Model Predictive Control for Wake Steering: a Koopman Dynamic Mode Decomposition Approach

12th October 2020

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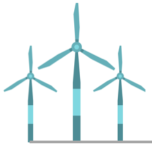
0 | Presentation outline

01 The problem

1. From wind turbine control to **wind farm control**: maximising the energy production collectively.
2. **Reduced Order Models** as a pathway to the design and implementation of controllers.

02 Methodology

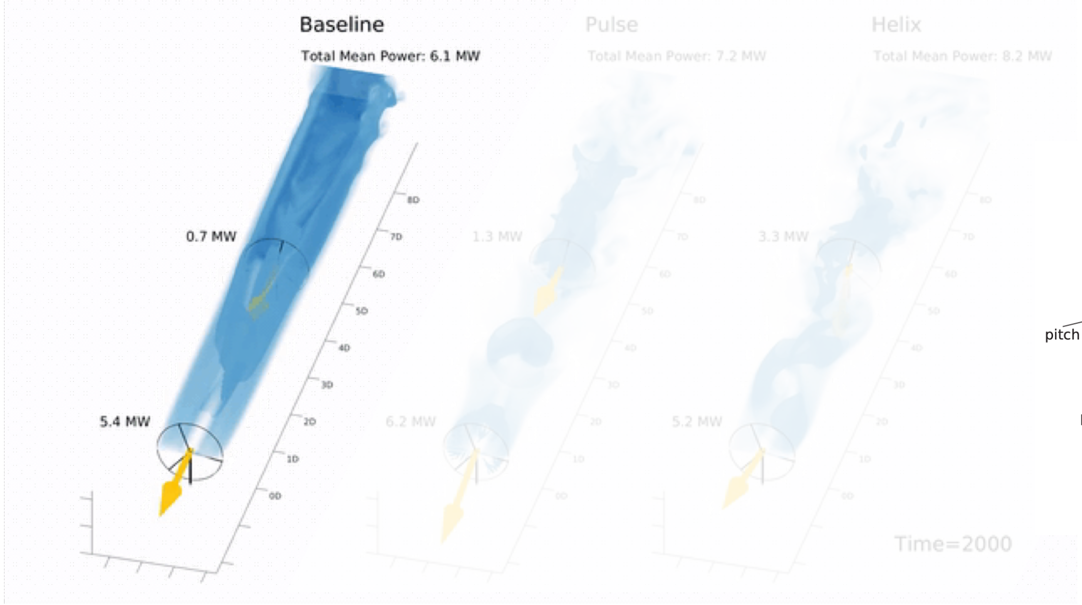
1. Use **Input Output Dynamic Mode Decomposition** to find a suitable Reduced Order Model.

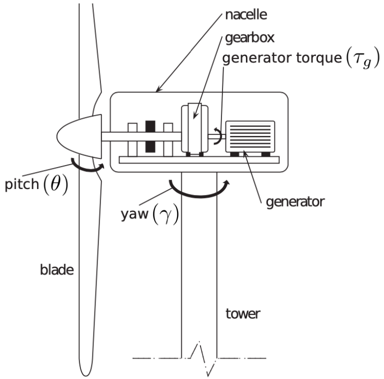


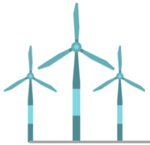
03 Results

1. How accurate are models in **reproducing the power generated** in each turbine and the flow between turbines?
2. Can a **predictive controller be designed** and implemented based on a Reduced Order Model?

1 | Considering aerodynamic interaction to maximise energy production



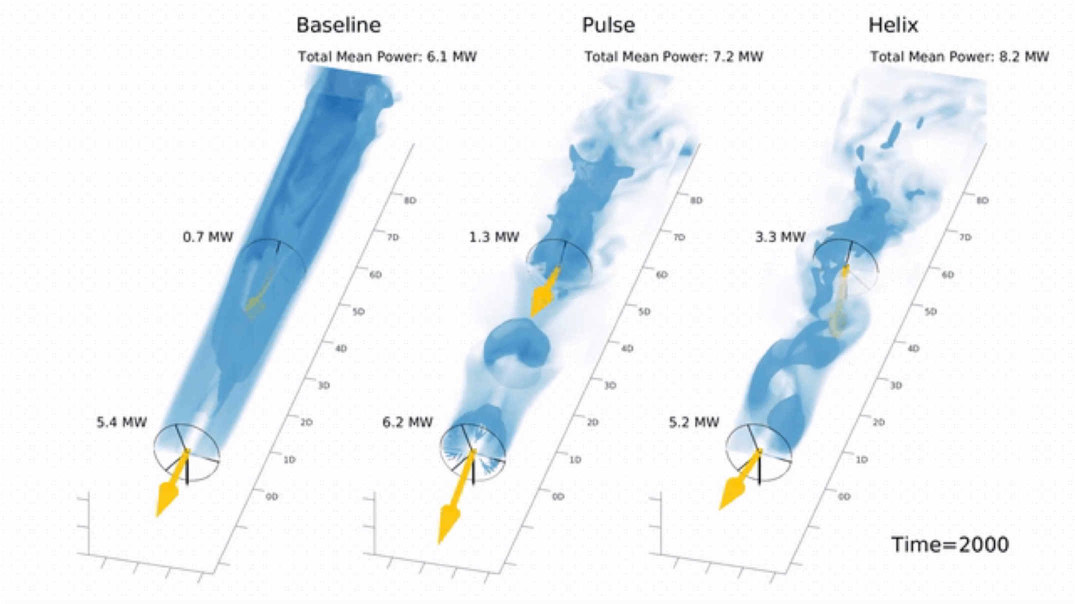




Sitting wind turbines together in wind farms is economically advantageous.

