

Home work 1 section1

1-The angle between the vectors $(1, 0, -1, 3)$ and $(1, \sqrt{3}, 3, -3)$ in R^4 is $a\pi$, where $a = \frac{3}{4}$.

Sol:

$$\cos \theta = \frac{v_1 \cdot v_2}{|v_1| |v_2|}$$

$$v_1 \cdot v_2 = \text{element wise multiplication} = -11$$

$$|v_1| = \sqrt{1^2 + 0 + (-1)^2 + 3^2} = \sqrt{11}$$

$$|v_2| = \sqrt{22}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\theta = 135^\circ \quad \theta = \frac{3}{4} \pi$$

2- Which of the angles (if any) of triangle ABC, with $A = (1, -2, 0)$, $B = (2, 1, -2)$, and $C = (6, -1, -3)$, is a right angle?

Answer: the angle at vertex AB.

$$\cos \theta = \frac{v_1 \cdot v_2}{|v_1| |v_2|}$$

$$\cos 90 = 0$$

So : $v_1 \cdot v_2$ must equal 0

$$A \cdot B = 2 - 2 + 0 = 0$$

3-Practice with numbers (if there is no answer, say so)

a. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 20 \\ 30 & 40 \\ 50 & 60 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 4 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 4 & 1 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

f. $\begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 7 \end{bmatrix}$

g. $\begin{bmatrix} 0 & 1 & 2 \\ 10 & -10 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

a. $\begin{bmatrix} 11 & 22 \\ 33 & 44 \\ 55 & 66 \end{bmatrix}$

b. $1*3+2*4=11$ c. $\begin{bmatrix} 11 & 2 \end{bmatrix}$

d. $\begin{bmatrix} 11 & 2 \\ 110 & 20 \end{bmatrix}$

e. no must col = rows of 2nd matrix

f. $\begin{bmatrix} 3 & 6 & 21 \\ 4 & 8 & 28 \end{bmatrix}$

g. $\begin{bmatrix} 18 & 21 & 24 \\ 5 & 10 & 15 \end{bmatrix}$

4-If one side of the triangle increases by 11 cm and the other side decreases by the same value, we get an equilateral triangle. When the first side is multiplied by four, it is 10 cm longer than three times the third side. Find the lengths of the triangle's sides. Write this in a matrix-vector form

Sol:

Length of triangles= a, b, c

$$a=b+11 \rightarrow 1 \quad a=c-11 \rightarrow 2 \quad 4b=10+3a \rightarrow 3$$

from 1 : $b=a-11$

$$4(a-11)=10+3a$$

$$a=54$$

$$b=54-11=43$$

$$c=a+11=65$$

Matrix form:

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -3 & 4 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 11 \\ -11 \\ 10 \end{bmatrix}$$

5-Two containers contain a water of different temperatures. If we mix 240 g of water from the first container with 260 g of water from the second container, the resulting water temperature will be 52°C. If we mix 180 g of water from the first container with 120 g of water from the second container, the resulting water temperature will be 46°C. What is the temperature of water in the containers?. Write this in a matrix-vector form.

Sol :

$$240 T_1 + 260 T_2 = (240 + 260) * 52$$

$$240 T_1 + 260 T_2 = 26000 \rightarrow 1$$

$$180 T_1 + 120 T_2 = 13800 \rightarrow 2$$

From 1 and 2 :

$$100 T_2 = 7600$$

$$T_2 = 76$$

$$T_1 = 26$$

Matrix form:

$$\begin{bmatrix} 240 & 260 \\ 180 & 120 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 26000 \\ 13800 \end{bmatrix}$$

6- If one dimension of the cuboid increases by 1 cm, the surface area of the cuboid increases by 54 cm². If the second dimension of the cuboid increases by 2 cm, the surface area of the cuboid increases by 96 cm². If the third dimension of the cuboid increases by 3 cm, its surface area increases by 126 cm². Find the dimensions of the cuboid. Write this in a matrix-vector form

Sol:

$$2lw + 2lh + 2wh = sa \rightarrow 1$$

$$2w(l+1) + 2h(l+1) + 2wh = sa + 54 \rightarrow 2$$

$$2l(w+2) + 2hl + 2h(w+2) = sa + 96 \rightarrow 3$$

$$2lw + 2l(h+3) + 2w(h+3) = sa + 126 \rightarrow 4$$

From 2 -1 :

$$2w + 2h = 54$$

$$W + h = 27 \rightarrow 6$$

From 3-1 :

$$4l + 4h = 96$$

$$L + h = 24 \rightarrow 7$$

From 4-1 :

$$6l + 6w = 126$$

$$L+w=21 \rightarrow 8$$

From 6 and 7 and 8 :

$$W= 12$$

$$l=9$$

$$h=15$$

Matrix Form:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ l \\ h \end{bmatrix} = \begin{bmatrix} 27 \\ 24 \\ 21 \end{bmatrix}$$