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Customers often stockpile reward points in linear loyalty programs (i.e., programs that do not explicitly reward stockpiling) despite several economic incentives against it (e.g., the time value of money). The authors develop a mathematical model of redemption choice that unites three explanations for why customers seem to be motivated to stockpile on their own, even though the retailer does not reward them for doing so. These motivations are economic (the value of forgone points), cognitive (nonmonetary transaction costs), and psychological (customers value points differently than cash). The authors capture the psychological motivation by allowing customers to book cash and point transactions in separate mental accounts. They estimate the model on data from an international retailer using Markov chain Monte Carlo methods and accurately forecast redemptions during an 11-month out-of-sample period. The results indicate substantial heterogeneity in how customers are motivated to redeem and suggest that the behavior in the data is driven mostly by cognitive and psychological incentives.

Keywords: prospect theory, mental accounting, loyalty programs, Bayesian estimation

Stockpiling Points in Linear Loyalty Programs

The way a loyalty program is designed influences when customers redeem their rewards. For example, certain loyalty programs, such as those offered in the airline industry, require customers to stockpile points to earn specific rewards (i.e., "reach 30,000 points to redeem a round-trip ticket"). Similar programs offer stockpiling goals such as "when you reach 5,000 points, we'll double your points." In these programs, as more points are stockpiled, the redeemable value of each point increases in a staggered way, as illustrated in Panel A of Figure 1. We refer to the design of these reward structures as "nonlinear."

In contrast, in "linear" reward structures (illustrated in Panel B, Figure 1), customers cannot increase the value of each point by stockpiling. Because each point is worth a fixed amount, typically a cash equivalent, customers can easily redeem as little as one point without the hassle of complicated rules. These retailers do not set stockpiling goals and do not require a minimum amount of points to redeem.
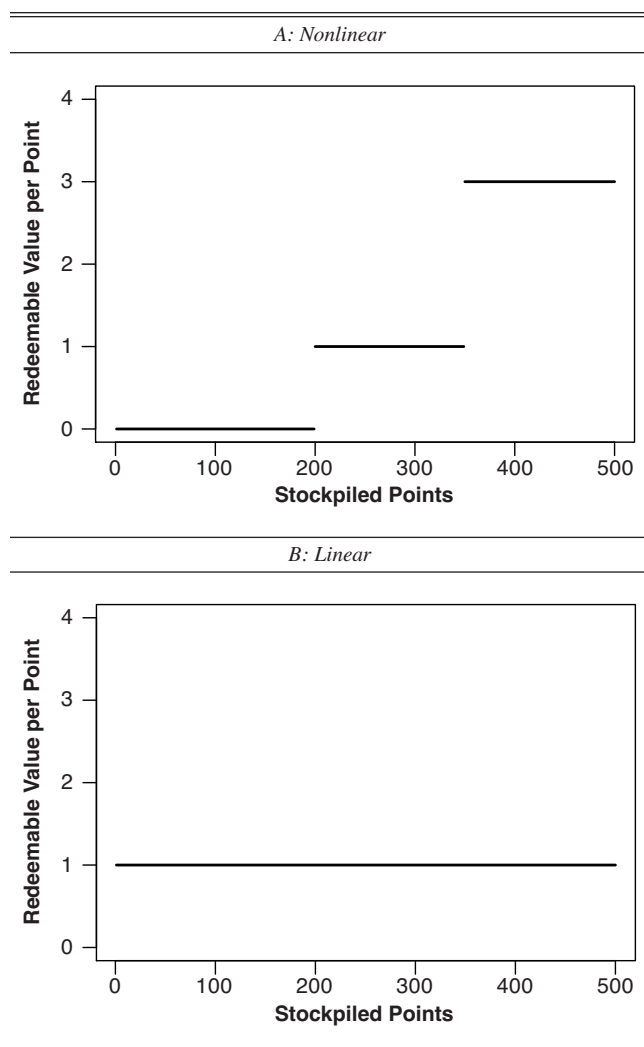
An example of a linear program is Capital One's Quicksilver Cash Rewards credit card. Cash rewards accumulate online and can be redeemed at any time. Amazon.com offers similar rewards in which redeemed points reduce the price that is paid. Tesco's successful Clubcard program began by rewarding every British pound spent with one point toward store vouchers (Humby, Hunt, and Phillips 2004). What these programs have in common is that they do not explicitly reward stockpiling points.

Indeed, customers participating in linear programs often face incentives *against* stockpiling. Unredeemed points can expire, and they can lose their value if the retailer enters bankruptcy or alters the program rules. Moreover, by delaying redemption, customers also forgo the time value of money from delayed rewards. Because linear programs do not reward stockpiling and unredeemed points may lose their value, we would expect customers to redeem regularly. Yet they do not! Even in controlled laboratory experiments,

Figure 1
EXAMPLE OF LINEAR VERSUS NONLINEAR REWARDS

A: Nonlinear

B: Linear

people are hesitant to redeem (Kwong, Soman, and Ho 2011).

As another example, in analyzing data from the linear program studied here, we found that only 3% of all purchases have redemptions associated with them. These customers rarely redeem even though doing so could reduce their basket price by nearly 30% on average. However, it is not the case that customers are completely ignoring opportunities to redeem: 40% of the customers in our panel eventually redeemed at least once over a 43-month observation period. Why do customers wait so long between redemptions and stockpile cash-like points in the process? This question is the focus of the article.

We present a model that unites three potential motivations for persistent stockpiling in a linear program:

•The opportunity cost of forgone points,
•Nonmonetary transaction costs, and
•How points are valued differently than cash.

The first is an economic incentive: customers forgo the opportunity to earn points on purchase occasions in which they redeem. The second is a cognitive incentive: customers

may find redeeming to be "costly," even if the process is as effortless as clicking a button at checkout. The third motivation is psychological, based on mental accounting (Thaler 1985). This explanation recognizes that customers may not perceive points and cash equally. For example, in interviews with customers from a linear program, one customer expressed sorrow when redeeming points: "It makes me feel sad because I don't have any points left on my card" (Smith and Sparks 2009, p. 545).

Drèze and Nunes (2004) propose a framework in which customers keep two mental accounts: one for cash and another for points. Customers may experience disutility when paying for a purchase (Prelec and Loewenstein 1998; Zellermayer 1996), and this "pain of paying" can vary by payment method (e.g., cash vs. check vs. card) (Soman 2003). Similarly, the pain of paying may also vary by the type of currency used to pay for a purchase (cash vs. points). Drèze and Nunes's objective is to enable firms to set prices using combinations of cash and points that will minimize customers' disutility of paying. This objective is not necessary in linear programs, because customers are always free to pay with countless combinations of cash and points using a fixed points-to-cash exchange rate. Nevertheless, their two-account framework provides an excellent foundation to study redemption behavior in linear programs.

We introduce several changes to Drèze and Nunes's (2004) framework to make it better suited for both the context (i.e., linear vs. nonlinear program) and the decision of interest here (i.e., when to redeem given the available stockpile vs. which currencies to use). First, we explicitly incorporate nonmonetary transaction costs of redeeming. Drèze and Nunes (2004, p. 71) recognize the existence of such costs but do not incorporate them into their utility model. Second, we add the consideration of forgone points by allowing customers to value the gains from the points earned in the program, not just the losses from spending cash and points. With these two changes, the model captures the first two motivations to stockpile. Third, we also allow customers to subjectively value points less than cash at a fixed conversion rate. Fourth, we do not restrict our attention to cases in which customers already have stockpiles large enough to cover the entire price (Drèze and Nunes 2004, pp. 62, 69). This restriction is reasonable for analyzing redemption behavior in nonlinear programs because, by design, customers are encouraged and sometimes restricted to wait until they have a large stockpile. This restriction is not needed to study linear programs because customers can easily redeem when stockpiles are small. Instead, we examine how redemption choice changes with the size of the available stockpile relative to total prices.

These changes lead to a multiple-accounts (MA) model that unites economic, cognitive, and psychological explanations for why customers of linear programs can be motivated to stockpile on their own. The MA model has two dimensions: (1) whether a customer evaluates gains and losses of cash in a separate mental account from those of points (i.e., multiple accounts vs. single account) and (2) whether these valuations within an account are made with value functions that are S-shaped (concave over gains and convex over losses) or instead linear over gains and losses. Table 1 shows that while the MA model allows for separate S-shaped mental accounts, it nests cases in which customers

Table 1
MA MODEL DIMENSIONS TO EVALUATE POINTS AND CASH

Value Functions	Multiple Accounts	Single Account
S-shaped	MA	SA
Linear	MLA	SLA

have a single S-shaped account (SA), multiple linear accounts (MLA), or a single linear account (SLA).

The MA model predicts that customers stockpile up to a latent threshold, which is not set by the retailer but is instead determined by the three motivations to stockpile: forgone points, transaction costs, and how each customer values points relative to cash. By allowing for two mental accounts, the relative shapes of the two value functions can motivate a customer to stockpile. The MA model can be considered structural (Chintagunta et al. 2006) in the sense that the estimated parameters directly determine the fundamental cost–benefit trade-off that governs redemption behavior. While structural models typically rely on standard expected utility theory (Von Neumann and Morgenstern 1944), the MA model is grounded in prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992), which governs the value function shapes, as well as mental accounting, which considers one versus two accounts.

This article examines the MA model in two fundamental ways. First, we analytically examine the model's implications for stockpiling. Afterward, we build an empirical version of the model, with a hierarchical Bayes structure, to reflect heterogeneity in how people perceive gains and losses of cash relative to gains and losses of points. We estimate the MA model along with the nested specifications given in Table 1 on observational data from a linear loyalty program of an international retailer and show how the three motivations to stockpile differ across customers. This analysis is useful to identify how customer segments may respond to alternative strategies for encouraging redemptions.

