Uncovering Consumer Decision Rules under Complex Dynamic Environments: The Case of Coalition Loyalty Programs

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Abstract

We propose a framework for measuring the impact of a coalition loyalty program on consumers network choice decisions. Unlike single-vendor loyalty programs, coalition loyalty programs generate a very complex dynamic environment in which future rewards from accumulated points influence customers current network choice. In this research, we explicitly model the potential forward-looking incentive of consumers in loyalty programs, while allowing for bounded rationality. Using individual-level data from a European coalition loyalty program, we apply the Bayesian classification algorithm and estimate heterogeneity in consumer decision rules. Our preliminary results indicate that there is a segment of consumers whose behavior is consistent with the prediction of dynamic programming models, but many consumers do not seem to care about accumulating points until their accumulated points get closer to the threshold level. We provide managerial implications for target marketing designs under such consumer behavior.

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1 Introduction

Coalition loyalty programs have become increasingly more popular recently. Such a program links a number of partner retailers together and provides a wider range of benefits to customers than the traditional single-vendor loyalty program. Customers can use one card to collect points from any vendors within the network. This allows them to accumulate points quicker.[1] Coalition loyalty programs may also be able to use marketing campaigns more cost effectively as one general campaign for the program could benefit all partner retailers, e.g., an email campaign that reminds customers about the program benefits may encourage them to shop within network and benefit all in-network partner retailers. When a partner retailer uses bonus points to attract customers to buy certain products, its impact would also likely be stronger in a coalition loyalty program because customers can redeem points at any partner retailers within the network.

The increased value of reward points due to the network effect (from both accumulating points and spending points viewpoints) should be able to attract more new customers to join the program, and make it more likely for existing customers to buy within the network. However, little research has been conducted to empirically measure the effectiveness of such a program. In this research, we develop and estimate a customer shopping decision model under a coalition loyalty program environment. The estimated model would allow us to understand how customers take into account different features of the program (including the portfolio of partners, the points accumulation schedule) and the future rewards when making their shopping decisions. Our approach takes both observed and unobserved customer heterogeneity into account, and it allows the data to infer their decision rules. It tells us which customers are more likely to behave in a way as predicted by a dynamic optimization problem, and which customers are more likely to use rule-of-thumb with different degree of sophistication. Uncovering the customers decision rules are important because it allows us to answer the following

[1] We refer "customer" to consumers who have joined the loyalty program.

questions. (1) To what extent can the program provider influence customers' decisions to shop within network via marketing campaign? (2) Which types of customers are more likely to provide higher lifetime value to the program? (3) Under what conditions would encouraging customers to redeem points lead to stronger customer loyalty to the network? (4) How does the portfolio of network partners affect customer's purchase and redemption behavior? (5) What is the value of a partner retailer in a coalition loyalty program?

The answers to these questions are important to improve our understanding about how to manage a coalition loyalty program. It should be highlighted that the coalition loyalty program is a two-sided market. From the retailer's viewpoint, whether it is worth joining the network depends on the number of consumers who enrol in the program, and their usage within network (i.e., how much they spend within network, or their share of wallet within network). To encourage customers to spend more within network, and encourage more new customers to enrol, it is also important to structure the portfolio of partner retailers within the network carefully to ensure they complement each other and maximize the value of reward points to customers.

Before we outline our framework, it is important to note the following challenges. Customers could be forward-looking in a loyalty program environment. Some researchers used the dynamic programming approach to model customer shopping behavior under the influence of a loyalty program. However, it is not clear if this would be the best approach to model customers' choice under the coalition loyalty program. In particular, the program that we analyze has three complications: (i) customers face an environment where there is a network of partner retailers with different characteristics, and (ii) one account could be shared by several customers within a household, making it difficult for each customer to track the total number of points accumulated and (iii) the incentive to earn points might also differ across customers within a household because often times only the lead account holder has the right to redeem points.

If there is only one customer in an account, this program is closer to a standard reward program studied in the literature from customer's viewpoint. It is relatively easier for the customer to keep track of the number of points accumulated. Arguably, the standard dynamic programming approach may be a reasonable way to model their decisions about whether to shop within the network or outside the network. However, as the number of customers per account grows, it becomes much harder for them to keep track of all of the relevant information.[2] For instance, it seems unlikely that a customer knows exactly how many points have been accumulated when he/she faces a purchase occasion (unless he/she checks the points balance right before making a purchase). This is because he/she may have incomplete information about what other customers sharing the account have purchased. In addition, in the coalition program that we study, only the main account holder will receive a voucher when the accumulated points passed a certain threshold. It is not clear if other customers of the account will obtain the full benefits of vouchers, and hence they may not value the points fully even if they know the accumulated points in every purchase occasion. To the extent that they do not recognize the full value of the points, their choice would deviate from the optimal strategies of shopping within network or not.

The above discussion suggests that we need an empirical framework that is flexible enough to accommodate both fully rational choice and bounded rational choice to different degree. We therefore propose to deviate from the standard dynamic programming approach when modeling the dynamic choice of customers. More specifically, we extend the approach developed by Geweke and Keane (2000) and Houser, Keane and McCabe (2004). Their approach of modeling forward-looking consumers behavior is flexible and does not impose the assumption that they solve the dynamic programming problem. We will explain how it works in the following sections.

[2]In dynamic programming, the relevant information is summarized by state variables which affect the current payoffs, or expected future payoffs.

2 Literature Review

Previous studies which study consumer forward-looking incentive in a single-vendor loyalty program have made use of the dynamic programming approach (e.g., Lewis 2004, Hartmann and Viard 2008, Kopalle et al. 2012). Such an approach assumes that consumers are fully rational and forward-looking, and make their purchase decisions optimally in order to maximize the expected future payoffs due to the reward points accumulated.

Although the dynamic programming approach provides a parsimonious way to model forward-looking behavior and has the advantage of conducting counterfactual experiments, solving for the optimal solution of a dynamic programming problem is computationally costly. Often times, this limits the number of state variables that can be incorporated into the problem. Moreover, this approach assumes consumers are fully rational and rule out the possibility of bounded rationality. As we discussed earlier, in a coalition loyalty program, it is not clear if consumers will be fully rational even if they take the impact of the expected future payoffs into account when they make their shopping decisions.