Wake interaction leads to power losses.

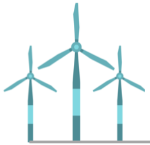
1 | Considering aerodynamic interaction to maximise energy production



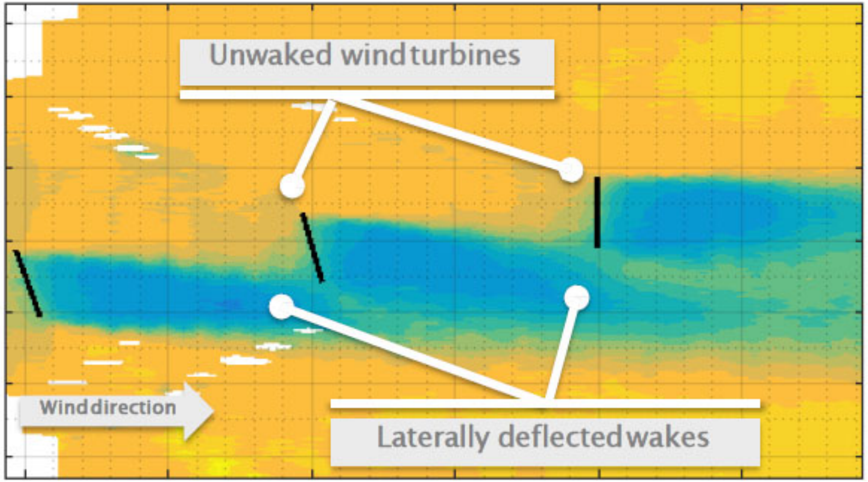
Sitting wind turbines together in wind farms is economically advantageous.

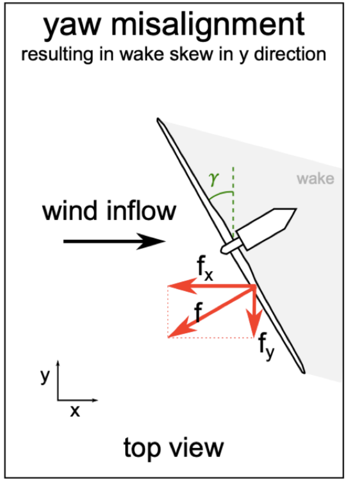
Wake interaction leads to power losses.

Wind Farm Control Strategies can mitigate losses.



1 | Redirecting the wake by misaligning upstream turbine rotor



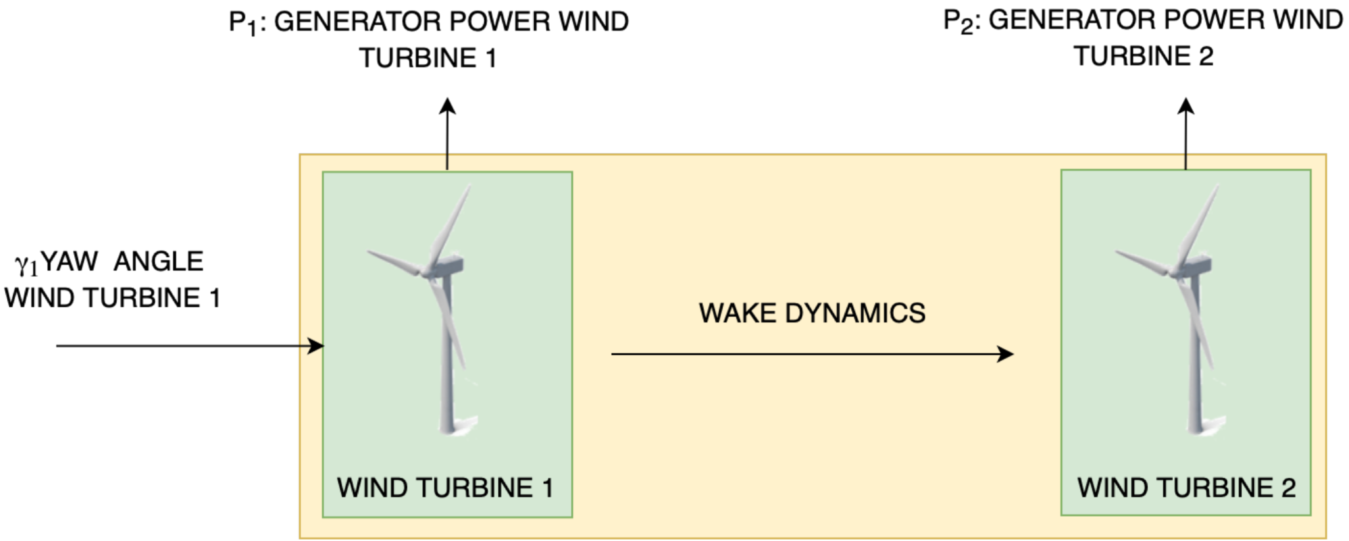


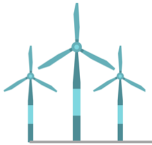
Wake Redirection Control (WRC) changes direction of the wake.

The goal is to avoid wake overlap with downstream turbine rotors.

The thrust force perpendicular to the rotor axis f_y causes the wake to be redirected.

2 | Linear state space reduced order model used to capture dynamics





Reduced Order Model maps the upstream turbine yaw angle to the power produced in each turbine.

The grey box model can be used to fully reconstruct the upstream turbine wake.

