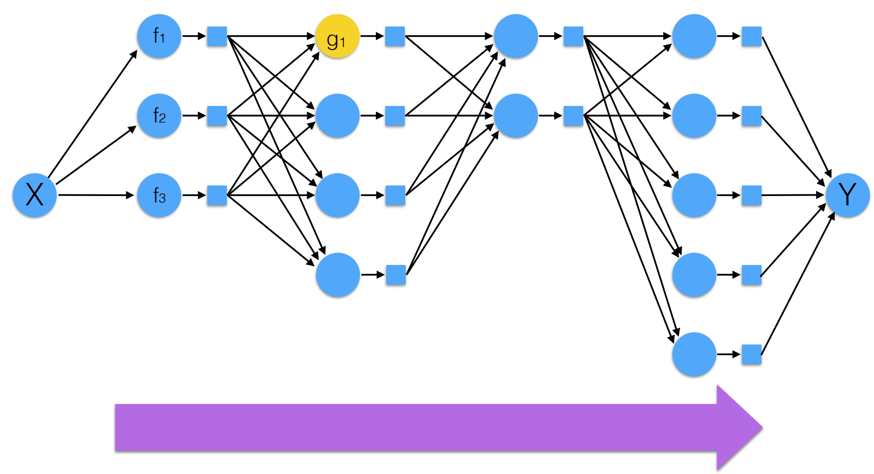


Forward propagation



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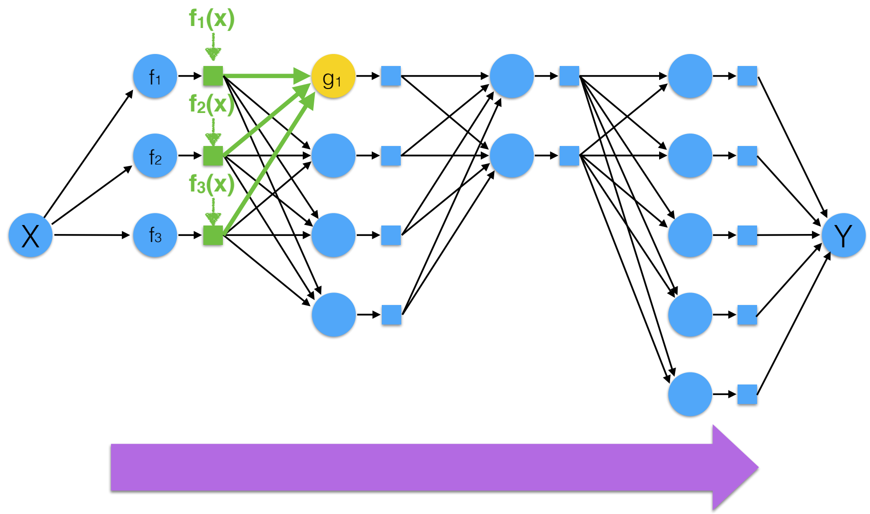
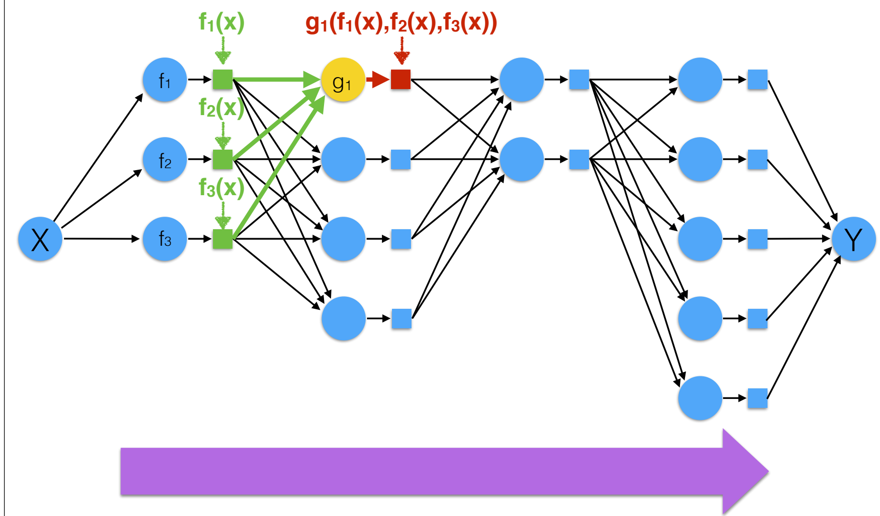
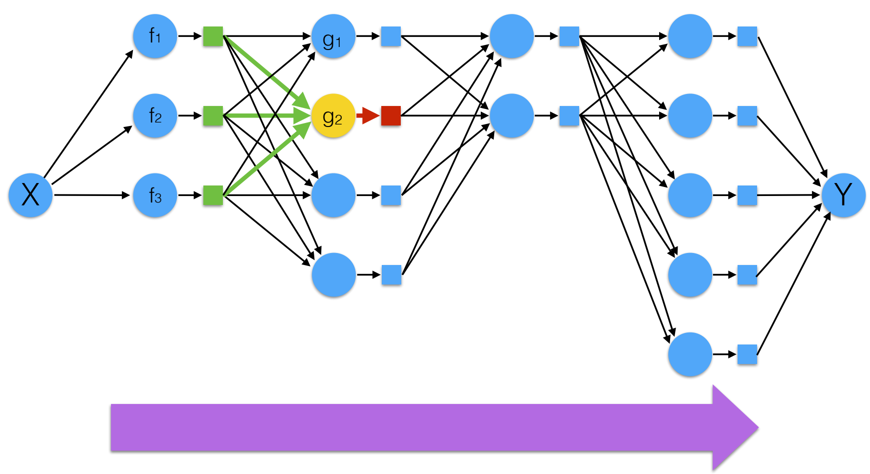


Forward propagation



Forward propagation



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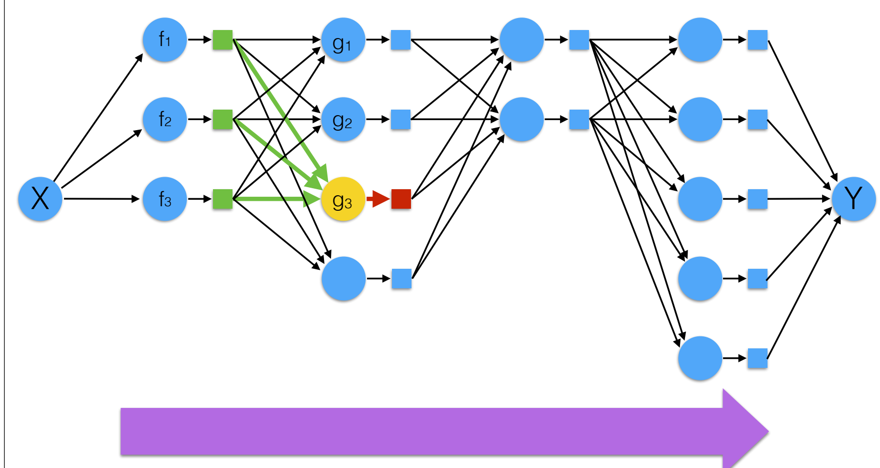

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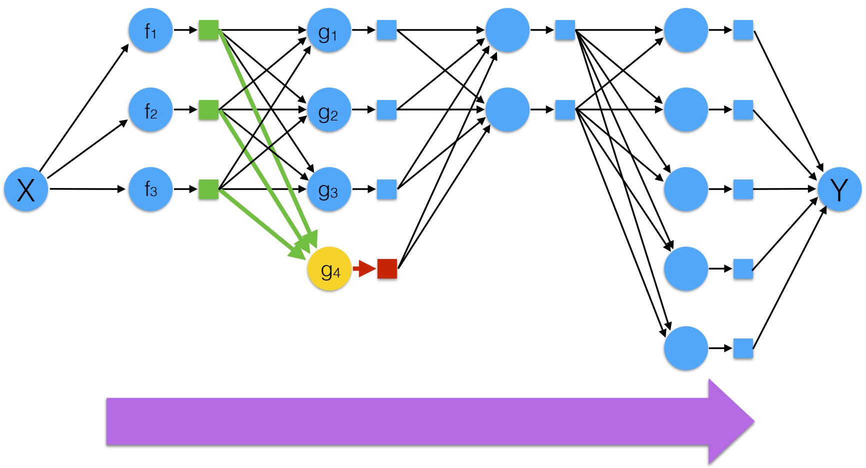

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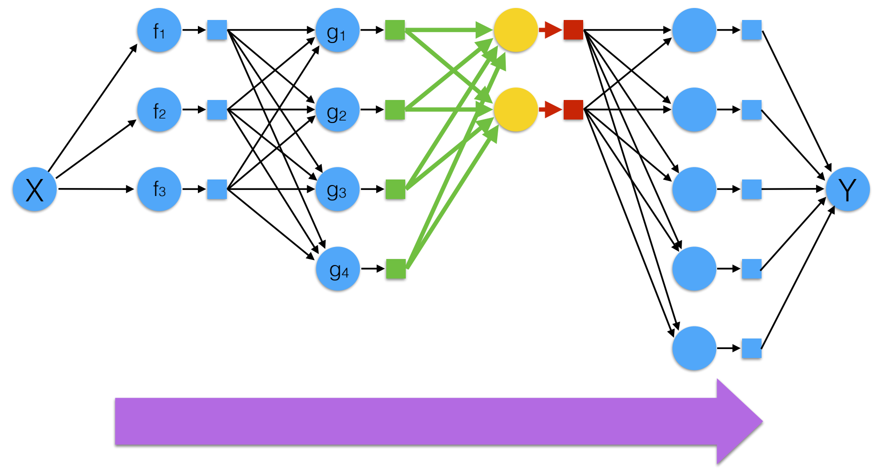