Our study will be the first paper that explicitly models the potential forward-looking incentive of members in loyalty programs, while allowing for bounded rationality. As far as we know, this will also be the first paper that study the forward-looking incentives in a coalition loyalty program.[3] The approach we take will be an extension of Houser, Keane and McCabe (2004) and Geweke and Keane (2000). Their approach is to approximate the expected future payoffs using a flexible functional form. To use their approach, one condition is that the dynamic problem needs to consist of state variables

[3] As pointed out by Dorotic et al. (2012), academic research about coalition loyalty programs is still very limited. Meyer-Waarden and Benavent (2006) find evidence that there is a positive network effect in two French retail partners in a coalition loyalty program. Dorotic et al. (2011) investigate the potential cross-vendor effects of promotional mailings across complementary partners within the network, but find such effects to be very small. None of these studies consider that members may have forward-looking incentives, and they may differ in terms of their bounded rationality.

that satisfy exclusion restrictions (Magnac and Thesmar 2002; Fang and Wang 2013; Ching, Erdem and Keane 2012). As pointed out in Ching and Ishihara (2012), the dynamic environment generated by loyalty programs provides such exclusion restrictions: the number of points accumulated (the main state variable) affects the expected future payoffs, but not the current payoffs faced by the consumers. These exclusion restrictions potentially allow us to identify the expected future payoffs in a flexible way. The Bayesian classification algorithm proposed by Houser, Keane and McCabe (2004) (HKM) can potentially infer the number of decision rules used in the population (which represents different degrees of bounded rationality). The advantage of their estimation algorithm is that there is no need to solve for the dynamic programming at all. But one challenge of using HKM's approach is that the data requirement can be very large because the number of parameters to estimate depends on the number of consumer types and the complexity of their decision rules. Fortunately, the data set we use is very rich. The large sample size and richness of the data set provide an excellent setting for us to apply and extend the HKM's approach in the actual field data.[4]

Another challenge of using HKM is that their approach still involves costly experimentation of the number of types of consumers, and the order of polynomials used to approximate each consumer type's expected future payoffs, and computing marginal likelihood for selecting the specification that describe the data best. To address this problem, we combine the Bayesian nonparametric approach with HKM.[5] The recent development of Bayesian nonparametric techniques allows us to infer the number of consumer types and the corresponding order of polynomials of the approximated value function in one step, avoiding the costly experimentation part of the HKM's approach.

[4] The dynamic environment studied in HKM is much simpler than the coalition loyalty program. Moreover, they use laboratory data instead of actual field data.

[5] Gershman and Blei (2012) provides an excellent survey of Bayesian nonparametric techniques.

3 A Proposed Model

We focus on modeling consumer choice of shopping in-network and out-of-network. We begin by describing the fully rational forward-looking consumers' decision problem. Then we explain how to apply and extend HKM in our setting.

Our framework models a member-level decision rather than an account-level decision. Let $n = 1, \ldots, N$ indexes account (or household), and $i \in A_n$ be a member of account n, where A_n is a set of members who share the account n.

The timing of the model is as follows. In each time period t (day, week, etc.), a purchase occasion arrives exogenously for each member. Each purchase occasion is defined by a pair: a single product category ($c = 0, 1, \ldots, C$ where $c = 0$ denotes no purchase), and purchase amount ($q > 0$ if $c > 0$; $q = 0$ if $c = 0$). Let $g(c, q|x_{int}, z_t, m_{nt})$ be the probability distribution that member i of account n faces a purchase occasion with (c, q) at time t. We allow the distribution to depend on (1) the characteristics of member i and account n (x_{int}) such as observed member/household demographics, unobserved propensity to shop, number of members in the account, set of vouchers held, etc., (2) time-varying factors common across accounts (z_t) such as seasonality, and (3) marketing communication for account n (m_{nt}). If no purchase occasion arrives (i.e., $c = 0$), consumers simply do nothing.

Suppose that a purchase occasion with (c, q) arrives at time t for member i of account n. Then, this member faces a decision of whether or not to complete the shopping within network. Let j denote the network option: in-network ($j = 1$) and out-of-network ($j = 0$). The single-period utility function from choosing in-network is given by

$$u_{i1t}(c, q) = Z_{ict}\alpha_i + \epsilon_{i1t},$$

where Z_{ict} is a vector of observed variables for member i of account n, category c at time t. For example, variables such as consumer demographics, purchase amount, past voucher usage, and number of in-network retailers for category c could be included in Z_{ict}. For identification purpose, we normalize the mean utility for out-of-network to be zero, i.e., $u_{i0t} = \epsilon_{i0t}$.

3.1 Dynamics

Our model defines two state variables: (1) number of points accumulated (p_{nt}), (2) number of past vouchers used (h_{nt}). We discuss each of them below.

The number of points accumulated at time t for account n (p_{nt}) is influenced by the network choice by all members. Let ρ_{in} and ρ_{out} be the earning ratio at in-network and out-of-network, respectively, and let $d_{nt} = \{d_{int}\}_{i \in A_{nt}}$ be a vector of network choices by members of account n who had a purchase occasion at time t and A_{nt} be the set of such members ($A_{nt} \subseteq A_n$). The decision variable d_{nt}^i takes one if member $i \in A_{nt}$ chooses in-network and zero otherwise. If the exogenously given purchase amount for member i at time t is $q_{it} = (q_{in,it}, q_{out,it})$, then p_{nt+1} will be given by

$$p'_{nt+1} = p_{nt} + \sum_{i \in A_{nt}} \left(d_{int} \rho_{in} q_{it} + (1 - d_{int}) \rho_{out} q_{it} \right).$$

