Dartmouth College

Amos Tuck School of
Business Administration

Working Paper No.
00-02

January 2001

The Economic Viability of Frequency Reward Programs in a Strategic Competitive Environment

Praveen K. Kopalle
Tuck School of Business at Dartmouth

Scott Neslin
Tuck School of Business at Dartmouth

THE ECONOMIC VIABILITY OF FREQUENCY REWARD PROGRAMS IN A STRATEGIC COMPETITIVE ENVIRONMENT

Praveen K. Kopalle

Scott A. Neslin*

January 2001

*Praveen Kopalle is an Associate Professor and Scott Neslin is the Albert Wesley Frey Professor of Marketing at the Amos Tuck School of Business Administration, Dartmouth College, Hanover, NH 03755, USA. The authors wish to thank Rajesh Aggarwal, Aradhna Krishna, Rajiv Lal, Don Lehmann, Paddy Padmanabhan, Kannan Srinivasan, and seminar participants at the Tuck School of Business, Harvard Business School, Hong Kong University of Science and Technology, and the Marketing Science Institute, for comments on this manuscript, Emily Bahl and Andrew Louis for research assistance. This research was supported by the Tuck Associates Program.

The Economic Viability of Frequency Reward Programs in a Strategic Competitive Environment

Abstract

We examine the conditions that enhance the economic viability of frequency reward programs in a strategic competitive environment. We focus particularly on conditions related to consumer behavior, namely the extent to which consumers value the future benefits offered by the reward, the expandability of the category, and consumers' preferences for competing brands. Consumers maximize utility over a long-term time horizon, taking into account the value of the reward. Two firms maximize profits over a long-term time horizon. They first decide between implementing a frequency reward program or a traditional pricing policy (a constant price), and then decide on the specific prices. We numerically solve for the sub-game perfect equilibrium for this two-stage game.

We find that a brand is more likely to find reward programs to be viable strategies if consumers value future benefits, if reward programs can expand the market, and if the brand has a higher preference. The market expandability finding is particularly interesting. If the sales increases generated by reward programs represent category growth, the power of frequency reward programs makes them an effective vehicle for generating profits. However, if gains come mainly from competitors, the power of frequency reward programs precipitates a strong competitive response that erodes profits in a classic prisoner's dilemma. We use the airline industry to explore our market expandability finding. We find evidence that the "major" airlines introduced reward programs to counter-act a stronger outside category (new entrants), and in doing so, they expanded their market.

[Keywords: Sales Promotion; Pricing; Frequency Reward Programs; Dynamic Decision Models]

I. INTRODUCTION

Frequency reward programs, also known as "loyalty programs" or "loyalty schemes," have become significant components of marketing strategy for many organizations. Companies such as AT&T, Saturn, Hilton, BarnesandNoble.com, and firms in industries ranging from airlines to supermarkets to financial services, all have made major investments in these programs (see Brookman 1998; Raphel 1998; Colloquy.Com Website). The core motivation for the company is to increase brand loyalty. While the specifics of the programs differ, the common theme is that frequency reward programs provide the customer a benefit for repeat purchasing the brand. For example, a typical award in the airline industry is a free trip anywhere in the United States if a customer flies 25,000 miles with an airline. This can be conceptualized as "Fly n times, Get $n+1^{th}$ flight free".

Frequency reward programs have their advocates as well as detractors. Advocates cite the potential to soften price competition by building loyalty (see Klemperer 1987), or extol spin-off benefits such as the accumulation of a database that can be used for direct marketing (Butscher 1998; Reynolds 1995). In addition, there is a widely held belief that the cost of customer acquisition is greater than the cost of customer retention, and frequency reward programs are a tool for customer retention (Orr 1995).

Detractors of frequency reward programs cite their high costs (Dowling and Uncles 1997; Lewis 1997) and complexity (Orr 1995). They even question whether in fact these programs create loyalty, either in a behavioral sense (Dowling and Uncles 1997; Sharp and Sharp 1997), or an attitudinal sense (Rosenspan 1998). Finally, skeptics raise concerns about whether competitive response neutralizes the value of frequency reward programs (Dowling and Uncles 1997; Lewis 1997).

The above discussion suggests that frequency reward programs offer a great deal of promise but it is not clear whether or not they are profitable activities. Our goal is to add insight to this debate by examining the factors that influence the economic viability of frequency reward programs in a strategic competitive environment. We focus on three factors related to consumer demand: consumers' intrinsic valuation of future rewards, the degree to which category demand is expandable, and consumers' relative preference for competing brands.

Our analysis is based on the interplay between consumer and firm decision-making. We assume that consumers maximize utility over a long-term time horizon. They take into account current and future benefits of purchasing a brand, and acknowledge that the value of future benefits is uncertain. We consider a duopoly where the two firms decide whether to institute a frequency reward program, taking into account competitive response as well as the demand that will result from utility-maximizing customers. Because the decision to offer a reward program is not easily reversed, we assume firms decide first whether to offer a frequency reward program (FP) and then decide on the specific prices. This sets up a two-stage game, and we numerically solve for the sub-game perfect Nash equilibrium for that game. We then vary the parameters relating to customer valuation of the future, expandability of the category, and relative preference for the two brands, in order to understand the role of these demand factors in making frequency reward programs viable.

Methodologically, we build on recent research on dynamic consumer utility maximization (Erdem and Keane 1996; Gonül and Srinivasan 1996). This yields a rich consumer behavior model, but one which is not amenable to closed form solution for firm behavior. We therefore follow researchers such as Ansari, Economides, and Ghosh (1994), Chintagunta and Jain (1992), Neslin, Powell, and Stone (1995), and Tellis and Zufryden (1995) in using numerical simulation to determine firm equilibrium.

Our findings can be outlined as follows:

- When consumers place high value on future benefits, frequency reward programs can be characterized as powerful multi-period promotions, and are more likely to be equilibrium outcomes under strategic competition.
- Frequency reward programs are more likely to be equilibrium outcomes when market demand is expandable.
- In markets where either consumers care less about future benefits or demand is not expandable, frequency reward programs can induce intense price competition, which results in lower industry and firm profits.
- Brands that command higher customer preference benefit more from reward programs compared to brands with lower customer preference.

We use the airline industry to explore some of the important implications of our model, particularly regarding the role of market expandability. We find evidence that the “major” airlines introduced reward programs to counter-act new entrants, and in doing so, they expanded their market.

In the rest of the paper, we review the literature, motivate and present our model, and examine results. We finish with an exploratory empirical analysis and a discussion of research and managerial implications.

II. PREVIOUS RESEARCH

Although there has not been much academic work pertaining specifically to frequency reward programs, the notion of consumer switching costs has been examined extensively (Beggs and Klemperer 1992; Klemperer 1987, 1995). For example, using a multi-period framework, Beggs and Klemperer (1992) examine equilibrium prices in a duopoly where consumers incur switching costs. The authors assume that it is too costly for a consumer to switch to buying from a firm other than the one from which she has previously bought. This implies that once a consumer buys a firm's product, say American Airlines, and joins their reward program, (s)he will not choose to fly any other airline in the future. In our framework, we allow consumers to switch, i.e., a member of the American Airlines frequent-flier program may choose to fly on United Airlines if the price and flight availability are right. This is more realistic since it is not uncommon for a consumer who has frequency reward points on one airline to travel on another airline since it may be more convenient. In a footnote, Beggs and Klemperer (1992) relax the assumption of infinite switching cost and state that their results hold when consumer switching costs are sufficiently high. Of course, this suggests that the results are unclear when switching costs are low. In our model, switching costs reach a natural equilibrium level since firms compete on price and can attract consumers even if those consumers have accumulated "points"on another firm's reward program.

Zhang, Krishna, and Dhar (2000) analyze the profit impact of using "front-loaded" incentives (price discounts), versus "rear-loaded" incentives (frequency rewards). They find that rear-loaded incentives are more likely to be profitable in markets where there is a substantial natural level of variety-seeking. This result is quite interesting and intuitive, because the benefit of rear-loaded incentives is to increase repeat purchasing, which is more of a concern in high variety-seeking markets. In "inertial" markets, rear-loaded incentives merely subsidize sales that would have occurred anyway.

Kim, Shi, and Srinivasan (1996) examine the type of reward most appropriate for a frequency reward program. In particular, they question whether rewards should be low cost “efficient” rewards such as discounts on subsequent purchases, or high cost “inefficient” rewards such as cash. The researchers find interestingly that inefficient rewards can in fact be optimal. The key condition is the existence of a heavy user segment that is price sensitive, and a light user segment that is not price sensitive. In that case, the firm uses an inefficient reward as a mechanism to commit to high prices. Then heavy users benefit from the reward while light users pay regular (high) prices. The result is a kind of price discrimination where light users are subsidizing profits lost to heavy users.

Among the few empirical papers in the area is an interesting study by Sharp and Sharp (1997). They examine a frequency reward program where several retailers offered a reward program good for purchases at any store within the consortium. The authors find that this program did not increase store loyalty. The reason has to do with the partnership nature of the loyalty program. Assume stores A and B are in the consortium. The reward program might make it more attractive for Store A customers to defect to Store B, since now they can do so and still gain a reward. This problem apparently offset any potential benefit of rewarding Store A’s current customers.

The above studies have made important progress in our understanding of how frequency reward programs work. Our research is unique in examining the role of critical consumer demand factors – valuation of future benefits, expandability of the market, and relative preference for brands – in determining the viability of frequency reward programs in a strategic competitive game. From a methodological standpoint, we build on a growing and very promising line of research in which consumers maximize their utility in a dynamic framework (Erdem and Keane 1996; Gonül and Srinivasan 1996). We include this feature in our analysis, but examine firm profit maximization as well.

III. QUALITIATIVE DESCRIPTION OF KEY COMPONENTS OF THE MODEL

Consumer Behavior

Multi-period decision process: Frequency reward programs essentially offer the consumer a discount for a future purchase. Consumers must take into account the future value of this discount in deciding whether to purchase the brand in the current period. In the words of Dowling and Uncles (1997), frequency reward programs "try to change the customer's choice process from operating in a spot market to operating in a multi-period, contractual relationship market". Our model explicitly assumes that the consumer makes a multi-period decision.

Uncertainty about future preferences: The issue of uncertainty arises any time a consumer attempts to take into account future benefits. The customer may not be sure whether he or she will want to take a particular airline on future trips, or how often he or she will want to shop at a particular store. It therefore is realistic that consumers will be uncertain about their future preferences for the product (Bulkley 1992). As a consequence, we assume that the customer is aware of his/her own distribution of future preferences, but does not know the exact preferences in the next period, when the frequency reward must be cashed in. The consumer therefore makes current period purchase decisions based on *expected* future preferences, but can switch to the competition if those expectations fail to materialize and the price of the competitor is right. This entices firms to compete for each other's customers on an ongoing basis.

Degree to which consumers care about the future: The degree to which consumers care about the future should play a key role in governing the viability of frequency reward programs. We represent this by the discount factor (β_c) consumers apply toward evaluating future benefits.

Utility maximization: In each period, consumers maximize their respective net discounted utility over a long time horizon. They do so by considering the impact of their current decisions on future utility.

Stochastic dynamic programming framework: The above four factors imply that the consumer decision process can be represented as a stochastic dynamic program. The optimization is dynamic because the frequency reward program requires them to consider future benefits; stochastic because the value of future benefits is uncertain.

<u>Firm Behavior</u>

Implementation of frequency reward program: A frequency reward program offers a benefit in the future if a purchase is made in the current period. A simple scheme that captures the essence of a frequency reward program is one which allows customers to obtain the product for free in the next period when the product is purchased at the offered price in the current period. Thus, we incorporate behavioral loyalty induced by the frequency reward program. That is, a customer who buys a product from an FP firm is more likely to "buy" the product from the same firm in the next period. We could make the structure more complex; however, we believe it would not change the qualitative nature of our results. The key notion, which we capture with our model, is to induce behavioral loyalty by providing the customer with a *future* reward based on *current* purchase behavior.

In addition, we incorporate an expiration date in the reward program. We assume that if the reward is not cashed in during the next period, it expires. In this assumption, we are capturing the common practice that frequency rewards have expiration dates. Good examples are airline award programs, clothing store programs etc. Again, we could add complexity by allowing the customer to hold the inventory for several periods before losing it, but we don't believe this would change our basic results.

Separation of whether to offer a frequency reward program versus specific prices: We assume the firms decide first whether to adopt a frequency reward program (FP) or a traditional pricing policy (TP) consisting of a constant price[1]. Next, the firms decide on the specific prices[2]. Our justification for this two-stage setup is that the decision to offer a frequency reward program or a traditional pricing program is less easily reversed than changing prices. Clearly, significant investment is required for setting up frequency reward programs. Even the traditional pricing program necessitates a significant investment of institutional resources (Rao 1991).