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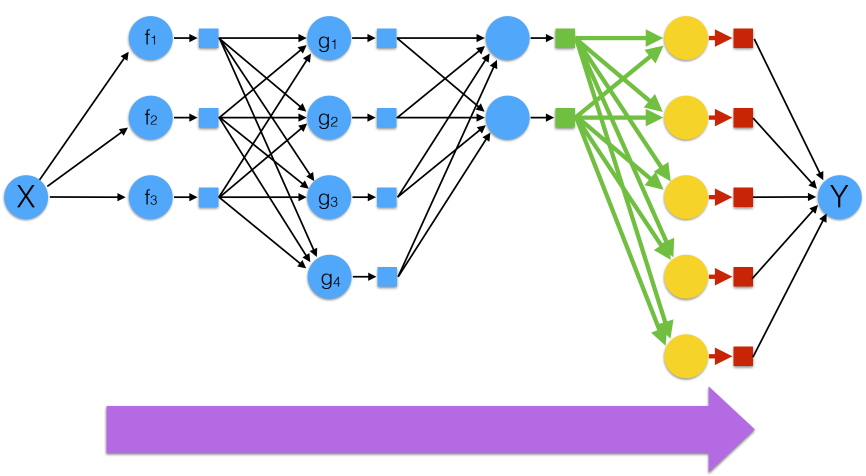
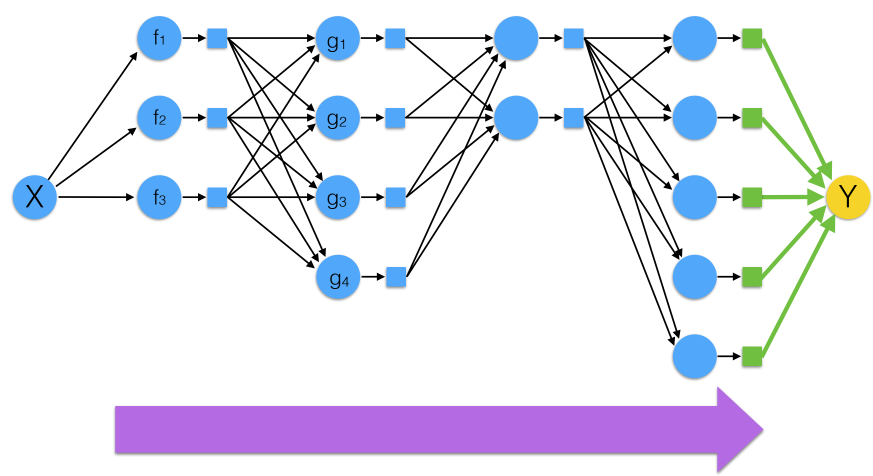
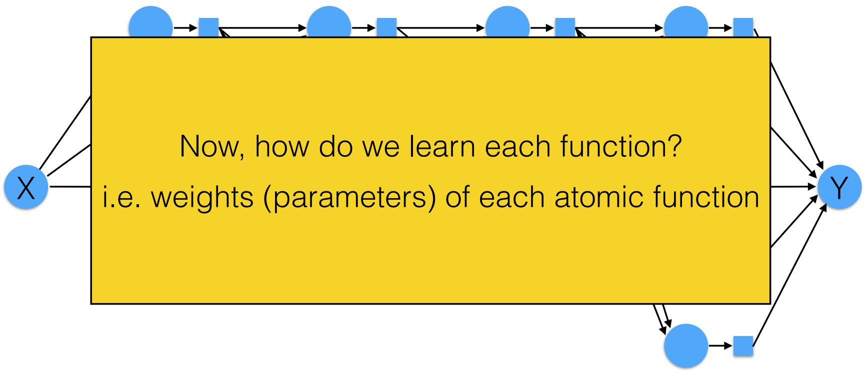


Forward propagation



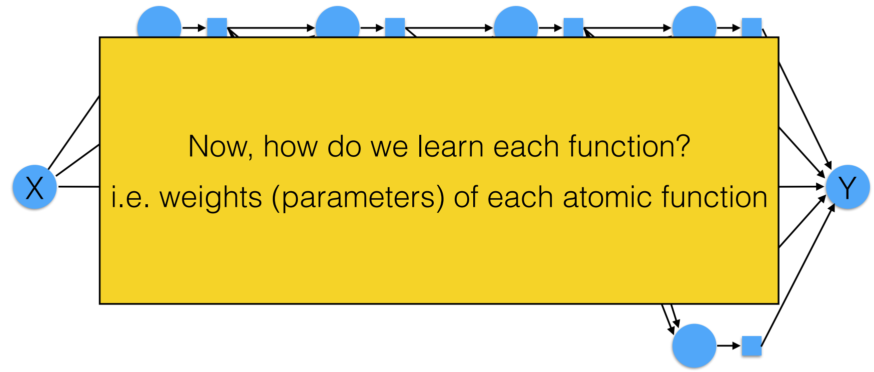
Neural Networks Example



Now, how do we learn each function?

i.e. weights (parameters) of each atomic function

Neural Networks Example



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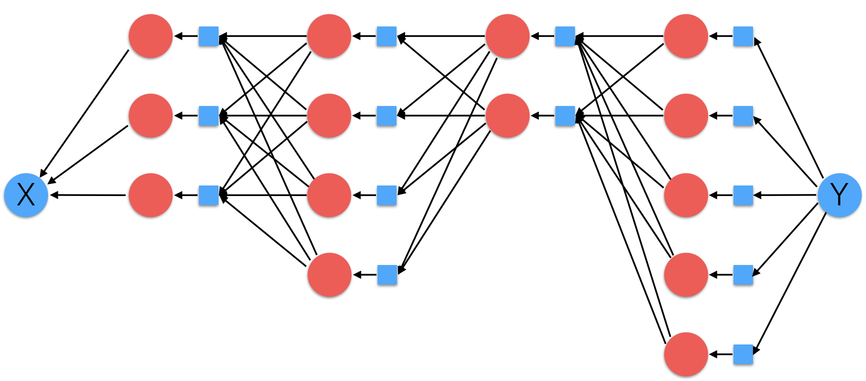
Backward propagation

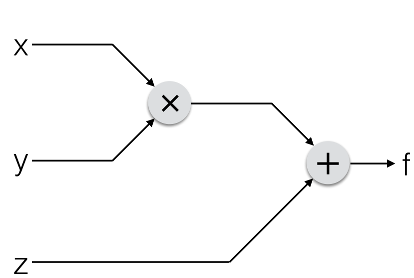


Computational Graph

Let’s step back.

Computational Graph

Function: f(x, y, z) = x*y+z



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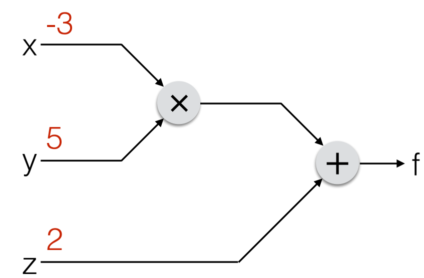

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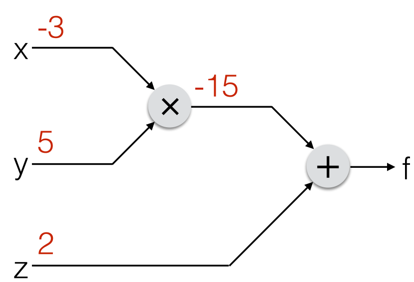

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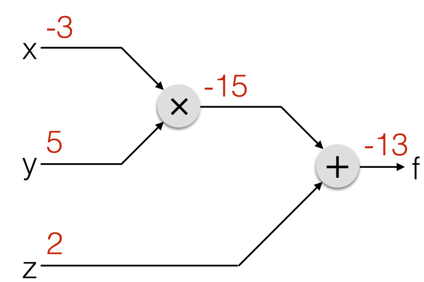
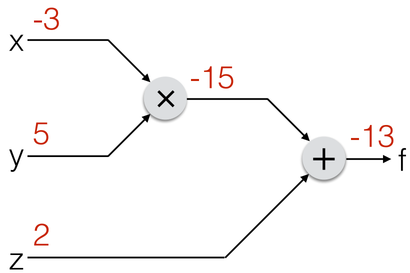


Computational Graph

Function: f(x, y, z) = x*y+z

Backpropagation



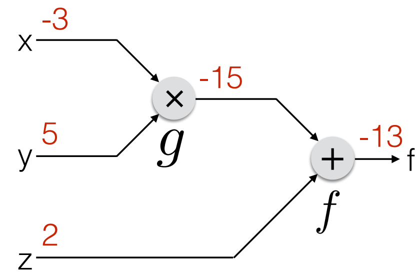
Computational Graph

Function: f(x, y, z) = x*y+z

Backpropagation

$g = x \times y$

$f = g + z$



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$g = x \times y$ $\quad \frac{\partial q}{\partial x} = y \quad \frac{\partial q}{\partial y} = x$

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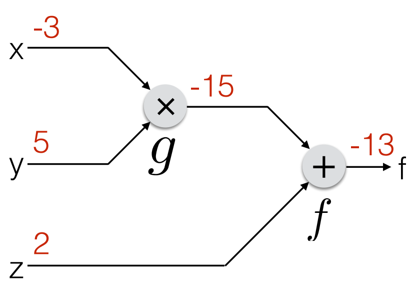

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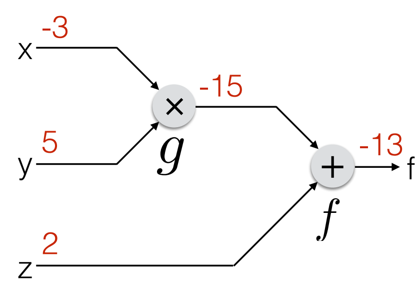

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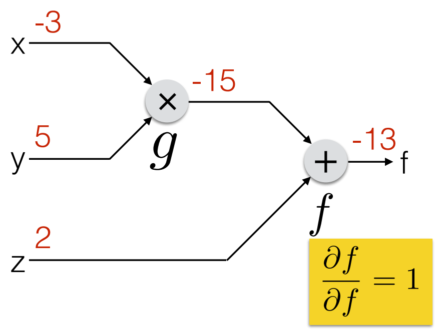