Note that the above evolution is when t is not the last date of an invoice cycle. If t is not the last date of an invoice cycle, $p_{nt+1} = p'_{nt+1}$. If t is the last date of an invoice cycle and p_{nt+1} exceeds the threshold points (multiple of 500 points in our application), then we need to subtract the number of points used for issuing vouchers from p'_{nt+1} to obtain p_{nt+1}. Those issued vouchers are added to the set of vouchers held $\bar{v}_{nt+1}$ at $t+1$. Let v_{it} be the set of vouchers used by member $i \in A_{nt}$ at time t, and let v_{nt}^{new} be the new set of vouchers issued for account n at the end of time t. Then $\bar{v}_{nt+1}$ evolves according to

$$\bar{v}_{nt+1} = \begin{cases} \bar{v}_{nt} \setminus \left(\bigcup_{i \in A_{nt}} v_{it} \right) & \text{if } t \text{ is not the last date of an invoice cycle,} \\ \left(\bar{v}_{nt} \setminus \left(\bigcup_{i \in A_{nt}} v_{it} \right) \right) \bigcup v_{nt}^{new} & \text{if } t \text{ is the last date.} \end{cases}$$

Finally, past voucher history h_{nt} (entire set of vouchers used in the past) might affect consumers' preferences towards shopping at in-network retailers. It evolves according to

$$h_{nt+1} = h_{nt} \bigcup (\bigcup_{i \in A_{nt}} v_{it})$$

Let $s_{nt} = (p_{nt}, \bar{v}_{nt}, h_{nt}, r_t)$ be the vector of state variables,[6] where r_t is the set of in-network partners in period t. Note that the in-network partners that do not belong to the category under consideration should not affect the current utility of whether to shop in-network or out-of-network. However, it can still influence the expected future payoffs because consumers' perceived value of reward points depend on where they can use the vouchers. Recall that d_{nt} is a vector of network choices by the members of account n who have a purchase occasion at time t. Then according to the Bellman principle, the value of member i of account n from choosing j at time t can be described as

$$V_{ijt}(s_{nt}) = u_{ijt} + \delta E[V_i(s_{nt+1})|s_{nt}, d_{nt}],$$

where δ is the discount factor and $E[V(s_{nt+1})|s_{nt}, d_{nt}]$ represents the expected future value from choosing j at time t. Note that unlike the standard dynamic programming problem, this expected future value depends on member i's choice AND the choices by all other members in A_{nt} at time t, because they influence the evolution of the state variables as well. Moreover, the E operator integrates out all future purchase occasions of all members in account n. Thus, a standard approach to solving for the value function is computationally very challenging. Also practically, it is very hard to imagine that all members of an account are able to (or willing to) solve such a complicated optimization problem. Moreover, although the main account holder may be more willing to invest time to take the future benefits into account, other members of the same account, who may only partially benefit from vouchers, would be less likely to use the optimal decision rule. Instead, they may apply rule-of-thumbs, with different degrees of sophistication.

[6] One tractable way to use h_{nt} in our econometric model is to convert it to the number of past vouchers used in total, or in any particular category.

Therefore, we do not assume that consumers are fully rational and solve for the above complex value function to make network choice decisions. Rather, we allow for the possibility that consumers are bounded rational and might use other decision rules. Depending on the type of decision rules used by different members, the profit-implication of the coalition loyalty program will be different. Thus, identifying the type of decision rules and the proportion of consumers using each type of decision rules will be crucial. To achieve these goals, we will apply and extend the framework of Houser, Keane, and McCabe (2004) to our context.

3.2 Modeling the Future Components

Following Houser, Keane, and McCabe (2004) and Geweke and Keane (2000), we rewrite the expected future value as follows

$$E[V(s_{nt+1})|s_{nt}, d_{nt}] = F(s_{nt}, d_{nt}|\pi_k) + \zeta_{ijt}, \qquad k = 1, .., K$$

where k denotes type of decision rules, $F(\cdot)$ denotes the future value polynomial, π_k denotes a finite vector of type-specific parameters, ζ_{ijt} is an idiosyncratic error made when attempting to implement decision rule k. As argued in Houser, Keane, and McCabe (2004), this specification nests many special cases such as nearly optimal, myopic, and completely random decision rules. Furthermore, we could also test if any superfluous factors not included in (s_{nt}, d_{nt}) affect consumers decision making by writing $F(\cdot)$ as a function of those factors.

In general, we could include the following factors when modeling the future value $F(\cdot)$: $I_{int} \equiv (p_{nt}, \bar{v}_{nt}, h_{nt}, r_t, d_{nt}, x_{int}, c_{it}, q_{it}, v_{it})$. where x_{int} is member i's characteristics, c_{it} is the purchase category for member i at time t, q_{it} is the (exogenous) purchase amount for member i at time t, and v_{it} is the number of vouchers used by member i at time t. $(x_{int}, c_{it}, q_{it}, v_{it})$ are the superfluous factors that we would like to include. We should make two remarks. First, estimating the interaction between r_t and other state variables are crucial for understanding how consumers value the partners network of the program. Second, as we discussed earlier, individual consumer's decision rule may depend on x_{int} because one may not fully benefit from a voucher if he/she is not the main account holder.

Since network choices will depend on the relative values of the future components for in-network and out-of-network, let us define

$$f(I_{int}|\pi) \equiv F(I_{int}|\pi, d_{it} = 1) - F(I_{int}|\pi, d_{it} = 0).$$

Given $f(I_{int}|\pi)$, the decision rule for member i of type-k decision rule at time t can be written as

$$\begin{aligned} \text{Choose in-network } (j = 1) \text{ iff } Y_{int}(I_{int}|k) &\equiv V_{i1t}(I_{int}|k) - V_{i0t}(I_{int}|k) \\ &= u_{i1t} - u_{i0t} + f(I_{int}|\pi_k) + \eta_{int} > 0, \end{aligned} \tag{1}$$

where $\eta_{int} = \zeta_{i1t} - \zeta_{i0t}$. We assume $\eta_{int} \sim N(0, \sigma_k^2)$ where σ_k^2 is the variance of the optimization error for decision rule k. Thus, our model is formally a mixture of probit models. Let θ_k be the population proportion of consumers who use decision rule k, and $\tau_i \in \{1, \ldots, K\}$ be the type of consumer i.