Alternative explanations to those encompassed by the MA model can be drawn from the literature on nonlinear programs. In particular, although research on nonlinear programs is vast (Bijmolt, Dorotic, and Verhoef 2010), the findings often cannot be easily translated to linear programs. For example, a psychological phenomenon called the "medium effect" (Hsee et al. 2003) exists when myopic consumers would make different stockpiling choices if rewards were denominated directly in dollars instead of in points. The effect is expected when points alter the perceived monetary return of stockpiling. One driver of the medium effect is a nonlinear exchange rate between points and cash. Nevertheless, "the mere presence of a medium (points) is not sufficient to produce a medium effect" (Hsee et al. 2003, p. 3). No medium effect is expected in linear loyalty programs that offer a fixed points-to-dollar exchange rate and reward customers with a fixed number of points for every dollar spent, as studied here. As a second example, the "goal-gradient hypothesis" (Kivetz, Urminsky, and Zheng 2006) finds that consumers tend to exert more effort (i.e., purchase faster) as they advance toward a redemption goal that is explicitly set by the retailer. Linear programs do not have such goals. Furthermore, the goal-gradient effect sheds light on purchasing rather than on the redemption behavior

in which we are interested; it assumes that redemption occurs when customers have stockpiled enough points to do so.

In summary, we use a model of mental accounting to empirically examine the potential motivations to persistently stockpile in linear programs and how they vary across customers. Our findings and documentation of stockpiling behavior in a linear program respond directly to a call in a recent article for research on how customers redeem: "Though redemptions are critical elements of loyalty programs, we do not really know why loyalty program members redeem, or why they do not" (Bijmolt, Dorotic, and Verhoef 2010, p. 224). Our results can be used to improve communication strategies to encourage redemptions and may also provide insight into why even in nonlinear programs, some customers persistently stockpile beyond explicit requirements.

The remainder of this article proceeds as follows: We first describe the data set to motivate both the theoretical and the empirical parts of the research. Then, we mathematically lay out the MA model and explain how it captures each motivation. We specify an empirical version of the utility model and demonstrate its performance, along with the other specifications of the 2×2 framework. We then apply the MA model to characterize the heterogeneity in customer motivations and assess policies that may potentially increase redemptions by addressing each of the three motivations. We conclude with a summary and directions for further research.

PROGRAM DESCRIPTION

Our empirical setting is a linear loyalty program that has operated for more than 20 years in a prominent supercenter chain in Latin America (the chain has asked to remain anonymous). It is the market leader in several countries for a range of high-end product categories similar to those offered by Target, Bed Bath & Beyond, and Home Depot. For example, the product categories it offers include flat-screen televisions, beds, hardware items, toys, kitchen appliances, home decor items, gardening tools, and camping equipment.

The data track the behavior of a cohort of customers who first signed up for the retailer's loyalty program in January 2008. For each visit of each individual, over a 43-month period, we observe the basket price, how many points were earned or redeemed, and the date. From these data, we are able to infer the available stockpile of points each person had at the time of each purchase.

Consistent with our introduction and motivating example, the loyalty program offers a linear reward policy. In our setting, customers cannot get the satisfaction of paying without any cash (i.e., they cannot pay 100% with points). Stockpiled points can be redeemed anytime to reduce up to 50% of the basket price (not including items on sale) at a constant and easy-to-remember points-to-cash exchange rate. We observe that this cap potentially affects 40% of redemptions—that is, those in which a customer had more points than 50% of the basket price. When the cap is nonbinding, customers, on average, redeemed points equal to 22% of their basket price.

Furthermore, redemption is a low-effort activity. Customers who want to redeem points simply show their loyalty card and tell the cashier they want to redeem. While

redemption does not require customers to keep track of their stockpiles, customers can easily check their balance at the cash register, online, or by phone. Point expiration is not an important motivation to redeem in this program because any purchase delays the expiration of a customer's entire stockpile by one year. Earning points is also simple: a customer presents his or her loyalty card to the cashier to earn 1% of the total purchase price in points.

In this program, points are not earned on shopping trips when a customer redeems. Thus, the redeeming customer sacrifices an opportunity to earn points on that trip. This opportunity cost is an economic motivation to delay redemption. Although this program feature is incorporated in our model, it is only one of three motivations to redeem. While some programs share this feature (i.e., as commonly occurs when using points to partially pay for a plane ticket), others reward customers based on the portion of the price paid with cash. Our model can easily be modified to capture these smaller forgone points by considering unearned points on only the redeemed amount instead of the full price.

Several features the program lacks suggest that some potential drivers (alternative explanations) of persistent stockpiling are unlikely to play a large role in this program. The first "missing" feature is that customers cannot increase the value of their points by purchasing larger baskets or by stockpiling many points. Recall that in a linear program, the economic value of points is constant over time, so it does not vary with the purchase price or with how many points are redeemed at once. Second, the program does not offer customers any special-tier status or any nonmonetary benefits based on their stockpiles. Thus, stockpiling in this program is not a way for customers to signal their status to others (Drèze and Nunes 2009). Third, when a customer forgets to bring his or her loyalty card for a given purchase, that purchase is not recorded in the data set, and the customer cannot earn or redeem points without the card. Consequently, for every purchase in our data set, the customer presented the loyalty program card, so our model estimation is not affected by purchases in which customers forget to bring their cards. This suggests that lack of interest in the program or forgetting about the program are possible but unlikely explanations for the observed stockpiling in this setting.

DATA DESCRIPTION

The data set contains 346 customers who signed up for the program in January 2008. We tracked their 10,219 purchase occasions from January 2008 through July 2011. We use each customer's first purchase occasion in January 2008 to calculate initial stockpile levels but then exclude it from our analysis because no redemptions could have taken place during the first purchase (i.e., customers did not have any points to redeem yet).

Redemptions are relatively rare in this program: customers only redeemed in 3% of purchases. Over the entire observation period, 60% of customers never redeemed. We call these customers "nonredeemers." It does not seem to be the case that the high number of nonredeemers are customers who simply stopped purchasing early during the observation period, because of the 244 customers who purchased at least once in 2011 (i.e., had more than three years of purchasing experience with the program), a majority (51.6%) were nonredeemers. Of the 137 redeemers, 69 redeemed once, 54 redeemed two to four times, 12 redeemed five to seven times, and 2 redeemed more than seven times. Customers who redeemed are also quite valuable. Relative to nonredeemers, customers who redeemed at least once over the 43 months had shopping baskets worth 25% more ($p = .008$), visited the retailer 141% more frequently ($p < .001$), and their recency since the last purchase observed in the data was 164 days shorter ($p < .001$). Table 2 further details how customer purchase behavior varies. It shows the distribution across customers of total redemptions, purchase frequency, total spend, and the maximum points stockpiled over the observation period. The large dispersions of these distributions suggest that people may differ in how they value cash relative to points.

Figure 2 shows how the main variables we examine in our analysis (redemptions, available stockpiles, and basket prices) evolve over time. It indicates that although mean monthly basket prices are highly variable, their level is steady over time. In contrast, stockpiles exhibit a positive trend. The low levels of redemption rates in the cohort's early months reflect that stockpiling behavior may be different during customers' early experiences with the program while they are building up their points.

In general, stockpiles are large enough to generate nearly 30% in cash savings. Figure 3 shows the distribution of the percentage of the basket price that can be reduced by redeeming stockpiled points. The distribution is capped because this program allows customers to cash in up to 50% of the basket during any single purchase occasion. Figure 4 plots the monthly averages of these savings. These averages tend to increase over time, with exceptions during the months of November and December.

Next, we illustrate in an exploratory way how the extent of stockpiling differs from what we would expect if customers were randomly deciding whether to redeem (i.e., flipping a coin). For every customer who redeemed at least once, we generate 1,000 simulations of their redemption choices from a sequence of Bernoulli trials with a probability of redemption equal to that of their observed redemption rate. We then compare the average run-lengths (i.e., average number of consecutive purchases with no redemption) of the observed and simulated sequences of redemption

Table 2

HETEROGENEITY IN CUSTOMER BEHAVIOR

Quantiles	0%	25%	50%	75%	100%
Total redemption occasions	0	0	0	1	15
Total purchase occasions	1	11	22	41	253
Total spend (currency units)	17	2,106	4,519	9,331	211,743
Maximum stockpile (currency units)	0	26	49	91	1,285

Figure 2
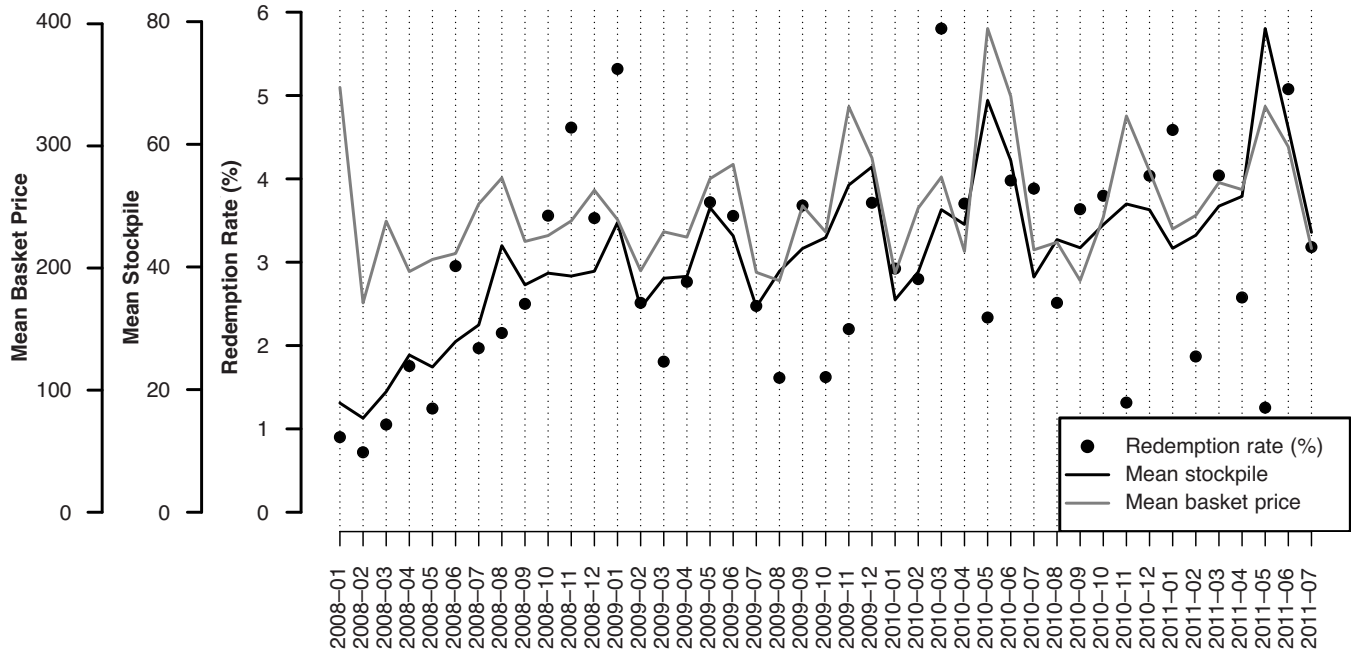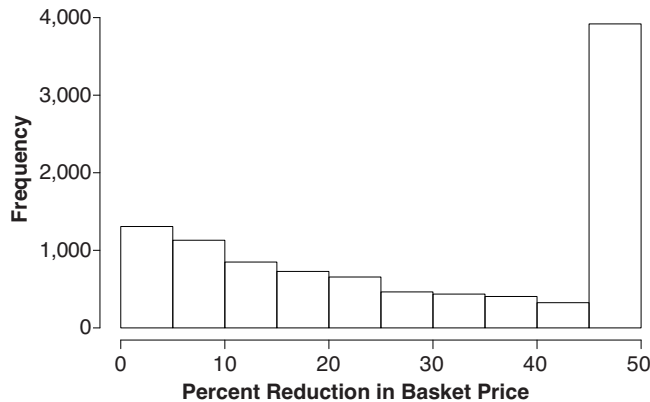REDEMPTIONS AND STOCKPILES OVER TIME