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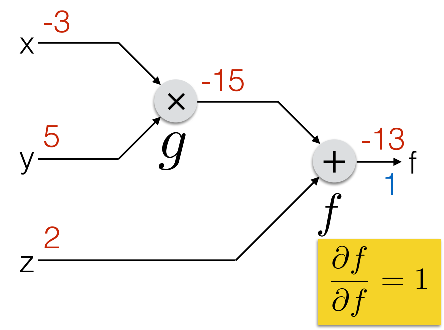

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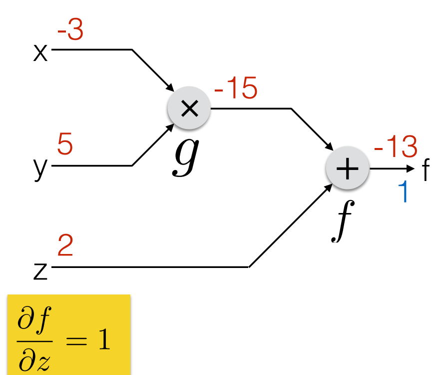

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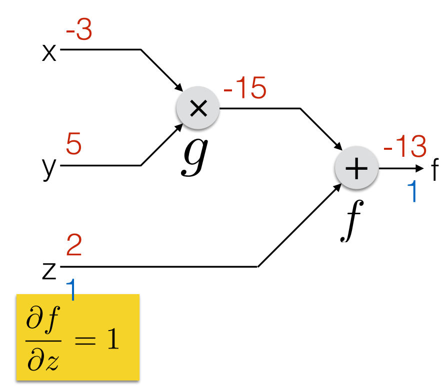

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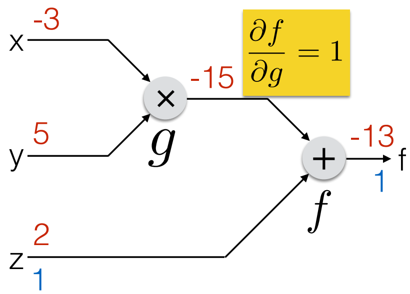

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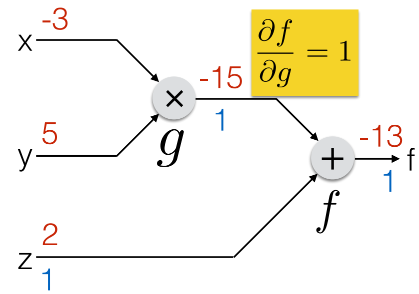

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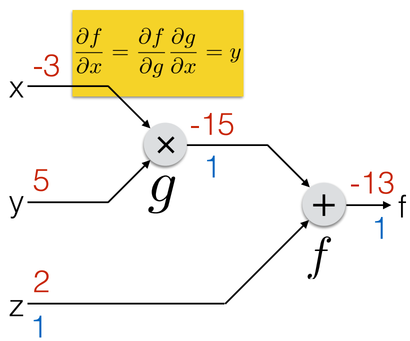

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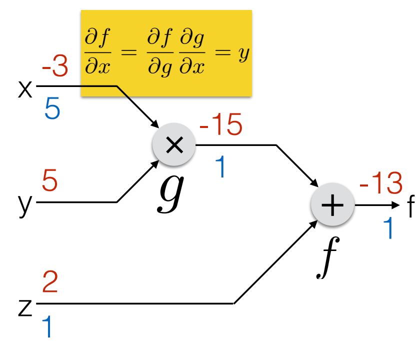

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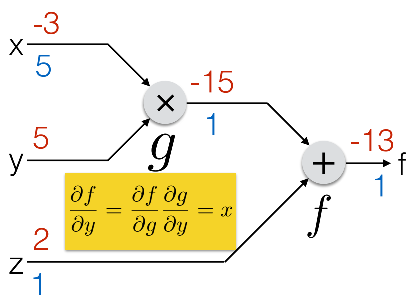


Computational Graph

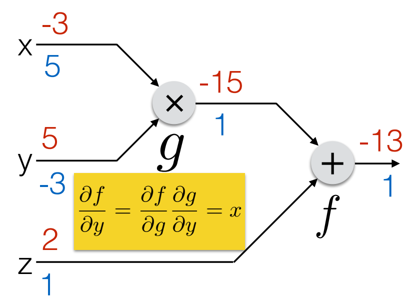
Function: f(x, y, z) = x*y+z

Backpropagation

$g = x \times y \qquad \frac{\partial q}{\partial x} = y \qquad \frac{\partial q}{\partial y} = x$

$f = g + z \qquad \frac{\partial f}{\partial g} = 1 \qquad \frac{\partial f}{\partial z} = 1$

We want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

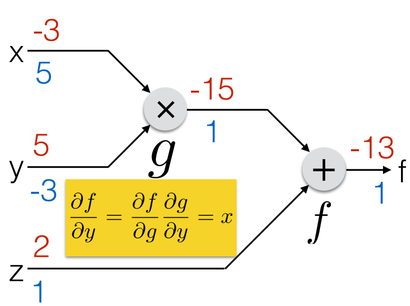


Computational Graph

Function: f(x, y, z) = x*y+z

Backpropagation

We get: $\frac{\partial f}{\partial x} = 5 \qquad \frac{\partial f}{\partial y} = -3 \qquad \frac{\partial f}{\partial z} = 1$



Computational Graph

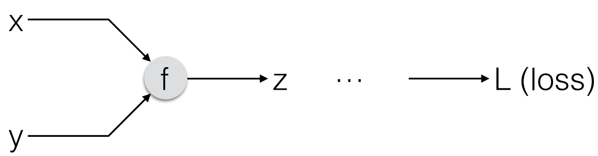
Note: a local view of it

Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Forward propagation

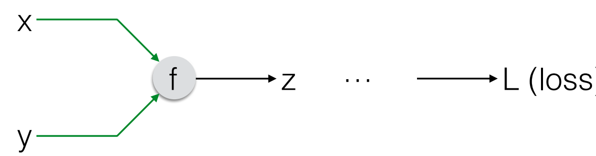


Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Forward propagation

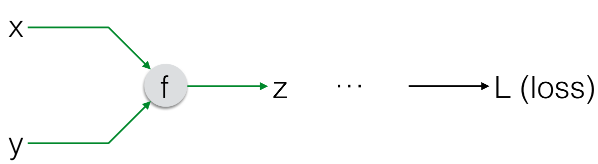


Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Forward propagation

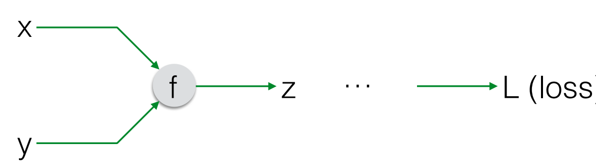


Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Forward propagation

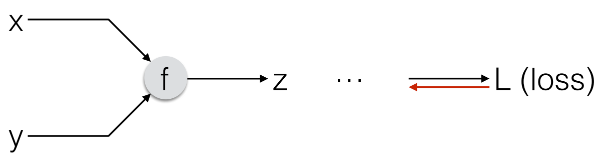


Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Backpropagation

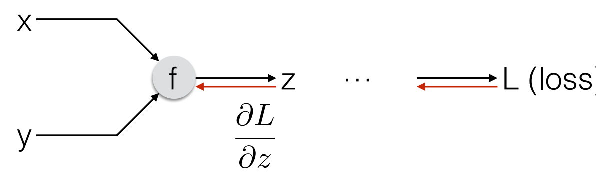


Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Backpropagation



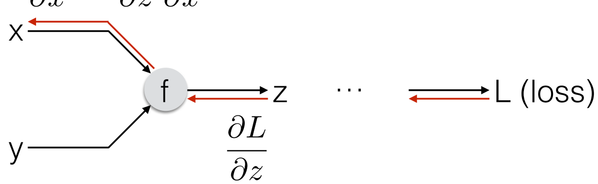
Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Backpropagation

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$$



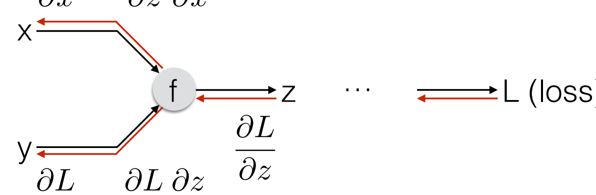
Computational Graph

Function: f(x, y, z) = x*y+z

A local view of backpropagation

Backpropagation

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$$



$$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$$

Computational Graph

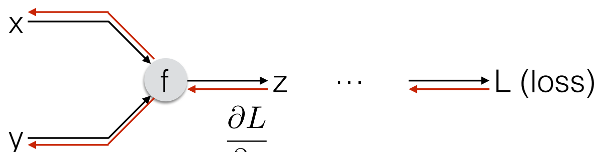
Function: f(x, y, z) = x*y+z

A local view of backpropagation

Requires a function to be differentiable

Backpropagation

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$$



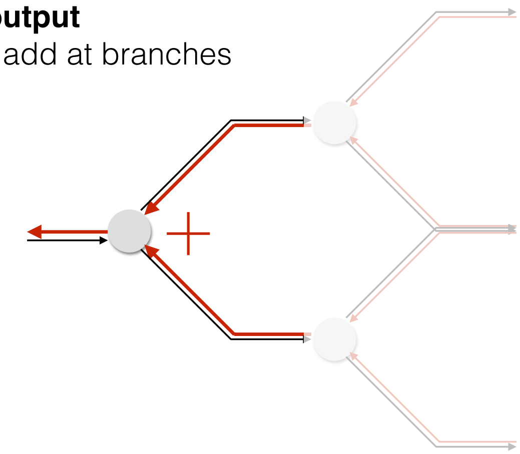
$$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$$