Finally, it is worth mentioning why the future components can be identified. If all state variables influence the current utility, then we are not able to separate the impact of those variables on the current utility and that on the future components. However, in our context, we have exclusion restrictions: variables that affect the future components but not the current utility. For example, the number of points accumulated, p_{nt}, does not directly affect the current utility, but affects the future components. Thus, it satisfies the exclusion restriction requirement. Also, r_t provides exclusion restrictions because other in-network retailers that do not belong to the category under consideration in the current period should only affect the members' decision via its expected future payoffs, i.e., members may take into consideration whether the points accumulated in the current period from category c (say clothing), can be redeemed in a vendor belonging to another category (e.g., supermarket) in the future.

3.3 Estimation Strategy

We use a Markov chain Monte Carlo algorithm for estimating the model. Recall that the likelihood is derived from the network choice made by customers, and that the choice probability is essentially a mixture of probit models. The estimation consists of the following five steps:

1. Draw latent utility values Y_{int}.
2. Draw decision rule coefficients π_k for all $k = 1, \ldots, K$.
3. Draw variance of optimization error σ_k^2 for all $k = 1, \ldots, K$.
4. Draw population type probabilities θ_k for all $k = 1, \ldots, K$.
5. Draw individual types τ_i for all $i \in A_n$, $n = 1, \ldots, N$.

In Houser, Keane, and McCabe (2004), they need to fix K and the functional form of $F(\cdot|\pi)$. They try models with different values of K and the functional form, and then use the marginal likelihood to choose among the models.

We extend their method by applying the Bayesian nonparametric approach. Instead of estimating models with different values of K and the functional form, we will make inferences about them using nonparametric techniques. This research will be the first to apply Bayesian nonparametric techniques for flexible consumers' dynamic decision problems and for identifying consumer heterogeneity in decision rules that might exist in the population of consumers.

4 Empirical Analysis

4.1 Estimation Models

Our estimation model is based on equation (1) and given by the following binary choice model:

$$\begin{aligned} &\text{Choose in-network } (j=1) \text{ iff} && (2) \\ Y_{int}(I_{int}|k) &\equiv V_{i1t}(I_{int}|k) - V_{i0t}(I_{int}|k) \\ &= u_{i1t} - u_{i0t} + f(I_{int}|\pi_k) + \eta_{int} \\ &= Z_{ict}\alpha_i + \epsilon_{i1t} - \epsilon_{i0t} + f(I_{int}|\pi_k) + \eta_{int} > 0. \end{aligned}$$

We assume that ϵ_{ijt}'s are normally distributed with zero mean and variance σ_ϵ^2. Let $\mu_{int} = \epsilon_{i1t} - \epsilon_{i0t} + \eta_{int}$. Then, $\mu_{int} \sim N(0, 2\sigma_\epsilon^2 + \sigma_k^2)$. As a result, our model boils down to a mixture of binary probit models. Note that we need to normalize the variance for one of the types. We set $2\sigma_\epsilon^2 + \sigma_k^2 = 1$ for $k = 1$.

For the future component $f(I_{int}|\pi_k)$, after experimenting with different variables and functional forms, we decide to include (1) number of points accumulated (p_{nt}) and (2) member i's observed characteristics (x_{int}). We should note that how sensitive customers are to the number of points accumulated has an important managerial implication for the loyalty program. A positive impact of p_{nt} on Y_{int} implies that customers are more likely to shop within network as they accumulate more points, and this is what we expect when customers are rational and forward-looking, and care about vouchers that they could obtain in the future (Ching and Ishihara 2012). If this is the case, then we expect that a marketing campaign that encourages a customer to shop within network today could have an amplified effect: the campaign does not only make him purchase within network today, but also increases the chance of in-network choice at the next purchase occasion, because his accumulated points will increase as a result of in-network purchase today. However, if a customer is completely insensitive to the number of points accumulated, we cannot expect such an amplified effect. Identifying which customers are sensitive thus could be potentially useful for designing target marketing.

Here, it is important to emphasize that p_{nt} is reset every time it reaches 500 points (and vouchers are issued). Thus, a positive impact of p_{nt} is not a simple reflection of the fact that customers who frequently shop within network are more likely to possess more points at the time of making the network choice. Furthermore, by including the interaction between observed characteristics (x_{int}) and the number of points accumulated, we are able to make inferences about how the sensitivity to the number of points accumulated varies across observed characteristics.

In what follows, we briefly describe the data sample and the variables we use for our empirical analysis.

4.2 Data

The data set comes from a European coalition loyalty program provider. It keeps track of its customers' purchase activities that are made with the loyalty program card. Among the information associated with each purchase activity, we use the following data for our empirical analysis: (1) date of purchase, (2) total purchase amount, (3) purchase categories, (4) retailer, (5) value of vouchers held and redeemed, (6) points earned, and (7) card number. From these data, we construct the following variables:

- Dependent variable: binary network choice (whether to shop within network or out of network).
- Independent variables:
  - Purchase amount (in CHF)
  - Cumulative value of vouchers redeemed
  - Number of in-network retailers belonging to the category under consideration

- Dummy variable that takes one if a customer can reach 500 points by shopping within network (but cannot reach by shopping out of network)
- Dummy variable that takes one on and after Sep. 1, 2009.
- Points accumulated, but not yet converted into vouchers

In addition to purchase activity data, we obtain demographic information for each account such as marital status, number of children, age, and income level. Our sample selection criteria are as follows:

- We drop all accounts that signed up prior to 2007. This is because purchases prior to August 2006 are all in-network purchases.
- We drop all accounts for which we do not have demographic information.
- We drop all accounts which never made in-network purchases.
- We drop all accounts which made less than 50 purchases a year (on average).

As a result of this screening, we have 233 accounts and 72,660 purchases.