Figure 3
PERCENTAGE OF THE BASKET THAT CAN BE REDUCED

choices. Figure 5 shows a histogram of the differences between each customer's observed and expected mean run-length. The majority of customers stockpile longer (i.e., tend to have longer run-lengths) than expected by a Bernoulli sequence with the same observed redemption rate. A one-sided bootstrap Kolmogorov–Smirnov test rejects the null hypothesis ($p < .001$) that customers' expected and observed mean run-lengths come from the same distribution, suggesting persistent stockpiling behavior.

REDEMPTION MODEL

In this section, we formalize the MA model described previously. We begin by developing the two dimensions (shape and number of accounts), and then we describe how

persistent stockpiling behavior is captured through economic, cognitive, and psychological motivations.

Dimension 1: The Shape of the Value Function

A value function determines customer i's perceived value of gains and losses. It is defined over deviations from a reference point (i.e., current wealth). A function that is linear in both gains and losses is the simplest specification. Equation 1 shows our linear specification of $w(x)$, which values gains linearly with a slope a_{iw} and losses with a slope that is steeper than gains by a loss aversion parameter $\lambda_{iw} > 1$.

$$(1) \qquad w(x) = \begin{cases} a_{iw}x & \text{if } x \geq 0 \\ -\lambda_{iw}a_{iw}(-x) & \text{if } x < 0 \end{cases}.$$

Alternatively, a value function can be S-shaped as proposed by prospect theory: concave over gains and convex over losses. Equation 2 shows an empirical specification of an S-shaped value function proposed by Tversky and Kahneman (1992). It exhibits diminishing marginal utility for gains and diminishing marginal disutility for losses. This is determined by the curvature parameter $a_{iw} \in (0, 1)$. The loss aversion parameter $\lambda_{iw} > 1$ allows losses to be steeper than gains.

$$(2) \qquad w(x) = \begin{cases} x^{a_{iw}} & \text{if } x \geq 0 \\ -\lambda_{iw}(-x)^{a_{iw}} & \text{if } x < 0 \end{cases}.$$

Dimension 2: The Number of Accounts

We describe customers as keeping both a "points account" and a "cash account" for evaluating possible redemptions
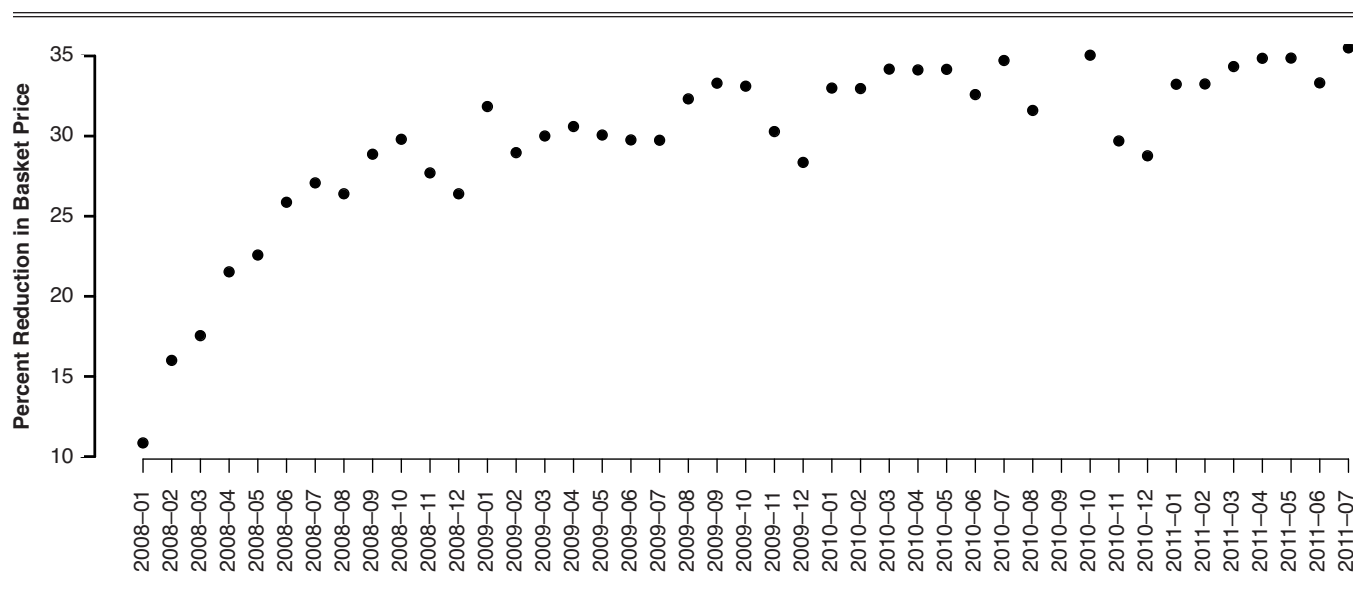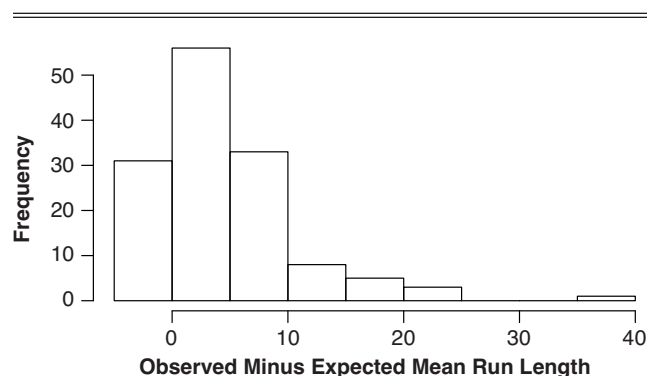
Figure 4
AVERAGE MONTHLY POTENTIAL SAVINGS

Figure 5
COMPARISON OF RUN-LENGTHS

and transactions. Consider a customer who cashes in $3 in points on a $10 item. This customer does so by paying the equivalent of $3 from his or her points account and $7 from his or her cash account. The $3 are a loss to "points wealth," and the $7 are a loss to "cash wealth." By redeeming, the customer also loses the opportunity to earn $.10 in points (i.e., 1% of the price). This $.10 is a "forgone gain" to the points account. Such debits and credits are evaluated separately as gains and losses in each mental account. In the MA and MLA models, we let customers have two different value functions, one for the cash account, $w(x)$, and another for the points account, $v(x)$.[1] To consider a customer who instead books transactions using a single mental account, let $w(x) = v(x) \; \forall \; x$.

Formally, consider customer i at the cash register on purchase occasion j, who purchases a basket of items with a total price m_{ij}. We model y_{ij}, the customer's decision to

redeem conditional on a purchase occurring. When the customer does not redeem ($y_{ij} = 0$), he or she pays the entire price with cash and earns points on that purchase. When the customer does redeem ($y_{ij} = 1$), he or she pays in part with stockpiled points s_{ij} and does not earn any points.[2] Next, we describe the utility for these payoffs mathematically.

Not redeeming ($y_{ij} = 0$). When a customer does not redeem, he or she pays the entire price m_{ij} in cash. Thus, cash wealth is reduced by $-m_{ij}$, and so the customer values this loss with a cash value function: $w(-m_{ij})$. The customer also earns points worth r% of the price, where r represents the retailer's reward rate. The gain of $m_{ij}r$ new points is valued with the points value function $v(x)$. In addition, a customer may subjectively value points differently than cash by a fixed subjective conversion rate $h_i > 0$, so this gain may be perceived as $m_{ij}rh_i$ dollars, with utility $v(m_{ij}rh_i)$.

Equation 1 lays out the utility of not redeeming. The term E_{ij} denotes the utility attributed to the basket of items. The error term $\varepsilon_{ij}^{y_{ij}}$ represents a shock to the customer's preferences that is observed to the customer but is unobserved by the researcher. The errors are independently distributed around zero, so $E[\varepsilon_{ij}^{y_{ij}}] = 0$.

$$(3) \quad u_{ij}\left(y_{ij} = 0\right) = E_{ij} + \underbrace{w\left(-m_{ij}\right)}_{\text{cash loss: full price}} + \underbrace{v\left(m_{ij}rh_i\right)}_{\text{gain: earned points}} + \varepsilon_{ij}^0.$$

Redeeming ($y_{ij} = 1$). When a customer redeems, he or she pays with a combination of points and cash. The maximum points the customer can redeem is naturally capped by the

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[1]For notational simplicity, we rescale the points to the same units as cash (i.e., 1 point = $1).

