

# Transpiling Programming Computable Functions to Answer Set Programs

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**Abstract.** Programming Computable Functions (PCF) is a simplified programming language which provides the theoretical basis of modern functional programming languages. Answer set programming (ASP) is a programming paradigm focused on solving search problems. In this paper we provide a translation from PCF to ASP. Using this translation it becomes possible to specify search problems using PCF.

## 1 Introduction

A lot of research aims to put more abstraction layers into modelling languages for search problems. One common approach is to add templates or macros to a language to enable the reuse of a concept [6,7,11]. Some languages such as HiLog [3] introduce higher order language constructs with first order semantics to mimic this kind of features. While the lambda calculus is generally not considered a regular modelling language, one of its strengths is the ability to easily define abstractions. We aim to shrink this gap by showing how the lambda calculus can be translated into existing paradigms. Our end goal is to leverage existing search technology for logic programming languages to serve as a search engine for problems specified in functional languages.

In this paper we introduce a transpiling algorithm between Programming Computable Functions, a programming language based on the lambda calculus and Answer Set Programming, a logic-based modelling language. This transpilation is a source-to-source translation of programs. We show how this can be the basis for a functional modelling language which combines the advantages of easy abstractions and reasoning over non-defined symbols. Transpiling Programmable Computable Functions to other languages like C [10], or a theorem prover like Coq [5] has been done before. But as far as the authors are aware, no approaches to translate it into logic programs have been done before.

In Section 2 we introduce the source language of our transpiler: Programming Computable Functions. In Section 3 we introduce the target language of our transpiler: Answer Set Programming. In Section 4 we describe the translation algorithm. Finally, in Section 5 we motivate why this kind of translation can be useful in practice. An implementation of the translator is made available online (<https://dtai.cs.kuleuven.be/krr/pcf2asp>), where it can be tested with your own examples.

## 2 Programming Computable Functions

Programming Computable Functions [8,15] (PCF) is a programming language based on the lambda calculus. It is not used as an end-user language; instead it provides a strong theoretical basis for more elaborate languages, such as Lisp, Caml or Haskell. There are many small variations of PCF, some extend it with booleans, tuples or arithmetic operators. One such variation is known as Mini-ML [4]. The particular flavor is irrelevant for the principles in this paper.

### 2.1 Syntax

The syntax of PCF relies heavily on the standard lambda calculus, extended with natural numbers, a selection construct and a fixpoint operator. We identify the following language constructs:

- function application  $e_1e_2$ , which is left associative,
- a lambda abstraction  $\lambda x.e$ , abstracting the variable  $x$  out of the expression  $e$ ,
- for each numeral  $n \in \mathbb{N}$ , a constant  $n$ ,
- constants **succ**, representing the successor function over  $\mathbb{N}$ , **pred** representing the predecessor function over  $\mathbb{N}$ ,
- a constant **fix**, representing the fixpoint operator, also known as the Y-combinator, and
- a ternary language construct **ifz**  $e_z$  **then**  $e_t$  **else**  $e_e$ , representing an if zero-then-else.

Suppose that  $\mathbb{I}$  is an infinite supply of identifiers. The syntax of PCF can be inductively defined as:

```
e = x ( $\in \mathbb{I}$ ) | e e |  $\lambda x.e$   
| n ( $\in \mathbb{N}$ ) | succ | pred | fix | ifz e then e else e
```

*Example 1.*  $(\lambda x. \text{succ} (\text{succ } x)) (\text{succ } 0)$  is a complicated way to write 3.

The expression **fix** allows us to write functions which would require recursive definitions in most programming languages. It takes a function  $f$  as argument and returns the fixpoint  $x$  of that function so that  $f(x) = x$ . From this it follows that **fix** satisfies the equation  $\text{fix } f = f (\text{fix } f)$ .

*Example 2.* A traditional recursive definition for the double of a number  $x$  could be:

```
double x = ifz x then 0 else 1 + 1 + double (x-1)
```

It is possible to rewrite this using **fix**, by abstracting both **double** and  $x$ , and using **pred** and **succ** for the increments and decrements:

```
fix (λdouble. λx. ifz x then 0 else succ (succ (double (pred x)))
```

The informal meaning of this expression is the doubling function.