Two-stage game: The above two assumptions imply a two-stage game. In the first stage, firms decide whether to adopt a frequency reward program or traditional pricing, and in the second stage, they decide on their respective steady state prices. These decisions have to take into account consumer dynamics because of the multi-period nature of the frequency reward program and the consumer utility maximization process over a long time horizon. Although the reward program is two-period, it is essentially a multi-period process from the firm's perspective. Consumers who bought the product in Period 1 will be ready to cash in during Period 2 and new consumers will be buying the product in Period 2, who in turn will be ready to cash in during Period 3 and so on. Although a two-period model would be simpler, it turns out that we cannot obtain our results in closed form even for a two-period game. Further, a two-period game ends in the second period and consumers would have no incentive to buy an FP firm's product in the second period; whereas, a multi-period model allows consumers to buy the product in any period. Also, the qualitative nature of our results did not differ between a two-period model and a multi-period model. Finally, a multi-period model would help determine the steady state result with respect to FP versus TP strategy and the particular pricing policy. Thus, we numerically solve for the sub-game perfect equilibrium of this two-stage game. This means

[1] We have also allowed the prices of the TP firm to vary probabilistically between two levels via a first order Markov process (Assunção and Meyer 1993). This represents a hi/lo promotional pricing strategy. The resulting equilibrium was constant low prices in all cases; hence, we use the simpler specification of constant prices.

that firms choose between frequency reward and traditional pricing strategies, cognizant of the equilibrium steady-state prices that will result depending on their and the competitor's actions.

Category Expandability

Category demand is not fixed: We assume a consumer chooses between two firms, but can also satisfy his or her needs by choosing from an alternative "outside category". The utility for this category will be denoted by U_0. For example, consumers can choose among competing airlines, or decide not to fly and instead use telephone or videoconference. Consumers can choose among grocery stores, or decide to purchase groceries from mass merchandisers such as Kmart or Wal-Mart. We adopt this feature first because it is realistic, but also because it allows us to study the impact of category expandability on the viability of frequency reward programs.

The alternative category is non-strategic: We assume the outside category provides a net utility U_0 to the consumer, but is non-strategic. It does not change its prices or product in response to activities in the focal category. For example, we assume that videoconference prices do not change as a result of frequency reward programs of airlines, or that Wal-Mart does not react strongly to price changes in grocery stores. In the long-run, video-conference prices as well as Wal-Mart might react to frequency reward programs, but in the intermediate term, we believe these markets are concerned with their own intra-category competition or cost structure. Wal-Mart has adopted an everyday low pricing strategy, its prices are driven primarily by its low costs, and it carries a much wider selection of products than grocery stores. We hence assume Wal-Mart does not react in the intermediate term to grocery store adoption of frequency reward programs. Another example is the airline industry; the focal category consists of the major airlines and the fringe/low-cost, small players form what is essentially an outside category.

---

[2] Note that we are not considering price promotions here, but a longer term pricing strategy. In most product categories, the regular price tends to remain fairly stable over time. Hence, price is considered a one-time decision

If in fact the outside category began to react to our focal category's pricing decisions, we would now have a three-firm competition and a new value of U_0 reflecting the strength of alternatives to this broader category. So we view the distinction between intra- and inter-category competition as an important characteristic of our model in that firms in any category face strategic as well as non-strategic players.

IV. CONSUMER BEHAVIOR MODEL

Consumers make purchase decisions to maximize total net discounted utility over a finite time horizon, [1,T]. In every period t, each consumer decides whether to purchase Firm 1's product, Firm 2's product, or neither. Let c_t denote the consumer's purchase decision in period t, i.e.,

$$c_t = \begin{cases} 0, \text{if neither Firm 1 nor Firm 2 is purchased in period } t \\ 1, \text{if Firm 1's brand is purchased in period } t \\ 2, \text{if Firm 2's brand is purchased in period } t \end{cases} \quad (1)$$

Let $U(c_t)$ denote the utility derived from c_t. The consumer's decision problem for each period t may be expressed as:

$$\max_{c_\tau} \sum_{\tau=t}^{T} \beta_c^{\tau-t} U(c_\tau) \quad (2)$$

where, $0 < \beta_c \leq 1$, denotes consumers' discount rate.

Let r_{it} denote a consumer's reservation price for firm i's product, i = 1, 2, in period t[3]. Consumers are heterogeneous with respect to their reservation prices and are represented by a density function, $f(r_i)$ (Kohli and Mahajan 1991); hence, r_{it} is a draw in period t from $f(r_i)$. At the

variable. We leave the inclusion of state-dependent, dynamic prices for future research.
[3] This reservation price should be interpreted as a single-period reservation price. Aside from future considerations, the consumer will not buy the product if price exceeds the reservation price. However, if future benefits are attractive, the consumer might actually purchase the product even if the current period price is greater than the reservation price. Note that sufficiently high values of a_i would ensure that reservations prices are not negative. Negative reservations prices would imply that a customer has to be paid to buy that product.

beginning of each period, consumers know their reservation prices for that period. Following Bulkley (1992), we assume that consumers are uncertain about their future reservation prices. Thus, when a consumer is deciding which airline to fly, she would know her reservation price for each airline in that period; however, she is uncertain about her future reservation prices since she may not be sure how much advance notice she will have and whether the flight schedules would meet her future needs. Consequently, consumers are aware that their future reservation price will be drawn from the distribution $f(r_i)$, but do not know the exact values of future draws of r_{it}. For tractability, we use a logistic distribution (which is similar to a normal distribution) for $f(r_i)$, with mean a_i. Thus, the probability density function of consumers' reservation prices for firm i, i=1,2, is given by,

$$f(r_i) = \frac{e^{a_i - r_i}}{(1+e^{a_i - r_i})^2} \qquad -\infty < r_i < \infty \tag{3}$$

In general then, consumer's utility in period t is given by,

$$U(c_t) = \begin{cases} r_{c_t t} - p_{c_t} & c_t = 1, 2 \\ U_0 & c_t = 0 \end{cases} \tag{4}$$

U_0 represents the utility the consumer can obtain by purchasing neither from Firm 1 or Firm 2. For consistency with our definition of reservation prices, we assume $U_0 > 0$. p_{c_t} is the price for c_t = Firm 1 or Firm 2.

Consumer Utility Maximization

Consumers maximize the sum of net discounted utility given the general model defined by equations (1)-(4). There are four cases depending on the pricing strategies undertaken by the firms, (FP,FP), (FP,TP), (TP,FP), or (TP,TP), where the first coordinate represents Firm 1's strategy and the second coordinate represents Firm 2's strategy. In this section we explore the

(FP,TP) case in some depth. We derive the specifics for this and for the other three cases in Appendix 1.

Assume Firm i offers a frequency reward program (FP) and Firm j follows traditional pricing (TP). Firm i sells its product at price p_i and if a consumer purchases Firm i's product in period t-1, she can cash in the frequency reward in period t and obtain Firm i's product for free in period t. If the reward is not cashed in during period t, it expires by the end of that period and cannot be carried forward to period t+1. This simple formulation captures the essence of frequency reward programs - they build up rewards that can be cashed in, and rewards that aren't cashed in become valueless at some point. Let I_{it} denote a consumer's inventory of Firm i's frequency reward in period t. We have,

$$I_{it+1} = \begin{cases} 0 & \text{if } c_t = 0, \text{or } c_t = j \\ 1 - I_{it} & \text{if } c_t = i \end{cases} \qquad (5)$$

Let p_j denote the prices of the traditional pricing Firm j. In the case under consideration now, where Firm i is FP and Firm j is TP, equation (4) can be rewritten as,

$$U(c_t) = \begin{cases} r_{it} - (1 - I_{it}) p_i & c_t = i \\ r_{jt} - p_j & c_t = j \\ U_0 & c_t = 0 \end{cases} \qquad (6)$$

where I_{it} is given by equation (5). Note that in any period consumers have a strong impetus to stay with the FP firm (Firm i) if they enter with an inventory of 1 in that period. However, the TP firm (Firm j) can always compete with price as consumer reservation prices for the two firms, r_{it} and r_{jt}, are heterogeneous, and consumers make decisions based on future *expected* preferences, not actual. Thus, it is still possible for the TP firm to capture some of the FP firm's "loyal" customers by charging a sufficiently low everyday price.

The consumer's utility maximization problem can now be reformulated as: in each period

t, given respective reservation prices for firms i and j in that period, r_{it} and r_{jt}, select c_t to maximize $\sum_{\tau=t}^{T} \beta_c^{\tau-t} U(c_\tau)$, where $U(c_\tau)$ is given by equation (6) and subject to prices p_i, p_j, and reward inventory defined by equation (5). We use stochastic dynamic programming to solve this problem, which is an optimal control problem in discrete time with I_{it} as the state variable, and c_t as the control variable. Following Bellman's principle of optimality (Stokey and Lucas, 1989), solving this problem is equivalent to solving the following recursive equation:

$$V_t(I_{it}) = \max_{c_t} \begin{cases} U(c_t) + \beta_c(E[V_{t+1}(I_{it+1})]) & t = 1,\ldots,T-1 \\ U(c_T) & t = T \end{cases} \tag{7}$$

where U(•) is defined in equation (6).

V. MODELING FIRM BEHAVIOR

As discussed in Section III, we model the competition between Firms 1 and 2 as a two-stage game. In the first stage, firms choose either the frequency reward program (FP) or the traditional pricing program (TP). In the second stage, both firms set prices. This results in four sub-games, namely, (FP,FP), (FP,TP), (TP,FP), and (TP,TP). We use sub-game perfect Nash equilibrium to determine the equilibrium FP or TP strategy (Fudenberg and Tirole 1992). For each sub-game, the firms determine the Nash equilibrium prices by maximizing the sum of net discounted expected profits over the time horizon [1,T].

We derive expressions for net discounted expected profits in Appendix 2. These follow directly from consumer utility maximization, since that determines demand. To calculate expected demand in each period, we first condition on the reward inventory level, which can equal either zero or one. Then we integrate over the distribution of reservation prices to determine the conditional percentage of customers who will buy from Firm 1, Firm 2, and the outside category. We next determine the unconditional probabilities of observing the two reward inventory levels and use these to weight the corresponding conditional purchase percentages to derive expected demand. The expected profit is obtained by multiplying the expected paying demand with unit contribution. Without loss of generality, we assume the variable cost to be zero.

VI. INSIGHTS FROM THE CONSUMER MODEL

We generate three insights from the consumer model that will be important for understanding our simulation results. We first draw on a two-period[4] version of equations 6-7 to derive an expression for the incremental value to the consumer of an FP firm's reward. Consider the case where Firm *i* pursues FP while Firm *j* pursues TP. In this case, the discounted expected value for the consumer who carries one unit inventory of Firm *i*'s frequency reward to Period 2 is:

$$\beta_c E[V_2(1)] = \beta_c E[\max(U_0, r_{i2}, r_{j2}\text{-}p_j)]$$

In other words, the expected value of carrying one inventory is the expected maximum utility derived from the three options the consumer faces in the second period: buying outside (U_0), staying with the firm whose inventory the consumer has (i.e., Firm *i*), or buying from the other firm. Similarly, the corresponding discounted expected value of carrying zero inventory of Firm *i*'s frequency reward to Period 2 is given by,

$$\beta_c E[V_2(0)] = \beta_c E[\max(U_0, r_{i2}\text{-}p_i, r_{j2}\text{-}p_j)]$$

Hence the discounted expected *gain* to the consumer for carrying one unit inventory of Firm *i*'s frequency reward, i.e., the value of the reward, is:

$$\beta_c\{E[V_2(1)] - E[V_2(0)]\} \qquad (8a)$$

$$= \beta_c\{E[\max(U_0, r_{i2}, r_{j2}\text{-}p_j)] - E[\max(U_0, r_{i2}\text{-}p_i, r_{j2}\text{-}p_j)]\} \qquad (8b)$$

Below we describe the insights generated from the consumer model.

[4] The insights in this section hold up numerically in the full multi-period model, but it is more clear if we motivate these insights using the two-period model.

RESULT 1: The value of a frequency reward is increasing in β_c. This can be seen easily from either equations 8a or 8b, and is quite intuitive. The FP firm's frequency reward is awarded in the future, so its value is proportional to β_c, the degree to which consumers value the future. The ramification is that frequency reward programs are much more powerful promotions when consumers value the future.

[Insert Table 1 About Here]

Table 1 shows how this affects demand for the FP firm. When $\beta_c = 1$, the FP firm garners much more demand than when $\beta_c = .5$.

