

# TTK4125 - Adaptive Control

## Lecture Notes

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# Lecture 1 Notes

## System identification and parameter estimation

System identification usually is an iterative process where you build a model from data. The usual steps are:

- Design an experiment
- Select an appropriate model structure
- Select an appropriate parameter estimation method, and apply it to the data
- Model validation
- Evaluate the model and go back if model is not good enough.

In this course we assume that the system is controllable and that its parameters are uncertain. We express this as:

$$\dot{x} = f(x, u; \theta) \quad (0.1)$$

$$y = h(x, u; \theta) \quad (0.2)$$

Where  $f$  and  $h$  are known functions, while the  $\theta$  vector constitutes the unknown parameters. The input  $u$  and output  $y$  are measurable, while the state  $x$  may or may not be measurable. For the majority of the course, we assume linear structure of the system:

$$\dot{x} = A(\theta)x + B(\theta)u \quad (0.3)$$

$$y = C(\theta)x + D(\theta)u \quad (0.4)$$

In most system estimation problems,  $\theta$  is assumed to be constant. In this case we can just estimate a model given a batch of data. In control applications, we get datapoints continuously, so having algorithms that can update the estimate is preferred.

The recursive algorithms usually expect constant parameters as well, but they pose some good qualities. 1. they are able to do real-time or online estimation. 2. they can handle slowly changing parameters, slow relative to system dynamics. 3. Can handle sudden but rare changes in parameters.

Adaptive control is therefore not only about estimating parameters of an model, but also about detecting faults. We can create parameters that we assume to be constant, but if we detect changes they signal that something is wrong in the system. This could also be an approach to making an adaptive controller.

## What is adaptive control, and why do we need it?

To illustrate adaptive control we will look at a simple example. Take the simplest possible linear system:

$$\dot{x}(t) = a \cdot x(t) + u(t), x(0) = 0 \quad (0.5)$$

A possible control law for this system, where the goal is to drive it to zero:

$$u(t) = -\hat{a}(t)x(t) - cx(t) \quad (0.6)$$

With  $c$  being a positive constant, and  $\hat{a}(t)$  being an estimate of the parameter  $a$ . The closed loop system is then:

$$\dot{x}(t) = -cx(t) + (a - \hat{a}(t))x(t) \quad (0.7)$$

We need to find a bounded but sufficiently good estimate of  $a$  for the regulation objective to be met. We start by setting  $\hat{a}(0) = a_0$  as some initial guess. As time passes, the signals  $x(t)$  and  $u(t)$  are measured, and we can use these to update our estimate  $\hat{a}(t)$ .

We define  $\tilde{a}(t) = \hat{a}(t) - a$ , this is the parameter estimation error. Now consider the Lyapunov function:

$$V(x, \tilde{a}) = x^2 + \tilde{a}^2 \quad (0.8)$$

Now, if  $V$  goes to zero, then both  $x$  and  $\tilde{a}$  go to zero. This means that  $\hat{a} \rightarrow a$ . We can differentiate  $V$ :

$$\dot{V} = -2cx^2(t) - 2\tilde{a}(t)x^2(t) + 2\tilde{a}(t)\dot{\hat{a}} \quad (0.9)$$

Now, looking at each term:

- The first term is always negative, and is the term that will make  $x(t)$  go to zero.
- The second term is indefinite, and could be positive or negative.
- The third term is also indefinite, but we can choose  $\dot{\hat{a}}$  ourselves.

By selecting

$$\dot{\hat{a}}(t) = x^2(t), \hat{a}(0) = a_0 \quad (0.10)$$

we get

$$\dot{V} = -2cx^2(t) \leq 0 \quad (0.11)$$

Now  $V$  is positive definite and  $\dot{V}$  is negative semi-definite.  $V$  is therefore bounded implying that  $x(t)$  and  $\tilde{a}(t)$  are bounded. And as  $V$  is bounded and not increasing, it converges to something, and  $\dot{V}$  converges to zero.

This is an adaptive control law, consisting of the input

$$u(t) = -(\hat{a}(t) + c)x(t) \quad (0.12)$$

and the adaptive law

$$\dot{\hat{a}}(t) = x^2(t), \hat{a}(0) = a_0 \quad (0.13)$$

Notice that we introduced nonlinearities in the linear system, so to analyze its properties we would need to use Lyapunov theory. Also notice that the adaptive law drives  $x$  to zero but it does not necessarily drive  $\tilde{a}$  to zero. But it does drive  $\tilde{a}$  to a sufficiently correct value to control the system.

In more complex systems, we take a modular approach to this problem. We separate the estimation process from the control design. Where the controller is design assuming known parameters.

## Signal Spaces

To use our adaptive laws, we need to know that the signals we use are bounded and integrable. We define  $\mathcal{L}_p$ -spaces,  $p = 1, 2, \dots, \infty$ . For a continuous signal  $f : [0, \infty) \rightarrow \mathcal{R}^n$

$$\int_0^\infty |f(t)|^p dt < \infty \Leftrightarrow f \in \mathcal{L}_p \quad (0.14)$$

Meaning that the  $p$ 'th power of the signal is integrable and bounded up to  $p$ . Then we define the  $\mathcal{L}_\infty$  space as:

$$\sup_{t \geq 0} |f(t)| < \infty \Leftrightarrow f \in \mathcal{L}_\infty \quad (0.15)$$

This is the class of bounded functions. We can norm these vectorspaces:

$$\|f\|_p = \left( \int_0^\infty |f(t)|^p dt \right)^{1/p}, p = 1, 2, 3, \dots \quad (0.16)$$

$$\|f\|_{\mathcal{L}_\infty} = \sup_{t \geq 0} |f(t)| \quad (0.17)$$

We have the equivalent for discrete time signals:

$$\sum_{k=0}^{\infty} |f(k)|^p < \infty \Leftrightarrow f \in l_p \quad (0.18)$$

$$\sup_{k \in \mathcal{Z}^+} |f(k)| < \infty \Leftrightarrow f \in l_{\infty} \quad (0.19)$$

$$\|f\|_p = \left( \sum_{k=0}^{\infty} |f(k)|^p \right)^{1/p}, p = 1, 2, 3, \dots \quad (0.20)$$

$$\|f\|_{l_{\infty}} = \sup_{k \in \mathcal{Z}^+} |f(k)| \quad (0.21)$$