*Example 3.*  $\text{fix } (\lambda \text{plus}. \lambda a. \lambda b. \text{ifz } a \text{ then } b \text{ else } \text{plus} (\text{pred } a) (\text{succ } b))$  of which the informal meaning is the binary sum function over natural numbers.

## 2.2 Operational Semantics

When considering expressions, we traditionally consider only those without free variables. However, when considering the operational semantics, we will generalise this to situations where free variables can occur. For this reason we introduce environments and closures through a mutually inductive definition.

**Definition 1.** An environment  $E$  is a mapping from identifiers to closures. A closure  $(E, e)$  consists of an environment  $E$  and an expression  $e$ , where the environment must interpret at least all the free variables in  $e$ .

We say an environment interprets an identifier  $x$  if it contains a mapping for  $x$ . The closure to which  $E$  maps an interpreted variable  $x$  is written as  $E[x]$ .

*Example 4.*  $(\text{succ } a, \{a \mapsto (\{\}, \text{succ } 0)\})$  is a valid closure which will evaluate to the number 2.

**Evaluation context** The evaluation relation  $\Downarrow$  is a relation between closures and values, which we will write as follows:

$$E, e \Downarrow V$$

$(E, e)$  is the closure that is being evaluated. When considering the evaluation of an expression without an explicit environment, we assume it has no free variables and we interpret this as the closure with the empty environment.  
 $V$  is the value that corresponds to the expression, this can either be a natural number or a closure. A natural number can be implicitly used as a closure with the empty environment.

**Notation** We will describe both the semantics of PCF and the translation algorithm using a set of inference rules. These are rules of the form

$$\frac{\text{Premise}_1 \quad \dots \quad \text{Premise}_n}{\text{Conclusion}}$$

An algorithmic interpretation of these rules will lead to a program which can evaluate/translate PCF. Most often, the easiest way to read this kind of rules is bottom up.

**Evaluation Rules** The following inference rules determine the operational semantics for PCF through the evaluation relation  $\Downarrow$ .

$$\frac{E[x] = (E_2, e) \quad E_2, e \Downarrow V}{E, x \Downarrow V}$$

$$\frac{\text{E}, e_1 \Downarrow (\text{E}_2, \lambda x. e_3) \quad \text{E}, e_2 \Downarrow V \quad \text{E}_2 \cup \{x \mapsto V\}, e_3 \Downarrow V_{ap}}{\text{E}, e_1 e_2 \Downarrow V_{ap}}$$

$$\overline{\text{E}, \lambda x. f \Downarrow (\text{E}, \lambda x. f)}$$

$$\overline{\text{E}, n(\in \mathbb{N}) \Downarrow n}$$

$$\frac{\text{E}, e \Downarrow n}{\text{E}, \text{succ } e \Downarrow n+1}$$

$$\frac{\text{E}, e \Downarrow n+1}{\text{E}, \text{pred } e \Downarrow n}$$

$$\frac{\text{E}, e_i \Downarrow 0 \quad \text{E}, e_t \Downarrow V}{\text{E}, \text{ifz } e_z \text{ then } e_t \text{ else } e_e \Downarrow V}$$

$$\frac{\text{E}, e_i \Downarrow n \quad n > 0 \quad \text{E}, e_e \Downarrow V}{\text{E}, \text{ifz } e_z \text{ then } e_t \text{ else } e_e \Downarrow V}$$

$$\frac{\text{E} \cup \{x \mapsto (\text{E}, \text{fix } (\lambda x. e))\}, e \Downarrow V}{\text{E}, \text{fix } (\lambda x. e) \Downarrow V}$$

These rules form an inductive definition of the evaluation relation  $\Downarrow$ . Note that this is a call-by-value semantics. This can be seen in the rule of applications, as the subexpression  $e_2$  is evaluated before adding it to the environment. A call-by-name semantics would just add the closure containing  $e_2$  instead of the evaluation of  $e_2$ .