RESULT 2: The value of the frequency reward is increasing in the price charged by the FP firm. This can be seen from equation 8b. Note that the first term of that equation is independent of p_i while the second term depends on p_i. The term, $E[\max(U_0, r_{i2}-p_i, r_{j2}-p_j)]$, is decreasing in p_i, since the maximum will obviously have to decrease as p_i increases. As a result, the value of the reward, represented by equation 8b, is increasing in p_i. This is a subtle finding but also makes good intuitive sense. The reward in this case is free product. Obviously that reward becomes more valuable if the product is worth more.

RESULT 3: The expandability of category demand is increasing in U_0. This can be seen by inspection of equation (6). If U_0 is low, (relative to a_1 and a_2), the outside category is not viable even at very high prices, p_i and p_j. The two firms already collectively "own" the customer and the market is large and cannot be expanded. When U_0 is high, the outside category is an attractive option for consumers. In this case, demand for the two firms is naturally quite low although it can be increased through lower prices – thus the market is expandable. So U_0 governs the expandability of the market. The demand ramifications of this are illustrated in Table 2 in the case where both firms follow FP.

[Insert Table 2 About Here]

When U_0=4, the outside category is unattractive and the two firms own the market. In this case, when Firm 1 decreases its price from $3 to $2.50, it gains six share points but that demand comes from Firm 2, not from growing the market. However, when U_0=10, the outside category is attractive and depresses demand for Firms 1 and 2. Now when Firm 1 decreases its price, it gains four share points but only 1 of those share points comes from Firm 2. The other 3 come from the outside category – the price decrease has expanded the category.

VII. SIMULATION METHOD

The model described in Sections IV and V was solved numerically using Mathematica. The proof for the existence of the equilibrium and the approach used mirrors Ansari et al. (1994)[5]. The major steps are as follows:

1. Specify a sub-game (e.g., (FP,TP)).
2. For a given set of prices for Firms *i* and *j*, calculate the expected value functions for the consumer utility maximization (Appendix 1).
3. Calculate expected demand (Appendix 2).
4. Calculate expected profit (Appendix 2).
5. Repeat steps 2-4 over a range of prices.
6. Search over all combinations of prices for the Nash Equilibrium to this sub-game.
7. Repeat steps 2-6 for the other three sub-games.
8. We now have a Nash solution for the prices in each sub-game. Next, search among these sub-games for the sub-game perfect Nash solution for whether to adopt FP or TP.

The parameter values chosen for the base case analysis are given in Table 3.

[Insert Table 3 About Here]

[5] Since the firms' pricing policies are constant in the time horizon considered, it makes it possible for us to follow Ansari et al. (1994).

Thus, in the base case we assume consumers value the next period as strongly as they value the first, that firms are equally preferred on average, and collectively are somewhat more attractive than the outside category available if neither of these firms' products are purchased. We will vary the discount factor (β_c), outside category preference (U_0), and relative preference (a_1 and a_2) to gain additional insights after we examine the base case.

VIII. SIMULATION RESULTS

Base Case

We first examine equilibrium strategies for the base case. To remove the beginning-game and end-game effects, we analyze profit in the steady state. The equilibrium is shown in Table 4 and the profits reported are expressed on a 100-period basis.

[Insert Table 4 About Here]

The sub-game perfect Nash equilibrium is for each firm to offer a frequency reward program (FP/FP). The equilibrium prices and profits for both firms are \$2.8 and \$70.6 respectively. The reader will readily recognize that in this case, reward programs have created a classic prisoner's dilemma. TP/TP is clearly not an equilibrium because each firm would have an incentive to adopt FP and earn higher profits. However, in the FP/TP solution, the TP firm's profit is lowered so much that it too has an incentive to switch to FP. Thus, the two firms end up in an FP/FP equilibrium.

The power of the FP program in instigating this prisoner's dilemma stems from our base case assumption that β_c=1, which means that the reward program is very attractive to consumers (Result 1). This enables the FP program to "lock up" customers for multiple periods. This is an aspect that TP cannot match. Further, the TP firm in the FP/TP sub-game is sufficiently hurt that it has an incentive to switch to FP.

The competitiveness of the FP/FP cell, which defines the prisoner's dilemma, occurs because in our base case, the source of demand generated by FP comes largely from the other firm, not from the outside category. Result 3 shows that when U_0 is lower, demand for the two firms is relatively weak and cannot be increased through lower prices, i.e., the focal category is not very expandable. In our base case, U_0=8 is in a middle range, judging from Table 3. For this value of U_0, enough of the incremental sales from the frequency program come from the competition that this evokes a strong competitive response, driving down profits for both firms.

In summary, the results we generated in Section VI strongly determined the results for our base case. The high value of β_c meant that the FP program would be a strong promotion (Result 1), and the moderate value of U_0 meant that much of the incremental sales from this promotion would come from the direct competitor (Result 3). As a result, we had the elements for a classic prisoner's dilemma.

Sensitivity Analysis

Varying β_c: Table 5 illustrates the equilibria depending on whether consumers highly value future benefits (β_c=1), versus place little value on future benefits (β_c=.4). Table 5 shows the following:

[Insert Table 5 About Here]

- As consumers put less value on future benefits, firms gain less from unilaterally defecting from a traditional pricing (TP) equilibrium. This is because the gain in demand is less since consumers don't value the future benefits promised by FP (Result 1).

- As consumers put less value on future benefits, an industry equilibrium of FP/FP becomes less attractive. This is because FP is not a strong promotion any more (Result 1) and it requires steep price decreases to gain any customers.

• Due to the above two factors, frequency reward programs are less likely to be an equilibrium if consumers care less about the future. If β_c is equal or less than .6, FP does not generate enough incremental demand to make it an attractive defection from the TP/TP cell. Thus, both firms follow TP.

Varying the expandability of the category (U_0): Result 3 shows that the potential for reward programs to expand the category increases as U_0 increases. The ramifications of this are illustrated in Table 6.

[Insert Table 6 About Here]

The specific findings of Table 6 are:

• Frequency reward programs are not an equilibrium if the outside category is unattractive ($U_0 \leq 6$). In this case, incremental sales gains from frequency reward programs must come from the direct competitor (Result 3). As a result, frequency reward programs stimulate significant competitor response and potential profits from incremental sales are competed away with price competition.

• If the outside category is more attractive ($U_0 \geq 8$), the focal category is potentially expandable (Result 3), and frequency reward programs are the sub-game perfect equilibrium. In this case, these programs are less threatening to direct competitors, and competition is less intense. When $U_0=8$, our prisoner's dilemma base case, there is enough competition that the FP/FP equilibrium does not improve profits over TP/TP. However, when $U_0=10$, much of the effect of the strong FP promotion is to expand the category. This invokes less competitive response and industry profits actually improve with FP/FP (compared to FP/TP or TP/TP when $U_0=10$).

Varying a_1 and a_2: Table 7 illustrates the equilibrium profits and prices depending on whether the mean reservation price for Firm 1's product is lower (a_1=9) versus when it is higher (a_1=11). (We would obtain symmetric results when a_2=9 or 11.)

[Insert Table 7 About Here]

Table 7 shows the following:

• There exist multiple equilibria (FP/TP or TP/FP). It turns out that if one firm chooses FP, the other firm would choose TP and vice versa.

• In moving from TP/TP to FP/TP, the firm that gains most is the one with the higher mean reservation price (a_i). This is a direct consequence of Result 2, that the value of a reward program is higher for a higher priced firm. This is because the "free" product received as the reward has a higher value. The firm with higher mean reservation price generally can charge a higher price (it has more monopoly power) and hence benefits more when it offers a reward program.

• Finally, to cross-check the results of varying U_0 and β_c, we varied U_0 and β_c for the different levels of a_1 and a_2. We find that the main effects of U_0 and β_c observed earlier still hold, i.e., when U_0 is low, the equilibrium solution is to offer TP/TP and when U_0 is high, the equilibrium shifts to FP/FP. Similarly, when β_c is low, the equilibrium is TP/TP.

IX. EXPLORATORY EMPIRICAL SUPPORT

The airline industry provides a rich context to explore some of the important implications of our model. In particular, this case provides evidence that the airlines introduced reward programs to counter-act a strong outside category (new entrants), and in doing so, they expanded their market. These findings are consistent with our model.

The Airline Deregulation Act was approved in 1978 and it opened the airline business to low-cost new entrants. Dozens stepped in (Air Transport Association 1999). In 1978 the number of scheduled passenger airlines was 26; by 1981 there were 86 (Air Transport Association's Annual Reports 1979, 1982). These new entrants provided stiff competition for the entrenched "major" airlines. The airline industry can thus be characterized by two categories, the "major" airlines serving nationwide, and the new entrants, which are primarily regional carriers. As a result of deregulation, the new entrants category grew, providing competition to the major airlines. This situation is reflected in our model by the outside category (low-cost new entrants) becoming more popular, i.e., increasing U_0. Our results suggest that high U_0 is a situation conducive to reward programs, and we would therefore predict conditions were right for the major airlines to introduce reward programs. Figure 1 shows this is precisely what occurred. Figure 1 shows (a) the increase in the number of new entrants from 1971 and 1986 with a dramatic jump between 1978 and 1981, and (b) the coincident introduction of frequency reward programs by the major airlines, and (c) the introduction of frequency programs by two fringe/smaller players after a considerable delay (about five years).

[Insert Figure 1 About Here]

Specifically, American Airlines introduced its frequent-flier program in May 1981. An article in Aviation Week and Space Technology (1983) notes,"*American introduced frequent flyer benefits to counter the rise of low-cost new entrants offering new services and low prices*". Within three months, the other 6 major airlines offered their respective frequency reward programs (Associated Press, November 19, 1982). We found no evidence of the immediate adoption of frequency reward programs by the new entrants. Five years later, two of the low cost airlines (People's Express and Southwest Airlines) offered a frequency reward program. They finally did so as a last effort to compete with the major airlines as they were losing passengers to the "majors" (Newsweek, May 12, 1986; Business Week, May 12, 1986; Aviation

Week and Space Technology, July 6, 1987).

Thus, we find that, consistent with our results, the airlines introduced reward programs when an outside category became more preferred (higher U_0). In this situation, we would also predict that reward programs would help grow the market for the major airlines. The question is whether this occurred. Indeed, there is some public-press evidence that the frequent-flier programs were proposed to overcome sluggish traffic (Aviation Week and Space Technology, November 14, 1983). It was reported that the frequent-flier programs were invented by the nation's airlines to buckle more people into their jets (New York Times, November 25, 1982). Domestic revenue passenger miles increased by 11.5 billion between 1981 and 1982 and another 16.8 billion the following year (Air Transport Association's Annual Reports 1982, 1983).

In order to collaborate these reports with hard data, we compiled a database from various sources (Air Transport Association's Annual Reports, Leading National Advertisers (LNA), Statistical Abstracts of the United States, and Standard and Poor's Industry Surveys). The database includes annual statistics from 1971 to 1997 on the following:

1. Annual revenue passenger miles of all the major airlines *(RPM)*
2. Inflation-adjusted price *(IP)*
3. Gross national product *(GNP)*
4. Number of passenger airlines *(NPA)*
5. Cumulative number of years since frequency reward programs were introduced (*CUMFP*)
6. Advertising dollars spent by the major airlines (*ADV*)

Although frequency reward programs were first instituted in 1981, their impact on demand may not be seen immediately due to the innovation diffusion effect (cf. Sultan, Farley, Lehmann 1990). Typical innovations diffuse gradually as only innovators adopt them at first, followed by the early and late "majority" who adopt later. We therefore capture the impact of frequency reward programs by the cumulative number of years since their introduction. Thus, to test whether the frequency reward programs increased demand for the airlines that instituted such programs (i.e., the major airlines), we estimated the following regression equation:

$$RPM_t = \alpha + + \beta_1 IP_t + \beta_2 GNP_t + \beta_3 NPA_t + \beta_4 CUMFP_t + \beta_5 ADV_t + \varepsilon_t \qquad (9)$$

where the subscript t indicates time and ε_t is the error term.

Table 8 presents the Yule-Walker estimates (Gallant and Goebel 1976) of equation (9) after correcting for first-order autocorrelation.

[Insert Table 8 About Here]

The findings in Table 8 suggest that the reward program increased demand for the major airlines. This is consistent with our theory that reward programs are useful when they can expand the market. Advertising and GNP were also key drivers of demand for the major airlines. The number of passenger airlines as well as real price have the correct negative signs, but are not statistically significant. The coefficients for these variables were highly correlated (-.86). Clearly there is multicollinearity in the data that makes it difficult to disentangle the effects of all the variables. However, the effect of the reward program comes through clearly[6].