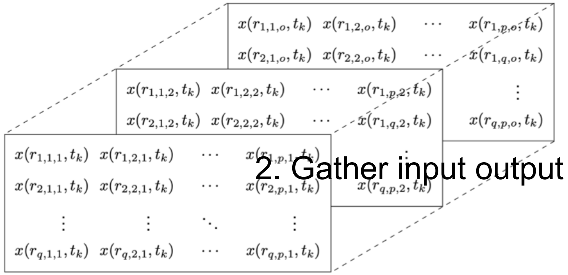
Compute a Reduced Order Model to design and implement a controller in a high fidelity simulator.

2 | Use Input Output Dynamic Mode Decomposition to obtain model

0. Final goal: obtain linear state space representation of wind farm system

$$x_{k+1} = \mathbf{A}\, x_k + \mathbf{B}\, u_k$$
$$y_k = \mathbf{C}\, x_k + \mathbf{D}\, u_k$$

1. Gather data for states in the time shifted snapshot matrices



$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_{m-1} \\ | & | & & | \end{bmatrix} \qquad \mathbf{X}' = \begin{bmatrix} | & | & & | \\ x_2 & x_3 & \cdots & x_m \\ | & | & & | \end{bmatrix}$$

2. Gather input output data matrices

$$\boldsymbol{\Upsilon} = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_{m-1} \\ | & | & & | \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} | & | & & | \\ y_1 & y_2 & \cdots & y_{m-1} \\ | & | & & | \end{bmatrix}$$

2 | Use Input Output Dynamic Mode Decomposition to obtain model

3. Use data only to find state space matrices

$$\begin{bmatrix} \mathbf{X}' \\ \boldsymbol{\Upsilon} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \boldsymbol{\Upsilon} \end{bmatrix}$$

4. Project onto low order subspace

   4.1. Perform Singular Value Decomposition of $\mathbf{X}$ (truncation value r)

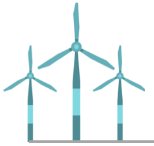
$$\mathbf{X} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^* = \begin{bmatrix} \mathbf{U} & \mathbf{U}_{\mathrm{rem}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\mathrm{rem}} \end{bmatrix} \begin{bmatrix} \mathbf{V}^* \\ \mathbf{V}^*_{\mathrm{rem}} \end{bmatrix}$$

   4.2. Project system into low order subspace

$$\tilde{\mathbf{A}} = \mathbf{U}^*\mathbf{A}\,\mathbf{U}, \qquad \tilde{\mathbf{B}} = \mathbf{U}^*\mathbf{B}, \qquad \tilde{\mathbf{C}} = \mathbf{C}\,\mathbf{U}, \qquad \tilde{\mathbf{D}} = \mathbf{D}$$

$$\min_{\begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}} & \tilde{\mathbf{D}} \end{bmatrix}} \left\| \begin{bmatrix} \mathbf{X}' \\ \boldsymbol{\Upsilon} \end{bmatrix} - \begin{bmatrix} \mathbf{U} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}} & \tilde{\mathbf{D}} \end{bmatrix} \begin{bmatrix} \mathbf{U}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \boldsymbol{\Upsilon} \end{bmatrix} \right\|_F^2$$

5. Obtain state space low order representation

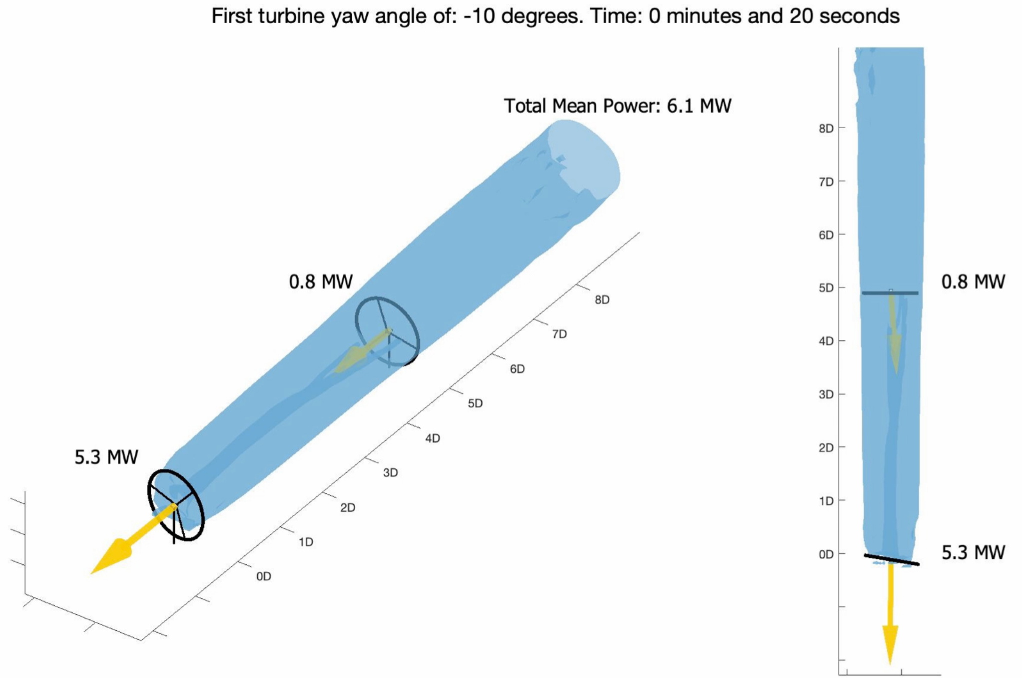


$$\Theta_{\mathrm{IODMD}} = \begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}} & \tilde{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^*\mathbf{X}' \\ \boldsymbol{\Upsilon} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}\,\mathbf{V} \\ \boldsymbol{\Upsilon} \end{bmatrix}^{\dagger}$$

3 | Gather states and input-output data for IODMD

Upstream turbine yaw angle varies from -10 to -30 degrees randomly throughout simulation.