*Example 5.* In the below tree you can follow the semantics of an expression using multiple inference rules. Every horizontal line represents the application of one evaluation rule.

$$\frac{\frac{\frac{\frac{\frac{\{(f \mapsto (\emptyset, \text{fix } (\lambda f. 4))\}, 4 \Downarrow 4}}{\emptyset, (\text{fix } (\lambda f. 4)) \Downarrow 4} \quad 4 > 0}{\emptyset, 2 \Downarrow 2} \quad \frac{\frac{\{\{x \mapsto 2\}, 2 \Downarrow 2}{\{x \mapsto 2\}, x \Downarrow 2}}{\{\{x \mapsto 2\}, \text{pred } x \Downarrow 1}}}{\emptyset, (\lambda x. \text{pred } x) \Downarrow 1}}}{\emptyset, \text{ifz } (\text{fix } (\lambda f. 4)) \text{ then } 3 \text{ else } (\lambda x. \text{pred } x) \Downarrow 1}$$

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**Listing 1** An example ASP program and its solutions

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(a) An ASP Program

```
1 p(1). p(2). p(3). p(4).
2 1 {q(X) : p(X)} 2.
3 r(X + Y) :- q(X), q(Y).
4 :- not r(5).
```

(b) The Answer Sets

```
Answer Set 1:  
p(1) p(2) p(3) p(4)  
q(1) q(4) r(2) r(5) r(8)  
Answer Set 2:  
p(1) p(2) p(3) p(4)  
q(2) q(3) r(4) r(5) r(6)
```

---

### 3 Answer Set Programming

Answer Set Programming [1] (ASP) is a modelling language with a strong basis in logic programming. It is mainly used as a language to specify NP-hard search problems [14]. There are a lot of different systems supporting a unified ASP standard[2]. An ASP program is essentially a logic program with some extra syntactic restrictions. An ASP solver computes the answer sets of the program under the stable semantics. An answer set consists of a set of atoms which together represent the solution of a problem. One program may have zero, one or multiple answer sets.

#### 3.1 Language

An ASP program is a set of rules of the form:

```
head :- body1, ..., bodyn, not bodyn+1, ..., not bodym
```

The first  $n$  body atoms are positive, the others are negative. The head and body atoms of the rules are of the form  $\text{id}(\text{term}_1, \dots, \text{term}_n)$ . Body atoms can also be comparisons ( $<$ ,  $>$ ,  $=$ ,  $\neq$ ) between terms. Terms can be either constants, variables, or arithmetic expressions over terms. Constants are numbers or named constants (strings starting with a lowercase character). Variables are represented as strings starting with an uppercase character. An ASP program is considered *safe* if all rules are safe. A rule is considered safe if all variables occurring in the rule, occur at least once in a positive body. If the head is omitted, the rule is considered a constraint. In this case no instantiations of the body of the rule should exist such that all the bodies are true.

Choice rules are a common syntactic extension for ASP. These allow heads of the form  $c_l \{a(X) : b(X)\} c_u$ , where  $c_l, c_u \in \mathbb{N}$  and  $c_l \leq c_u$ . This head is considered true if between  $c_l$  and  $c_u$  instances of  $a(X)$  are true, given  $b(X)$ . They allow to easily introduce symbols that are not uniquely defined. We can for instance declare  $p$  to be a singleton containing a number between 1 and 10 with the choice rule:  $1 \{p(X) : X = 1..10\} 1$ . The ASP program containing only this line has 10 answer sets, one for each possible singleton.

*Example 6.* In Listing 1 you can see an example ASP program together with its answer sets. The first line of the program defines the predicate  $p$  as the numbers between 1 and 4. The second line is a choice rule with no bodies. It states that  $q$  is a subset of  $p$  and contains 1 or 2 elements. The third line says that  $r$  is the sum of any two elements (possibly the same one) from  $q$ . The fourth line asserts that  $r$  should contain 5.

### 3.2 Grounding (and Solving)

To understand the details of the translation mechanism, basic knowledge of how an ASP system constructs an answer set is needed. Constructing answer sets happens in two phases: grounding and solving [12]. The grounding process transforms the ASP program to an equivalent propositional program. The solver then constructs the actual answer sets from this propositional format. The translation from PCF described in this paper will produce a fully positive, monotone theory without choice rules or constraints. ASP grounders produce the actual (unique) answer set for this kind of programs. Note that not all ASP systems use the same algorithms, but the information presented here is common to most systems.

