

Home work 3&4

$$1- \quad M = \begin{bmatrix} 17 & -11 \\ 6 & -3 \end{bmatrix}$$

$$\text{Det}(M) = (17 \cdot -3) - (6 \cdot -11) = 15$$

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 4 & -5 \end{bmatrix} = 1(-15-4) - 1(-10-3) + 2(8-9) \\ = -8$$

$$2- \text{ a) } A = \begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$$

$$\text{Det}(A) = (-9 + 6) = -3$$

$$A^{-1} = 1/\text{det}(A) * \text{adj}(A)$$

$$1/-3 * \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{2}{3} \\ 1 & 1 \end{bmatrix}$$

$$\text{b) } A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{Det}(A) = 1(1-1) - 0(0-1) + 1(-1) = -1$$

$$A^{\text{minor}} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{\text{adj}} = \text{transpose}(A^{\text{minor}}) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = A^{\text{adj}} * 1/\text{det}(A) = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\begin{aligned}
 3- \quad M &= \begin{bmatrix} 3 & 1 & 0 & -1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1/3 & 0 & -1/3 \\ 2 & 4 & 3 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1/3 & 0 & -1/3 \\ 0 & 10/3 & 3 & 4 \end{bmatrix}
 \end{aligned}$$

Rank=2

$$\begin{aligned}
 M &= \begin{bmatrix} 5 & 2 & 3 \\ 7 & 2 & 2 \\ 9 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2/5 & 3/5 \\ 7 & 2 & 2 \\ 9 & -1 & 1 \end{bmatrix} = \\
 \begin{bmatrix} 1 & 2/5 & 3/5 \\ 0 & -4/5 & -11/5 \\ 0 & -23/5 & -22/5 \end{bmatrix} &= \begin{bmatrix} 1 & 2/5 & 3/5 \\ 0 & 1 & 11/4 \\ 0 & -23/5 & -22/5 \end{bmatrix} = \\
 \begin{bmatrix} 1 & 0 & 17/10 \\ 0 & 1 & 11/4 \\ 0 & 0 & 33/4 \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 17/10 \\ 0 & 1 & 11/4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

Rank=3

$$4- \quad A) \begin{bmatrix} 1 & 4 & 3 & -1 & | & 5 \\ 1 & -1 & 1 & 2 & | & 6 \\ 4 & 1 & 6 & 5 & | & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & -1 & | & 5 \\ 0 & -5 & -2 & 3 & | & 1 \\ 0 & -15 & -6 & 9 & | & -11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & -1 & | & 5 \\ 0 & 1 & 2/5 & -3/5 & | & -1/5 \\ 0 & -15 & -6 & 9 & | & -11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & -1 & | & 5 \\ 0 & 1 & 2/5 & -3/5 & | & -1/5 \\ 0 & 0 & 0 & 0 & | & -14 \end{bmatrix}$$

The equations is inconsistent (have no solutions)

B)

$$\begin{bmatrix} 1 & -2 & 1 & -1 & | & 3 \\ 2 & -4 & 1 & 1 & | & 2 \\ 1 & -2 & -2 & 3 & | & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & -1 & | & 3 \\ 0 & 0 & -1 & 3 & | & -4 \\ 0 & 0 & -3 & 4 & | & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & -1 & | & 3 \\ 0 & 0 & 1 & -3 & | & 4 \\ 0 & 0 & 0 & -5 & | & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 2 & | & -1 \\ 0 & 0 & 1 & -3 & | & 4 \\ 0 & 0 & 0 & 1 & | & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 & | & 1 \\ 0 & 0 & 1 & 0 & | & -2 \\ 0 & 0 & 0 & 1 & | & -2 \end{bmatrix}$$

Infinite number of solutions:

$$x_4 = -2 \quad x_3 = -2 \quad \text{let } x_2 = c_1 \quad x_1 - 2x_2 = 1$$

$$x_1 = 2c_1 + 1$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ -2 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

C)

$$\begin{bmatrix} 1 & 2 & 3 & | & 1 \\ 2 & -1 & 1 & | & 2 \\ 3 & 1 & 1 & | & 4 \\ 0 & 5 & 2 & | & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 1 \\ 0 & -5 & -5 & | & 0 \\ 0 & -5 & -8 & | & 1 \\ 0 & 5 & 2 & | & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 1 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & -3 & | & 1 \\ 0 & 0 & -3 & | & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & | & 1 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & | & -1/3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 4/3 \\ 0 & 1 & 0 & | & 1/3 \\ 0 & 0 & 1 & | & -1/3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$x_1 = 4/3$$

$$x_2 = 1/3$$

$$x_3 = -1/3$$

$$5- A) A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2 - \lambda & -1 \\ -1 & 2 - \lambda \end{bmatrix} = 0$$

$$(2 - \lambda)^2 - 1 = 0$$

$$\lambda^2 + 4 - 4\lambda - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda = 3 \quad \lambda = 1$$

