Home work 2

1- Express the vector (9; 6) as a linear combination of the vectors (1; 2) and (1;-4).

Sol:

$$\begin{bmatrix} 1 & 1 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

a+b=9

2a-4b=6

So : a=7 b=2

$7\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

2- Determine whether the vector x1 = (2; 1;3) lies in the span of the vectors x2 = (1; 2; 3) and x3 = (2; 3; 1).

Sol:

Lie in span mean : it can be written as linear compination of them :

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

a+2b=2

2a+3b=1

3a+b=3

a=-4 b=3

but in 3rd equation that is not correct so it doesn't lie in the equation

3- a:

v1 = [−2 3], w = [−8 12]

w=4[-2 3]=4*v1

b:

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 10 \end{bmatrix}$$

2a=4

2b=-6

5a=10

a=2 b=-3

w=2v1-3v2

4- 1) e
2) d
3) b
4) f
5) a
6) c

5. (2 points) Let S = {v1,v2,v3,v4,v5} where,

$$V1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix}, V2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, V3 = \begin{bmatrix} 1 \\ 5 \\ -1 \\ 5 \end{bmatrix}, V4 = \begin{bmatrix} 1 \\ 1 \\ 4 \\ -1 \end{bmatrix}, V5 = \begin{bmatrix} 2 \\ 7 \\ 0 \\ 2 \end{bmatrix}$$

Find a basis for the span Span(S).

Sol:

By matrix reduction

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 2 & 3 & 5 & 1 & 7 \\ 2 & 1 & -1 & 4 & 0 \\ -1 & 1 & 5 & -1 & 2 \end{bmatrix}$$

R2→-2R1+R2

and R3→-2R1+R3

and R4→R1+R4

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & -1 & -3 & 2 & -4 \\ 0 & 2 & 6 & 0 & 4 \end{bmatrix}$$

R3→R2+R3

And R4 → -2R2+R4

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

R4→-2R3+R4

R2→R3+R2

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R1→-R3+R1

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R1→-R2+R1

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From matrix the independents and who are basis is : V1,V2,V4