[2]A utility-maximizing customer with two mental accounts chooses either to redeem as many points as possible ($y_{ij} = 1$) or not to redeem at all ($y_{ij} = 0$). This binary choice between "redeem as many points as possible versus not redeem at all" is an implication derived from our model (i.e., not an assumption) when both cost functions grow at a decreasing rate (Drèze and Nunes 2004, p. 63). The binary consideration set is equivalent to modeling a consumer searching for the optimal redemption amount.

available points s_{ij} and a cap κ on how much of the basket price m_{ij} can be redeemed (as described previously, we study a program that caps the amount redeemed at $\kappa = 50\%$ of the price). Thus, by redeeming, the customer spends $\tilde{s}_{ij} = \min(s_{ij}, m_{ij}\kappa)$ points and uses $m_{ij} - \tilde{s}_{ij}$ dollars in cash to pay for the remainder of the price. The customer perceives the $\tilde{s}_{ij}$ points spent as a loss to the points account worth $\tilde{s}_{ij}h_i$, valued with his or her points value function as $v(-\tilde{s}_{ij}h_i)$. Separately, the cash loss of $m_{ij} - \tilde{s}_{ij}$ dollars is valued by the customer's cash value function: $w(-[m_{ij} - \tilde{s}_{ij}])$. Equation 2 summarizes the utility of redeeming. The customer also incurs a nonmonetary transaction cost c_{ij}, reflecting perceived time and effort required to redeem. Note that this customer is forward-looking to the extent that he or she considers how redeeming affects points available for future use.

$$(4) \qquad u_{ij}(y_{ij} = 1) = E_{ij} + \underbrace{w(-m_{ij} + \tilde{s}_{ij})}_{\text{cash loss: discounted price}}$$

$$+ \underbrace{v(-\tilde{s}_{ij}h_i)}_{\text{points loss: redeemed points}} - c_{ij} + \varepsilon_{ij}^1.$$

Subtracting Equation 3 from Equation 4, the customer is expected to redeem when net utility $z(s_{ij}, m_{ij})$ (Equation 5) is greater than zero. Equation 6 rewrites this inequality to distinguish the expected benefit of redeeming on the left-hand side from the expected cost on the right-hand side.

$$(5) \qquad z(s_{ij}, m_{ij}) = w(-m_{ij} + \tilde{s}_{ij}) - w(-m_{ij}) + v(-\tilde{s}_{ij}h_i)$$
$$- v(m_{ij}rh_i) - c_{ij}, \text{ and}$$

$$(6) \qquad \underbrace{w(-m_{ij} + \tilde{s}_{ij}) - w(-m_{ij})}_{\text{cash saved}}$$

$$\geq \underbrace{\left[v(m_{ij}rh_i) \right.}_{\text{forgone new points}} + \underbrace{c_{ij}}_{\text{transaction cost}} \left. \right] \underbrace{- v(-\tilde{s}_{ij}h_i)}_{\text{points spent}}.$$

The benefit of redeeming denotes the value of the cash saved: the customer only pays $m_{ij} - \tilde{s}_{ij}$ dollars instead of the full price of m_{ij} dollars. These savings are perceived as a reduced loss (rather than a gain) in the cash account. The right-hand side shows three costs to redeem: forgone points, a transaction cost, and redeemed points.

Three Types of Motivations

As we previously mentioned, forgone points are an economic incentive to persistently stockpile up to a point, while transaction costs are a cognitive one. Note that these two incentives remain fixed as the customer accumulates more points (i.e., fixed with respect to $\tilde{s}_{ij}$). The third psychological incentive is captured by differences in how a customer values points relative to cash. The model allows customers to value points differently than cash in two ways: (1) through the subjective conversion rate, h_i, and (2) by having separate mental accounts for cash and point transactions.

Stockpiling driven by the conversion rate. For a customer who subjectively values points less than cash by $h_i < 1$, the incentive to redeem "inferior points" for "superior cash" grows as the customer stockpiles increasingly more points (regardless of the number of mental accounts). Consider a

customer who has a single linear account: $v(x) = w(x)$. For this customer, the cash saved from redeeming is $w(-m_{ij} + \tilde{s}_{ij}) - w(-m_{ij}) = w(\tilde{s}_{ij})$ (because $w(x)$ is linear), and the costs are $-w(-\tilde{s}_{ij}h_i) + [w(m_{ij}h_ir) + c_{ij}]$, so the customer's net expected utility is $z(\tilde{s}_{ij}, m_{ij}) = w[\tilde{s}_{ij}(1 - h_i)] - [w(m_{ij}h_ir) + c_{ij}]$. Notice that the value of redeeming $w[\tilde{s}_{ij}(1 - h_i)]$ is positive and grows with $\tilde{s}_{ij}$, while the costs do not vary with $\tilde{s}_{ij}$. Thus, the customer will be increasingly motivated to redeem as the stockpile grows. The customer will stockpile until $w[\tilde{s}_{ij}(1 - h_i)]$ surpasses the value of forgone points and transaction costs. When points are not considered inferior to cash (i.e., $h_i = 1$), this incentive to redeem disappears, but a customer with multiple accounts can still be expected to eventually redeem.

Stockpiling driven by booking transactions in multiple accounts. To observe how the relative shapes of the two accounts alone can motivate stockpiling, we temporarily set the conversion rate $h_i = 1$ and illustrate the incentives to redeem when the two accounts are either both linear in gains and losses (the MLA model) or both S-shaped (the MA model).

First, we compare the costs and benefits from Equation 6 for the MLA model. The value functions $w(x)$ and $v(x)$ are linear in x, so as redeemable points grow, the benefits grow at a rate equal to the loss slope of the cash value function $w(x)$, and the costs grow at a rate equal to the loss slope of the points value function $v(x)$. With the linear functional form shown in Equation 1, the benefits are

$$w(-m_{ij} + \tilde{s}_{ij}) - w(-m_{ij}) = \lambda_{iw}a_{iw}\tilde{s}_{ij},$$

and the costs are (when $h_i = 1$)

$$-v(-\tilde{s}_{ij}) + [v(m_{ij}r) + c_{ij}] = \lambda_{iv}a_{iv}\tilde{s}_{ij} + [a_{iv}(m_{ij}r) + c_{ij}].$$

When $\tilde{s}_{ij}$ is close to 0, a customer has little incentive to redeem: $z(0, m_{ij}) = -[a_{iv}(m_{ij}r) + c_{ij}]$. As the customer accumulates points, the benefits can grow faster than the costs, depending on the parameter values. Net utility $z(s_{ij}, m_{ij})$ increases with s_{ij} when the slope of losses is greater for cash than for points: $\lambda_{iw}a_{iw} > \lambda_{iv}a_{iv}$. Figure 6 illustrates how the

Figure 6
COST–BENEFIT TRADE-OFF FOR MLA
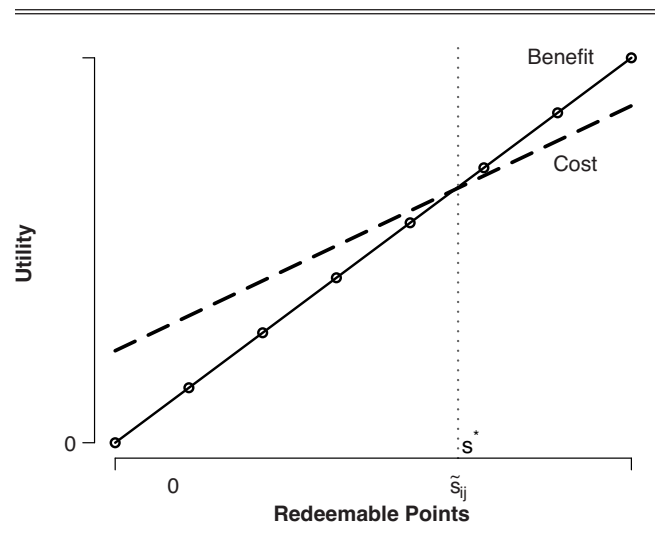
costs and benefits of redeeming evolve with $\tilde{s}_{ij}$ when this condition is met. In the figure, the net utility of redeeming is positive for any stockpile value greater than s*, the point at which a customer becomes indifferent between redeeming and not.

Second, we consider the multiple S-shaped accounts (MA) model. When the accounts are S-shaped, a customer will eventually redeem as points are accumulated if the slope of losses is sufficiently steeper for cash than for points. In addition, the current basket price now influences how many redeemable points are needed to entice redemption. In other words, $\partial z/\partial \tilde{s}$ is a function of not only $\tilde{s}_{ij}$ but also of m_{ij} due to the nonlinearity of the value functions. Using Equation 6 and the S-shaped functional form shown in Equation 2 with $h_i = 1$, we define the benefits of redeeming as

$$w\left(-m_{ij} + \tilde{s}_{ij}\right) - w\left(-m_{ij}\right) = -\lambda_{iw}\left(m_{ij} - \tilde{s}_{ij}\right)^{a_{wi}} + \lambda_{iw}\left(m_{ij}\right)^{a_{wi}},$$

and the costs as

$$-v\left(-\tilde{s}_{ij}\right) + \left[v\left(m_{ij}r\right) + c_{ij}\right] = \lambda_{iv}\left(\tilde{s}_{ij}\right)^{a_{vi}} + \left[\left(m_{ij}r\right)^{a_{vi}} + c_{ij}\right].$$

and so the net expected utility of redeeming is

$$(7) \qquad z\left(s_{ij}, m_{ij}\right) = -\lambda_{wi}\left(m_{ij} - \tilde{s}_{ij}\right)^{a_{wi}} + \lambda_{wi}\left(m_{ij}\right)^{a_{wi}}$$
$$- \lambda_{vi}\left(\tilde{s}_{ij}\right)^{a_{vi}} - \left(m_{ij}r\right)^{a_{vi}} - c_{ij}.$$

As $\tilde{s}_{ij}$ increases, the benefits grow at an increasing rate, and the costs grow at a decreasing rate. A growing stockpile will lead to a positive net expected utility of redeeming if the slope of losses is sufficiently steeper for cash than for points (i.e., $a_{iw} > a_{iv}$). Figure 7 illustrates how the cost–benefit trade-off evolves with $\tilde{s}_{ij}$ when $a_{iw} > a_{iv}$. In the figure, the benefits surpass the costs for all stockpile levels above s*, the point at which a customer becomes indifferent between redeeming or not.