Computational Graph

Function: f(x, y, z) = x*y+z

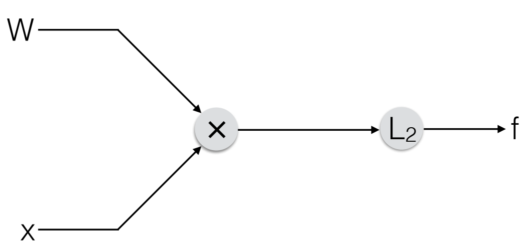
Multiple output

Gradients add at branches



Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n} (W \cdot x)_i^2$



Computational Graph

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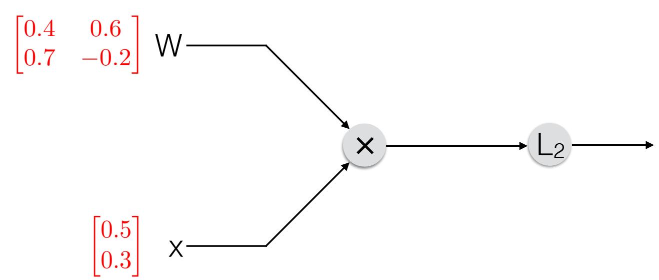

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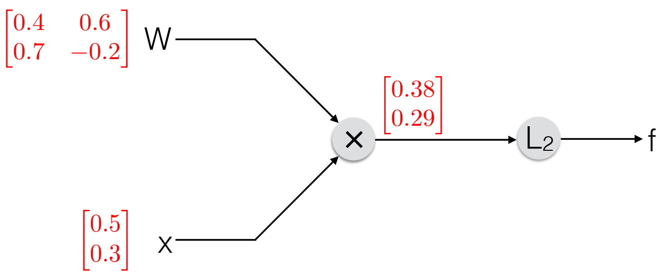

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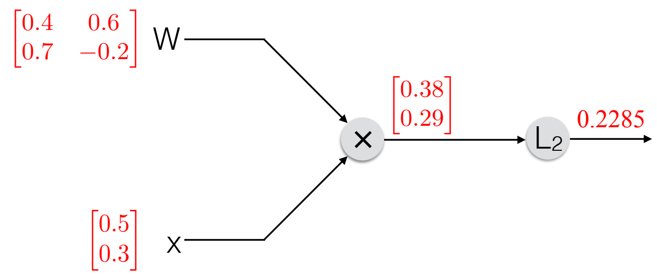

Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n} (W \cdot x)_i^2$

We want: $\frac{\partial f}{\partial W_{i,j}}, \frac{\partial f}{\partial x_i}$

Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n} (W \cdot x)_i^2$

We want: $\frac{\partial f}{\partial W_{i,j}}, \frac{\partial f}{\partial x_i}$

$$q = W \cdot x = \begin{pmatrix} W_{1,1x1} + ... + W_{1,nxn} \\ ... \\ W_{n,1x1} + ... + W_{n,nxn} \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + ... + q_n^2$$

Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n}(W \cdot x)_i^2$

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$$f(q) = ||q||^2 = q_1^2 + ... + q_n^2$$

$$\frac{\partial q_k}{\partial W_{i,j}} = 1_{k=i} x_j$$

$$\frac{\partial f}{\partial W_{i,j}} = \Sigma_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial W_{i,j}} = \Sigma_k (2q_k)(1_{k=i} x_j) = 2 q_i x_j$$

Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n}(W \cdot x)_i^2$

We want: $\frac{\partial f}{\partial W_{i,j}}, \boxed{\frac{\partial f}{\partial x_i}}$

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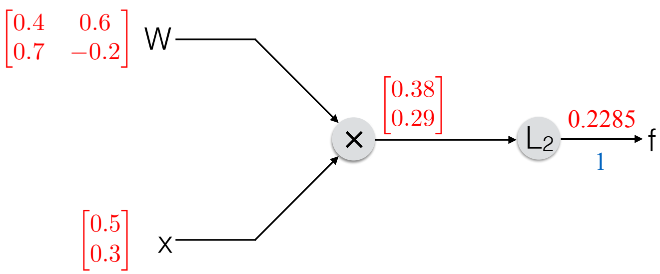
$$f(q) = ||q||^2 = q_1^2 + ... + q_n^2$$

$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$

$$\frac{\partial f}{\partial x_i} = \Sigma_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial x_i} = \Sigma_k 2 q_k W_{k,i}$$

Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n}(W \cdot x)_i^2$



Computational Graph

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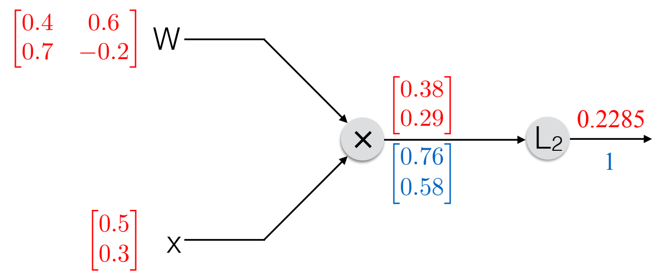
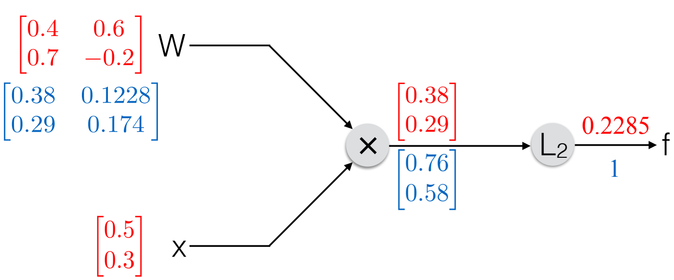


$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \ldots + W_{1,n}x_n \\ \ldots \\ W_{n,1}x_1 + \ldots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \ldots + q_n^2$$

Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n}(W \cdot x)_i^2$

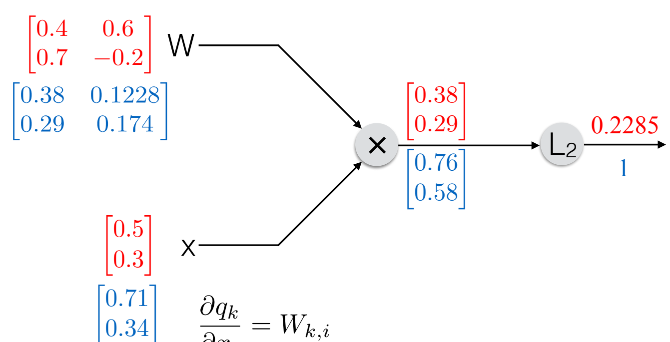


$$\frac{\partial q_k}{\partial W_{i,j}} = 1_{k=i}x_j$$

$$\frac{\partial f}{\partial W_{i,j}} = \Sigma_k \frac{\partial f}{\partial q_k}\frac{\partial q_k}{\partial W_{i,j}} = \Sigma_k(2q_k)(1_{k=i}x_j) = 2q_i x_j$$

Computational Graph

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \Sigma_{i=1}^{n}(W \cdot x)_i^2$



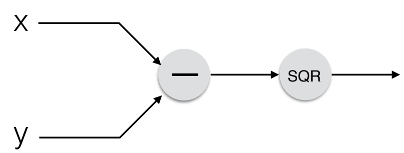
$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$

$$\frac{\partial f}{\partial x_i} = \Sigma_k \frac{\partial f}{\partial q_k}\frac{\partial q_k}{\partial x_i} = \Sigma_k 2q_k W_{k,i}$$

Layers with Computational Graph

L2-loss

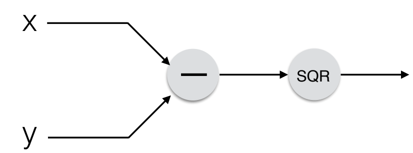
$L_2(x, y) = (x - y)^2$



Layers with Computational Graph

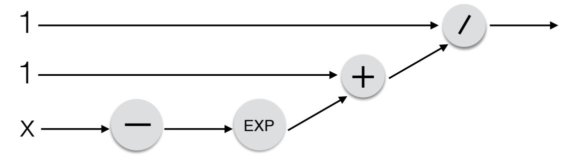
L2-loss

$L_2(x, y) = (x - y)^2$



Sigmoid

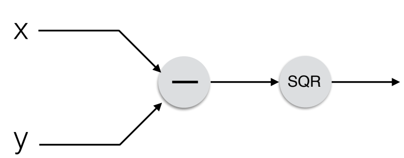
$\mathrm{sigmoid}(x) = \frac{1}{1 + e^{-x}}$



Layers with Computational Graph

L2-loss

$L_2(x, y) = (x - y)^2$



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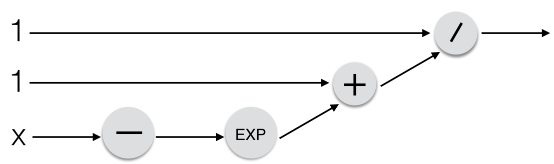
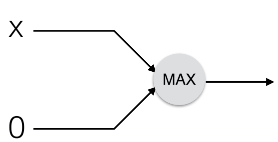
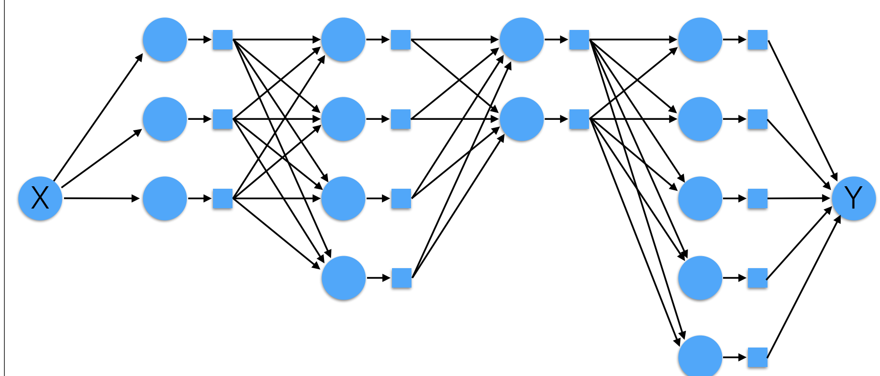