Table 1 shows the summary statistics of the variables. In our sample, about 10% of purchases were made within network. One thing to highlight is that we create two different measures for the number of points accumulated. One is defined as the number of points accumulated prior an invoice date. Thus this variable could take more than 500 points. The other is defined as the number of points accumulated up to 500 points. In other words, this measure will be reset every time it reaches 500 points, regardless of whether it has been converted into vouchers or not. The latter measure may make more sense if customers use the 500-point threshold as the timing of attaining a voucher in their mind, rather than the invoice date as the timing.

We also create the correlation matrix to gain some insights about the relationships between variables (Table 2). It turns out that many variables are uncorrelated. For example, we might

Table 1: Summary Statistics

Variable	# obs	Mean	S.D.	Min	Max
In-network choice	72260	0.095	0.293	0	1
Amount spent (CHF)	72260	125.0	256.8	0.01	9169.8
Cumulative value of vouchers redeemed	72260	72.2	102.4	0	560
# in-network partners belonging to the catengory	72260	22.4	14.7	0	56
Will reach 500 points if purchase in-network	72260	0.077	0.266	0	1
Dummy for purchases after Sep. 1, 2009	72260	0.828	0.377	0	1
Points accumulated prior to invoice	72260	562.6	452.9	0	4940
Points accumulated with 500 pts cap	72260	246.9	144.3	0	499.9
Primary cardholder	72260	0.871	0.336	0	1
Married	233	0.459	0.499	0	1
# children	233	0.618	0.998	0	4
Age (2012)	233	42.4	12.7	20	81
Annual income (in 1000 CHF)	233	78.9	22.8	10	100

expect that customers who have redeemed many vouchers tend to be those who are loyal to the program, and are more likely to shop within network. However, the correlation between in-network choice and cumulative value of vouchers redeemed is very small (0.03). These small correlations seem to indicate that there could be many factors that make customers shop out of network, such as factors that make out-of-network purchases more convenient for customers than in-network purchases.

4.3 Empirical Results

Given the data set, our final estimation specification is given by a probit model with the latent variable Y_{int} (equation 2) specified as

$$Y_{int} = \alpha_n + x'_{nt}\beta + (\pi_k + w'_{nt}\gamma)p_{nt} + \mu_{int}, \tag{3}$$

where customers choose in-network if $Y_{int} > 0$; α_n is an account fixed effect for account n; x_{nt} is a vector of variables in the current utility function: purchase amount, cumulative value of vouchers redeemed, number of in-network retailers belonging to the category under consideration, a dummy variable that takes one if a customer can reach 500 points by shopping within network, and a dummy

Table 2: Correlation Matrix

Variable	var 1	var 2	var 3	var 4	var 5	var 6	var 7	var 8	var 9	var 10	var 11	var 12
1. In-network choice	1											
2. Amount spent (CHF)	0.017	1										
3. Cumulative value of vouchers redeemed	0.028	0.044	1									
4. # in-network partners belonging to the catengory	0.096	0.094	0.044	1								
5. Will reach 500 points if purchase in-network	0.007	0.135	-0.008	0.041	1							
6. Dummy for purchases after Sep. 1, 2009	-0.075	-0.022	0.182	-0.044	-0.017	1						
7. Points accumulated prior to invoice	0.043	0.031	0.220	0.059	-0.111	-0.017	1					
8. Points accumulated with 500 pts cap	-0.003	0.000	0.025	-0.006	0.283	0.022	0.085	1				
9. Primary cardholder	0.004	0.007	-0.085	-0.020	0.010	0.002	-0.045	0.003	1			
10. married	0.013	0.023	0.038	-0.007	0.009	0.002	0.046	-0.014	-0.120	1		
11. nchildren	0.008	0.041	-0.014	0.023	0.030	0.018	-0.014	-0.006	-0.121	0.306	1	
12. age	0.046	0.036	0.025	0.002	0.025	0.094	0.025	0.007	-0.086	0.291	0.156	1
13. income	-0.005	0.078	0.205	0.045	0.013	-0.020	0.214	0.018	-0.115	0.160	0.257	0.218

variable that takes one on and after Sep. 1, 2009, and β is a vector of parameters associated with these variables; p_{nt} is the number of points accumulated, and π_k captures the heterogeneity in the sensitivity to p_{nt}; w_{nt} is a vector of observed characteristics for account n such as marital status, number of children, age, income, and a dummy variable that takes one if the purchase was made by the primary cardholder of the account, and γ is a vector of parameters associated with the interaction effects between W_{nt} and p_{nt}; μ_{int} is an $i.i.d.$ normally distributed optimization error.

We estimate the model using several different specifications. Table 3 and 4 summarize the estimation results. Each table includes three estimation specifications: (1) two-segment model, (2) three-segment model, and (3) nonparametric model. The first two models are similar to Houser, Keane, and McCabe (2004) in that we fix the number of segments and estimate (π_k, σ_k^2) for each segment. The last model extends Houser, Keane, and McCabe (2004), and estimate π_k nonparametrically.[7] The difference between Table 3 and 4 is that we use different measures for the number of points accumulated, as we described earlier. In Table 3, we use the number of points accumulated prior to an invoice date, and in Table 4, we use the number of points accumulated with the 500-point

[7]At this moment, we have not allowed for heterogeneity in σ_k^2 for the nonparametric models. For the results presented in Table 3 and 4, we assume that $\sigma_k^2 = 1$ for all k.