Figure 7
COST–BENEFIT TRADE-OFF FOR MA
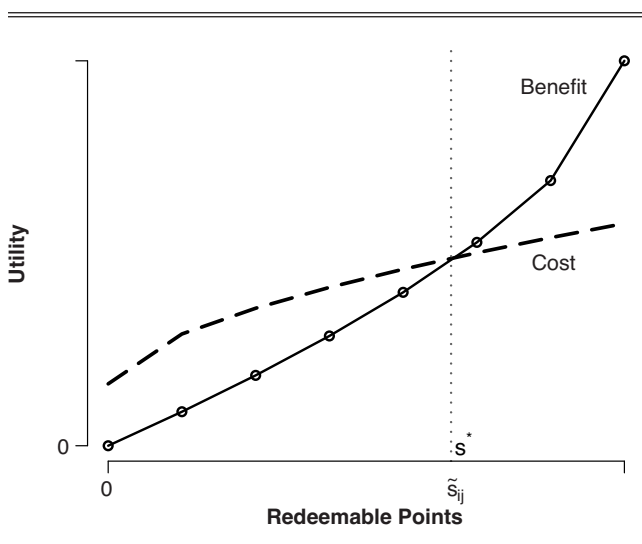
Because the value functions are S-shaped, it may be possible that w(x) is steeper than v(x) for large losses but not for small losses. This occurs when $a_{iw} > a_{iv}$ and $\lambda_{iw} < \lambda_{iv}$ (i.e., the points account has a greater degree of loss aversion than the cash account). An indifference point s* can still be reached as long as w(x) is steeper than v(x) for large losses (i.e., when there are many points available to redeem). See Appendix A for a formal proof.

Having shown how the MA model explains persistent stockpiling behavior through the economic, cognitive, and psychological motivations, we complete the empirical specification. The propensity to redeem can be written in closed form by assuming that the errors $\varepsilon^{y_{ij}}$ are independently and identically distributed Gumbel.[3] As a result, we model the redemption choice y_{ij} from a Bernoulli distribution with a probability p_{ij}, where $logit(p_{ij}) = z(s_{ij}, m_{ij})$. We also let transaction costs c_i be fixed across the purchases of each individual. We summarize the empirical MA model and its special cases in Table 3. Due to the linearity of the value functions, the MLA and SLA models lead to the same empirical model, which we refer to only as the SLA model.

The logit of the redemption propensity of the MA model is equivalent to the net utility shown in Equation 7. The model allows a_{iw} to be greater than, equal to, or less than a_{iv} (so it does not restrict the slope of losses to be steeper for cash than for points). Note that the MA model is able to capture an interaction between redeemable points $\tilde{s}_{ij}$ and basket price m_{ij} and can reflect framing effects. For example, because the loss function is concave, a $5 savings on a $10 purchase is valued more than a $5 savings on a $20 purchase (Thaler 1985; Tversky and Kahneman 1981). The SA model sets w(x) = v(x) (i.e., $\lambda_{iw} = \lambda_{iv}$ and $a_{iw} = a_{iv}$). The SLA model further restricts w(x) to be linear instead of S-shaped.

Bayesian Specification

Next, we discuss the Bayesian specification. We complete the empirical specification by modeling individual differences through a hierarchical Bayes framework, which allows for heterogeneity across customers in how they value points relative to cash (Rick, Cryder, and Loewenstein 2008). A hierarchical Bayes model is ideally suited to analyze behavior at the individual level in this case because it leverages data on rare redemption occasions across customers. A prior distribution on the individual-level parameters allows the model to partially pool data across individuals. For the MA model, let β_i represent a vector of the transformed individual-level parameters: $\beta_i = [\log(\lambda_{wi} - 1),$ $\log(\lambda_{vi} - 1), logit(a_{wi}), logit(a_{vi}), logit(h_i), c_i]$. These transformations restrict the loss-aversion parameters to be greater than one and the curvature parameters as well as the conversion rate h_i to be between 0 and 1. The vector β_i is

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[3]The error term $\varepsilon^{y_{ij}}$ corresponds to ε^1_{ij} and ε^0_{ij} from Equations 1 and 2.

Table 3
SUMMARY OF EMPIRICAL SPECIFICATIONS

Model	$logit(p_{ij})$
MA	$-c_i - \lambda_{wi}(m_{ij} - \tilde{s}_{ij})^{a_{wi}} + \lambda_{wi}m_{ij}^{a_{wi}} - \lambda_{vi}(\tilde{s}_{ij}h_i)^{a_{vi}} - (m_{ij}rh_i)^{a_{vi}}$
SA	$-c_i - \lambda_i(m_{ij} - \tilde{s}_{ij})^{a_i} + \lambda_i m_{ij}^{a_i} - \lambda_i(\tilde{s}_{ij}h_i)^{a_i} - (m_{ij}rh_i)^{a_i}$
SLA/MLA	$-c_i + \beta_{si}\tilde{s}_{ij} + \beta_{mi}m_{ij}$

assumed to follow a multivariate normal prior distribution with mean μ and a precision matrix Ω (Equation 8).

$$(8) \qquad \beta_i \sim MVN(\mu, \Omega).$$

The prior mean μ is chosen to be a vector of zeros. Because this prior distribution governs the transformed parameters β_i, the prior means of the untransformed parameters are 2 for λ_{wi} and λ_{vi} and .5 for a_{wi}, a_{vi}, and h_i. The prior precision matrix Ω is a diagonal matrix, meaning that on the untransformed scale, 99.7%, (i.e., within three standard deviations) of the prior draws for λ_{wi} and λ_{vi} lie between 1.05 and 21.08 and 99.7% of the prior draws for a_{wi}, a_{vi}, and h_i lie between .05 and .95.

To complete our Bayesian model specification, as is standard (Gelman et al. 2003), let μ follow a conjugate multivariate normal with mean μ_0 and precision Ω_0, and let Ω^{-1} follow a conjugate Wishart distribution with ρ degrees of freedom and an inverse scale matrix R. The hyperparameters are chosen as identity matrices for Ω_0 and R^{-1}, a zero vector for μ_0 and the dimension of the covariance matrix for ρ to make it proper (i.e., 5 for the MA model). We use the same specifications of the prior and hyperprior distributions for the SA and SLA models. For the SA model, β_i excludes λ_v and a_v. For the SLA model, $\beta_i = [\beta_{si}, \beta_{mi}, c_i]$, and Ω is chosen to be a diagonal matrix times .5.

We estimate these specifications using a Markov chain Monte Carlo (MCMC) sampler with OpenBUGS software. Population-level parameters are sampled from their marginal posterior distributions. These parameters can be directly sampled due to their conjugate hyperpriors. Individual-level transformed parameters β_i are sampled from $p(\beta_i|\mu, \Omega, y)$ (Equation 8) with an adaptive Metropolis block sampler.

Next, we describe how the net utility for each specification evolves with observed stockpile levels s_{ij} and basket prices m_{ij}. These differences provide alternative and empirically testable stockpiling mechanisms.

EMPIRICAL IDENTIFICATION

Due to the novelty of the MA model and its highly nonlinear nature, we conducted a simulation study to demonstrate parameter recovery for data sets of the size and sparsity used here (see Appendix B). Analytically, the parameters are identified by the nonlinearity of the value functions. Empirically, the parameters are identified by the variation in both basket prices and stockpiles across purchases associated with and without redemptions. To elaborate on this, we examine how redemption propensity is influenced by an increase in both points and basket price.

The equations in Table 4 delineate when additional points may lead to an increase or a decrease in a customer's net utility to redeem (z). For simplicity, we omit the subscripts on m and $\tilde{s}$. The predictions of each specification differ by how $\partial z/\partial \tilde{s}$ varies with prices and stockpiles. For SLA, the

simplest specification, an increase in redeemable points can increase or decrease net redemption utility in a constant manner, depending on the sign of β_{si}. For the MA and SA models, $\partial z/\partial \tilde{s}$ varies with both $\tilde{s}_{ij}$ and m_{ij}, meaning that an increase in points influences redemption propensity differently depending on stockpiled points and basket price. The SA model, as previously explained, requires points to be perceived as inferior to cash to predict redemptions ($h_i < 1$).

For the MA model, when the relative shapes of the two accounts meet the condition that $\lambda_{wi}a_{wi} > \lambda_{vi}a_{vi}$, additional points can allow $\partial z/\partial \tilde{s}$ to become positive when $\tilde{s}$ is sufficiently large relative to price. Consider two customers who are each about to purchase a \$10 basket of goods. Customer A has an \$8 stockpile, and Customer B has a \$1 stockpile. If the company gave each customer one additional point, under the MA model, we could expect this point to increase the propensity to redeem only for Customer A.

Next, we examine how an increase in basket price influences redemption propensity. Table 5 shows $\partial z/\partial m$ for each model. The SLA model predicts that an increase in price either increases or decreases net expected utility in a constant manner, depending on the sign of β_{mi}. For the MA and SA models, $\partial z/\partial m$ is negative regardless of the parameter values, meaning that at the individual level, these models expect larger prices to be associated with fewer redemptions conditional on $\tilde{s}$. In the next section, we discuss the empirical results and examine how different types of consumers seem to vary by how strongly they consider the economic, cognitive, and psychological motivations to persistently stockpile.

EMPIRICAL RESULTS

We separate the data of the linear loyalty program into in-sample and out-of-sample data sets. We used the in-sample data (January 2008–August 2010; 7,557 purchases) to estimate the models. We ran three independent MCMC chains from different starting values, thinning every 50 samples. We determined convergence using the Gelman and Rubin (1992) diagnostic of between-to-within chain variance. We ran each model for 5,000 iterations, using the last 3,000 iterations of each chain (9,000 draws in total) for analysis.