Undoubtedly there were several reasons for the introduction of reward programs in the airline industry, and we do not purport to have made a comprehensive study. However, we have provided strongly suggestive evidence that the major airlines introduced reward programs at least in part in reaction to the growing popularity of an outside category, represented in our model by a higher U_0, and in so doing, they expanded their market. Market expansion is an important

[6] We also added a dummy variable equal to one beginning in the year the reward programs were introduced, and zero in preceding years. This would capture any step change in demand that happened immediately as a result of the reward program. This variable, however, was insignificant, while the *CUMFP* variable was still significant. We

condition for reward programs, as shown by our model.

X. SUMMARY AND DISCUSSION

We have conducted an economic analysis of frequency reward programs to understand the conditions under which such programs are a result of strategic competition in a duopoly. Our model includes utility maximizing customers who take into account current as well as future utility. Firms compete for these customers in a two-stage game, where in the first stage they decide whether to offer frequency reward or traditional pricing program – and in the second stage select prices. We solve this game numerically for sub-game perfect equilibrium and vary parameters to gain insights. Following are our major conclusions:

- *Frequency reward programs are fundamentally powerful, multi-period promotions.* They are promotions in the sense that they offer customers a financial incentive to change their behavior (cf. Blattberg and Neslin 1990 pp. 1-3). They are multi-period in that they force customers to think of future as well as current purchases. This is unique to frequency reward programs in comparison with typical promotions. The unique power of frequency reward programs is that if the firm wins a customer, it can lock in the customer for more than one purchase occasion. Most promotions simply attract a customer for the current purchase occasion – any post-promotion usage effect is not explicit to the promotion[7].
- *The power of frequency reward programs as promotions, and their likelihood of being equilibrium outcomes, is a function of the extent to which consumers value future benefits.* Frequency reward programs offer the customer benefits that actualize in the future. It is therefore not surprising that they are much more powerful, and much more likely to enhance

therefore dropped the dummy variable from the model. The results suggest that the reward program affected demand gradually as is consistent with the diffusion of an innovation.

[7] Although see Kopalle, Mela, and Marsh (1999) for an empirical and economic analysis of promotions that do have long-term effects.

firm profits, if consumers care more about the future.

- *A key factor determining the economic viability of frequency reward programs is the extent to which they can be used to expand the category.* To the extent that frequency reward programs draw mainly from competitors, they can accentuate price competition and decrease profits. The reason is because they are such powerful multi-period promotions. They therefore invite strong competitive response and the resulting equilibrium is lower per-unit prices and lower profits from the same market size. If, however, frequency reward programs can expand the category, their power as a promotion device makes them especially attractive. In this case, they are the equilibrium outcome. Indeed, our exploratory empirical analysis of the airline industry provided evidence that the "major" airlines introduced reward programs to counter-act a stronger outside category (new entrants), and in doing so, they expanded their market.
- *The view emerging from our research is that frequency reward programs are most viable in expandable markets where customers do not highly discount future benefits.* However, if category demand is inelastic or if customers discount future benefits a lot, they can stimulate intense price competition and will not be useful marketing instruments. In our model, frequency reward programs enhance behavioral loyalty, not attitudinal loyalty. They mechanically entice the customer to purchase the product several times, and are not designed to change their fundamental attitudes or preferences for the brand. Under these conditions, frequency reward programs just become a higher stakes form of promotion. As customers use up their "points," they become vulnerable to competitive offers again, and firms compete heavily for customers' multi-period behavioral loyalty.
- *Frequency reward programs are especially effective for strong brands.* The reason is that

strong brands command higher prices, and this accentuates the value of a reward program. The key assumptions here are: (1) the reward program does not degrade the brand, and (2) the reward is in the form of free or discounted product, not cash or gifts (cf. Kim et al. 1996).

Our work has several implications for managers, as follows:

- *Frequency reward programs should be considered when customers do not highly discount future benefits.* Good examples are the airlines and grocery industries. Certainly the frequent traveler in the airline industry highly values the future benefits of a free upgrade, trip, etc. In the grocery industry, the rewards are not as high, but the frequency of shopping makes the frequency reward program attractive to customers. Durable goods products such as automobile products are probably not good markets for frequency reward programs. The benefit is so far in the future that it will be highly discounted by the customer.
- *Frequency reward programs should be considered when the program can expand the category.* The U.S. supermarket and airline industries might be examples where this condition holds. The supermarket industry in the U.S. has recently been under attack from mass merchandisers such as Wal-Mart and Kmart. It therefore makes sense for supermarkets to adopt frequency reward programs because at least part of the effect of the program is to attract customers back from the mass merchandisers. Note the important presumption that the mass merchandisers will not retaliate with their own reward program. This is particularly true in Wal-Mart's case because of its cost driven pricing strategy that does not rely too much on price promotions, and since its product line is so much broader than those of the U.S. supermarkets. Interestingly, the U.K. supermarket industry is an arena where reward programs may have been a mistake. In this case, two chains – Sainsbury's and Tesco - dominated the market and found themselves in "points wars" once they started frequency

reward programs (Benady and Brierley 1996; Fletcher 1996). A category seemingly ripe for a reward program would be the cellular telephone usage market. This is an expandable market since many consumers now own cell phones. Some of the demand generated from a reward program would undoubtedly come from existing phone service, but this would be a prime opportunity for generating category demand from new usage situations (Wansink and Ray 1996).

- *Managers should not automatically increase the reward for an underperforming reward program.* It is tempting to "fix" an under-performing reward program by increasing the reward. This may be played out in higher point rewards, stronger partners, or highly targeted programs. However, this may simply be playing out the stiff price competition we found in markets where customers care less about the future benefits or when category demand is inelastic. Managers should be careful before increasing frequency program rewards. The key questions are the extent to which consumers discount the future benefits and whether the resultant incremental demand is hurting direct competitors.
- *Strong brands should consider frequency reward programs as a promotional vehicle.* Strong brands have a built-in advantage if the rewards from their programs are free or discounted product, since that product has intrinsically high value. It is interesting to note that indeed many strong brands have embraced reward programs – e.g., the top tier airlines, Hilton and Marriott Hotels, etc. The airlines use of first-class upgrades is an interesting illustration. Airlines have in recent years charged higher and higher prices for first class or business class seats. The classic explanation for this of course would be price discrimination. However, our analysis suggests an additional explanation – the airlines are just increasing the value of the rewards they offer through their frequent-flier programs.

- *One possible avenue for using frequency reward programs effectively, especially in mature markets, is to think of them as "loss leaders" contributing toward a more strategic goal.* That goal might be the accumulation and "mining" of a customer database (Reynolds 1995). Or perhaps frequency reward programs can be part of an overall loyalty program that includes direct selling, cross-selling, and other relationship-building activities.

We believe the analysis in this paper helps to contribute toward our understanding of frequency reward programs. However, there is more work to be done. For example, one of our assumptions is that the behavioral loyalty induced by frequency reward programs does not lead to fundamental attitude or preference changes. This could be investigated through survey or experimental data. Empirical work also needs to be undertaken on the types of categories where consumers are likely to value future rewards. In fact, we need to learn the extent to which future reward valuation is an individual difference trait rather than a category characteristic. Finally, future research could relax the assumption of constant prices and examine the impact of a dynamic pricing policy on frequency reward programs. We hope this work encourages others to continue working on this managerially important and conceptually rich marketing strategy.

Table 1

Demand Assuming Different Values for Future Discount Parameter, β_c

		$\beta_c = 1$ (FP price = 3.10 TP price = 1.70)	$\beta_c = .5$ (FP price = 3.10 TP price = 1.70)
Demand	FP Firm:	.446	.356
	TP Firm:	.389	.425
	Outside Category:	.165	.219

Table 2

Effectiveness of Frequency Reward Program Assuming Outside Category Is Unattractive Versus Attractive

		U_0 = 4 (Outside Category is unattractive)	U_0 = 10 (Outside category is attractive)
Demand	Firm 1, FP	.50	.11
(Firm 1's FP Price=$3.00,	Firm 2, FP	.50	.11
Firm 2's FP Price=$3.00)	Outside Category	0	.78
Demand	Firm 1, FP	.56	.15
(Firm 1's FP Price=$2.50,	Firm 2, FP	.44	.10
Firm 2's FP Price=$3.00)	Outside Category	0	.75

Table 3

Parameter Values for the Base Case

Parameter Description	**Value**
Consumer discount rate, β_c	1.0
Firms' discount rate, β_f	1.0
Mean reservation price for Firm 1's product, a_1	10.0
Mean reservation price for Firm 2's product, a_2	10.0
Utility of outside category, U_0	8.0
Time horizon, T	120

Table 4

Base Case Strategies

		Firm 2	
		Frequency Program (FP)	Traditional Pricing (TP)
Firm 1	Frequency Program (FP)	Profit = 70.6, 70.6 Price = 2.8, 2.8	Profit = 80.0, 65.0 Price = 3.1, 1.7
	Traditional Pricing (TP)	Profit = 65.0. 80.0 Price = 1.7, 3.1	Profit = 73.60, 73.60 Price = 1.9, 1.9

Note: In any pair, the first number refers to Firm 1, and the second number refers to Firm 2

Table 5

Impact of β_c on the Equilibrium Strategy

	β_c=.4			β_c=.8		
		Firm 2			Firm 2	
	Firm 1	FP	TP	Firm 1	FP	TP
Profits Prices	FP	56.8, 56.8 2.3, 2.3	64.7, 67.2 2.6, 1.8	FP	**66.1, 66.1** **2.6, 2.6**	74.4, 65.2 2.9, 1.7
Profits Prices	TP	67.2, 64.7 1.8, 2.6	**73.6, 73.6** **1.9, 1.9**	TP	65.2, 74.4 1.7, 2.9	73.6, 73.6 1.9, 1.9
	β_c=.6			β_c=1		
		Firm 2			Firm 2	
	Firm 1	FP	TP	Firm 1	FP	TP
Profits Prices	FP	61.9, 61.9 2.5, 2.5	70.5, 66.8 2.8, 1.8	FP	**70.6,70.6** **2.8, 2.8**	80.1, 65.0 3.1, 1.7
Profits Prices	TP	66.8, 70.5 2.8, 1.8	**73.6, 73.6** **1.9, 1.9**	TP	65.0. 80.1 1.7, 3.1	73.60, 73.60 1.9, 1.9

NOTE: Bold cells are the sub-game perfect equilibria. The first number in any pair refers to the row firm; the second number in any pair refers to the column firm.

Table 6

Impact of U_0 on the Equilibrium Strategy

	U_0=4			U_0=8		
		Firm 2			Firm 2	
	Firm 1	FP	TP	Firm 1	FP	TP
Profits Prices	FP	102.7, 102.7 3.7, 3.7	130.1, 106.9 4.6, 2.2	FP	**70.6,70.6** **2.8, 2.8**	80.1, 65.0 3.1, 1.7
Profits Prices	TP	106.9, 130.1 2.2, 4.6	**149.7, 149.7** **3, 3**	TP	65.0. 80.1 1.7, 3.1	73.60, 73.60 1.9, 1.9
	U_0=6			U_0=10		
		Firm 2			Firm 2	
	Firm 1	FP	TP	Firm 1	FP	TP
Profits Prices	FP	98.9, 98.9 3.6, 3.6	121.5, 98.7 4.3, 2.1	FP	**26.1, 26.1** **1.6, 1.6**	27.4, 24.0 1.7, 1.3
Profits Prices	TP	98.7, 128.5 2.1, 4.3	**128.8, 128.8** **2.7, 2.7**	TP	24.0, 27.4 1.3, 1.7	24.8, 24.8 1.3

NOTE: Bold cells are the sub-game perfect equilibria. The first number in any pair refers to the row firm; the second number in any pair refers to the column firm.