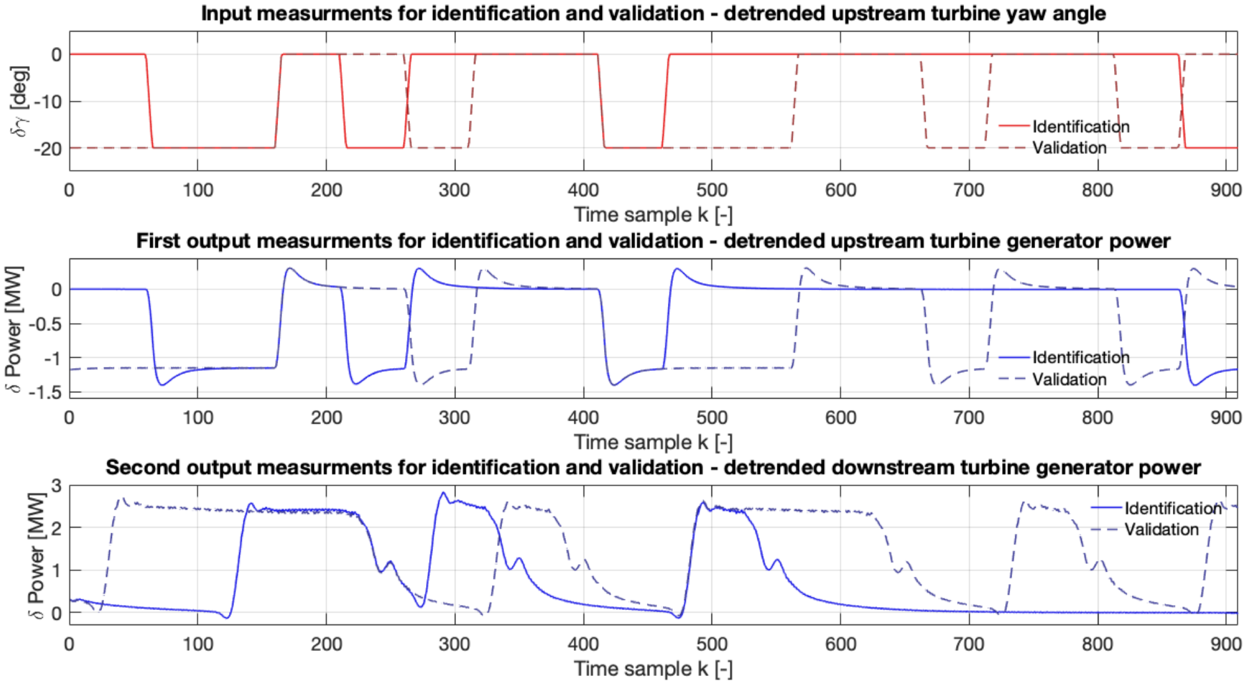
0. Gather data in SOWFA
1. Pre process data
2. Compute $IODMD_u$
3. Validate model
4. Non linear observables
5. Design predictive controller
6. Tune controller parameters
7. Test controller in SOWFA

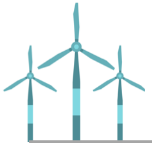


3 | Remove mean from input-output measurements

Two simulations are performed for identification and validation purposes.

Data is pre-processed and offsets removed.





3 | Use streamwise velocity component *u* as state

The streamwise velocity component u is firstly used to derive the IODMD model.

A truncational value r is used and r models of increasing size computed.

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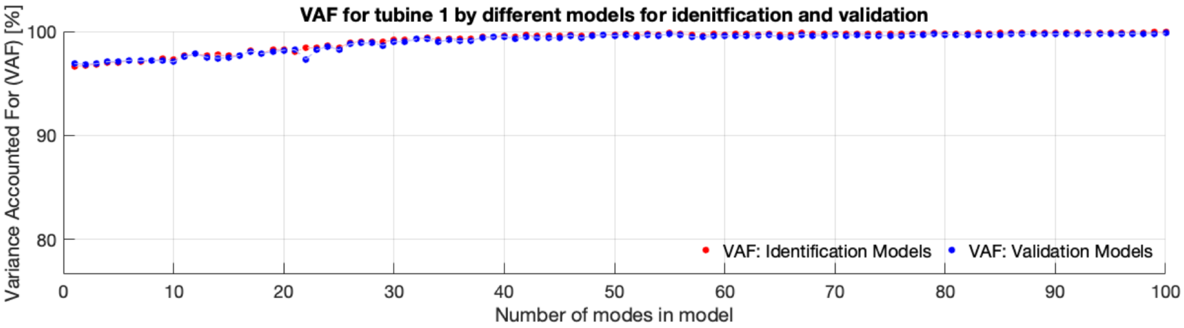