After estimating each model using the in-sample data, we used the parameter draws to generate predictive distributions (and posterior point estimates) for a substantial 11-month out-of-sample period (September 2010–July 2011; 2,662 purchases). Obtaining accurate out-of-sample forecasts was challenging because they were not generated using the observed out-of-sample redemption choices. Instead, we used a recursive forecasting approach to ensure that forecasts were generated from stockpiles that were consistent with previously predicted redemption choices. For each draw, we update the stockpiles at each out-of-sample purchase occasion using a customer's forecasted previous

Table 4
SUMMARY OF $\partial z/\partial \tilde{s}$ FOR SPECIFICATIONS

Model	$\partial z/\partial \tilde{s}$
MA	$[\lambda_{wi}a_{wi}(m-\tilde{s})^{a_{wi}-1}] - [\lambda_{vi}a_{vi}h_i^{a_{vi}}(\tilde{s})^{a_{vi}-1}]$
SA	$\lambda_i a_i[(m-\tilde{s})^{a_i-1} - h_i^{a_{vi}}(\tilde{s})^{a_i-1}]$
SLA	β_{si}

Table 5
SUMMARY OF $\partial z/\partial m$ FOR SPECIFICATIONS

Model	$\partial z/\partial m$
MA	$\lambda_{wi}a_{wi}[m^{a_{wi}-1} - (m-\tilde{s})^{a_{wi}-1}] - a_{vi}(rh_i)^{a_{vi}}m^{a_{wi}-1}$
SA	$\lambda_i a_i[m^{a_i-1} - (m-\tilde{s})^{a_i-1}] - a_i(rh_i)^{a_i}m^{a_i-1}$
SLA	β_{mi}

redemption choice, as shown in Equation 9. This approach is analogous to how Erdem, Imai, and Keane (2003) predict ketchup purchases by updating households' latent ketchup inventories with previously forecasted purchases.

$$(9) \qquad s_{i(j+1)} = \begin{cases} s_{ij} + rm_{ij} & \text{if } y_{ij}^{\text{predicted}} = 0 \\ s_{ij} - \tilde{s}_{ij} & \text{if } y_{ij}^{\text{predicted}} = 1 \end{cases}.$$

Sustaining accurate forecasts during the long 11-month out-of-sample time frame is particularly challenging in our setting for two reasons. First, errors compound over time through the stockpiles. Early errors in redemption predictions are carried over through the stockpile levels. Second, the models must extrapolate customer behavior as the stockpiles of some customers grow beyond their in-sample levels. Recall Figure 2, which shows mean monthly prices and stockpiles across the in-sample and out-of-sample periods. While stockpiles tend to grow smoothly over time, basket prices are more volatile and do not exhibit a simple time trend. In contrast to most longitudinal analyses, we are more reliant on using the out-of-sample period to assess model validation because redemption behavior seems substantially different at the beginning of the calibration period and then gradually evolves to a steadier pattern (see Figure 2). To assess model fit across the empirical models, we break our assessment into fit at the aggregate and individual levels. The former provides an overall assessment, while the latter is done to reflect the heterogeneity that may exist in customer motivations to persistently stockpile.

Overall Model Fit

We evaluated overall model fit with the deviance information criterion (DIC)[4] (Spiegelhalter et al. 2002) and the negative of the log-marginal density (LMD). We calculated the LMD using the harmonic mean of the likelihood values evaluated at the posterior draws (Newton and Raftery 1994; Rossi, Allenby, and McCulloch 2005). For both DIC and –LMD, a lower measure indicates a better model fit. In our context, DIC may be the more reliable measure because the harmonic mean of the likelihood values evaluated at the posterior draws can be heavily influenced by a few small outlying draws. Table 6 shows these measures for three periods in the data: 2008 (the cohort's first year in the program), the remainder of the in-sample period (January 2009–August 2010), and the out-of-sample period (September 2010–July 2011). These measures are supplemented with Figure 8, which compares actual and expected bimonthly

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[4]We used the median rather than the mean to compute DIC due to a few outlying draws. This may be due to the highly nonlinear nature of the MA model.

Table 6
DEVIANCE INFORMATION CRITERION AND
(–LOG-MARGINAL DENSITY)

	In-Sample (2008)	*In-Sample (2009+)*	*Out-of-Sample*
MA	698.0 (352.9)	1,139.5 (577.5)	945.9 (872.5)
SA	734.8 (372.3)	1,187.0 (597.3)	866.8 (1,209.7)
SLA	646.3 (310.6)	1,065.2 (506.2)	2,998.8 (2,052.3)

redemptions.[5] As we explain in the "Data Description" section, the different patterns in behavior during 2008 may be due to larger psychological transaction costs of redeeming when a customer joins the program, which may decrease as customers become more familiar with the retailer.

In-sample, all three specifications perform similarly in aggregate. However, during the 11-month out-of-sample period, the MA and SA models sustain predictions that closely track observed redemptions across time, while the SLA model is less successful.

We now examine each model's ability to forecast the aggregate out-of-sample distribution of redemptions across customers. The sum of squared errors between the actual and expected number of customers who made zero to five redemptions during the out-of-sample period (five redemptions was the maximum observed) are 25 for the MA model, 101 for the SA model, and 271 for the SLA model. In summary, the MA model followed by the SA model are better able to forecast aggregate patterns in redemption behavior compared with the SLA model. However, despite SLA's poor aggregate forecasts, a simple one-account model may still be the appropriate model for some consumers; we explore this possibility next.

Differences in Motivations Across Customers

The MA model's ability to capture economic, cognitive, and psychological motivations to stockpile can provide insight to managers seeking ways to better manage their programs. Many retailers closely monitor redemptions because they not only determine a program's costs and liabilities but also indicate a program's effectiveness and help the retailers identify valuable customers. MasterCard advisor Bob Konsewicz suggests that retailers should encourage redemptions: "encouraging and driving redemptions allows members to engage in and experience the value proposition of the program, and the sooner they do that, the better!" (Konsewicz 2007; see also Kwong, Soman, and Ho 2011). Colloquy and Swift Exchange echo Konsewicz after finding that more than one-third of the $48 billion in rewards issued in the United States each year are never redeemed (Hlavinka and Sullivan 2011). As an example, Amazon.com automatically prompts Discover card customers to redeem at checkout. Should Amazon.com change the way it highlights the reward balance (see Figure 9) and instead frame rewards in terms of gains and losses, such as: "Save up to $22.07! Apply rewards to reduce your price"? Are high-value customers who have not redeemed yet likely to respond more favorably to monetary incentives?

Customers may respond differently to various incentives depending on what motivates them to stockpile. As a first step to evaluate how each of the three motivations (economic, cognitive, and psychological) differs at the individual level, we examine the posterior distributions of the MA model's parameter values (Table 7). In the discussion that follows, parameters without the i subscript refer to the untransformed population-level parameters μ, and those with the subscript refer to individual-level estimates.

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[5]The MA model can partially capture seasonal holiday variation (see Figure 8) without the use of holiday dummies because it conditions on purchase behaviors (i.e., frequency and prices), which also tend to systematically vary during the holiday season.
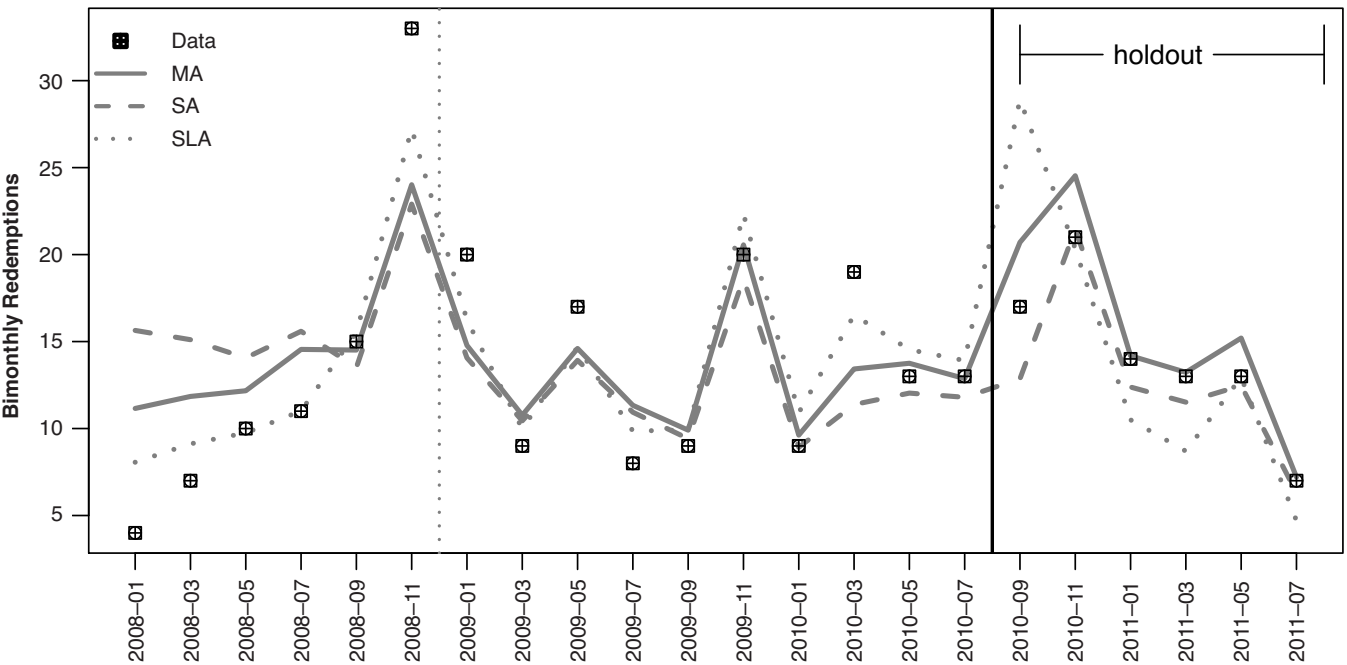
Figure 8
BIMONTHLY REDEMPTIONS

Figure 9
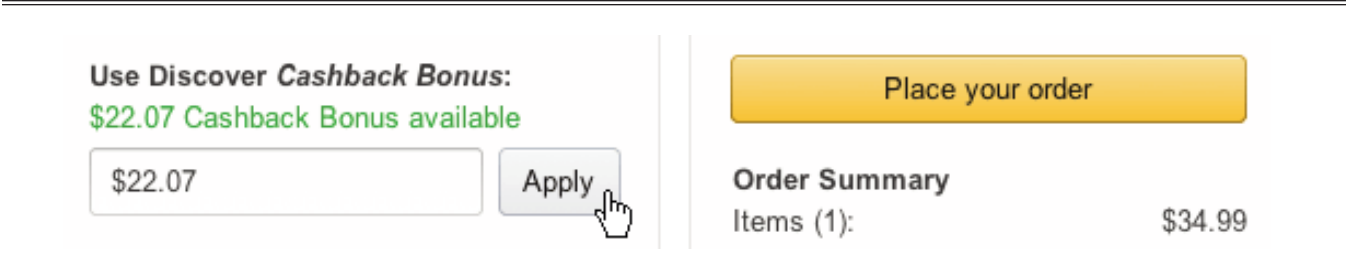REDEEMING CREDIT CARD'S POINT STOCKPILE AT AMAZON.COM CHECKOUT

Table 7
POSTERIOR DISTRIBUTION OF THE MA MODEL'S
POPULATION-LEVEL PARAMETERS

Parameter	M	2.5% Bound	97.5% Bound
λ_w	1.21	1.07	1.45
λ_v	1.78	1.28	2.74
a_w	.347	.273	.415
a_v	.020	.007	.042
h	.523	.172	.865
c	1.79	.937	2.50

Recall that persistent stockpiling can arise from two ways of valuing points differently than cash. The first is a fixed conversion rate h_i. Note from Equation 7 that h_i only appears in the MA model likelihood as $h_i^{a_{vi}}$. Across individuals, the 95% posterior interval for $h_i^{a_{vi}}$ ranges from .978 to .984, suggesting some impact in how cash is valued relative to points but one that is unlikely to be a major determinant of redemption choice. The second is differences in accounts.