The grounding process uses a bottom-up induction of the program. At any point in time, the grounder contains a set of atoms which are possibly part of an answer set. This set starts empty, and by the end of the process this set contains an overapproximation of all answer sets. The grounder tries to instantiate rules using this set of atoms. Whenever a rule is instantiated, the instantiated head is added to this set, and the ground instantiation of the rule is added to the grounding of the program. ASP grounders require that all variables occur in a positive body atom, this is the so-called safety requirement on rules. Safe rules have the property that only the positive part of the program is essential for finding all rule instantiations and current grounding approaches heavily rely on this property.

*Example 7.* Consider the rule  $d(X-1) :- d(X), X > 0$  and the current set of grounded atoms is just the singleton  $\{d(1)\}$ . The grounder can now instantiate the body atom  $d(X)$  with  $X=1$ . The other body atom  $1 > 0$  can be statically evaluated to be true. This leads to the newly ground rule  $d(0) :- d(1)$  and  $d(0)$  is added to the set of grounded atoms. The grounder can now try to instantiate the rule with  $X=0$ , but the comparison  $0 > 0$  prevents the rule to be added to the ground program.

After the grounding phase an ASP solver can produce the actual answer sets based on the grounding. An ASP solver typically uses a SAT solver extended with some ASP specific propagators. The inner workings of these programs are not needed to understand the contents of this paper.

## 4 Translation

In this section we explain the core of the translation mechanism. Section 4.1 defines the relation between the translation and the PCF semantics. Section 4.2 introduces some conventions which explain the structure of the resulting program. Finally, Section 4.3 explains the static part of the translation. Section 4.4 defines the translation relation between PCF expressions and the dynamic part of the translation.

### 4.1 Characterisation of the translation

**Translation relation** The translation is characterised using a relation  $\rightsquigarrow$  which we will write as follows:

$$(E, S_1), e \rightsquigarrow A, (t, S_2)$$

**E** is a mapping from PCF-variables to ASP-terms for at least the free variables in  $e$ . This works analogously to the environment of the PCF semantics, which was the mapping from PCF-variables to closures.

**S<sub>1</sub>** is a set of ASP atoms ensuring the ASP-terms in E are safe, and constraints enforcing the ifzero-semantics.

**e** is the PCF expression that is translated.

**A** is the ASP program consisting of a set of safe ASP rules, this is the program that contains all the helper rules to translate  $e$

**t** is the ASP term which represents the translation of  $e$

**S<sub>2</sub>** is the set of ASP atoms ensuring that  $t$  is safe

It can be unintuitive that there are ASP-terms occurring on both sides of the translation relation. The explanation for this lies in the handling of free variables. The translation relation will be defined structurally, this means that for the translation of a composite term, the translation of its subterms are needed. This implies that when translating the expression  $(\lambda x. x)$ , the subterm  $x$  needs to be translated as well. The translation needs some context to interpret this  $x$  and the context of a translation environment will be some information about the parts which are already translated.

A PCF expression corresponds to a single value, but a logic program corresponds to an answer set with a lot of atoms. We need a way to indicate the actual value that is meant with the logic program. The *result*-predicate is used to indicate the resulting value of the program.

**Definition 2.** *The ASP translation of a PCF expression  $e$  determined by  $\rightsquigarrow$  is the ASP program  $A$  such that  $(\emptyset, \emptyset), e \rightsquigarrow A_2, (t, S)$  and  $A = A_2 \cup \{\text{result}(t) :- S\}$ .*

**Soundness of the translation** PCF inherently works on expressions which evaluate to a particular value, ASP programs define relations. A certain equivalence criterion is needed to validate the translation. For this we use the *result*-predicate. For ease of defining the correspondence the soundness criterion is restricted to programs with a numeric evaluation.

**Definition 3.** A sound translator for PCF to ASP maps every PCF expression  $e$  to an ASP program  $A$  with a unique answer set. This answer set contains at most one atom for the *result*-predicate. If  $\emptyset, e \Downarrow n \in \mathbb{N}$ , then  $\text{result}(n)$  must be an element of the answer set of  $A$ .

*Claim.* The translation of PCF expressions determined by  $\rightsquigarrow$  is a sound translator.

In this paper we will not prove this claim. We state it here to give the reader an intuition about the correspondence between a program and its translation.