Table 7

Impact of Varying a_1 and a_2 on the Equilibrium Strategy

			Firm 2 (a_2=10)	
			FP	TP
Profits Prices	Firm 1 (a_1=9)	FP	17.26, 37.71 2.0, 3.1	**22.01, 38.99** **2.2, 1.8**
Profits Prices		TP	**18.11, 46.64** **1.4, 3.4**	20.22, 41.30 1.5, 1.9
			Firm 2 (a_2=10)	
			FP	TP
Profits Prices	Firm 1 (a_1=11)	FP	48.52, 23.86 3.7, 2.5	**63.40, 26.40** **4.2, 1.6**
Profits Prices		TP	**53.01, 33.72** **2.1, 2.9**	57.88, 32.24 2.4, 1.8

Table 8

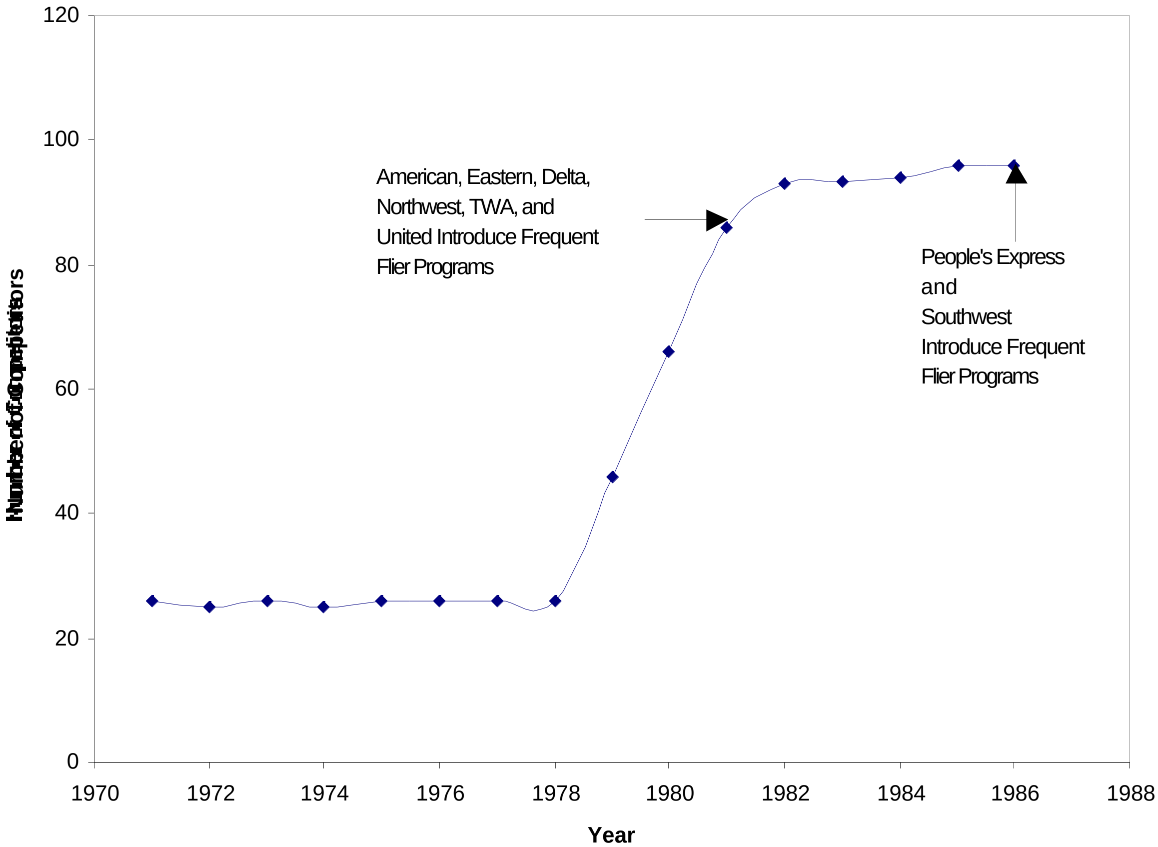
Regression Results (t-values in parentheses)

Dependent Variable: *RPM*, revenue passenger miles of all major airlines

Independent Variables	Parameter Estimate
Intercept	-86627 (-.98)
Number of passenger airlines, *NPA*	-18.33 (-.05)
Gross national product, *GNP*	2138.44 (4.12)
Inflation-adjusted price, *IP*	-361.24 (-.75)
Cumulative years since reward programs were introduced, *CUMFP*	11327.00 (3.79)
Advertising by major airlines, *ADV*	.099 (2.15)
Sample size	27
Adjusted R-square	0.88
Durbin-Watson	1.75
Autoregressive coefficient	-0.21

FIGURE 1

New Entrants and the Introduction of Reward Programs by the Major Airlines



APPENDIX 1

Expected Value Function for Consumer Utility Model

(FP,TP) Case:

Firm *i* follows frequency reward program (FP), Firm *j* follows a traditional pricing program (TP). Following Bellman's principle of optimality (Stokey and Lucas, 1989), solving Equation (2) is equivalent to solving the following recursive equation:

$$V_t(I_{it}) = \max_{c_t}\{U(c_t) + \beta_c(E[V_{t+1}(I_{it+1})])\} \qquad \text{(A1)}$$

Thus,

$$V_t(I_{it}) = \max_{0,i,j} \begin{matrix} U(0) + \beta_c E[V_{t+1}(0)] \\ U_t(i) + \beta_c E[V_{t+1}(I_{it+1})] \\ U_t(j) + \beta_c E[V_{t+1}(0)] \end{matrix} \qquad \text{(A2)}$$

where, $U(0) = U_0$, $U_t(i) = r_{it} - (1-I_{it})\, p_i$, and $U_t(j) = r_{jt} - p_j$.

Backward Induction, Period T case:

When $t = T$, the last period, we have,

$$E[V_T(I_{iT})] = E[\max_{0,i,j} \begin{matrix} U_T(0) \\ U_T(i) \\ U_T(j) \end{matrix} ?] = E[\max_{0,i,j} \begin{matrix} U_0 \\ r_{iT} - (1-I_{iT})\,p_i \\ r_{jT} - p_j \end{matrix} ?] \qquad \text{(A3)}$$

Let $Y_T = \max(r_{iT}-(1-I_{it})p_i,\ r_{jT}-p_j)$. By conditioning on whether Y_T is greater than U_0, i.e., whether the more attractive of the two firms is better or worse than the purchase-neither option, we obtain, $E[V_T(I_{iT})] = E(Y_T \mid Y_T > U_0)P(Y_T > U_0) + U_0 P(Y_T \le U_0)$. The cumulative density function of Y_T is given by,

$$G(Y_T) = P(Y_T \le y_T) = P(r_{iT} - (1-I_{iT})\,p_i \le y_T)P(r_{jT} - p_j \le y_T)$$

Thus, $G(Y_T) = \dfrac{1}{(1+e^{a_i-(1-I_{it})p_i-y_T})(1+e^{a_j-p_j-y_T})}, -\times < y_T < \times$, and the p.d.f. is:

$$g(y_T) = \frac{1}{(1+e^{a_i-(1-I_{it})p_i-y_T})(1+e^{a_j-p_j-y_T})}\left[\frac{e^{a_i-(1-I_{it})p_i-y_T}}{1+e^{a_i-(1-I_{it})p_i-y_T}} + \frac{e^{a_j-p_j-y_T}}{1+e^{a_j-p_j-y_T}}\right], -\times < y_T < \times$$

We also need the conditional p.d.f. for Y_T. This is as follows:

$$g(y_T \mid y_T > U_0) = \frac{g(y_T)}{1 - G_{y_T}(U_0)} =$$

$$[\frac{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})}{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})-1}][\frac{1}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})}$$

$$[\frac{e^{a_i-(1-I_{iT})p_i-y_T}}{1+e^{a_i-(1-I_{iT})p_i-y_T}} + \frac{e^{a_j-p_j-y_T}}{1+e^{a_j-p_j-y_T}}]], \qquad U_0 < y_T < \times$$

Let $K = \frac{1}{1 - G_{Y_T}(U_0)}$. We have $\mathrm{E}(\mathrm{Y_T} \mid \mathrm{Y_T} > \mathrm{U_0}) = ,$

$$\overset{\times}{\underset{U_0}{}} y_T g(y_T \mid y_T > U_0) = K[\overset{\times}{\underset{U_0}{}}\frac{y_T e^{a_j-p_j} dy_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})^2} + \overset{\times}{\underset{U_0}{}}\frac{y_T e^{a_i-(1-I_{iT})p_i} dy_T}{(1+e^{a_j-p_j-y_T})(1+e^{a_i-(1-I_{iT})p_i-y}\ )^2}]$$

$$= \mathrm{K}[\mathrm{I_1} + \mathrm{I_2}\ \overset{\times}{\underset{\mathrm{U_0}}{]}}$$

$$\text{Now, } I_1 + I_2 = [\frac{y_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})} - \frac{1+e^{a_i-(1-I_{iT})p_i-y_T} + y_T e^{a_i-(1-I_{iT})p_i-y_T}}{(1+e^{a_j-p_j-y_T})(1+e^{a_i-(1-I_{iT})p_i-y_T})^2} dy_T +$$

$$\frac{y_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})} - \frac{1+e^{a_j-p_j-y_T} + y_T e^{a_j-p_j-y_T}}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})^2} dy_T\ ]$$

$$= \frac{2y_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})} - 2\ \frac{dy_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})} - I_1 - I_2$$

This implies, $I_1 + I_2 = \frac{y_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})} - \frac{dy_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})}$

Further, $\frac{dy_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})} = \frac{e^{a_j-p_{jT}} Log(e^{y_T} + e^{a_j-p_j}) - e^{a_i-(1-I_{iT})p_i} Log(e^{y_T} + e^{a_i-(1-I_{iT})p_i})}{e^{a_j-p_j} - e^{a_i-(1-I_{iT})p_i}}$

Hence,

$$I_1 + I_2 = \frac{y_T}{(1+e^{a_i-(1-I_{iT})p_i-y_T})(1+e^{a_j-p_j-y_T})} - [\frac{e^{a_j-p_{jT}} Log(e^{y_T} + e^{a_j-p_j}) - e^{a_i-(1-I_{iT})p_i} Log(e^{y_T} + e^{a_i-(1-I_{iT})p_i})}{e^{a_j-p_j} - e^{a_i-(1-I_{iT})p_i}}]$$

$$= \frac{y_T e^{2y_T}}{(e^{y_T} + e^{a_i-(1-I_{iT})p_i})(e^{y_T} + e^{a_j-p_j})} - [\frac{e^{a_j-p_{jT}} Log(e^{y_T} + e^{a_j-p_j}) - e^{a_i-(1-I_{iT})p_i} Log(e^{y_T} + e^{a_i-(1-I_{iT})p_i})}{e^{a_j-p_j} - e^{a_i-(1-I_{iT})p_i}}]$$

$$= y_T - \frac{y_T e^{2(a_j-p_j)}}{(e^{a_j-p_j} - e^{a_i-(1-I_{iT})p_i})(e^{y_T} + e^{a_{jT}-p_j})} + \frac{y_T e^{2(a_i-(1-I_{iT})p_i)}}{(e^{a_j-p_j} - e^{a_i-(1-I_{iT})p_i})(e^{y_T} + e^{a_i-(1-I_{iT})p_i})} -$$

$$[\frac{e^{a_j-p_j} Log(e^{y_T} + e^{a_j-p_j}) - e^{a_i-(1-I_{iT})p_i} Log(e^{y_T} + e^{a_i-(1-I_{iT})p_i})}{e^{a_j-p_j} - e^{a_i-(1-I_{iT})p_i}}]$$

But by construction, $y_T = \dfrac{e^{a_j - p_j}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(e^{y_T}) - \dfrac{e^{a_i - (1-I_{iT}) p_i}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(e^{y_T})$

Hence, upon substitution and simplification,

$$I_1 + I_2 = \frac{e^{a_j - p_j}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(\frac{e^{y_T}}{e^{y_T} + e^{a_j - p_j}}) - \frac{e^{a_i - (1-I_{iT}) p_i}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(\frac{e^{y_T}}{e^{y_T} + e^{a_i - (1-I_{iT}) p_i}}) -$$
$$\frac{y_T e^{2(a_j - p_j)}}{(e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i})(e^{y_T} + e^{a_j - p_j})} + \frac{y_T e^{2(a_i - (1-I_{iT}) p_i)}}{(e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i})(e^{y_T} + e^{a_i - (1-I_{iT}) p_i})}$$

Thus, $[I_1 + I_2]_{U_0}^{\times} = -\dfrac{e^{a_j - p_j}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(\dfrac{e^{U_0}}{e^{U_0} + e^{a_j - p_j}}) + \dfrac{e^{a_i - (1-I_{iT}) p_i}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(\dfrac{e^{U_0}}{e^{U_0} + e^{a_i - (1-I_{iT}) p_i}}) +$

$$\frac{U_0 e^{2(a_j - p_j)}}{(e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i})(e^{y_T} + e^{a_j - p_j})} - \frac{U_0 e^{2(a_i - (1-I_{iT}) p_i)}}{(e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i})(e^{y_T} + e^{a_i - (1-I_{iT}) p_i})}$$

$$= -\frac{e^{a_j - p_j}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(\frac{e^{U_0}}{e^{U_0} + e^{a_j - p_j}}) + \frac{e^{a_i - (1-I_{iT}) p_i}}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} Log(\frac{e^{U_0}}{e^{U_0} + e^{a_i - (1-I_{iT}) p_i}}) +$$
$$\frac{U_0 e^{2(a_j - p_j)}}{(e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i})(e^{y_T} + e^{a_j - p_j})} - \frac{U_0 e^{2(a_i - (1-I_{iT}) p_i)}}{(e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i})(e^{y_T} + e^{a_i - (1-I_{iT}) p_i})}$$

$$= \frac{e^{a_j - p_j} Log(e^{U_0} + e^{a_j - p_j}) - e^{a_i - (1-I_{iT}) p_i} Log(e^{U_0} + e^{a_i - (1-I_{iT}) p_i})}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} - \frac{U_0}{(1 + e^{a_i - (1-I_{iT}) p_i - U_0})(1 + e^{a_j - p_j - U_0})}$$