$$\text{tr}(A) = \lambda_1 + \lambda_2 = 4$$

$$\det(A) = \lambda_1 * \lambda_2 = 3$$

for $\lambda = 3$:

$$|A - \lambda I| X = 0$$

$$\begin{bmatrix} 2 - 3 & -1 \\ -1 & 2 - 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & | & 0 \\ -1 & -1 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$$x_1 + x_2 = 0$$

$$\text{let } x_2 = -c_1 \quad x_1 = c_1$$

$$x_1 = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ if } c_1 = 1 \quad x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$|A - \lambda I| x_2 = 0$$

$$\begin{bmatrix} 2 - 1 & -1 \\ -1 & 2 - 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 = 0 \quad \text{let } x_2 = c_1 \quad x_1 = c_1$$

$$x_2 = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ if } c_1 = 1 \quad x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

B) let $A^2 = M$

$$M = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$|M - \lambda I| = 0$$

$$\begin{bmatrix} 5 - \lambda & -4 \\ -4 & 5 - \lambda \end{bmatrix} = 0$$

$$(5 - \lambda)^2 - 16 = 0$$

$$\lambda^2 + 25 - 10\lambda - 16 = 0$$

$$\lambda^2 - 10\lambda + 9 = 0$$

$$\lambda = 9 \quad \lambda = 1$$

$$\text{tr}(A^2) = \lambda_1 + \lambda_2 = 10$$

$$\det(A^2) = \lambda_1 * \lambda_2 = 9$$

for $\lambda = 9$:

$$|A - \lambda I| X_1 = 0$$

$$\begin{bmatrix} 5 - 9 & -4 \\ -4 & 5 - 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & -4 & | & 0 \\ -4 & -4 & | & 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_1 + x_2 = 0 \quad \text{let } x_2 = -c_1 \quad x_1 = c_1$$

$$x_1 = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

for $\lambda = 1$:

$$|A - \lambda I| x_2 = 0$$

$$\begin{bmatrix} 5 - 1 & -4 \\ -4 & 5 - 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|c} 4 & -4 & 0 \\ -4 & 4 & 0 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_1 - x_2 = 0 \quad \text{let } x_2 = c_2 \quad x_1 = c_2$$

$$x_2 = c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C) \det(A) = 2 \cdot 2 - 1 = 3$$

$$A^{-1} = \frac{1}{3} * \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad \text{let } A^{-1} = M$$

$$|M - \lambda I| = 0$$

$$\begin{bmatrix} 2/3 - \lambda & 1/3 \\ 1/3 & 2/3 - \lambda \end{bmatrix} = 0$$

$$(2/3 - \lambda)^2 - 1/9 = 0$$

$$\lambda^2 - 4/3 \lambda + 1/3 = 0$$

$$\lambda = 1 \quad \lambda = 1/3$$

$$\text{tr}(A^{-1}) = \lambda_1 + \lambda_2 = 4/3$$

$$\det(A^{-1}) = \lambda_1 * \lambda_2 = 1/3$$

for $\lambda = 1$:

$$|A - \lambda I| X_1 = 0$$

$$\begin{bmatrix} 2/3 - 1 & 1/3 \\ 1/3 & 2/3 - 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1/3 & 1/3 & | & 0 \\ 1/3 & -1/3 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$$x_1 - x_2 = 0 \quad \text{let } x_2 = c_1 \quad x_1 = c_1$$

$$X_1 = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

for $\lambda = 1/3$:

$$(A - \lambda I) X_1 = 0$$

$$\begin{bmatrix} 2/3 - 1/3 & 1/3 \\ 1/3 & 2/3 - 1/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1/3 & 1/3 & | & 0 \\ 1/3 & 1/3 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$$x_1 + x_2 = 0 \quad \text{let } x_2 = -c_2 \quad x_1 = c_2$$

$$X_2 = c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$D) A+4I = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} \quad \text{let } A+4I=M$$

$$|M-\lambda I|=0$$

$$\begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix} = 0$$

$$(6-\lambda)^2 - 1 = 0$$

$$\lambda^2 - 12\lambda + 36 - 1 = 0$$

$$\lambda^2 - 12\lambda + 35 = 0$$

$$\lambda_1 = 7 \quad \lambda_2 = 5$$

$$\text{tr}(A+4I) = \lambda_1 + \lambda_2 = 12$$

$$\det(A+4I) = \lambda_1 * \lambda_2 = 35$$

for $\lambda=7$:

$$|A-\lambda I|X_1=0$$

$$\begin{bmatrix} 6-7 & -1 \\ -1 & 6-7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & -1 & 0 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_1 + x_2 = 0 \quad \text{let } x_2 = -c_1 \quad x_1 = c_1$$

$$x_1 = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

for $\lambda = 5$:

$$|A - \lambda I| x_1 = 0$$

$$\begin{bmatrix} 6 - 5 & -1 \\ -1 & 6 - 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_1 - x_2 = 0 \quad \text{let } x_2 = c_2 \quad x_1 = c_2$$

$$x_2 = c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$