## 4.2 Conventions

In the translation, all PCF expressions  $e$  correspond to a tuple  $(t, S)$  where  $t$  is an ASP term and  $S$  is a set of ASP bodies. Natural numbers have constants in both PCF and ASP which have a natural correspondence. PCF functions are identified by an ASP term  $t_f$ , so that for every ASP term  $t_x$ , the tuple  $(Y, \{\text{inter}(t_f, t_x), Y\})$  denotes the image of the function  $t_f$  applied to  $t_x$ . All functions have infinite domains, and thus the full function cannot be represented in a finite answer set. The *domain* predicate serves the purpose of making a finite estimate of the relevant parts of the function. If at some point in the evaluation of  $e$ , the function  $t_f$  is applied to the value  $t_x$ ,  $\text{domain}(t_f, t_x)$  should be true. The *inter*-predicate only needs to be defined for the domain of the function, resulting in a finite answer set containing the relevant parts of the interpretation of the function.

Remember that *result* predicate is used as the predicate determining the final result of the program. So the translation of a PCF expression is an ASP program, defining only 3 predicates:

1. *inter*: determines the interpretation of functions
2. *domain*: determines the (relevant) domain of functions
3. *result*: determines the end result

**Magic Set Transformation** In these conventions a link with the magic set transformations [13] of logic programs appears. The magic set transformation allows us to transform a query, which is traditionally executed top-down, to a program, which can be executed bottom-up. It uses the *magic* predicates to indicate which subqueries need to be performed. As explained in Section 3.2, ASP uses a bottom-up grounding process. So, the translation from PCF to ASP also converts a top-down query (the evaluation of PCF) to a bottom-up process (the ASP grounding). The *domain*-predicate has a function similar to the *magic* predicates: it indicates for which arguments a function needs to be calculated.

---

**Listing 2** Static preamble of the ASP translation

---

```
1 inter((pred,X),X-1) :- domain(pred,X), X > 0.
2 inter((succ,X),X+1) :- domain(succ,X).
3 inter((fix,F),Y)      :- domain(fix,F), inter((F,f(F)),Y).
4 inter((f(F),X),Y)    :- domain(f(F),X), inter((F,f(F)),FIX),
5                                inter((FIX,X),Y).
6 domain(F,f(F))      :- domain(fix,F).
7 domain(FIX,X)        :- domain(f(F),X), inter((F,f(F)),FIX).
```

---

### 4.3 Static Preamble

The translation of any PCF expression consists of a dynamic part and a static part. The static part ensures that the interpretation of the `succ`, `pred` and `fix` builtins is taken care of. The dynamic part is produced by the translation algorithm and takes care of the actual PCF expression. The static part is the same for every translation and can be seen in Listing 2. The first two lines of the static part ensure the right translation of the `pred` and `succ` terms. E.g. the PCF-term `succ` correctly corresponds to the ASP tuple  $(\text{succ}, \{\})$  according to the conventions defined in Section 4.2. For instance, if somewhere the PCF-term `succ 0` needs to be evaluated. The term will translate to  $(Y, \{\text{inter}((\text{succ}, 0), Y)\})$  which will result in  $Y$  being equal to 1 in the answer set.

Just like `pred` and `succ`, the PCF- and ASP-term of `fix` are the same. But the required rules in the preamble are more complex. A naive translation could look like this:

```
inter((fix,F),Z) :- inter((fix,F),Y), inter((F,Y),Z).
```

This rule most closely represents  $\text{fix } f = f(\text{fix } f)$ , but in the stable semantics this equation is not correctly represented by the above rule. Instead, an intermediate term `f(F)` is introduced to symbolically represent the fixpoint of `F` in ASP. Now we are able to write the fixpoint as the function `F` applied to the symbolic function `f(F)` as can be seen on line 3. If the fixpoint is a function, we need to be able to apply it to arguments. Line 4 serves this purpose: to apply `X` to a fixpoint of a function, you can apply `F` to this fixpoint (to ensure we do not have the symbolic representation) and then apply `X` to the result. Finally, lines 6 and 7 ensure that the function applications performed in lines 3 and 4 are all well-defined through the `domain` predicates.

### 4.4 Translation Algorithm

In this section we present the translation algorithm as a definition for the translation relation  $\rightsquigarrow$  using inference rules. Sometimes new ASP constants or variables are needed in the translation. We suppose there is some global supply of those. We use the notation  $\text{head} \leftarrow B$  for the ASP rule where `head` is the head atom and `B` is the set of body atoms.