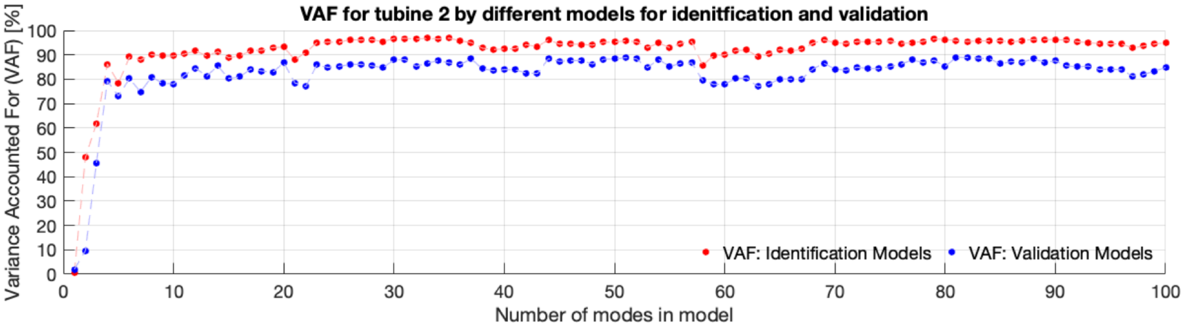

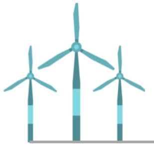

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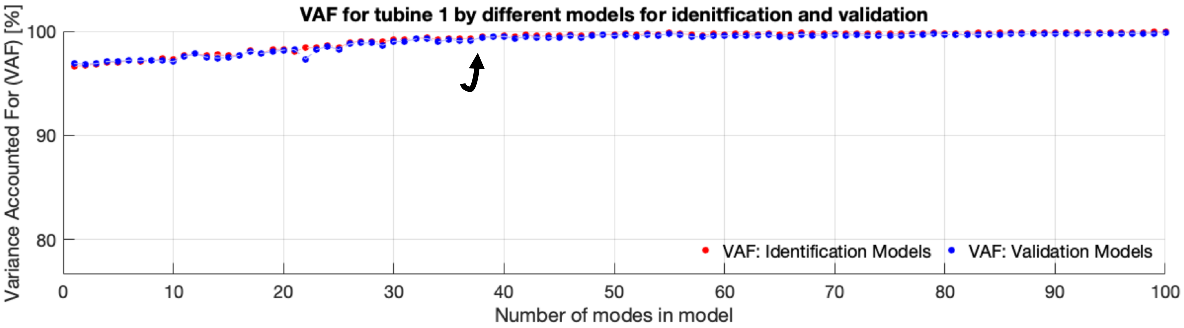
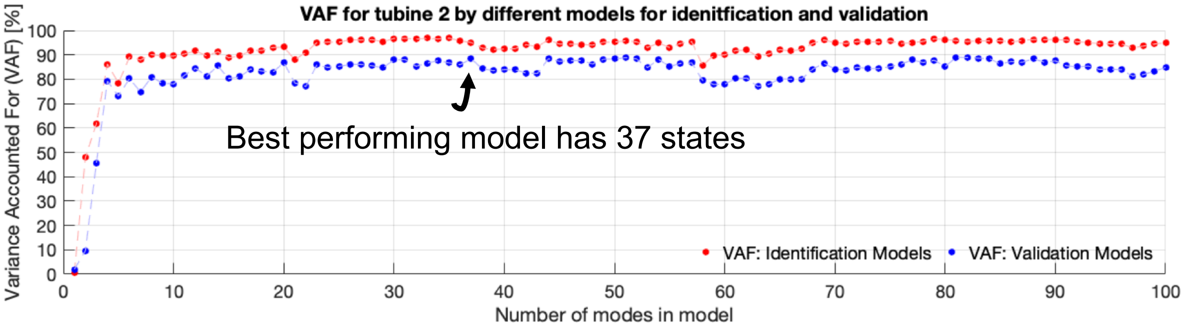
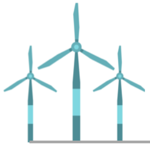
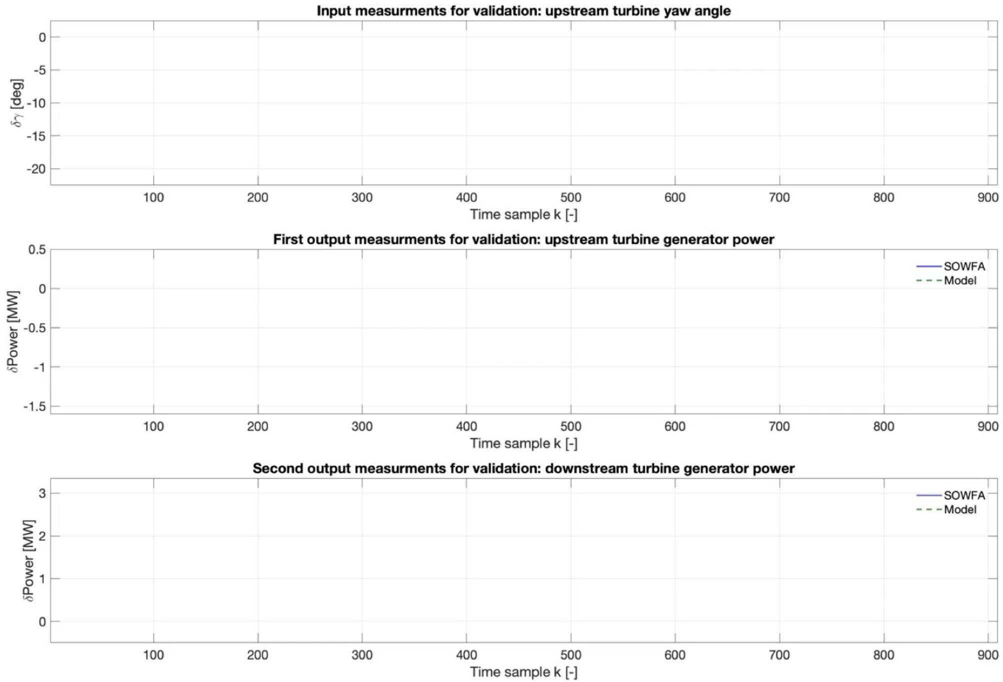


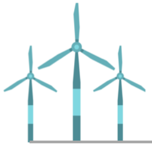




3 | Validate fit of the model in terms of predicting turbines power

Best performing model reproduces generator power of first turbine with a fit of 99% and generator power of second turbine with a fit of 88%.