Since, $K = \dfrac{1}{1 - G_{Y_T}(U_0)}$, we get,

$$E[y_T \mid y_T > U_0] = \int_{U_0}^{\times} g(y_T \mid y_T > U_0) =$$

$$[\frac{(1 + e^{a_i - (1-I_{iT}) p_i - U_0})(1 + e^{a_j - p_j - U_0})}{(1 + e^{a_i - (1-I_{iT}) p_i - U_0})(1 + e^{a_j - p_j - U_0}) - 1}][\frac{e^{a_j - p_j} Log(e^{U_0} + e^{a_j - p_j}) - e^{a_i - (1-I_{iT}) p_i} Log(e^{U_0} + e^{a_i - (1-I_{iT}) p_i})}{e^{a_j - p_j} - e^{a_i - (1-I_{iT}) p_i}} -$$
$$\frac{U_0}{(1 + e^{a_i - (1-I_{iT}) p_i - U_0})(1 + e^{a_j - p_j - U_0})}]$$

Further, $P(Y_T > U_0) = 1 - P(Y_T \le 0) = 1 - \dfrac{1}{(1 + e^{a_i - (1-I_{iT}) p_i - U_0})(1 + e^{a_j - p_j - U_0})}$

Thus, E[$V_T(I_{iT})$] =

$$[\frac{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})-1}{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})}][\frac{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})}{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})-1}][$$

$$\frac{e^{a_j-p_j}Log(e^{U_0}+e^{a_j-p_j})-e^{a_i-(1-I_{iT})p_i}Log(e^{U_0}+e^{a_i-(1-I_{iT})p_i})}{e^{a_j-p_j}-e^{a_i-(1-I_{iT})p_i}}-\frac{U_o}{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})}]+$$

$$\frac{U_o}{(1+e^{a_i-(1-I_{iT})p_i-U_0})(1+e^{a_j-p_j-U_0})}$$

$$=\frac{e^{a_j-p_j}Log(e^{U_0}+e^{a_j-p_j})-e^{a_i-(1-I_{iT})p_i}Log(e^{U_0}+e^{a_i-(1-I_{iT})p_i})}{e^{a_j-p_j}-e^{a_i-(1-I_{iT})p_i}}$$

Special Case:

When a_i-$(1$-$I_{iT})p_i = a_j - p_j = a$, we have,

$$\mathrm{E}[y_T \mid y_T > U_0] = {}_{U_0}^{\times} g(y_T \mid y_T > U_0) =$$

$$[\frac{(1+e^{a-U_0})^2}{(1+e^{a-U_0})^2-1}]\,{}_{U_0}^{\times}\,\frac{2y_T e^{a-y_T}}{(1+e^{a-y_T})^3}=[\frac{(1+e^{a-U_0})^2}{(1+e^{a-U_0})^2-1}]\,{}_{U_0}^{\times}\,\frac{2y_T e^{a}e^{2y_T}}{(e^{y_T}+e^{a})^3}$$

$$=2e^{a}[\frac{(1+e^{a-U_0})^2}{(1+e^{a-U_0})^2-1}][\frac{-1-2y_T}{2(e^{a}+e^{y_T})}+\frac{y_T}{2e^{a}}+\frac{ye^{a}}{2(e^{a}+e^{y_T})^2}-\frac{Log(e^{a}+e^{y_T})}{2e^{a}}\,]_{U_0}^{\times}$$

To verify the integral, we find upon simplification,

$$\frac{d}{dy_T}[\frac{-1-2y_T}{2(e^{a}+e^{y_T})}+\frac{y_T}{2e^{a}}+\frac{ye^{a}}{2(e^{a}+e^{y_T})^2}-\frac{Log(e^{a}+e^{y_T})}{2e^{a}}]=\frac{2y_T e^{2y_T}}{(e^{y_T}+e^{a})^3}$$

$$\text{Thus,}\quad {}_{U_0}^{\times}\,g(y_T \mid y_T > U_0)=$$

$$2e^{a}[\frac{(1+e^{a-U_0})^2}{(1+e^{a-U_0})^2-1}][\frac{-1-2y_T}{2(e^{a}+e^{y_T})}+\frac{Log(e^{y_T})}{2e^{a}}+\frac{ye^{a}}{2(e^{a}+e^{y_T})^2}-\frac{Log(e^{a}+e^{y_T})}{2e^{a}}\,]_{U_0}^{\times}$$

$$=2e^{a}[\frac{(1+e^{a-U_0})^2}{(1+e^{a-U_0})^2-1}][\frac{e^{2a}+e^{a+U_0}-U_0e^{2U_0}+(e^{a}+e^{U_0})^2Log(e^{a}+e^{U_0})}{2e^{a}(1+e^{a-U_0})^2}]$$

$$=\frac{e^{2a}+e^{a+U_0}-U_0e^{2U_0}+(e^{a}+e^{U_0})^2Log(e^{a}+e^{U_0})}{(e^{U_0}+e^{a})^2-e^{2U_0}}$$

$$E[y_T \mid y_T > U_0]=\frac{e^{2a}+e^{a+U_0}-U_0e^{2U_0}+(e^{a}+e^{U_0})^2Log(e^{a}+e^{U_0})}{(e^{a}+e^{U_0})^2-e^{2U_0}}$$

$$P(Y_T > U_0) = 1 - P(Y_T \le U_0) = 1 - \frac{1}{(1+e^{a-U_0})^2} = \frac{(e^a + e^{U_0})^2 - e^{2U_0}}{(e^a + e^{U_0})^2}$$

$$\text{Hence, } \mathrm{E}[V_T(I_{iT}, p_{jT})] = [\frac{e^{2a} + e^{a+U_0} - U_0 e^{2U_0} + (e^a + e^{U_0})^2 Log(e^a + e^{U_0})}{(e^a + e^{U_0})^2 - e^{2U_0}}][$$

$$\frac{(e^a + e^{U_0})^2 - e^{2U_0}}{(e^a + e^{U_0})^2}] + U_0 \frac{1}{(1+e^{a-U_0})^2}$$

$$= \frac{e^{2a} + e^{a+U_0} + (e^a + e^{U_0})^2 Log(e^a + e^{U_0})}{(e^a + e^{U_0})^2}$$

Backward Induction, General Period *t* case:

Proceeding now to periods earlier than period T, Equation (7) can be rewritten fully as,

$$V_t(I_{it}) = \max_{0,i,j} \begin{array}{l} U_0 + \beta_c E[V_{t+1}(0)] \\ r_{it} - (1 - I_{it}) p_i + \beta_c E[V_{t+1}(I_{it+1})] \\ r_{jt} - p_j + \beta_c E[V_{t+1}(0)] \end{array} \tag{A4}$$

By conditioning in the same way we did for period *T*, we obtain,

$$E[V_t(I_{it})] = \frac{e^{b_{jt} - p_j} Log(e^{U_0 + b_{0t}} + e^{b_{jt} - p_j}) - e^{b_{it} - (1-I_{it}) p_i} Log(e^{U_0 + b_{0t}} + e^{b_{it} - (1-I_{it}) p_i})}{e^{b_{jt} - p_j} - e^{b_{it} - (1-I_{it}) p_i}} \tag{A5}$$

where,

$$b_{it} = a_i + \beta_c E[V_{t+1}(1 - I_{it+1})],$$
$$b_{jT-1} = a_j + \beta_c E[V_{t+1}(0)]\},$$
$$b_{0T-1} = \beta_c E[V_{t+1}(0)]$$

(FP,FP) Case - Backward Induction, General Period t:

When both firms follow a frequency reward program, let I_{it} and I_{jt} be consumers' inventory for firms *i* and *j*. Given the nature of the reward program, note that when $I_{it} = 1$, $I_{jt} \neq 1$ and vice-versa, i.e., for any consumer, $I_{it} = 1$ or 0, $I_{jt} = 1$ or 0, and $I_{it} + I_{jt} = 1$ or 0. Further,

$$I_{it+1} = \begin{array}{l} 0, \text{if } c_t = 0 \text{ or } c_t = j \\ 1 - I_{it}, \text{if } c_t = i \end{array}$$

$$I_{jt+1} = \begin{array}{l} 0, \text{if } c_t = 0 \text{ or } c_t = i \\ 1 - I_{jt}, \text{if } c_t = j \end{array}$$

For a consumer with inventories, I_{it} and I_{jt}, the utility levels for the two firms are given by,

$$U_t(0) = U_0 + \beta_c E[V_{t+1}(0,0)]$$
$$U_t(i) = r_{it} - (1 - I_{it})p_i + \beta_c E[V_{t+1}(I_{it+1},0)]$$
$$U_t(j) = r_{jt} - (1 - I_{jt})p_j + \beta_c E[V_{t+1}(0, I_{jt+1})]$$

where,

$$V_t(I_{it}, I_{jt}) = \max_{0,i,j} \begin{array}{l} U(0) + \beta_c E[V_{t+1}(0,0)] \\ U_t(i) + \beta_c E[V_{t+1}(1 - I_{it},0)] \\ U_t(j) + \beta_c E[V_{t+1}(0,1 - I_{jt})] \end{array}$$

$U(0) = U_0$, $U_t(i) = r_{it} - p_i$, and $U_t(j) = r_{jt} - p_j$. By following the same procedure as for (FP,TP), we get,

$$E[V_t(I_{it}, I_{jt})] = \frac{e^{b_{jt}-(1-I_{jt})p_j} Log(e^{U_0+b_{0t}} + e^{b_{jt}-(1-I_{jt})p_j}) - e^{b_{it}-(1-I_{it})p_i} Log(e^{U_0+b_{0t}} + e^{b_{it}-(1-I_{it})p_i})}{e^{b_{jt}-(1-I_{jt})p_j} - e^{b_{it}-(1-I_{it})p_i}}$$

where,

$$b_{0t} = \beta_c E[V_{t+1}(0,0)]$$
$$b_{it} = a_i - (1 - I_{it})p_i + \beta_c E[V_{t+1}(I_{it+1},0)]$$
$$b_{jt} = a_j - (1 - I_{jt})p_j + \beta_c E[V_{t+1}(0, I_{jt+1})]$$

(TP,TP) Case - Backward Induction, General Period t:

Let both firms, *i* and *j*, follow a traditional pricing program. In any period *t*, the utility levels of consumers for the two firms are given by,

$$U_t(i) = r_{it} - p_i + \beta_c E[V_{t+1}],$$
$$U_t(j) = r_{jt} - p_j + \beta_c E[V_{t+1}],$$
$$U_t(0) = U_0 + \beta_c E[V_{t+1}]$$

By following the same procedure as for (FP,TP), we get,

$$E[V_t] = \frac{e^{b_{jt}-p_j} Log(e^{U_0+b_{0t}} + e^{b_{jt}-p_j}) - e^{b_{it}-p_i} Log(e^{U_0+b_{0t}} + e^{b_{it}-p_i})}{e^{b_{jt}-p_j} - e^{b_{it}-p_i}}$$

where,

$$b_{it} = a_i + \beta_c E[V_{t+1}],$$
$$b_{jt} = a_j + \beta_c E[V_{t+1}],$$
$$b_{0t} = \beta_c E[V_{t+1}]$$

APPENDIX 2

Expected Demand and Profit Calculations

(FP,TP) Case:

Let Firm *i* follow FP and Firm *j* follow TP, $i, j = 1,2, i \neq j$. Firm *i*'s objective function is given by,

$$\max_{p_i} \sum_{t=1}^{T} \beta^{t-1} E[\pi_{it}] \tag{A6}$$

Firm j's objective function is given by,

$$\max_{p_j} \sum_{t=1}^{T} \beta^{t-1} E[\pi_{jt}] \tag{A7}$$

In period *t*, let $d_{it}(I_t, p_j)$ be the expected proportion of demand for Firm *i*, conditional on the customer having a reward inventory I_t. Similarly, let $d_{jt}(I_t, p_j)$ be the conditional expected proportion of demand for Firm *j*, and $d_{0t}(I_t, p_j)$ be the conditional expected proportion who do not buy either brand. By definition, $d_{it}(I_t, p_j) + d_{jt}(I_t, p_j) + d_{0t}(I_t, p_j) = 1$. Conditioning on $U_t(j)$, we have,

$$d_{it}(I_t, p_j) = P[U_t(i) > U_t(j) \mid U_t(j) > U_t(0)]P[U_t(j) > U_t(0)] +$$
$$P[U_t(i) > U_t(0)]P[U_t(j) \leq U_t(0)]$$

$$P[U_t(j) > U_t(0)] = 1 - P[U_t(j) \leq U_t(0)] = 1 - \frac{1}{1 + e^{a_j - p_j - U_0}}$$

$$P[U_t(i) > U_t(0)] = 1 - \frac{1}{1 + e^{a_i - (1 - I_t) p_i - U_0}}$$

Let $U_t(j) \mid U_t(j) > U_t(0) = Z_t(j)$. Then, let $f(z_t(j))$ denote the probability density function of $Z_t(j)$ and z_{tj} denote a random variable corresponding to $f(z_t(j))$. Following Hogg and Craig (1978),

$$f(z_t(j)) = \frac{f_{u_t(j)}(z_t(j))}{1 - F_{u_t(j)}(U_0)} = \left[\frac{e^{a_j - p_j - z_{tj}}}{(1 + e^{a_j - p_j - z_{tj}})^2}\right]\left[\frac{1 + e^{a_j - p_j - U_0}}{e^{a_j - p_j - U_0}}\right], \quad U_0 < z_{tj} < \infty$$

$$P[U_t(i) \le z_t(j)] = \int_{U_0}^{\infty}\int_{-\infty}^{z_{Tj}} f(u_t(j)) f(z_{tj}) du_{tj} dz_{tj}$$

$$= \int_{U_0}^{\infty}\int_{-\infty}^{z_{Tj}} [\frac{e^{a_i-(1-I_t)p_i-u_{tj}}}{(1+e^{a_i-(1-I_t)p_i-u_{tj}})^2}][\frac{1+e^{a_j-p_j-U_0}}{e^{a_j-p_j-U_0}}][\frac{e^{a_j-p_j-z_{tj}}}{1+e^{a_j-p_j-z_{tj}}}]du_{tj}dz_{tj}$$

$$= [\frac{1+e^{a_j-p_j-U_0}}{e^{a_j-p_j-U_0}}]\int_{U_0}^{\infty}[\frac{1}{1+e^{a_i-(1-I_t)p_i-u_{tj}}}]_{-\infty}^{z_{Tj}}\frac{e^{a_j-p_j-z_{tj}}}{(1+e^{a_j-p_j-z_{tj}})^2}dz_{tj}$$

$$= [\frac{1+e^{a_j-p_j-U_0}}{e^{a_j-p_j-U_0}}]\int_{U_0}^{\infty}\frac{e^{a_j-p_j}e^{2z_{tj}}}{(e^{z_{tj}}+e^{a_j-p_j})^2(e^{z_{tj}}+e^{a_i-(1-I_t)p_i})}dz_{tj}$$

$$= -[\frac{[1+e^{a_j-p_j-U_0}]e^{U_0}}{e^{a_i-(1-I_t)p_i}-e^{a_j-p_{jt}}}][\frac{e^{a_j-p_j}}{e^{a_j-p_j}+e^{U_0}}+\frac{e^{a_i-(1-I_t)p_i}}{e^{a_i-(1-I_t)p_i}-e^{a_j-p_j}}Log[\frac{e^{a_j-p_j}+e^{U_0}}{e^{a_i-(1-I_t)p_i}+e^{U_0}}]]$$

Hence,

$$d_{it}(I_t, p_j) = [1-P[U_t(i) \le z_t(j)]]\frac{e^{a_j-p_j-U_0}}{1+e^{a_j-p_j-U_0}}+\frac{e^{a_i-(1-I_t)p_i-U_0}}{1+e^{a_i-(1-I_t)p_i-U_0}}[\frac{1}{1+e^{a_j-p_j-U_0}}]$$

$$= [1+\{\frac{[1+e^{a_j-p_j-U_0}]e^{U_0}}{e^{a_i-(1-I_t)p_i}-e^{a_j-p_j}}\}\{\frac{e^{a_j-p_j}}{e^{a_j-p_j}+e^{U_0}}+\frac{e^{a_i-(1-I_t)p_i}}{e^{a_i-(1-I_t)p_i}-e^{a_j-p_j}}Log(\frac{e^{a_j-p_j}+e^{U_0}}{e^{a_i-(1-I_t)p_i}+e^{U_0}})\}]$$

$$\frac{e^{a_j-p_j-U_0}}{1+e^{a_j-p_j-U_0}}+\frac{e^{a_i-(1-I_t)p_i-U_0}}{1+e^{a_i-(1-I_t)p_i-U_0}}(\frac{1}{1+e^{a_j-p_j-U_0}})$$

Regarding the expected demand for brand *j*, we have,

$$d_{jt}(I_t, p_j) = P[U_t(j) > U_t(i) \mid U_t(i) > U_t(0)]P[U_t(i) > U_t(0)] +$$
$$P[U_t(j) > U_t(0)]P[U_t(i) \le U_t(0)]$$

Following the same method of derivation as for the demand of Firm *i*, we get,

$$d_{jt}(I_t, p_j) = [1+\{\frac{[1+e^{a_i-(1-I_t)p_i-U_0}]e^{U_0}}{e^{a_j-p_j}-e^{a_i-(1-I_t)p_i}}\}\{\frac{e^{a_i-(1-I_t)p_i}}{e^{a_i-(1-I_t)p_i}+e^{U_0}}+\frac{e^{a_j-p_j}}{e^{a_j-p_j}-e^{a_i-(1-I_t)p_i}}Log(\frac{e^{a_i-(1-I_t)p_i}+e^{U_0}}{e^{a_j-p_j}+e^{U_0}})\}]$$

$$\frac{e^{a_i-(1-I_t)p_i-U_0}}{1+e^{a_i-(1-I_t)p_i-U_0}}+(\frac{e^{a_j-p_j-U_0}}{1+e^{a_j-p_j-U_0}})(\frac{1}{1+e^{a_i-(1-I_t)p_i-U_0}})$$

Finally, the expected proportion of consumers not buying either Firm *i* or Firm *j* is,

$$d_{0t}(I_t, p_j) = P[U_t(i) \le U_t(0)]P[U_t(j) \le U_t(0)]$$
$$= \frac{1}{(1+e^{a_i-(1-I_t)p_i-U_0})(1+e^{a_j-p_j-U_0})}$$

We now have demand, conditional on inventory levels. We proceed to calculate the unconditional probability distributions for inventory levels and then use these to weight the conditional demand distributions to derive unconditional demand. First the unconditional probabilities of observing various inventory levels: For any set of prices in period *t*, p_i and p_j, let $PI_{0t}(p_i,p_j)$ be the expected proportion of consumers entering period t with 0 inventory of Firm i's frequency reward. Let $PI_{1t}(p_i,p_j)$ be the expected proportion of consumers entering period t with 1 inventory of Firm i's frequency reward.

$$PI_{1t}(p_i,p_j) = PI_{0t-1}(p_i,p_j)d_{it-1}(0)$$

Similarly, $PI_{0t}(p_i,p_j)$ is the expected proportion of consumers entering period t with 0 inventory of Firm i's frequency reward. $PI_{0t}(p_i,p_j) = 1- PI_{1t}(p_i,p_j)$.

Hence, the expected total demand for Firm i in period t is,

$$E[D_{it}(p_i,p_j)] = PI_{0t}(p_i,p_j)d_{it}(0) + PI_{1t}(p_i,p_j)d_{it}(1)$$

The expected total demand for Firm j in period t is,

$$E[D_{jt}(p_i,p_j)] = PI_{0t}(p_i,p_j)d_{jt}(0) + PI_{1t}(p_i,p_j)d_{jt}(1)$$

Finally, the expected proportion of consumers not purchasing from either firms is,

$$E[D_{0t}(p_i,p_j)] = 1 – E[D_{it}(p_i,p_j)] – E[D_{jt}(p_i,p_j)]$$

Thus the expected profit for Firm i in period t is,

$$E[\pi_{it}(p_i,p_j)] = \{p_i- v\}\{PI_{0t}(p_i,p_j)d_{it}(0)\}$$

Similarly, the expected profit for Firm j in period t is,

$$E[\pi_{jt}(p_i,p_j)] = [p_j- v][PI_{0t}(p_i,p_j)\, d_{jt}(0) + PI_{1t}(p_i,p_j)d_{jt}(1)]$$

(FP,FP) Case:

Let both firms, i and j, follow frequency reward program. In any period t, let a consumer's inventories of Firm i's and Firm j's respective FP programs be denoted by (I_{it}, I_{jt}). Note that when $I_{it} = 1$, $I_{jt} \neq 1$ and vice-versa, i.e., for any consumer, $I_{it} = 1$ or 0, $I_{jt} = 1$ or 0, and $I_{it} + I_{jt} = 1$ or 0. In period t, let $d_{it}(I_{it}, I_{jt})$ be the conditional expected proportion of demand for Firm i. Similarly, let $d_{jt}(I_{it}, I_{jt})$ be the conditional expected proportion of demand for Firm j, and $d_{0t}(I_{it}, I_{jt})$ be the conditional expected proportion who do not buy either brand. By definition, $d_{it}(I_{it}, I_{jt}) + d_{jt}(I_{it}, I_{jt}) + d_{0t}(I_{it}, I_{jt}) = 1$.

Following the procedure used in the (FP,TP) case, we obtain,

$$d_{it}(I_{it},I_{jt}) = [1+\{\frac{[1+e^{b_{jt}-(1-I_{jt})p_j-U_0-b_{0t}}]e^{U_0+b_{0t}}}{e^{b_{it}-(1-I_{it})p_i}-e^{b_{jt}-(1-I_{jt})p_j}}\}\{\frac{e^{b_{jt}-(1-I_{jt})p_j}}{e^{b_{jt}-(1-I_{jt})p_j}+e^{U_0+b_{0t}}}+\frac{e^{b_{it}-(1-I_{it})p_i}}{e^{b_{it}-(1-I_{it})p_i}-e^{b_{jt}-(1-I_{jt})p_j}}$$

$$Log(\frac{e^{b_{jt}-(1-I_{jt})p_j}+e^{U_0+b_{0t}}}{e^{b_{it}-(1-I_{it})p_i}+e^{U_0+b_{0t}}})\}]\frac{e^{b_{jt}-(1-I_{jt})p_j-U_0-b_{0t}}}{1+e^{b_{jt}-(1-I_{jt})p_j-U_0-b_{0t}}}+(\frac{e^{b_{it}-(1-I_{it})p_i-U_0-b_{0t}}}{1+e^{b_{it}-(1-I_{it})p_i-U_0-b_{0t}}})(\frac{1}{1+e^{b_{jt}-(1-I_{jt})p_j-U_0-b_{0t}}})$$

$$d_{jt}(I_{it},I_{jt}) = [1+\{\frac{[1+e^{b_{it}-(1-I_{it})p_i-U_0-b_{0t}}]e^{U_0+b_{0t}}}{e^{b_{jt}-(1-I_{jt})p_j}-e^{b_{it}-(1-I_{it})p_i}}\}\{\frac{e^{b_{it}-(1-I_{it})p_i}}{e^{b_{it}-(1-I_{it})p_i}+e^{U_0+b_{0t}}}+\frac{e^{b_{jt}}}{e^{b_{jt}-(1-I_{jt})p_j}-e^{b_{it}-(1-I_{it})p_i}}$$ where,

$$Log(\frac{e^{b_{it}-(1-I_{it})p_i}+e^{U_0+b_{0t}}}{e^{b_{jt}-(1-I_{jt})p_j}+e^{U_0+b_{0t}}})\}]\frac{e^{b_{it}-(1-I_{it})p_i-U_0-b_{0t}}}{1+e^{b_{it}-(1-I_{it})p_i-U_0-b_{0t}}}+(\frac{e^{b_{jt}-(1-I_{jt})p_j-U_0-b_{0t}}}{1+e^{b_{jt}-(1-I_{jt})p_j-U_0-b_{0t}}})(\frac{1}{1+e^{b_{it}-(1-I_{it})p_i-U_0-b_{0t}}})$$

$$d_{0t}(I_{it},I_{jt}) = \frac{1}{(1+e^{b_{jt}-(1-I_{jt})p_j-U_0-b_{0t}})(1+e^{b_{it}-(1-I_{it})p_i-U_0-b_{0t}})}$$

$$b_{0t} = \beta_c E[V_{t+1}(0,0)]$$
$$b_{it} = a_i - (1-I_{it})p_i + \beta_c E[V_{t+1}(I_{it+1},0)]$$
$$b_{jt} = a_j - (1-I_{jt})p_j + \beta_c E[V_{t+1}(0,I_{jt+1})]$$