**Scoping** When translating an expression, the free variables in this expression need to be filled in. As we translate nested expressions level per level, we need to pass these values along the expression tree. For this reason, we do not just associate an identifier with a function but a tuple containing an identifier and the current scope. The current scope is a tupling of the full codomain of the translation environment  $E$ . We will refer to it as  $\text{scope}_E$ .

### Builtins (numbers, pred, succ, fix)

$$\overline{(E,S), b \rightsquigarrow \emptyset, (b,S)}$$

Builtins are relatively easy to translate. The hard work is taken care of by the static preamble described in Section 4.3. A builtin produces no new ASP rules and is translated by itself. Safety is however taken into account, not for the scoping of variables, but for the handling of the if-zero constraints.

### Variable

$$\overline{(E,S), x \rightsquigarrow \emptyset, (E[x],S)}$$

The algorithm carries around a mapping that represents how variables should be translated. This makes translating it a simple variable easy: just look it up in the mapping and combine it with the required safety.

### Application

$$\frac{(E,S), e_1 \rightsquigarrow A_1, (t_1, B_1) \quad (E,S), e_2 \rightsquigarrow A_2, (t_2, B_2)}{(E,S), e_1e_2 \rightsquigarrow A_1 \cup A_2 \cup \text{rule}_{\text{domain}}, (X, \text{body}_{\text{inter}} \cup B_1 \cup B_2)}$$

$$\begin{aligned} X &= \text{a new ASP variable} \\ \text{rule}_{\text{domain}} &= \{\text{domain}(t_1, t_2) \leftarrow B_1 \cup B_2\} \\ \text{body}_{\text{inter}} &= \{\text{inter}((t_1, t_2), X)\} \end{aligned}$$

Applications are translated by independently translating the two subexpressions. The produced ASP programs need to be combined, with the additional rule that  $t_2$  should be added to the domain of the function  $t_1$ . To obtain the resulting value, we use the *inter*-predicate according to the conventions explained in Section 4.2.

*Example 8.* The rule below shows how the application rule can be used to translate the successor of 1. The static part of the translation ensures that the *inter*-relation for **succ** is interpreted correctly so that in any solution. The  $X$  gets evaluated to 2.

$$\frac{(\emptyset, \emptyset), \text{succ} \rightsquigarrow \emptyset, (\text{succ}, \emptyset) \quad (\emptyset, \emptyset), 1 \rightsquigarrow \emptyset, (1, \emptyset)}{(\emptyset, \emptyset), \text{succ } 1 \rightsquigarrow \{\text{domain}(\text{succ}, 1)\}, (X, \text{inter}((\text{succ}, 1), X))}$$

## Lambda

$$\frac{(\mathbf{E} \cup (\mathbf{x}, X), S \cup \text{body}_{\text{domain}}), e \rightsquigarrow A, (t, B)}{(\mathbf{E}, S), \lambda \mathbf{x}. e \rightsquigarrow A \cup \text{rule}_{\text{inter}}, ((l, \text{scope}_E), S)}$$

$$\begin{aligned} X &= \text{a new ASP variable} \\ l &= \text{a new ASP constant} \\ \text{rule}_{\text{inter}} &= \{\text{inter}(((l, \text{scope}_E), X), t) \leftarrow B\} \\ \text{body}_{\text{domain}} &= \{\text{domain}((l, \text{scope}_E), X)\} \end{aligned}$$

Lambda expressions bring a new variable into scope, so they modify the  $(E, S)$ -environment before recursively translating the body of the expression. The freshly scoped variable needs to be put into the scoping function  $E$ , for this we assign it a new ASP variable ( $X$  in the rule). This variable should have a finite range, we invent a new name for our function ( $l$  in the rule) and use the *domain* predicate to restrict  $X$  to the domain of the function. The resulting translation  $(t, B)$  represents the image of the function, so the rule  $\text{rule}_{\text{inter}}$  is added to couple the representation of the function to its interpretation.

*Example 9.*  $(\emptyset, \emptyset)(\lambda \mathbf{x}. 2) \rightsquigarrow \{\text{inter}(((l,()), X), 2) \leftarrow \text{domain}((l,()), X)\}, ((l,()), \emptyset)$   
 This can be read as follows: The translation of the constant function to 2 in an empty environment is represented by the constant  $(l,())$ . The interpretation of  $(l,())$  when applied to any term  $X$  in the domain of  $(l,())$  is 2.