ReLU

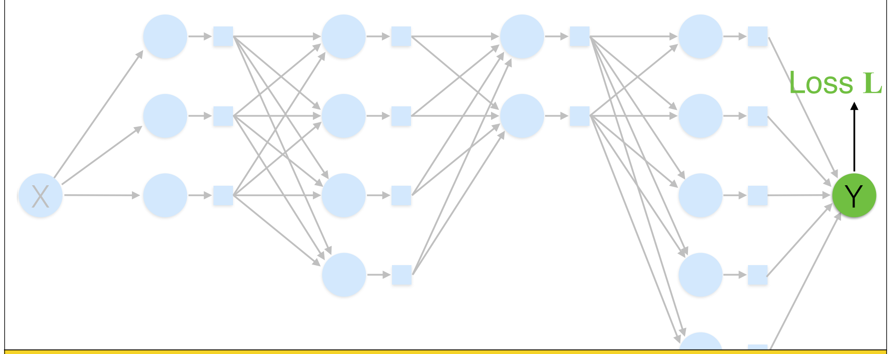
$\mathrm{ReLU}(x) = \max(x, 0)$



Neural Networks Example



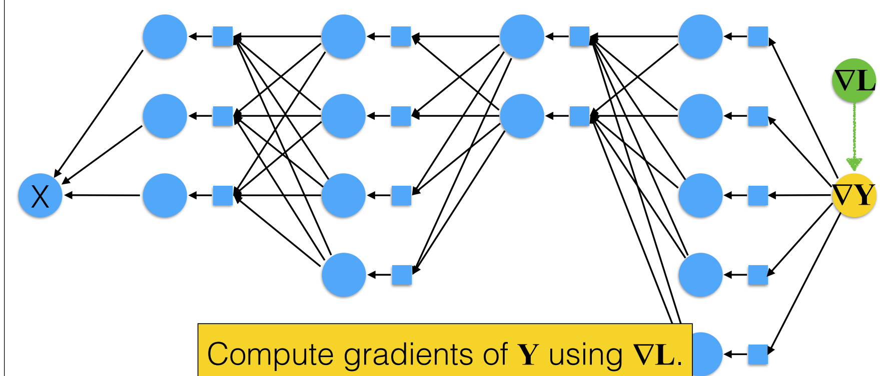
Backward propagation



Compute a loss using an estimated Y and the ground truth Y*.

e.g., $L(W) = \| f(X; W) - Y^* \|_2$

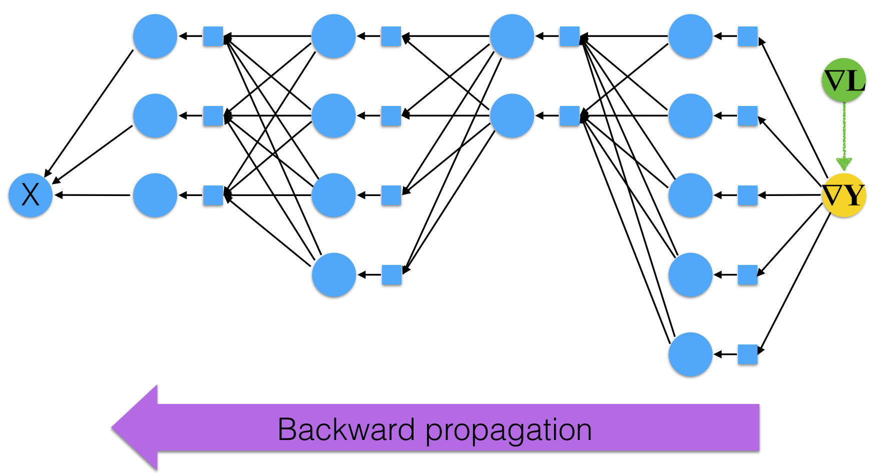
Backward propagation



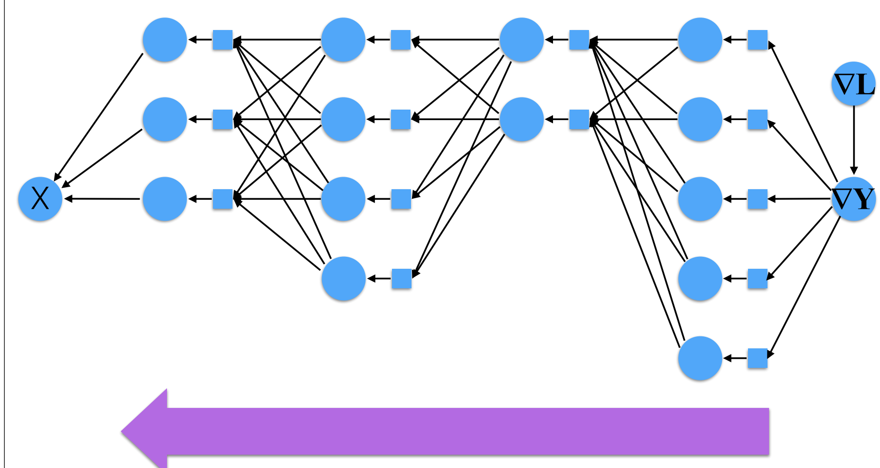
Compute gradients of **Y** using **∇L**.

