NUMBERTHEORY

AleXtreme Bootcamp

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What is Number Theory?

Number theory is a branch of pure mathematics that deals with the properties and relationships of numbers, particularly **Integers**.

Integers:

Whole numbers without any decimal or fractional parts.

Agenda

- Integer Operations
 - Division /
 - Modulus %
- Integer Relationships
 - Divisors
 - Prime Factors
 - GCD / LCM

- Special Integers
 - Primes
 - Perfect Squares
- Algorithms
 - Sieve of Eratosthenes

Numbers in Computer

Integers, Floats and Doubles

- Think of them as different containers to store a number each with its own properties.
- Float and double seem to be interchangeable with only precision difference.
- We can interchange types easily in C++. But where does the fraction go?

ceil() and floor()

- Ceiling goes up and flooring goes down.
- What about negative numbers?

Division Operation

5 / 2 = 2 ?

- Division outcome between 2 integers will result in an integer too.
- The computer floors the outcome to make it an integer.
- Where Did The Remainder Go?

Modulus Operator

Getting the Division Remainder

- What's the range of the outcome?
- Think of the modulus like a clock circle.
- What if the remainder is zero?
- What about negative numbers?
- Problem (<u>Game with Integers</u>)

Divisors

What are they?

- Each number has at least 2 divisors.
- Problem (<u>Two Regular Polygons</u>)

Calculating Divisors of N

- Can we do it faster?
- What about 36?

Divisors

ECPC Problem

$$aX^3 + bX^2 + cX^1 + d = 0$$

Find an integer X that satisfies the given constants a, b, c and d.

Multiples

- What are multiples?
- Calculating multiples of a number N up to M

Prime Numbers

What are They?

- How to check if a number is prime?
- Can we do it faster?
- Sieve of Eratosthenes
- Problem (<u>T-primes</u>)
- Can we modify Sieve to get all divisors too?

Prime Factors

The Fundamental Theorem of Arithmetic

- Problem (Odd Divisor)
- Calculating prime factors for n?
- Problem (<u>k-Factorization</u>)

GCD

Greatest Common Divisor

- How to calculate it? Time Complexity?
- GCD Usage (Reducing fractions, Flouriest problem)
- Special cases, GCD(A,B) = A or 1
- Problem (Relatively prime pairs)
- GCD for more than 2 numbers?
- GCD from factorization

LCM

Least Common Multiplier

- Problem (Trains Meeting)
- LCM from factorization
- LCM from GCD
- Can LCM be 1? LCM(A,B) = A?
- Problem (<u>LCM Problem</u>)

What's Next?

- Binary Exponentiation
- Modular Arithmetic
- Extended Euclidean Algorithm
- Chinese Remainder Theorem
- More...

THANKYOU!