## If zero-then-else

$$\frac{\begin{aligned} &(\mathbf{E}, S), e_{\text{ifz}} \rightsquigarrow A_{\text{ifz}}, (t_{\text{ifz}}, B_{\text{ifz}}) \\ &(E, B_{\text{ifz}} \cup \{t_{\text{ifz}} = 0\}), e_{\text{then}} \rightsquigarrow A_{\text{then}}, (t_{\text{then}}, B_{\text{then}}) \\ &(E, B_{\text{ifz}} \cup \{t_{\text{ifz}} \neq 0\}), e_{\text{else}} \rightsquigarrow A_{\text{else}}, (t_{\text{else}}, B_{\text{else}}) \end{aligned}}{(\mathbf{E}, S), \text{if } e_{\text{ifz}} \text{ then } e_{\text{then}} \text{ else } e_{\text{else}} \rightsquigarrow A_{\text{ite}} \cup \text{rule}_{\text{ite}}, (X, S \cup \text{body}_{\text{ite}})}$$

$$\begin{aligned} X &= \text{a new ASP variable} \\ \text{ite} &= \text{a new ASP constant} \\ \text{rule}_{\text{ite}} &= \{\text{inter}((\text{ite}, \text{scope}_E), t_{\text{then}}) \leftarrow B_{\text{then}} \\ &\quad , \text{inter}((\text{ite}, \text{scope}_E), t_{\text{else}}) \leftarrow B_{\text{else}}\} \\ \text{body}_{\text{ite}} &= \{\text{inter}((\text{ite}, \text{scope}_E), X)\} \\ A_{\text{ite}} &= A_{\text{ifz}} \cup A_{\text{then}} \cup A_{\text{else}} \end{aligned}$$

If zero expressions are translated using the translations of its three subexpressions. But we need to alter the safety to ensure that the “then”-part is only evaluated if the “if”-part is 0 (and the analog for the “else” part). To construct the value of the full expression we define an intermediate symbol ( $\text{ite}$  in the rule)

---

**Listing 3** Translation of  $(\lambda x. \text{if } z \neq 0 \text{ then succ else pred}) 2 4$ 

---

```
1 inter((ite1,(X0)),succ):-domain((10,()),X0),X0=0.
2 inter((ite1,(X0)),pred):-domain((10,()),X0),X0<>0.
3 inter(((10,()),X0),X1):-domain((10,()),X0),inter((ite1,(X0)),
   X1).
4 domain((10,()),2).
5 domain(X2,4):-inter(((10,()),2),X2).
6 result(X3):-inter(((10,()),2),X2),inter((X2,4),X3).
7 % omitted static part visible in Listing 1
```

---

to represent the union of the “then” and the “else” part. Because the extra safety ( $= 0, \neq 0$ ) is mutually exclusive, only one of those terms will have a denotation, so the interpretation of *ite* will be unique.

*Example 10.* The translation of  $(\lambda x. \text{if } z \neq 0 \text{ then succ else pred}) 2 4$  is visible in Listing 3. The static part is omitted. Lines 1 and 2 are result of the if zero-then-else translation. Line 3 is the result of the lambda translation. Lines 4 and 5 are the result of the application. And in line 6 the end result can be seen. This rule can be read as follows: Let  $X_2$  be the application of the function to 2. Let  $X_3$  be application of  $X_2$  to 4. The final result is  $X_3$ .

#### 4.5 Optimisations

The translation algorithm which is given in the previous section is not an optimal translation. A lot of optimisations are possible, for instance, not all variables in scope need to be present in  $\text{scope}_E$ , only the ones which are actually used in the subexpression. Applying such optimisations can significantly reduce the size of the grounding of the ASP program. The possibilities here are very interesting research topics, but are considered out of scope for this paper.

#### 4.6 Implementation

An implementation was made in Kotlin. The runtime uses Clingo [9] to run the resulting ASP files, but the resulting specifications could be used with any ASP-Core-2 [2] compliant system. On <https://dtai.cs.kuleuven.be/krr/pcf2asp> you can find a tool on which you can try out the translation. A few example PCF formulas are provided, but you can ask for translations of arbitrary PCF formulas and see their corresponding answer set.