The MA model indicates a generally steeper loss curve for cash than for points because the curvature parameters are greater for cash than for points (i.e., $a_w > a_v$) at the population level. At the individual level, the 95% posterior interval for $a_{iw} - a_{iv}$ is also positive, ranging from .296 to .376 (i.e., does not include 0 for any customer).

The following indicators use the posterior means computed from the MCMC sampler draws to compare the three motivations to stockpile. The last ratio summarizes differences between the cash and points accounts. The closer this "account differences" ratio is to 0, the more similar the accounts. We evaluate the ratio at $x = 10$.

- Economic motivation: mean $[(m_{ij}r)^{a_{vi}}]$ (i.e., the mean value of forgone points for each individual);
- Cognitive motivation: c_i; and
- Psychological motivation: h_i and $\lambda_{wi}(x)^{a_{wi}}/\lambda_{vi}(x)^{a_{vi}} - 1$.

These indicators are used to segment customers using K-means clustering. Table 8 shows the standardized indicator means of three segments of size 142, 117, and 87, respec-

Table 8
SEGMENT MEANS FOR EACH STANDARDIZED MOTIVATION
INDICATOR

Segment	Economic Forgone Points	Cognitive Fixed Costs	Psychological Account Differences	Exchange Rate
1	−.67	.44	−.43	−.25
2	.73	.51	−.57	.77
3	.12	−1.41	1.47	−.62

tively. Although we used the exchange rate h_i to segment customers, removing it did not affect the segmentation, which is consistent with our previous finding suggesting that this factor is unlikely to be a major determinant of redemption choice.

The three scatterplots in Figure 10 show how the standardized indicators vary across customers. As the third panel shows, fixed costs and account differences are highly correlated: customers in Segment 3 have both low fixed costs and accounts that are very different from one another, while customers in Segments 1 and 2 have high fixed costs and accounts that are more similar to one another. An SA model may be adequate for members of Segments 1 and 2, who have relatively similar accounts and are strongly motivated by cognitive fixed costs. Segments 1 and 2 differ in the magnitude of their forgone points. Adding a fourth segment does not add further insight (Segments 1 and 2 are partitioned into three groups, with high, medium, and low forgone points).

Figure 11 compares individuals by their total redemptions and maximum accumulated points. The first panel shows that customers in Segments 2 and 3 have greater stockpiles than those in Segment 1. The second panel shows that although these two groups have comparably high levels of stockpiles, every Segment 3 customer redeemed at least once, while only a few redeemed in Segment 2.

Thus, differences in redemption behavior seem to be driven mostly by the two cognitive and psychological motivations (fixed costs and differences between accounts), which are correlated. Most redeemers are in Segment 3. Recall from the scatterplots that Segment 3 differs mostly from Segments 1 and 2 on the basis of cognitive and psychological motivations, and less so for the economic motivation.

Analysis of Policies That Target Each Motivation

Having examined differences in motivations across customers, we illustrate how the MA model can help managers economically evaluate potential policy changes to the program that could lead to higher redemption rates. We consider three hypothetical policy changes to the linear program studied here. Each aims to "lift redemptions" by addressing a particular motivation, so we refer to them as the economic, cognitive, and psychological policies.

Consider the following three hypothetical policies. The economic policy rewards points for every basket price (i.e., rm_{ij}), regardless of redemption choices. Removing the opportunity costs of redeeming mitigates the economic incentive to stockpile. The cognitive policy automates redemptions. It mitigates cognitive costs by automatically reducing a customer's basket price if his or her stockpile is greater than 15

Figure 10
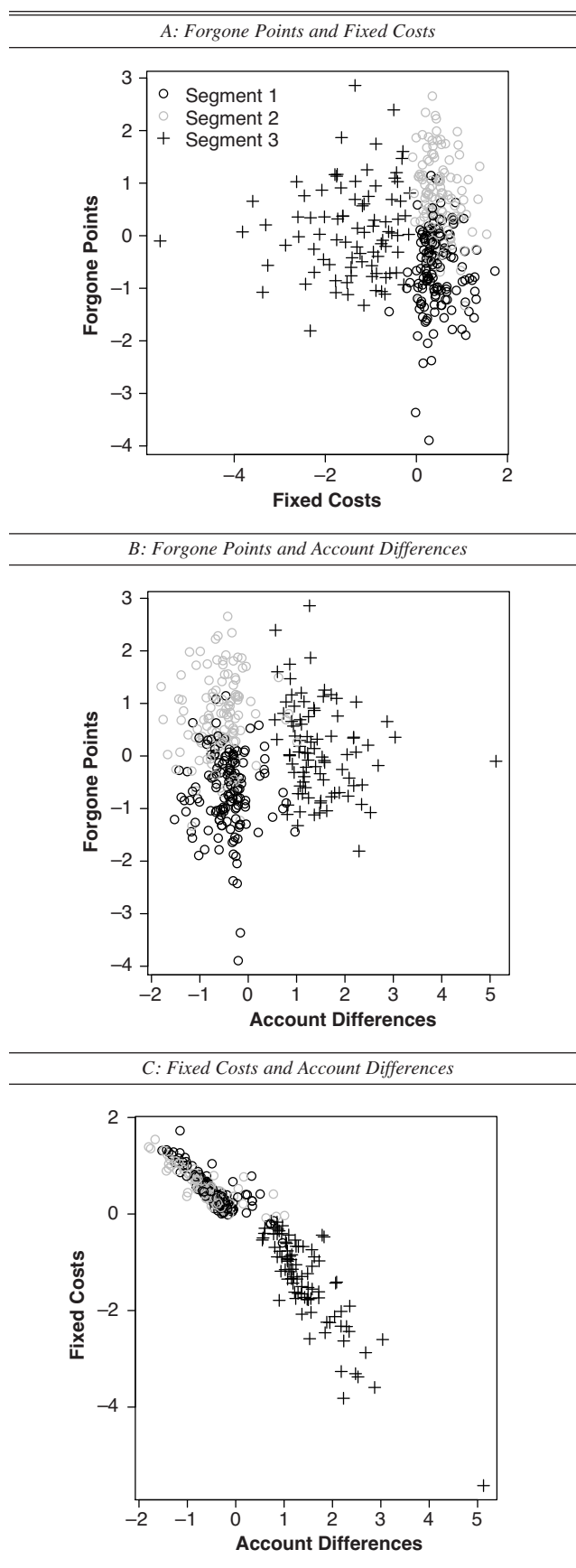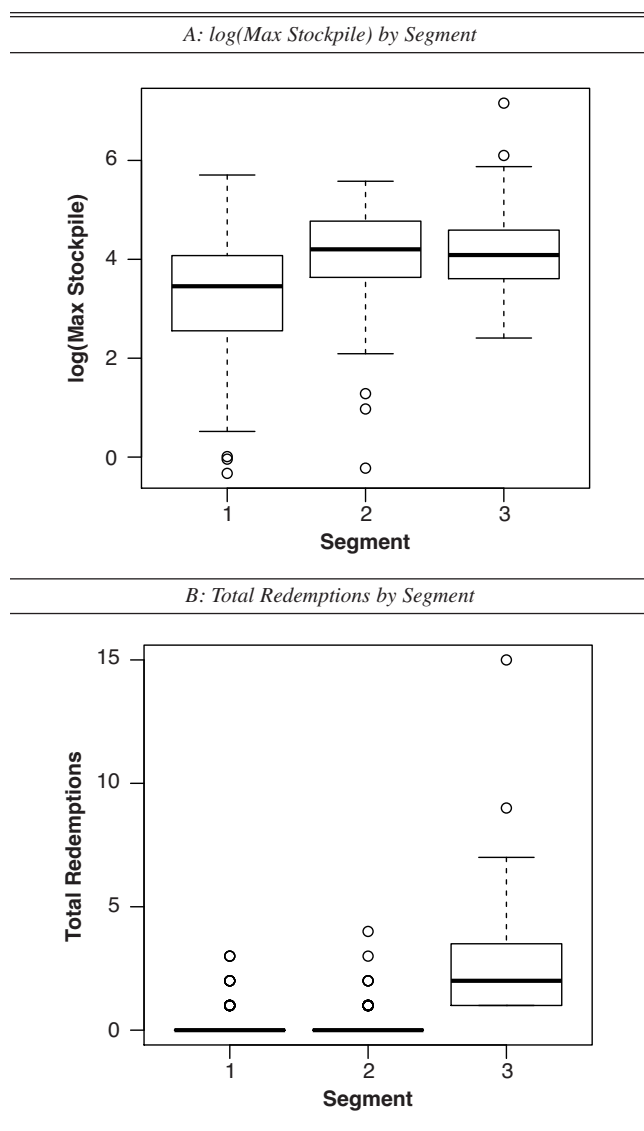HOW MOTIVATIONS VARY BY SEGMENTS

A: Forgone Points and Fixed Costs

B: Forgone Points and Account Differences

C: Fixed Costs and Account Differences

Figure 11
REDEMPTIONS AND STOCKPILES ACROSS SEGMENTS

A: log(Max Stockpile) by Segment

B: Total Redemptions by Segment

Table 9
FINANCIAL ANALYSIS OF POLICY CHANGES

Policy	Rate	Points Redeemed	Points Outstanding	Change in Redeemed Points/ −Change in Outstanding Points
Current	3.5%	4,549	15,966	N.A.
Economic	4.1%	5,499 (+21%)	14,892 (−7%)	.75
Cognitive	21.0%	16,382 (+260%)	2,360 (−85%)	.87
Psychological	6.6%	9,643 (+112%)	10,439 (−35%)	.92

Notes: Redeemed and outstanding points are scaled by some constant. N.A. = not applicable.