Table 3: Parameter estimates: # points accumulated prior to an invoice date

	2 seg		3 seg		nonparametric	
variables	mean	s.d.	mean	s.d.	mean	s.d.
Current utility parameters						
Amount spent	6.67E-06	2.57E-05	7.62E-06	2.60E-05	1.15E-06	2.55E-05
Cumulative value of vouchers redeemed	-0.001	1.46E-04	-0.001	1.88E-04	-0.001	1.29E-04
# in-network partners for relevant catengory	0.008	4.42E-04	0.008	4.80E-04	0.008	4.48E-04
Will reach 500 points if purchase in-network	0.040	0.026	0.040	0.026	0.052	0.026
Dummy for purchases after Sep. 1, 2009	-0.101	0.028	-0.083	0.029	-0.097	0.026
Dummy for primary cardholder	-0.271	0.043	-0.240	0.047	-0.257	0.042
Future components						
Points accumulated but not converted						
type 1	-1.79E-05	1.21E-04	2.86E-07	1.4E-04		
type 2	3.40E-04	1.92E-04	3.72E-05	3.3E-04		
type 3			-4.3E-04	3.8E-04		
variance of optimization error						
type 1	1	0	1	0		
type 2	0.30	0.052	0.357	0.097		
type 3			0.308	0.089		
population proportion of type						
type 1	0.975	0.013	0.913	0.035		
type 2	0.025	0.013	0.053	0.031		
type 3			0.034	0.017		
Interactions btw future components and demographic variables						
Points accumulated X						
Married	1.72E-05	4.46E-05	1.44E-05	5.24E-05		
# children	9.49E-06	1.82E-05	2.39E-05	1.98E-05		
Age	1.06E-06	1.87E-06	1.56E-06	2.09E-06		
Income	-2.12E-06	1.21E-06	-2.57E-06	1.34E-06		
Primary cardholder	2.00E-04	5.18E-05	1.76E-04	5.52E-05	1.96E-04	4.80E-05
# observations	72,260		72,260		72,260	

cap. We find that the estimation results are similar in both cases. Thus below, we only discuss the results in Table 3.

The first specification of equation (3) we estimated is a two-segment model, where we allow two types of consumers who differ in their sensitivity to the number of points accumulated (π_k) and the variance associated with the optimization error (σ_k^2). The estimates are reported under "2 seg" in Table 3.

Parameters under *Current utility parameters* are β in equation (3). We find that purchase amount is positive but not significant. Indeed, throughout the three specifications, the impact of the purchase

Table 4: Parameter estimates: # points accumulated with 500-point cap

variables	mean	s.d.	mean	s.d.	mean	s.d.
Current utility parameters						
Amount spent	1.44E-05	2.52E-05	-7.67E-06	2.55E-05	2.17E-06	2.60E-05
Cumulative value of vouchers redeemed	-0.001	1.40E-04	-0.001	2.38E-04	-0.001	1.30E-04
# in-network partners for relevant catengory	0.007	4.86E-04	0.007	5.86E-04	0.008	4.45E-04
Will reach 500 points if purchase in-network	0.051	0.026	0.054	0.028	0.037	0.027
Dummy for purchases after Sep. 1, 2009	-0.117	0.027	-0.107	0.025	-0.112	0.023
Dummy for primary cardholder	-0.162	0.046	-0.147	0.038	-0.092	0.043
Future components						
Points accumulated with 500 pts cap						
type 1	6.21E-05	2.55E-04	1.45E-05	2.73E-04		
type 2	2.75E-04	2.85E-04	-2.15E-04	3.91E-04		
type 3			6.22E-05	4.02E-04		
variance of optimization error						
type 1	1	0	1	0		
type 2	0.388	0.028	0.320	0.082		
type 3			0.377	0.063		
population proportion of type						
type 1	0.942	0.019	0.872	0.032		
type 2	0.058	0.019	0.047	0.027		
type 3			0.081	0.035		
Interactions btw future components and demographic variables						
Points accumulated X						
Married	1.31E-04	1.29E-04	1.43E-04	1.22E-04		
# children	-1.53E-05	5.29E-05	-3.65E-05	5.61E-05		
Age	-4.48E-07	4.13E-06	6.62E-07	4.35E-06		
Income	-2.80E-06	2.54E-06	-2.72E-06	2.71E-06		
Primary cardholder	4.50E-05	1.39E-04	6.15E-05	1.38E-04	-1.76E-04	1.50E-04
# observations	72,260		72,260		72,260	

amount on in-network choice was not significant. Cumulative value of vouchers redeemed is negative. One possible reason for the negative coefficient is that out-of-network retailers might have become gradually more attractive to customers over time. If consumers who used to shop within network gradually increase out-of-network purchases over time, then we expect to observe a higher cumulative value of vouchers redeemed and less in-network choices for these customers, resulting in a negative coefficient. As we have captured via a dummy for purchases after Sep. 1, 2009, the in-network choice probability indeed decreases significantly after September 1, 2009. This is mainly because the earning ratio for out-of-network purchases was doubled from 0.1 to 0.2. Now if customers are not fully aware of the change on Sep. 1, 2009, this negative effect could happen at a different point in time for different customers, depending on when they began recognizing the increased attractiveness of out-of-network retailers. Such a time-varying individual unobserved heterogeneity could create the negative correlation between the cumulative value of vouchers redeemed and in-network choice. At this point, we are trying to find variables that could capture such an effect.

The next variable in the current utility function is the number of in-network partners that belong to the category under consideration at each purchase occasion. Each purchase occasion is associated with a category, and we might expect that customers are more likely to shop within network if there are more in-network partners that belong to the category. One reason is because a larger number of in-network retailers make it more convenient for customers to shop within network (e.g., easier to access to the retailers that sell the category). As expected, we find that the coefficient is positive and significant throughout the three specifications. While this finding seems obvious, it is important to note that when there are many in-network partners that belong to a category, there could also be many out-of-network retailers that belong to the category. If that is the case, a positive effect of the variable could be capturing something beyond the convenience factor. For example, it may be the case that the presence of the loyalty program for a category becomes stronger in consumers' mind when more in-network partners exist for the category.

Finally, we examine a purchase situation where consumers are close to reaching 500 points. We created a dummy that takes one when, given a purchase amount, consumers can reach 500 points if they make the purchase within network, but cannot reach 500 points if they make the purchase out of network. For example, suppose that a consumer has accumulated 450 points and need to purchase a product that costs 100 CHF today. Assuming 1 or 0.5 earning ratio for in-network purchase (so points earned will be 50 or 100) and 0.1 or 0.2 earning ratio for out-of-network purchase (points earned will be 10 or 20), we know that if this customer completes the purchase within network, he can reach 500 points today. However, if he completes the purchase out of network, he cannot reach 500 points.