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X}$$

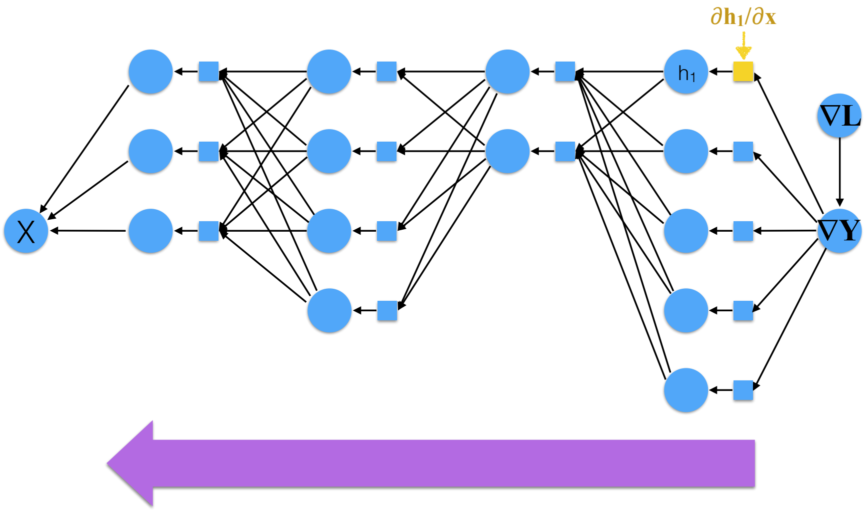
Backward propagation



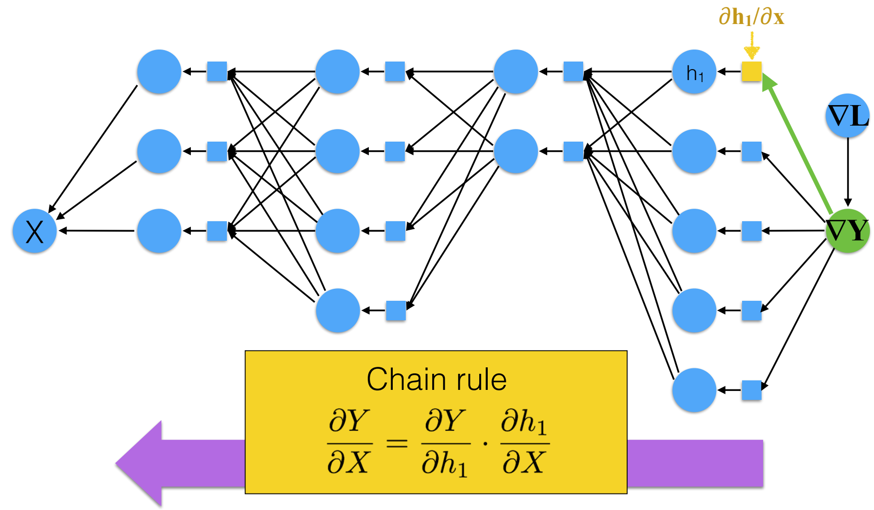
Backward propagation



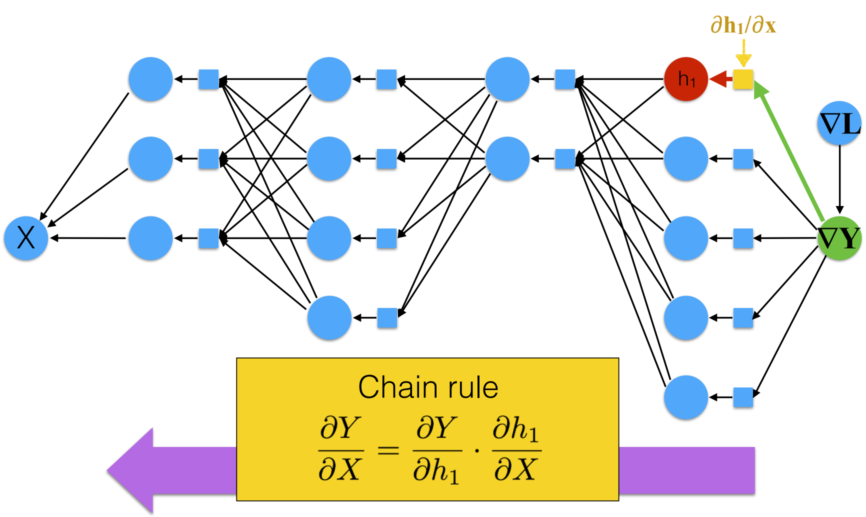
Backward propagation



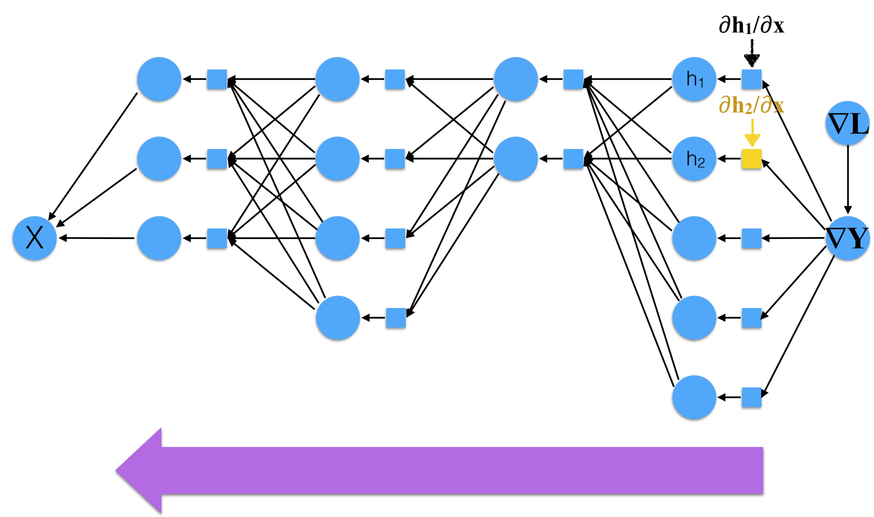
Backward propagation



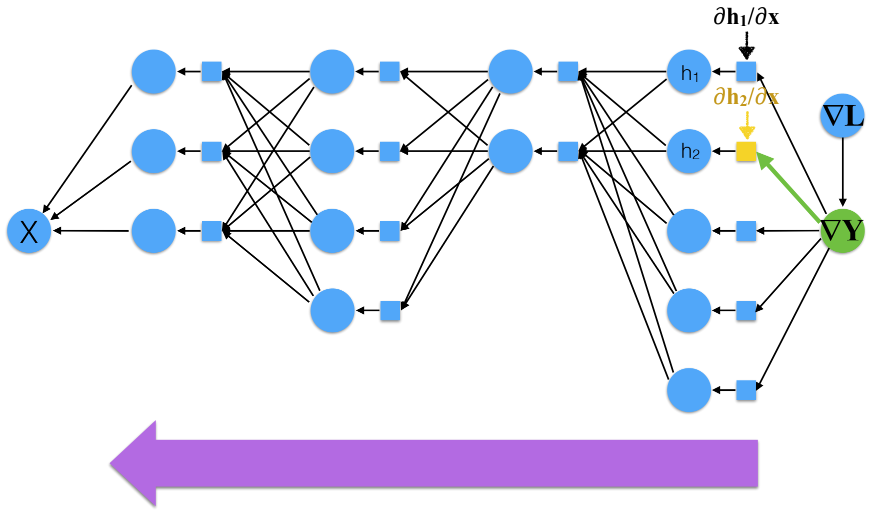
Backward propagation



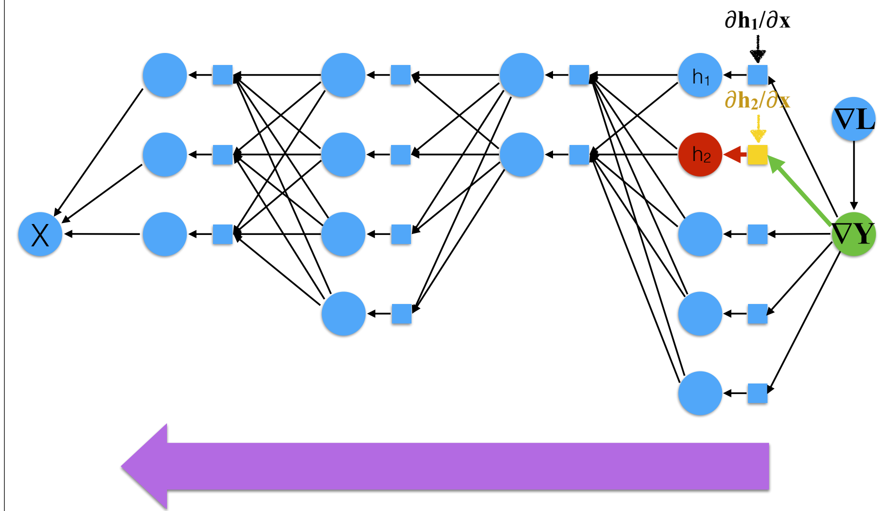
Backward propagation



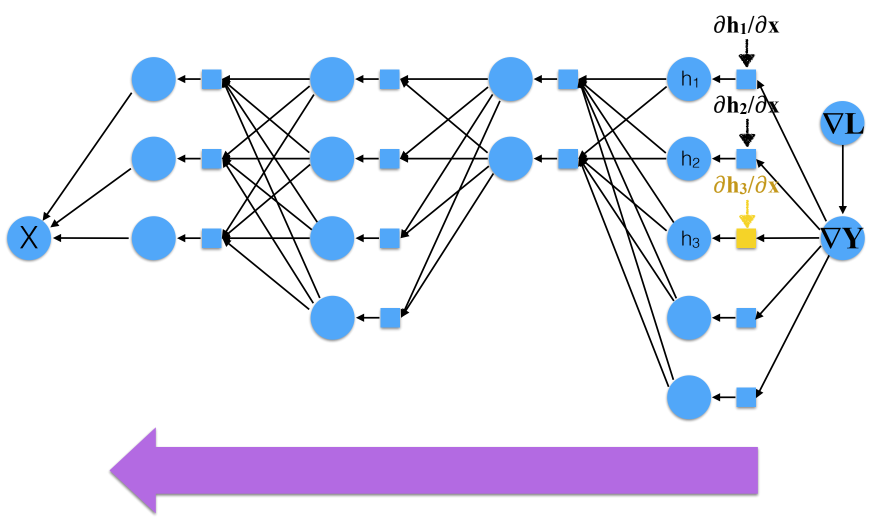
Backward propagation



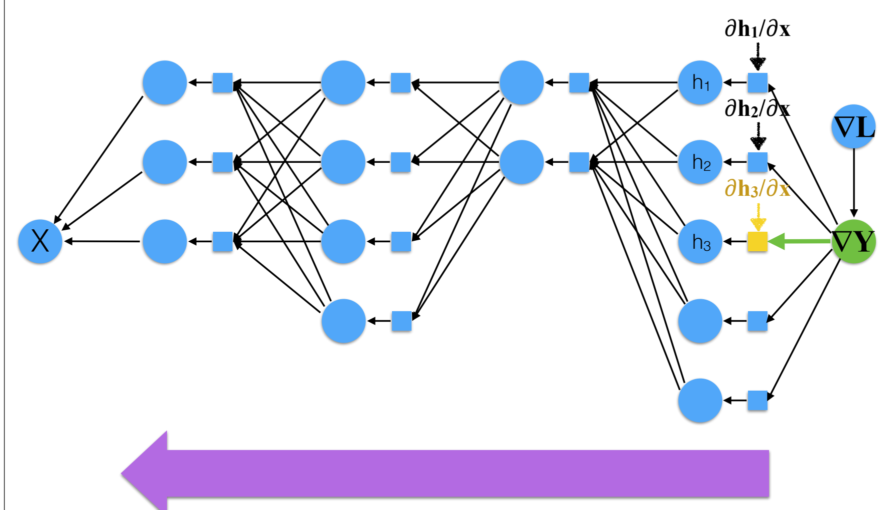
Backward propagation



Backward propagation



Backward propagation



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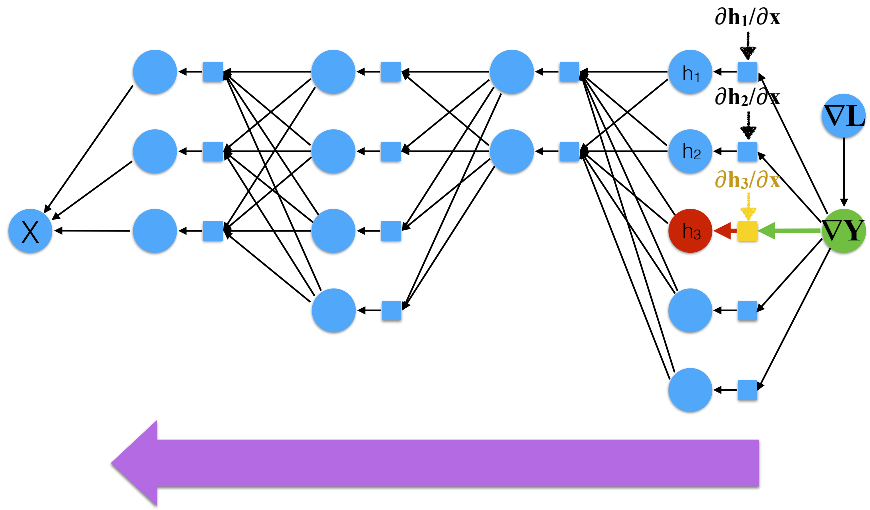
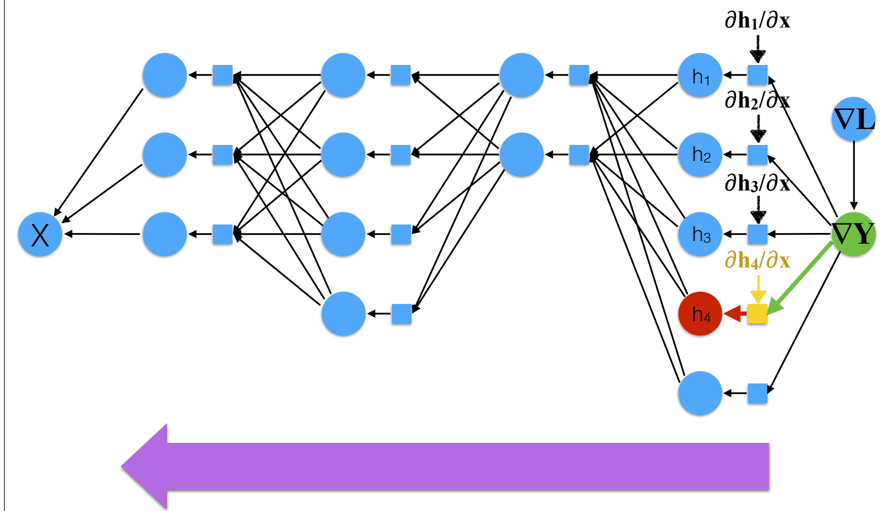
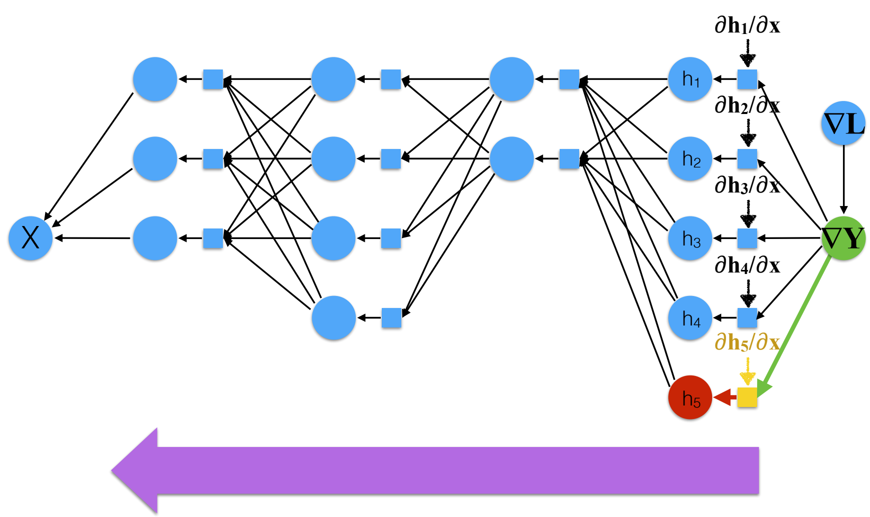