These changes in redemption rates correspond to the previous results, suggesting that behavior in this data set is driven mostly by cognitive and psychological incentives.

Table 9 also compares the financial consequences of the policies. In particular, it compares lost revenue from redeemed points and liabilities from stockpiles outstanding at the end of the out-of-sample period. The last column of Table 9 compares the costs of reducing liabilities by one currency unit. Specifically, it shows the ratio of the change in additional points redeemed (relative to the current policy) divided by the negative of the change in outstanding points (relative to the current policy). A smaller ratio indicates a more cost-efficient policy. All else being equal (e.g., purchase behaviors), policies that lead to more redemptions are intuitively more costly overall because more points are redeemed, but they also lead to larger reductions in liabilities. Notably, however, in this data set, policies that lead to more redemptions are not necessarily the "cheapest" way to reduce firm liabilities. The economic policy is expected to reduce liabilities at the lowest per-unit cost even though it is the least successful at increasing redemptions. Note also that even though the cognitive autoredemption policy effectively reduces the most liabilities, its cost ratio is comparable to the psychological policy, so it seems to be a relatively cost-efficient approach to limit the firm's point liabilities.

The predictions illustrate how the three policies can influence redemptions and profitability. However, the MA redemption model conditions on purchase behavior, so its predictions do not account for the possibility that the policies themselves may lead to changes in how frequently or how much customers purchase. To the extent that greater redemptions increase customer satisfaction, and thus purchase frequency, the forecasted redemption rates are an underestimate. Analogously, the forecasts for the psychological policy do not consider potential additional utility that customers may experience from purchasing a basket for "free" by paying entirely with points (Shampanier, Mazar, and Ariely 2007).

GENERAL DISCUSSION

Complicated program rules and undesirable rewards are often blamed for low redemption rates. However, these explanations cannot describe why, even in a linear program with simple rules, redemptions are relatively rare. More generally, research to date has not successfully addressed why more than $48 billion rewards issued in the United States each year are never redeemed (Hlavinka and Sullivan

points (a 25% discount for the average basket price). Customers still retain the option to redeem when their stockpile is below 15 points, but because redeeming is automatic, customers can enjoy rewards more frequently without incurring cognitive costs. Finally, the psychological policy allows customers to redeem up to 100% of the basket price, instead of the current actual policy of capping rewards at 50%. Removing the cap increases redeemable points, and as they increase, the differences in the mental accounts of cash and points lead the benefits of redeeming to grow faster than the costs, so customers may redeem more frequently.

We apply the MA model to analyze how implementing each of these policies at the start of the out-of-sample period would affect redemptions as well as the firm's finances. Relative to the current policy, all policies are expected to increase total out-of-sample redemptions over the 11-month period compared with the current policy. The economic, cognitive, and psychological policies increase redemptions by .6, 17.5, and 3.1 percentage points, respectively (Table 9).

2011). This article provides insight into redemption behavior in linear programs. An advantage of studying linear programs is that we can isolate motivations to redeem that do not depend on the explicit incentives to stockpile present in nonlinear programs.

We model how economic, cognitive, and psychological incentives can motivate customers to stockpile up to a point even though the retailer does not explicitly reward point accumulation. We estimate the model on observational data from a linear loyalty program and use it to describe how these distinct motivations differ across customer segments. We find that behavior across individuals is mostly driven by cognitive and psychological motivations to redeem (fixed costs and separate accounts) and less so by economic incentives (forgone points). For retailers seeking to improve their strategies to manage redemptions, our findings provide insight into how customers are likely to respond to communication strategies, promotions, and policy changes (i.e., changing the maximum redeemable points). Further research could consider the extent to which these three motivations can explain why even in nonlinear programs some customers stockpile beyond the retailer's explicit incentives to do so.

Although we consider three motivations, we recognize that the observed stockpiling behavior may be consistent with other alternatives.[6] Studies with more detailed data are better suited to explore them. For example, it is plausible to posit that the types of items purchased may influence the redemption decision. Customers may stockpile to redeem as much as possible on luxury items because paying in points instead of cash can reduce the guilt of indulging on such items (Kivetz and Simonson 2002). Assessing empirical evidence for this rival explanation would be challenging in our setting because customers redeem on an entire basket of items rather than on any single item. Survey data, if appropriately collected and matched, could be used to learn which item(s) in a basket prompted a customer to redeem. Redemption choice may also influence the types of items that customers purchase. Finally, although we do not rule out that prices can be dependent on the redemption choice, our MA model predicts a negative relationship between redemptions and basket prices (Table 5), and the data seem consistent with these predictions (the estimated price coefficients of the SLA model are negative for most customers). Although this research is not a complete account of all possible stockpiling alternatives, we hope it is a good step toward a more integrated understanding of redemption behavior in linear loyalty programs.

APPENDIX A: PROOF OF EXISTENCE OF AN INDIFFERENCE POINT

Appendix A presents a theorem showing that when there are two S-shaped value functions $w(x)$ and $v(x)$, an indifference point s^* can be reached as long as $w(x)$ is steeper than $v(x)$ for large losses (i.e., when there are many points available to redeem). The subscripts are dropped in the proofs for simplicity.

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[6]The Web Appendix investigates a rational forward-looking alternative. The results of the MA model as well as exploratory statistical analyses indicate a lack of empirical support for the alternative in this data set.

We begin with brief propositions that characterize how the variable benefits and costs of redeeming vary with redeemable points. These proofs stem directly from two properties of an S-shaped value function: over the loss domain ($x < 0$), a value function $u(x)$ is increasing (Property 1: $u' > 0$) and convex (Property 2: $u'' > 0$).

> **Proposition 1.** The benefit of redeeming $b(\tilde{s}_{ij}, m_{ij}) = w(-m_{ij} + \tilde{s}_{ij}) - w(-m_{ij})$ is a strictly increasing and strictly convex function of redeemable points $\tilde{s}_{ij}$: $b' > 0$; $b'' > 0$.

Proof. The first derivative $b'(\tilde{s}) = w'(-m + \tilde{s}) > 0$ comes from Property 1 of the value function. The second derivative $b''(\tilde{s}) = w''(-m + \tilde{s}) > 0$ comes from Property 2 of the value function.

> **Proposition 2.** Denote the variable cost of redeeming by $c(\tilde{s}_{ij}, m_{ij}) = -v(-\tilde{s}_{ij})$. The variable cost is a strictly increasing and strictly concave function of redeemable points $\tilde{s}_{ij}$: $c' > 0$; $c'' < 0$.

Proof. The first derivative $v'(-\tilde{s}) < 0$ comes from Property 1 of the value function. Thus, $c'(\tilde{s}) = -v'(-\tilde{s}) > 0$. The second derivative $c''(\tilde{s}) = v''(-\tilde{s}) < 0$ comes from Property 2 of the value function.

Now we introduce a condition that formalizes when $w(x)$ is steeper in losses than $v(x)$ (i.e., spending cash should be "more painful" than spending points, at least for large prices): in the "asymmetry condition," there exists a price m' such that for all $m > m'$, $|w(-m)| > |v(-m)|$.

> **Theorem 1.** Given the asymmetry condition, a threshold $s^* > 0$ such that $z(s^*, m_{ij}) = 0$ is guaranteed to exist for prices m_{ij} greater than some finite level $\overline{m}_{ij}$.

Proof. The net utility of redeeming is decomposed into a variable component and a fixed component, $z(s) = q(\tilde{s}) - f$, where $q(\tilde{s}) = b(\tilde{s}) - c(\tilde{s})$, and the fixed costs include the opportunity cost of forgone points and the transaction cost, $f = v(mr) + c$.

When $\tilde{s} = 0$, $q(0) = 0$, and thus, $z(\tilde{s}) = -f < 0$. When $\tilde{s} = m$, $q(m) = v(-m) - w(-m) > 0 \, \forall \, m > m'$ by the asymmetry condition. Because $f''(m) < 0$ and $q''(m) > 0 \, \forall \, m > m'$, there exists a price $\tilde{m}$ such that $q(m) > f(m) \, \forall \, m > \tilde{m}$. When $q(m) > f(m)$, then $z(m) > 0$, and thus, a threshold s^* such that $z(s^*) = 0$ is guaranteed to exist by the continuity of $z(s)$.

APPENDIX B: PARAMETER RECOVERY

Appendix B illustrates an example of parameter recovery for the MA model's population-level parameters based on our observed data. We use the independent variables in the complete data set (10,219 purchase occasions from 346 individuals) to simulate a new set of redemption choices. We begin by generating the β_i parameters for each individual according to Equation 8, where μ is set close to values estimated from our data set: $[-1.7, -.3, -.64, -3, .1, 1.8]$, and Ω is set to an identity matrix times 2.9. We then use each individual's simulated β_i parameters, their observed prices, and their observed stockpiles to generate a new sequence of redemption choices for each of their observed purchase occasions, according to the MA model (Equation 7). The redemption rate in the simulated data set is 2.5%.

We ran three independent chains from different starting values and assess their convergence using the Gelman and

Rubin (1992) diagnostic of between-to-within chain variance. We ran the model for 5,000 iterations, using the last 3,000 iterations of each chain (9,000 draws in total) for analysis. Table B1 compares the actual and estimated untransformed population-level parameters. Each of the true population-level parameters is contained within 95% posterior intervals. The errors for the individual-level parameters λ_{wi}, λ_{vi}, a_{wi}, a_{vi}, h_i, and c_i are, on average, .31, $-$.13, $-$.05, .04, .03, and .04, respectively, and the mean absolute percentage errors are 25.9%, 12.5%, 20.4%, 104.9%, 14.6%, and 16.6%, indicating good model fit.

Table B1
PARAMETER RECOVERY EXAMPLE

Parameter	Actual	Estimated	Error	2.5% Bound	97.5% Bound
λ_w	1.18	1.40	.22	1.08	2.18
λ_v	1.74	1.51	$-$.23	1.18	2.00
a_w	.35	.28	$-$.07	.20	.35
a_v	.05	.08	.03	.03	.14
h	.52	.56	.03	.20	.89
c	1.80	1.86	.06	1.29	2.36

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