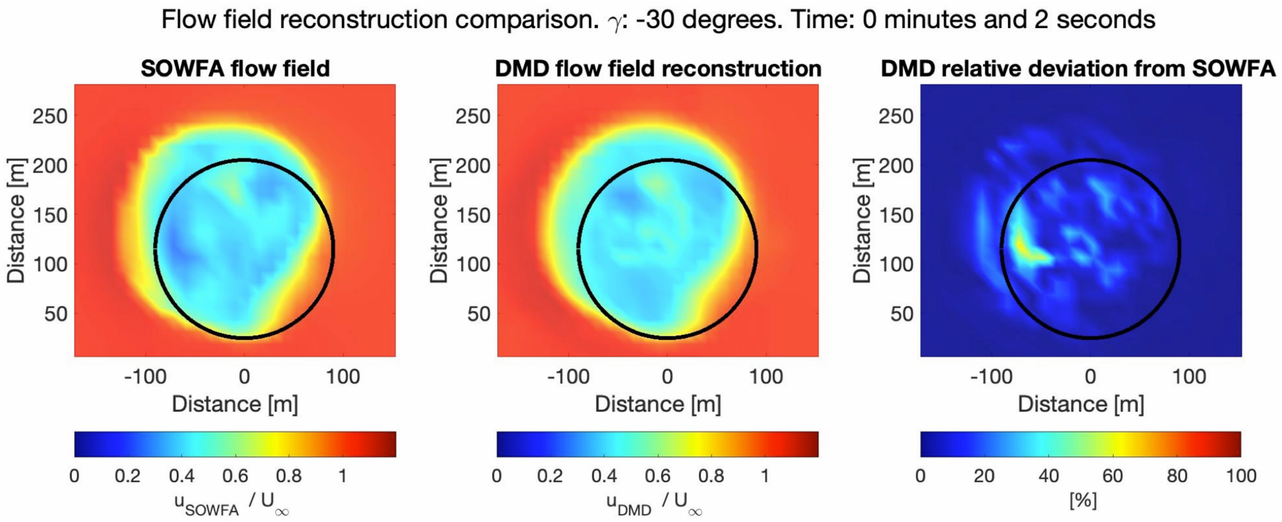


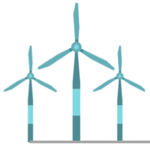


3 | Validate fit of the model in terms of predicting wake behaviour

At every time instant k the normalised root mean squared error (NRMSE) is computed for all grid points used in IODMD.

The average value throughout the simulation is taken, corresponding to 4%.





3 | Validate fit of the model in terms of predicting wake behaviour

Reconstruction of wake at hub height plane.

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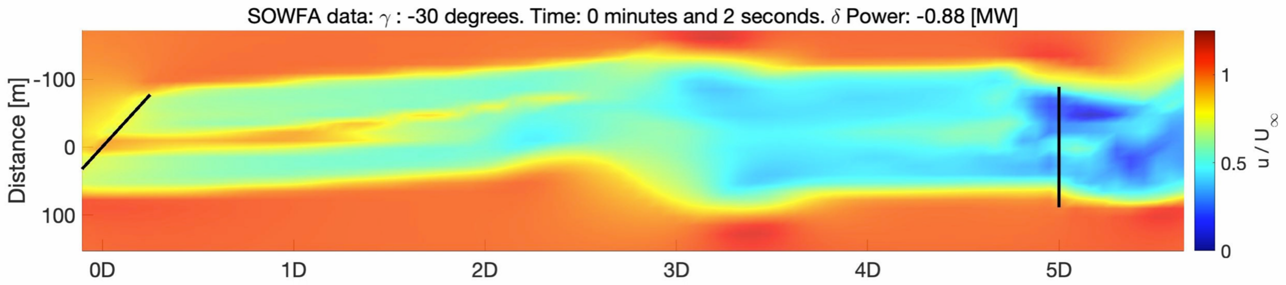
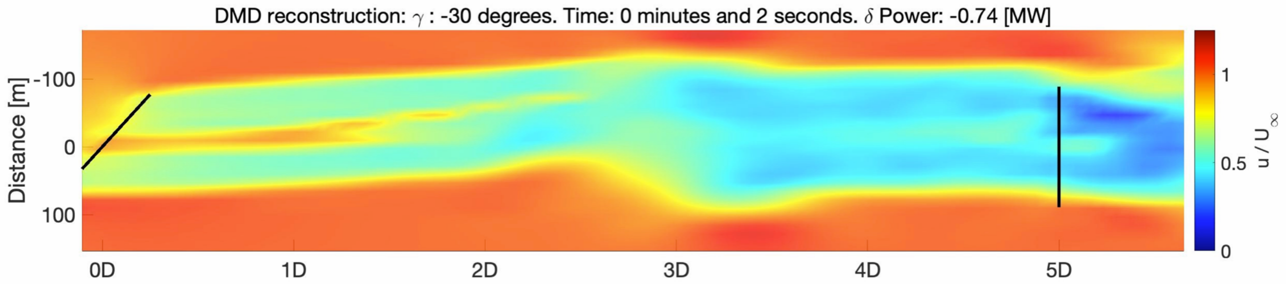




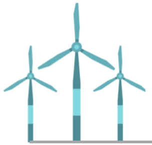
3 | Test other states to compute reduced order models

Use other information to build data matrices (termed observables).

Increases in model's fit by using non linear observables.

Less number of states needed to reproduce dynamics.