For any set of prices, p_{it} and p_j, in period t, let $PI_{00t}(p_i,p_j)$ be the expected proportion of consumers entering period t with 0 inventory of Firm i's frequency reward and 0 inventory of Firm j's frequency reward. Similarly, let $PI_{01t}(p_i,p_j)$ be the expected proportion of consumers entering period t with 0 inventory of Firm i's frequency reward and 1 inventory of Firm j's frequency reward. Finally, $PI_{10t}(p_i,p_j)$ denotes the expected proportion of consumers entering period t with 1 inventory of Firm i's frequency reward and 0 inventory of Firm j's frequency reward. Given that the frequency reward expires in one period, consumers will not be able to carry an inventory of both firm's frequency reward at the same time. Thus, we get,

$PI_{00t}(p_i,p_j) = PI_{00t-1}(p_i,p_j)d_{0t}(0,0) + PI_{10t-1}(p_i,p_j)[d_{1t}(1,0)+d_{0t}(1,0)] +$
$PI_{01t-1}(p_i,p_j)[d_{jt}(0,1)+d_{0t}(0,1)]$

$PI_{10t}(p_i,p_j) = PI_{00t-1}(p_i,p_j)d_{it}(0,0) + PI_{01t-1}(p_i,p_j)d_{it}(0,1)$

$PI_{01t}(p_i,p_j) = PI_{00t-1}(p_i,p_j)d_{jt}(0,0) + PI_{10t-1}(p_i,p_j)d_{jt}(1,0)$

Thus, the expected total demand for Firm i in period t is,

$E[D_{it}(p_i,p_j)] = PI_{00t-1}(p_i,p_j)\ d_{it}(0,0) + PI_{01t-1}(p_i,p_j)\ d_{it}(0,1) + PI_{10t-1}(p_i,p_j)\ d_{it}(1,0)$

Similarly, the expected total demand for Firm j in period t is,

$E[D_{jt}(p_i,p_j)] = PI_{00t-1}(p_i,p_j)\ d_{jt}(0,0) + PI_{10t-1}(p_i,p_j)\ d_{jt}(1,0) + PI_{01t-1}(p_i,p_j)\ d_{jt}(0,1)$

Finally, the expected proportion of consumers not purchasing from either firms is,

$E[D_{0t}(p_i,p_j)] = 1 – E[D_{it}(p_i,p_j)] – E[D_{jt}(p_i,p_j)]$

Hence the expected profit for Firm i in period t is,

$E[\pi_{it}(p_i,p_j)] = \{p_i - v\}\{PI_{00t-1}(p_i,p_j)\, d_{it}(0,0) + PI_{01t-1}(p_i,p_j)\, d_{it}(0,1)\}$

Similarly, the expected profit for Firm j in period t is,

$E[\pi_{jt}(p_i,p_j)] = \{p_j - v\}\{PI_{00t-1}(p_i,p_j)\, d_{jt}(0,0) + PI_{10t-1}(p_i,p_j)\, d_{jt}(1,0)\}$

(TP,TP) Case:

Let both firms, i and j, follow price promotion program. In period t, let $d_{it}(p_i, p_j)$ be the conditional expected proportion of demand for Firm i. Similarly, let $d_{jt}(p_i, p_j)$ be the conditional expected proportion of demand for Firm j, and $d_{0t}(p_i, p_j)$ be the conditional proportion who do not buy either brand. By definition, $d_{it}(p_i, p_j) + d_{jt}(p_i, p_j) + d_{0t}(p_i, p_j) = 1$. Following the backward induction procedure in the expected value computation (Appendix 1), we obtain,

$$d_{it}(p_i, p_j) = [1 + \{\frac{[1 + e^{b_{jt} - p_j - U_0 - b_{0t}}]e^{U_0 + b_{0t}}}{e^{b_{it} - p_i} - e^{b_{jt} - p_j}}\}\{\frac{e^{b_{jt} - p_j}}{e^{b_{jt} - p_j} + e^{U_0 + b_{0t}}} + \frac{e^{b_{it} - p_i}}{e^{b_{it} - p_i} - e^{b_{jt} - p_j}}$$
$$Log(\frac{e^{b_{jt} - p_j} + e^{U_0 + b_{0t}}}{e^{b_{it} - p_i} + e^{U_0 + b_{0t}}})\}]\frac{e^{b_{jt} - p_j - U_0 - b_{0t}}}{1 + e^{b_{jt} - p_j - U_0 - b_{0t}}} + (\frac{e^{b_{it} - p_i - U_0 - b_{0t}}}{1 + e^{b_{it} - p_i - U_0 - b_{0t}}})(\frac{1}{1 + e^{b_{jt} - p_j - U_0 - b_{0t}}})$$

$$d_{jt}(p_i, p_j) = [1 + \{\frac{[1 + e^{b_{it} - p_i - U_0 - b_{0t}}]e^{U_0 + b_{0t}}}{e^{b_{jt} - p_j} - e^{b_{it} - p_i}}\}\{\frac{e^{b_{it} - p_i}}{e^{b_{it} - p_i} + e^{U_0 + b_{0t}}} + \frac{e^{b_{jt}}}{e^{b_{jt} - p_j} - e^{b_{it} - p_i}}$$
$$Log(\frac{e^{b_{it} - p_i} + e^{U_0 + b_{0t}}}{e^{b_{jt} - p_j} + e^{U_0 + b_{0t}}})\}]\frac{e^{b_{it} - p_i - U_0 - b_{0t}}}{1 + e^{b_{it} - p_i - U_0 - b_{0t}}} + (\frac{e^{b_{jt} - p_j - U_0 - b_{0t}}}{1 + e^{b_{jt} - p_j - U_0 - b_{0t}}})(\frac{1}{1 + e^{b_{it} - p_i - U_0 - b_{0t}}})$$

$$d_{0t}(p_i, p_j) = \frac{1}{(1 + e^{b_{jt} - p_j - U_0 - b_{0t}})(1 + e^{b_{it} - p_i - U_0 - b_{0t}})}$$

$$b_{it} = a_i + \beta_c E[V_{t+1}],$$
$$b_{jt} = a_j + \beta_c E[V_{t+1}],$$
$$b_{0t} = \beta_c E[V_{t+1}]$$

The expected total demand for Firm i, i=1,2, in period t is $d_{it}(p_i,p_j)$, where i,j=1,2 and j ? i.

Finally, the expected proportion of consumers not purchasing from either firm is,

$d_{0t}(p_i,p_j) = 1 - d_{it}(p_i,p_j) - d_{jt}(p_i,p_j)$

Hence the expected profit for Firm i in period t is,

$E[\pi_{it}(p_i,p_j)] = (p_i - v)[d_{it}(p_i,p_j)]$

where i,j=1,2 and j ? i.

REFERENCES

Air Transport Association (1999), *Airline Handbook*.

Air Transport Association (1979, 1982, 1983), *Annual Reports*.

Ansari, Asim, Nicholas Economides, and Avijit Ghosh (1994), "Competitive Positioning in Markets with Nonuniform Preferences," *Marketing Science*, 13 (3) 248-273.

Associated Press (1982), "Consumer Scorecard: A Bonus for Repeat Business," November 19.

Assunção, João and Robert J. Meyer (1993), "The Rational Effect of Price Promotions on Sales and Consumption," *Management Science*, 39 (5) 517-535.

Aviation Week and Space Technology (1983), "Airlines Gear for New Challenges," November 14, p.48.

Aviation Week and Space Technology (1987), "Airline Observer," July 6, p.37.

Benady, David and Sean Brierley (1996), "Few Benefits on the Cards," *Marketing Week*, 19 (15) 28-32.

Beggs, Alan and Paul Klemperer (1992), "Multi-Period Competition With Switching Costs," *Econometrica*, 60 (3) 641-666.

Blattberg, Robert C., and Scott A. Neslin (1990), *Sales Promotion: Concepts, Methods, and Strategies*, Englewood Cliffs, NJ: Prentice-Hall, Inc.

Brookman, Faye (1998) "Companies Invest in Customer Loyalty," *Marketing News*, 32 (March 2) 14.

Business Week (1986), "People Express Wants All the Frills It Can Get," May 12, p.31.

Butscher, Stephan A. (1998) "Limited Loyalty Programs Create Strategic Databases," *Marketing News*, 31 (October 27) 13.

Bulkley, George (1992), "The Role of Loyalty Discounts When Consumers are Uncertain of the Value of Repeat Purchases," *International Journal of Industrial Organization*, 10, 91-101.

Chintagunta, Pradeep K. and Dipak Jain (1992), "A Dynamic Model of Channel Member Strategies for Marketing Expenditures," *Marketing Science*, 11 (2) 168-188.

Dowling, Grahame R., and Mark Uncles (1997) "Do Customer Loyalty Programs Really Work?" *Sloan Management Review*, 38 (Summer) 71-82.

Erdem, Tulin and Michael P. Keane (1996) "Decision-making Under Uncertainty: Capturing

Dynamic Brand Choice Processes in Turbulent Consumer Goods Markets," *Marketing Science*, 15 (1) 1-20.

Fletcher, Karen (1996), "Blunting the Points," *Marketing*, August 29, 30-33.

Fudenberg, Drew and Jean Tirole (1992), *Game Theory*, MIT Press, Cambridge: MA.

Gallant, A. R. and J. J. Goebel (1976), "Non-linear Regression with Autoregressive Errors," *Journal of American Statistical Association*, 71, 961-967.

Gonül, Fusun and Kannan Srinivasan (1996), "Estimating the Impact of Consumer Expectations of Coupons on Purchase Behavior: A Dynamic Structural Model, *Marketing Science*, 15 (3) 262-279.

Hogg, Robert V. and Allen T. Craig (1978), *Introduction to mathematical statistics*, Macmillan Press, New York: NY.

Klemperer, Paul D. (1987), "Markets with Consumer SwitchingCost," *Quarterly Journal of Economics*, 102 (May), 375-394.

Klemperer, Paul (1995), "Competition When Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade," *Review of Economic Studies*, 62, 515-539.

Kim, Byungdo, Mengze Shi, and Kannan Srinivasan (1996), "Reward Programs and Tacit Price Collusion," Paper presented at the *1996 Marketing Science Conference* (March), Berkeley, CA.

Kohli, Rajeev, and Vijay Mahajan (1991), "A Reservation-Price Model for Optimal Pricing of Multiattribute Products in Conjoint Analysis," *Journal of Marketing Research*, 28 (August), 347-354.

Kopalle, Praveen, Carl Mela, and Lawrence Marsh (1999) "The Dynamic Effect of Discounting on Sales: Empirical Analysis and Normative Pricing Implications," *Marketing Science*, 18 (3), 317-332.

Lewis, Herschell Gordon (1997), "Does Your "Loyalty" Program Inspire any Loyalty?" *Direct Marketing*, 60 (June) 46-48.

Neslin, Scott A; Powell, Stephen G; Stone, Linda Schneider (1995), "The effects of retailer and consumer response on optimal manufacturer advertising and trade promotion strategies," *Management Science*, 41 (5) 749-766.

Newsweek (1986), "People's New Flight Plan," May 12, p.60.

New York Times (1982), "The Frequent Flier Bonuses," November 25, p.1.

Orr, Alicia (1995), “Customers for Life!” *Target Marketing*, 18 (March) 20.

Rao, Ram C. (1991), “Pricing and Promotions in Asymmetric Duopolies,” *Marketing Science*, 10 (2) 131-144.

Raphel, Neil (1998), “The Future of Loyalty Marketing,” *Supermarket Business*, 53 (April) 23-30.

Reynolds, Jonathan (1995) “Database Marketing and Customer Loyalty: Examining the Evidence,” *European Retail Digest*, Oxford, UK: Oxford Institute of Retail Management, Issue 7 (July) 31-38.

Rosenspan, Alan (1998) “Delusions of Loyalty: Where Loyalty Programs Go Wrong,” *Direct Marketing* 60 (February) 24-27.

Sharp, Byron, and Anne Sharp (1997), “Loyalty Programs and Their Impact on Repeat-Purchase Loyalty Patterns,” *International Journal of Research in Marketing*, 14, 473-486.

Stokey, Nancy L., and Robert E. Lucas (1989), *Recursive Methods in Economic Dynamics*, Harvard University Press: Cambridge, MA.

Sultan, Fareena, John U. Farley, and Donald R Lehmann (1990), “A Meta-Analysis of Applications of Diffusion Models,” *Journal of Marketing Research*, 27 (February), 70-77.

Tellis, Gerard J., and Fred S. Zufryden (1995), “Tackling the Retailer Decision Maze: Which Brands to Discount, How Much, When and Why?” *Marketing Science*, 14 (1 of 2) 271-299.

Wansink, Brian and Michael L. Ray (1996), “Advertising Strategies to Increase Usage Frequency,” *Journal of Marketing*, 60 (1) 31-46.

Zhang, Z. John, Aradhna Krishna, and Sanjay K. Dhar (2000), “The Optimal Choice of Promotional Vehicles: Front-Loaded or Rear-Loaded Incentives?” *Management Science*, 46 (3), 348-362.