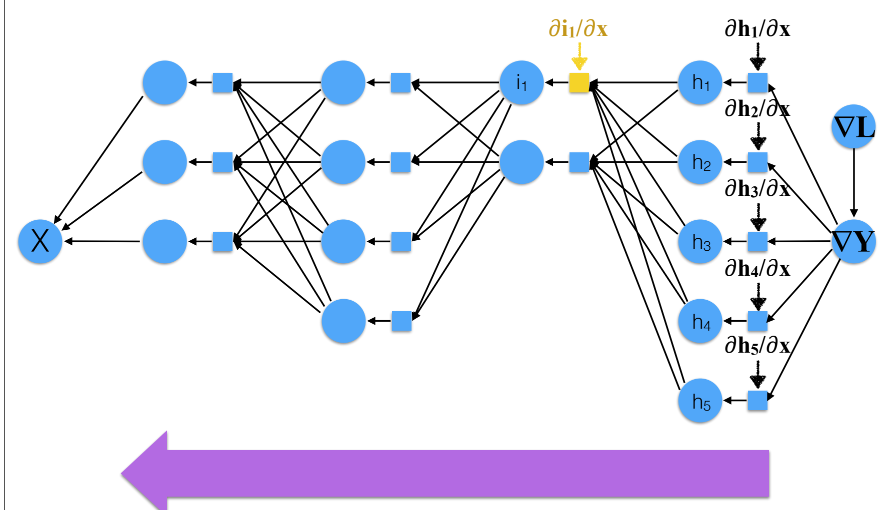
Backward propagation



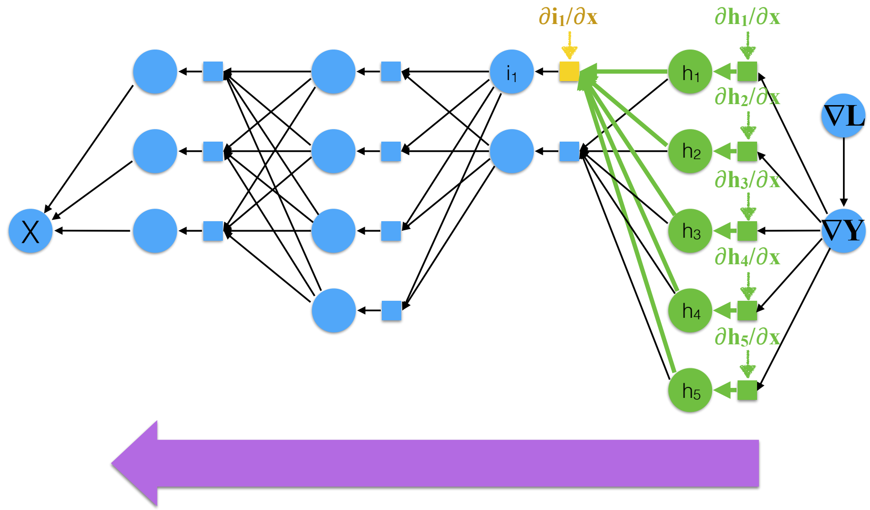
Backward propagation



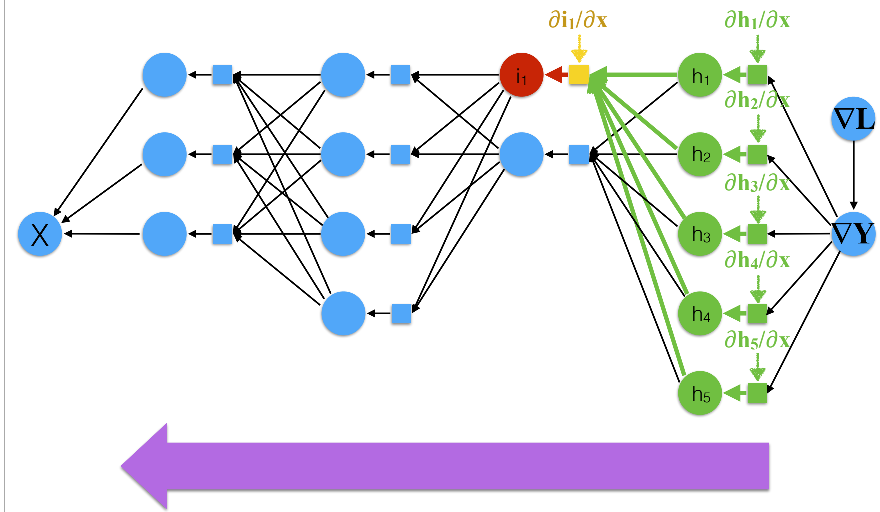
Backward propagation



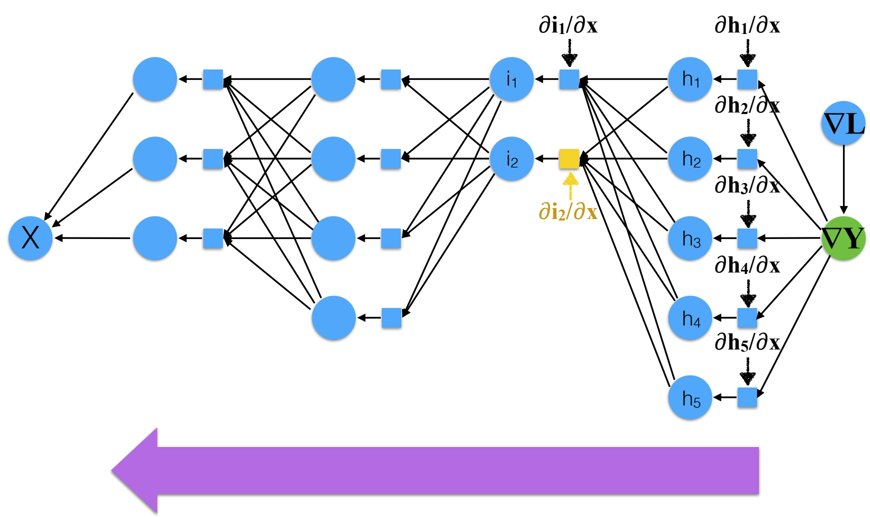
Backward propagation



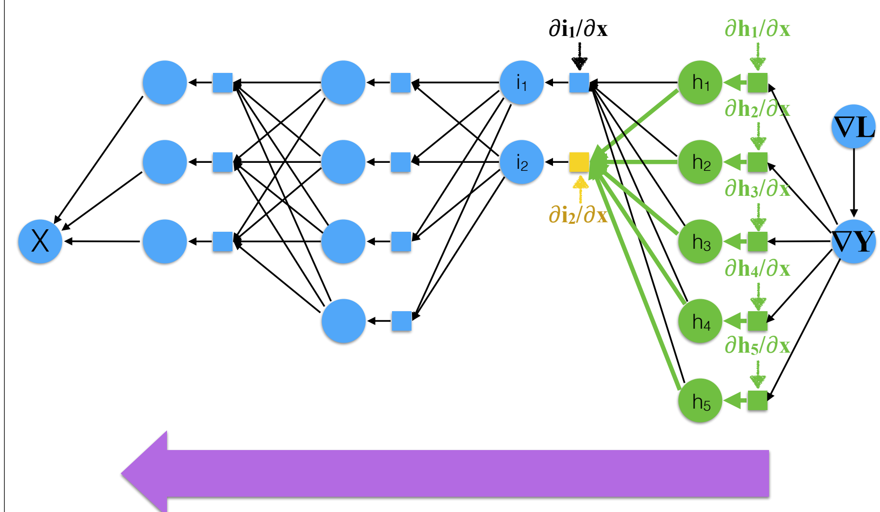
Backward propagation

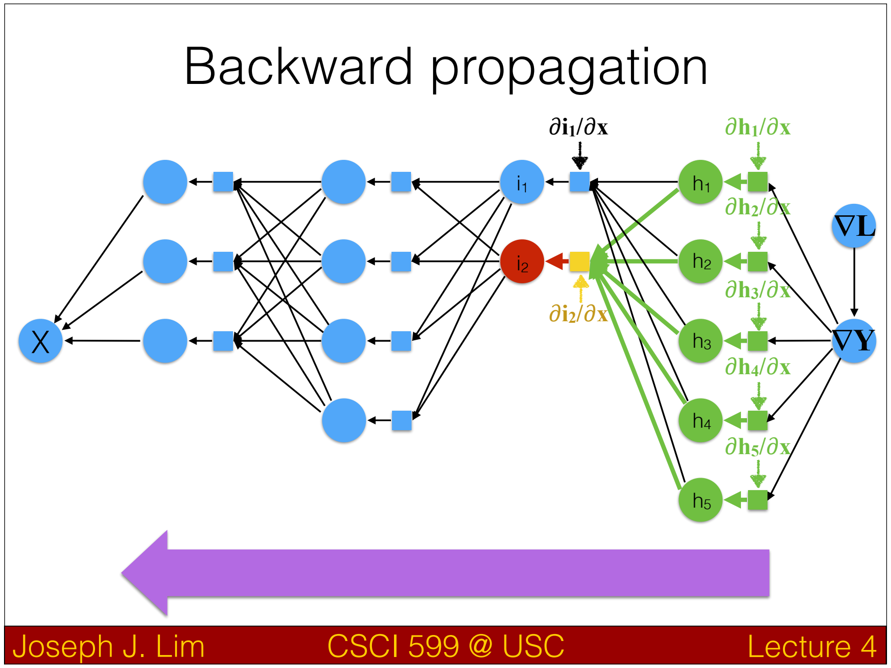


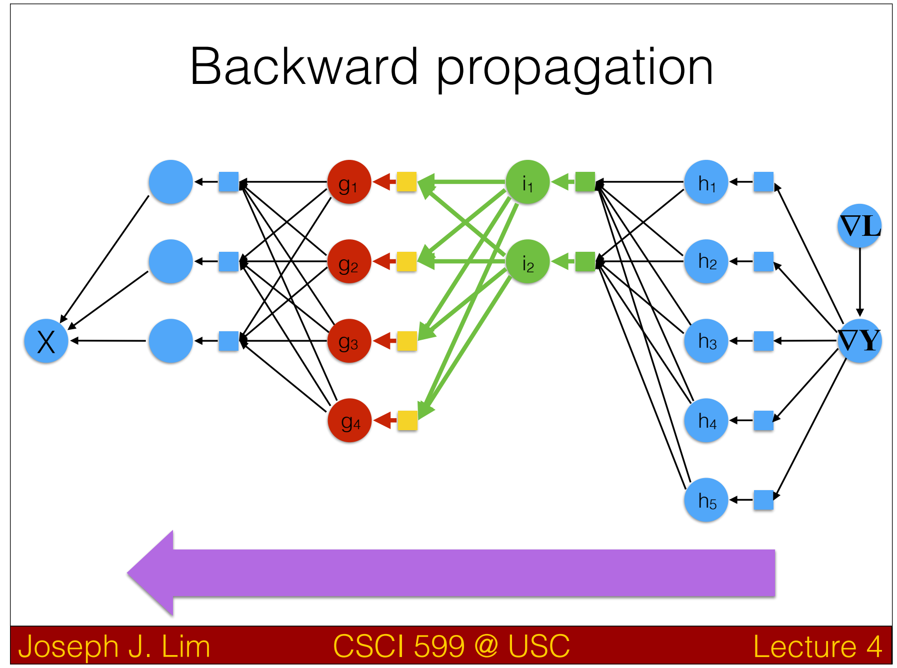
Backward propagation



Backward propagation




Backward propagation
∂i1/∂x
∂h1/∂x
∂h2/∂x
∂h3/∂x
∂h4/∂x
∂h5/∂x
∂i2/∂x
i1
i2
h1
h2
h3
h4
h5
X
∇L
∇Y
Joseph J. Lim
CSCI 599 @ USC
Lecture 4


Backward propagation
g1
g2
g3
g4
i1
i2
h1
h2
h3
h4
h5
X
∇L
∇Y
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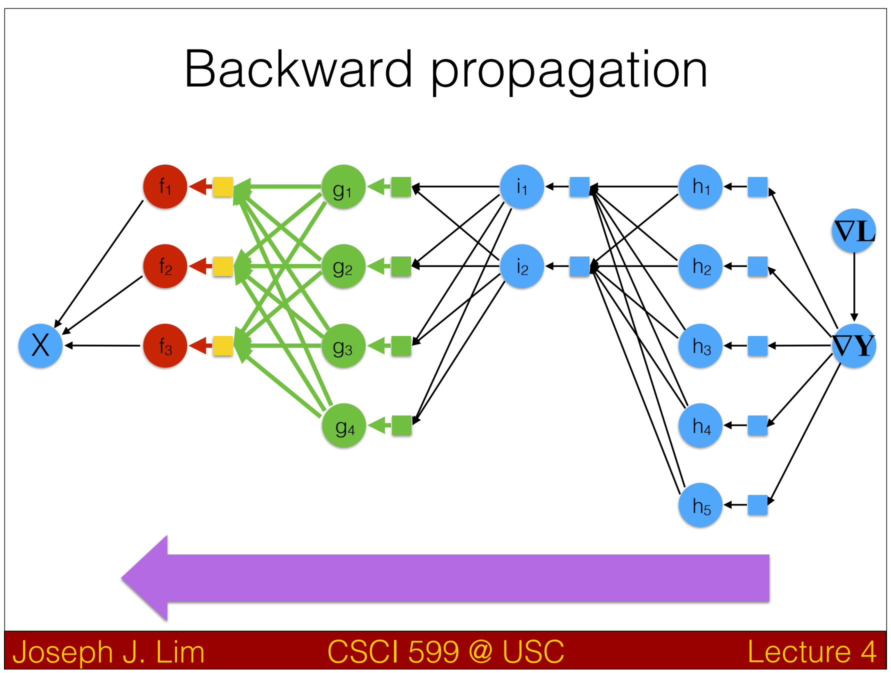
Lecture 4


Backward propagation
f1
f2
f3