### 5 Applications

#### 5.1 Multiple interpretations for one variable

Directly translating PCF gives us little more than a traditional interpreter of PCF would do, but based on this translation we can provide extra functionality, leveraging the existing ASP solvers. Traditional PCF does not support the

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**Listing 4**  $a + b = c$  in PCF

---

```
1 ( $\lambda \text{eq} . \lambda \text{plus} .$ 
2    $\text{eq} (\text{plus } a \ b) \ c$ )
3
4 ( $\text{fix} (\lambda \text{eq} . \lambda x . \lambda y . \text{ifz } x \ \text{then} (\text{ifz } y \ \text{then} 0 \ \text{else} 1)$ 
5    $\text{else} (\text{ifz } y \ \text{then} 1 \ \text{else} \text{eq} (\text{pred } x) \ ($ 
6    $\text{pred } y)))$ )
6 ( $\text{fix} (\lambda \text{plus} . \lambda x . \lambda y . \text{ifz } y \ \text{then} x \ \text{else} \text{plus} (\text{succ } x) \ (\text{pred } y))$ )
```

---

**Listing 5**  $a + b = c$  in ASP

---

```
1  $1\{a(X)\}1 :- X=1..10.$ 
2  $1\{b(X)\}1 :- X=1..10.$ 
3  $1\{c(X)\}1 :- X=1..10.$ 
4  $:- \text{not result}(0).$ 
5 ...
6  $\text{domain}(X1, A) :- \text{domain}((10,()), X0), \text{domain}((11, (X0)), X1), a(A).$ 
7 ...
```

---

possibility that the interpretation of a term is not uniquely defined, but we can extend PCF so we can declare the variable  $a$  as a number between 1 and 10 without defining its specific value. In that case we can get (at most) 10 different evaluations of our program, one for each interpretation of  $a$ . It is easy to extend the translation to encode this in ASP.

Traditional interpreters solve the question: “What is the evaluation of this program?”. But using these variables another question can be interesting: what value(s) for  $a$  should I choose so that the program evaluates to 0. We can leverage the strengths of ASP solvers to find the solutions. Expressing that the evaluation should be zero can be done through a simple ASP constraint:

```
 $:- \text{not result}(0).$ 
```

When this constraint is added, the resulting answer sets will now all have the same interpretation (0) for the  $\text{result}$  predicate, but we are interested in the interpretation for  $a$ .

*Example 11.* In Listing 4 you can see a PCF expression representing that  $a + b = c$ . If we now use choice rules in ASP to translate these variables to the domain of natural numbers between 0 and 10, we can use ASP to find multiple solutions of this equation. An example of how this would look in ASP can be seen in Listing 5.

The problem in Example 11 can easily be generalised to arbitrarily complex polynomials to model mixed integer problems. A graph coloring problem can be represented by using a new constant for each node that needs to be colored and writing down an expression that evaluated to 0 if the graph is colored correctly.

An important thing to note here is that ASP does not naively calculate the result for all possible values of the choice rules. It uses a CDCL-based solving algorithm to explore the search space in an intelligent way.

### 5.2 Towards a more expressive language

PCF is not intended to be an end-user language, but it serves as a basis for many real world programming languages. Analogously, we are developing a more expressive language based on the principles of PCF. This language includes more complex data types for representations which are more elegant than possible in PCF. Together with the multiple-model semantics of ASP this leads to an interesting modelling language. Using these ideas the new Functional Modelling System (FMS) is being developed. On the website <https://dtai.cs.kuleuven.be/krr/fms> a demonstration of this new system can be found. This system is an extension of PCF with some more practical language constructs and uses the translation principles described in this paper to use ASP as a solver engine for this new language. However as indicated in Section 4.5, a lot of optimisations are needed to be competitive with native ASP encodings. The efficiency of these translators have not been formally investigated yet.

## 6 Conclusion

We presented a translation from PCF to ASP programs. A basic translation is easily implemented, and many optimisations are possible. With only small changes, we can exploit the search power of ASP to solve problems expressed in PCF. This translation can serve as a basis to use functional programming techniques in modelling languages for search problems, or even tighter integrations between functional and logical languages. FMS is under development now and uses the techniques described in this paper as a basis for its language.

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