Model properties	Model obtained by IODMD			Δ IODMD_u	
Observable	$\text{VAF(WT2)}_{\text{max}}$	Model size	$\text{VAF(WT1)}\vert_r$	Δ VAF $(\text{WT2})_{max}$	Δ size
v	87.12 %	40	99.39 %	-1.28%	+8.11%
w	87.54 %	50	99.35 %	-0.86%	+35.14%
u'	78.14 %	38	98.08 %	-10.26%	+2.70%
v'	61.30 %	31	97.13 %	-27.10%	-16.22%
u^2	89.63 %	26	98.83 %	**+1.23%**	**-29.73%**
v^2	89.58 %	12	97.52 %	**+1.18%**	**-67.57%**
w^2	87.85 %	47	97.67 %	-0.55%	+27.03%
$u \cdot v$	69.40 %	15	97.58 %	-19.00%	-59.46%
$u^2 + v^2$	89.42 %	26	98.84 %	+1.02%	-29.73%
$u^2 + v^2 + w^2$	89.36 %	49	99.75%	+0.96%	+32.43%



3 | Design a predictive controller to produce pre-specified power

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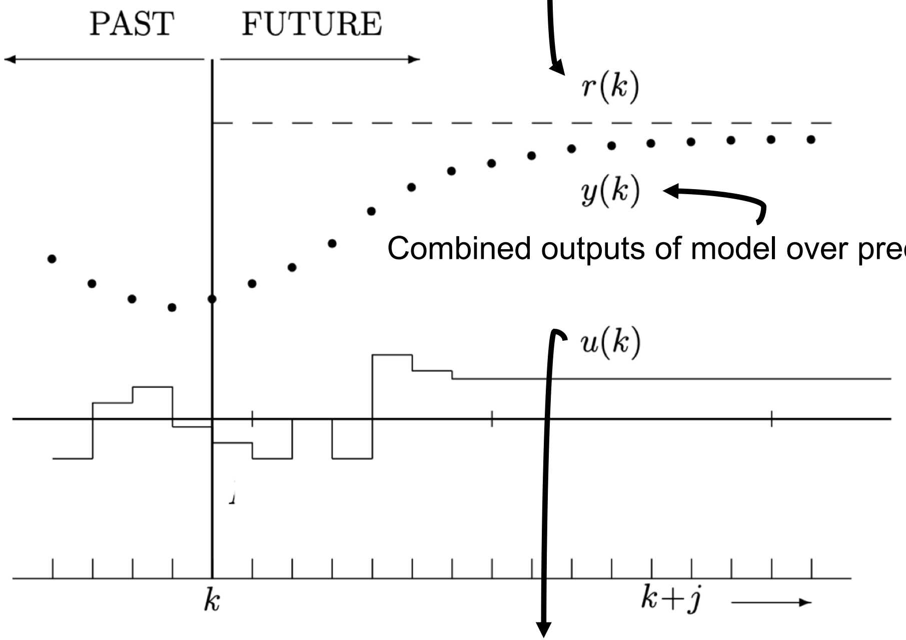
Reference for collective power to be produced by both turbines



Combined outputs of model over prediction horizon H_p

Optimised control action over control horizon H_c

3 | Formulate model predictive control mathematically

- 0. Gather data in SOWFA
- 1. Pre process data
- 2. Compute IODMD_u
- 3. Validate model
- 4. Non linear observables
- 5. Design predictive controller
- 6. Tune controller parameters
- 7. Test controller in SOWFA

1. Use extended state space model to predict outputs throughout prediction horizon

$$\bar{\mathbf{P}} = \bar{\mathbf{H}}_e \Delta\bar{\mathbf{U}} + \mathbf{\Gamma}_e \boldsymbol{x}_k^e$$

$$\begin{bmatrix} \mathbf{P}_{k+1} \\ \vdots \\ \mathbf{P}_{k+H_p} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_e\mathbf{B}_e & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_e\mathbf{A}_e^{H_p-1}\mathbf{B}_e & \cdots & \mathbf{C}_e\mathbf{B}_e \end{bmatrix} \begin{bmatrix} \Delta u_k \\ \vdots \\ \Delta u_{k+H_p-1} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_e\mathbf{A}_e \\ \vdots \\ \mathbf{C}_e\mathbf{A}_e^{H_p} \end{bmatrix} \boldsymbol{x}_k^e$$

2. Formulate a cost function which penalises deviations from reference and control effort

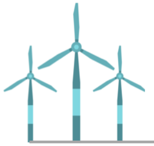
$$J = (\hat{\mathbf{P}} - \mathbf{P}_{\text{ref}})^{\mathrm{T}}\,\mathbf{Q}\,(\hat{\mathbf{P}} - \mathbf{P}_{\text{ref}})^{\mathrm{T}} + \Delta\bar{\mathbf{U}}^{\mathrm{T}}\,\mathbf{R}\,\Delta\bar{\mathbf{U}}$$

3. Impose restrictions on yaw angle

$$-35 \leq \gamma \leq 0 \quad \text{[degrees]}$$

4. Solve optimisation problem as a Quadratic Programming problem

$$\Delta\bar{\mathbf{U}}^* = \min_{\Delta\bar{\mathbf{U}}} \left\{\tfrac{1}{2}\Delta\bar{\mathbf{U}}^{\mathrm{T}}\,\mathcal{H}\,\Delta\bar{\mathbf{U}} + \boldsymbol{f}^{\mathrm{T}}\Delta\bar{\mathbf{U}}\right\}, \text{ subject to } \mathbf{M}\,\Delta\bar{\mathbf{U}} \leq \mathbf{\Lambda}$$



3 | Design an estimator to update states

Final test of controller: states are updated based on output measurements from SOWFA