g1
g2
g3
g4
i1
i2
h1
h2
h3
h4
h5
X
∇L
∇Y
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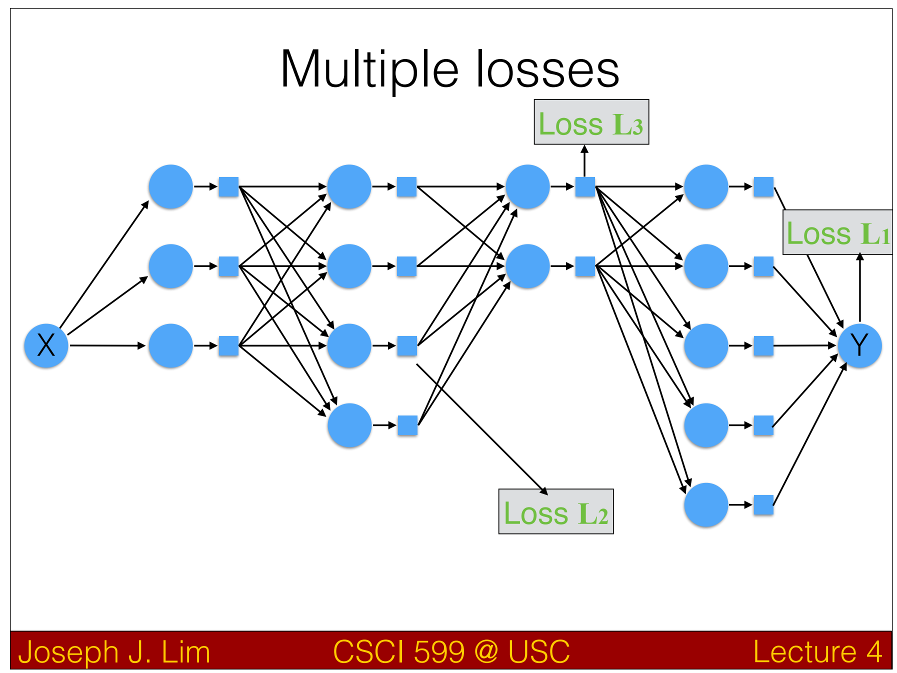
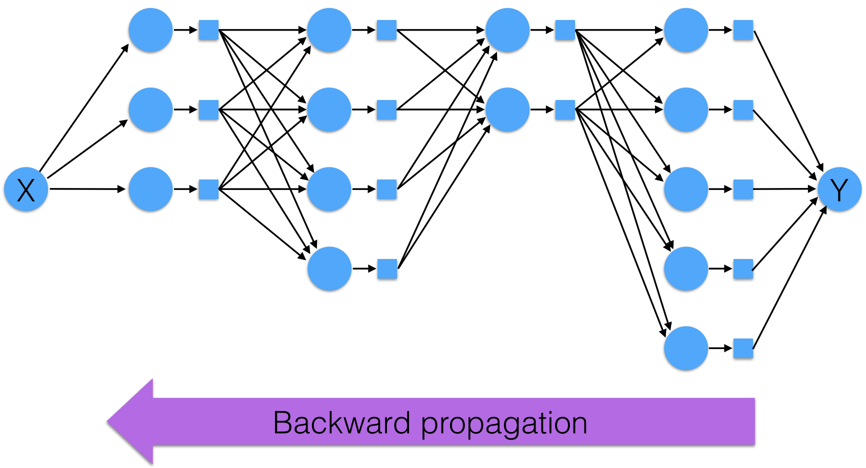

Multiple losses
Loss L3
Loss L1
Loss L2
X
Y
Joseph J. Lim
CSCI 599 @ USC
Lecture 4

Neural Networks Example



Summary

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- A choice of **loss function** matters.
- Neural networks is essentially **wired neurons** (small functions).
- Weights can be learned using **backward propagation** (e.g. computational graph).

Break Time

See you in 15 mins!

Project Discussion

Next week

- Convolutional Neural Networks
- Training Neural Networks

Todo

- Midterm on week 7
- Assignment #1 & Project proposal due week 8
- Project Meeting with TA #1 by week 8

Questions?