If he cares about getting vouchers, he may be more likely to complete the purchase within network. Note that this variable is related to the number of points accumulated, which we consider as a factor in the future component. The difference is that if the 500-point threshold works as the timing of attaining a voucher in customers' mind (not the invoice date), then this dummy variable captures an immediate incentive to shop within network, rather than an incentive via the future component. Our result shows that the coefficient is positive and significant in the nonparametric model, and also marginally significant in the two- and three-segment models. We find this result interesting, as it shows that customers do seem to increase in-network purchases if the number of points accumulated is closer to the 500 point threshold. Using this result, we could design a marketing campaign that targets customers with a relatively large number of points accumulated, but still away from 500 points. By encouraging such customers to earn more points, it could bring their points closer to 500 points. Once they get closer to 500 points, they become more likely to shop within network.

Here, two things are worth noting. First, this dummy variable partly captures the impact of the number of points accumulated (again, this is the main variable of our interest) on in-network choices. The difference here is that this dummy concerns only with a specific situation: (1) the number of points accumulated is closer to 500 points, and (2) current purchase can bring the points to 500 points only if it's completed within network. The number of points accumulated in the future component (p_{nt}) examines the general tendency that as customers accumulate more points, they are more likely to shop within network, regardless of whether the accumulated points are close to 500 points or away from 500 points. As Ching and Ishihara (2012) show, if consumers discount the future significantly (i.e., they care more about what they get today than what they will get in the future), we tend to see less of this general tendency and more of the "closer to completion" effect. That is, if consumers are very far from reaching 500 points (say, 50 points accumulated), an increase in the number of points accumulated (say from 50 to 100) will hardly have an effect on in-network choices.

However, as consumers get closer to reaching 500 points (say, points accumulated changed from 400 to 450), the chance that they shop within network increases significantly. We will come back to this point when we discuss the estimates for π_k (under the future component).

Finally, we might suspect that consumers who are closer to reaching 500 points might adjust the purchase amount appropriately so that they can reach 500 points. If that is the case, purchase amount is endogenous. Since we only observe the actual amount spent in the data set, we cannot fully address this issue. However, according to the correlation matrix in Table 2, we do not find any significant correlation between purchase amount and points accumulated with 500-point cap. That is, at least customers do not seem to increase purchase amount when they are closer to reaching 500 points.

The last variable in the current utility function is the dummy for whether a choice was made by the primary cardholder or not. We find a negative and significant coefficient.

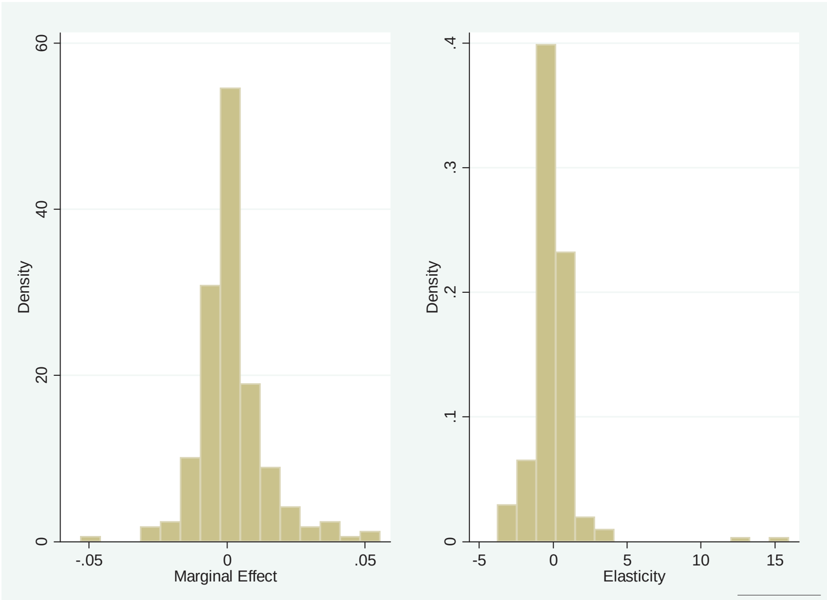
Now we examine the estimates for the future component. In the two-segment model, we allow for two types of customers who have different sensitivities to the number of points accumulated. In addition, we allow for the interaction terms between observed demographic characteristics and the number of points accumulated. The latter will be informative of who (in terms of observed characteristics) is more sensitive to the number of points accumulated, so the marketing campaign can be customized by segmenting customers based on those demographic characteristics. As we discussed earlier, the sensitivity to the number of points accumulated has an important implication for marketing campaigns. Suppose that a customer is (positively) sensitive to the number of points accumulated, i.e., the in-network choice probability increases as he accumulates more points. Then, we expect that a marketing campaign that encourages him to shop within network today could have an amplified effect: the campaign does not only make him purchase within network today, but also increases the chance of in-network choice at the next purchase occasion, because his accumulated points will increase as a result of in-network purchase today. However, if a customer is completely insensitive to the number of points accumulated, we cannot expect such an amplified effect. Identifying which customers are sensitive thus could be potentially useful for target marketing.

Our estimation results from the two-segment model show, however, that the effects are small and even negative. In the row labeled as "Points accumulated but not converted", we see that one of the types has a positive and significant coefficient (type 2) and the other has a negative but non-significant coefficient (type 1). We also find that the proportion of type 2 consumers is only 2.5%. This result suggests that there is a small proportion of consumers who are sensitive to the number of points accumulated, but the majority of consumers do not care about it. One possibility is that most customers discount the future significantly, so the dummy variable "Will reach 500 points if purchase in-network" that we discussed above largely captures the effect of the number of points accumulated on in-network choice. Moreover, it is possible that since consumers can earn points from out-of-network retailers as well, the incentive to shop within network might be much smaller as compared to a single-vendor loyalty program.