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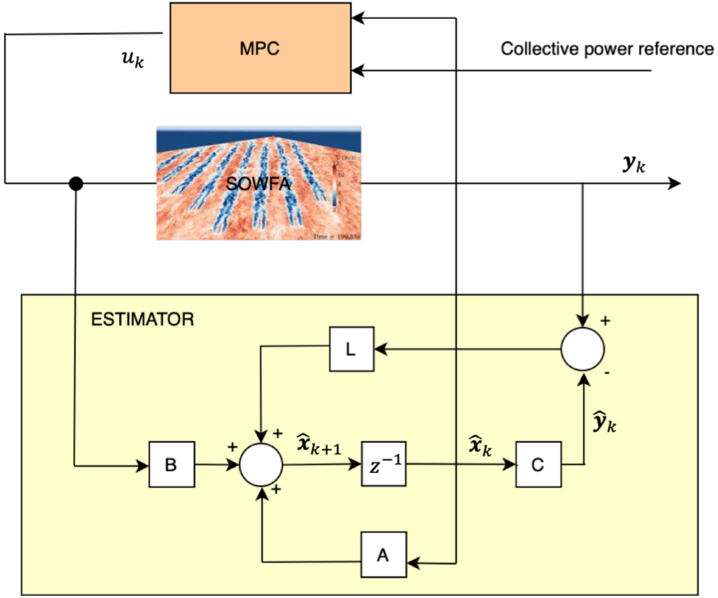

Final controller settings: $H_p = H_c = 350$. **Q**=10. **R**=1.

3 | Test controller in SOWFA

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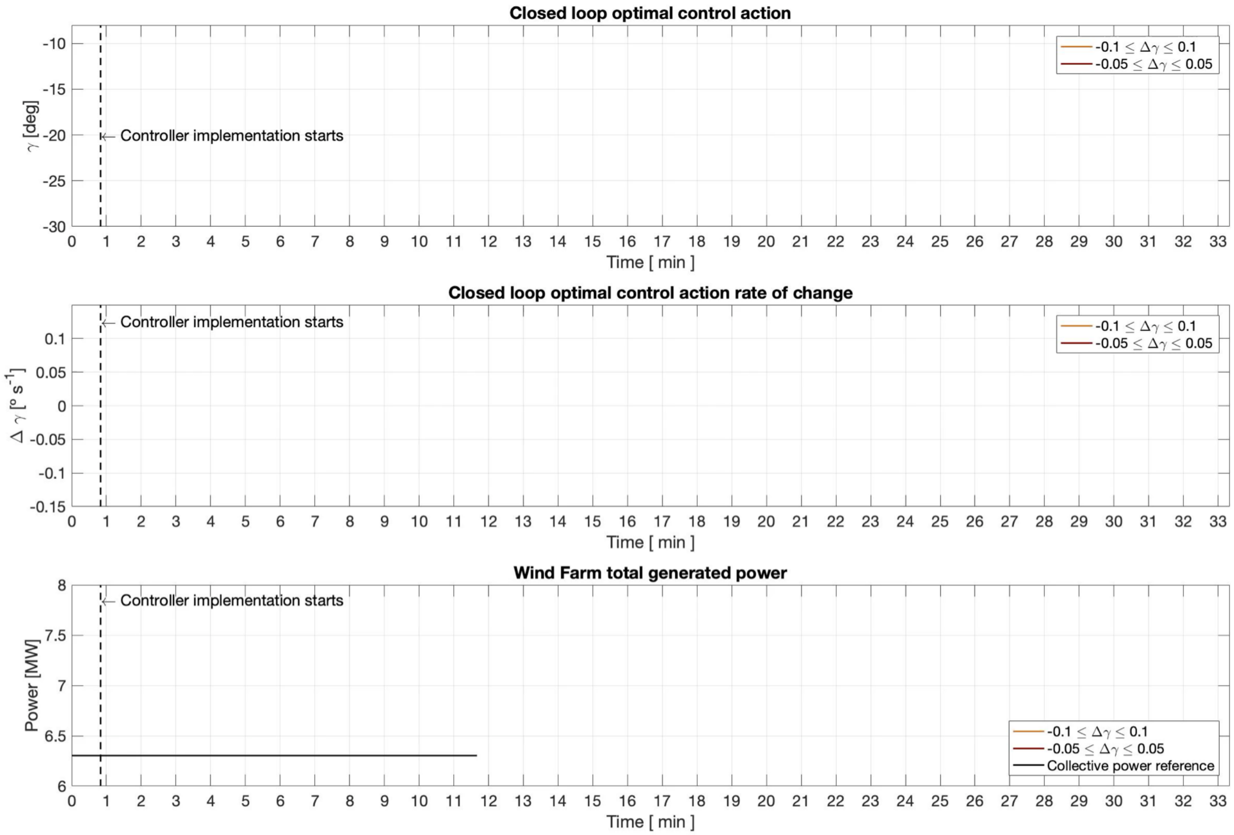

4 | Conclusions

1. **Reduced Order Models are able to reproduce the dynamics of the wind farm system**
    1. IODMD_u accounts for 99% of upstream turbine generator power variance and 88% of downstream turbine generator power variance.
    2. Wake is reconstructed with an average NRMSE of 4%.

2. **Using other flow variables leads to higher fits of model outputs.**
    1. Using non linear variables, such as u^2 or $u^2 + v^2$, leads to increases in fit of outputs in the order of 1.23% and decreases in number of states of the model.

3. **Model Predictive Controller based on the Reduced Order Models can be designed and implement in high fidelity solvers for a reference tracking problem.**
    1. Slow dynamics of wind farm are embedded in the controller