Note that these coefficients capture the main effect of the number of points accumulated for each type. We have also included the interaction effects between the variable and observed demographic characteristics, and thus we need to examine the overall effects. In both two- and three-segment models, we find that consumers with lower income and primary cardholders tend to care more about the number of points accumulated. We do not find any significant effect of marital status, number of children, or age. Thus, we decide to use more flexible distribution to capture customer heterogeneity that might not be captured via demographic variables.

We extend Houser, Keane, and McCabe (2004) and estimate π_k nonparametrically. Using Bayesian nonparametric approaches, we estimate the sensitivity for each account. Thus, target marketing can be customized at an account-level. Also, unlike a standard random-coefficients spec-

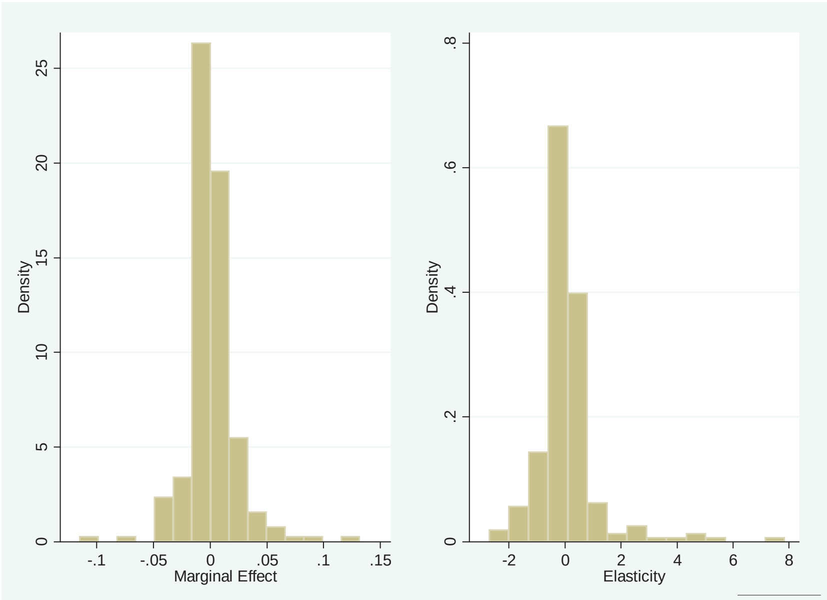
Figure 1: Marginal effect and elasticity of # points accumulated (prior to an invoice date)



ification with normal distribution, it does not impose a symmetric assumption. Thus, there is a potential that we can capture customers who are very sensitive to the number of points accumulated. Figures 1 and 2 show the distribution of the marginal effect and elasticity of the number of points accumulated across accounts, when the number of points accumulated increases by the amount equal to the average purchase amount. Other variables are set at their average values (demographic variables are set according to each account's demographics, the dummy for purchases after Sep. 1, 2009 is set to one, and the dummy for primary cardholder is set to 1). We set the increase in the number of points to the average purchase amount, in order to examine the marginal effect and elasticity of additional in-network purchase.

The left panel in Figure 1 is the marginal effect (an increase in the choice probability due to an increase in the number of points accumulated). It shows that most customers exhibit close to zero sensitivity to the number of points accumulated. The distribution is close to symmetric, and there are a small portion of customers who exhibit relatively large marginal effects. The most sensitive

Figure 2: Marginal effect and elasticity of # points accumulated (with 500-point cap)



customer exhibits a 5-percentage point increase in the in-network choice probability. The marginal effects might seem small, but note that the average choice probability is 0.095 (Table ??). To control for the base choice probability, we also compute the elasticity. On the right panel, we plot the distribution of the elasticities across accounts. The distribution becomes skewed, and we still find that a very small portion of customers have reasonably large elasticities (larger than 10 percent change in the choice probability). Finally, similar to two- and three-segment models, we observe that some customers exhibit a negative marginal effect and elasticity. We note that we do not account for the effect of the "dummy for reaching 500 points if purchases in-network" (this effect is positive and significant under the nonparametric model). Thus, if we account for this effect, the overall marginal effect or elasticity could be positive.

5 Implications for Decision-Making

Our analysis develops a framework for identifying heterogeneity in consumers' decision rules. We specifically examine how the number of points accumulated plays a role in affecting in-network purchase decisions, and identify customer heterogeneity in the sensitivity to this variable. Both economic and psychology theories suggest that consumers are more likely to choose in-network purchases (over out-of-network purchases) as their accumulated points get closer to the threshold level (500 points in our application). Built on these theories, we examine the data and estimate consumer heterogeneity in the sensitivity to the number of points accumulated. This heterogeneity has an important implication for target marketing. For customers who are (positively) sensitive to the number of points accumulated, we can expect an amplified effect of marketing efforts. That is, by encouraging them to shop in-network today, the number of points accumulated increases for them. This in turn increases the chance that they shop in-network at the next purchase occasion.

We first examine the heterogeneity in two and three-segment models. Also we examine how observed demographic characteristics affect the sensitivity to the number of points accumulated. It turned out that income and cardholder status have a significant effect, but the effects are small. However, we find that customers who are closer to reaching 500 points are more likely to shop in-network if current purchase at an in-network retailer can bring the points to 500 points. This result seems to suggest that customers are not very forward-looking. Rather, they start caring about accumulating points only when they are "one-step" away from reaching the threshold. This indicates that the loyalty program could use the current number of points accumulated for each customer as an indicator for effective target marketing. For example, it could design a marketing campaign that targets customers with a relatively large number of points accumulated, but still away from 500 points. Encouraging such customers to earn more points will bring their points closer to 500 points.

Once they get closer to 500 points, their chance of making the next purchase in-network increases.

Second, in the two-segment model and nonparametric model, we identified a small portion of customers who are sensitive to the number of points accumulated. For those customers, we might expect an amplified effect of marketing efforts as well. The use of the nonparametric approach to identify each account's heterogeneity has an advantage in that we can customize marketing efforts individually. That is, the loyalty program could estimate each customer's heterogeneity nonparametrically, identify accounts that have a higher (positive) sensitivity to the number of points accumulated, and then target those customers in designing marketing campaigns.

6 Conclusion

To be written.

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