

1. Holes in inference patterns

- Terms and identity

(1a) 31 is prime. $\varphi[31] [= P(\underline{31})]$
The number of persons in this room is 31. $n = 31$
 \therefore The number of persons in this room is prime. $\varphi[n] [= P(\underline{n})]$

(b) It is fact of elementary arithmetic that 31 prime.
The number of persons in this room is 31.
 \therefore It is fact of elementary arithmetic that the number of persons in this room is prime.

(2a) John's salary is higher than Mary's. $\varphi[j, \underline{m}] [= s(j) > s(m)]$
 John is the dean. $j = d$
Mary is the vice dean. $m = v$
 \therefore The dean's salary is higher than the vice dean's. $\varphi[\underline{d}, \underline{v}]$

(b) Bill knows that the dean's salary is higher than the vice dean's.
 John is the dean.
Mary is the vice dean.
 \therefore Bill knows that John's salary is higher than Mary's.

- Problems with existential quantification

(3a) Urs is a Swiss millionaire. $\varphi[M] [= S(u) \ \& \ \underline{M}(u)]$
 All millionaires admire Scrooge McDuck. $(\forall x) [M(x) \rightarrow A(x)]$
[Only millionaires admire Scrooge McDuck.] $(\forall x) [A(x) \rightarrow M(x)]$
 \therefore Urs is a Swiss admirer of Scrooge McDuck. $\varphi[A] [= S(u) \ \& \ A(u)]$

Urs is an alleged millionaire.
 All millionaires admire Scrooge McDuck.
Only millionaires admire Scrooge McDuck.
 \therefore Kim is an alleged admirer of Scrooge McDuck.

(4a) Paul is wearing a pink shirt with green sleeves.
 All pink shirts with green sleeves have striped collars and gold buttons.
[Only pink shirts with green sleeves have striped collars and gold buttons.]
 \therefore Paul is wearing a shirt with striped collars and gold buttons.

(b) Paul is looking for a pink shirt with green sleeves.
 All pink shirts with green sleeves have striped collars and gold buttons.
Only pink shirts with green sleeves have striped collars and gold buttons.
 \therefore Paul is looking for a shirt with striped collars and gold buttons.

- (5a) Susan is entering a restaurant on Main Street.
The only restaurants on Main Street are *La Gourmande* and *Le Gourmet*.
 \therefore Susan is entering *La Gourmande*, or [Susan is entering] *Le Gourmet*.
- (b) Susan is looking for a restaurant on Main Street.
The only restaurants on Main Street are *La Gourmande* and *Le Gourmet*.
 \therefore Susan is looking for *La Gourmande*, or [Susan is looking for] *Le Gourmet*.
- (6a) Paul is wearing a pink shirt with green sleeves.
 \therefore There are pink shirts with green sleeves.
- (b) Paul is looking for a pink shirt with green sleeves.
 \therefore There are pink shirts with green sleeves.
- (7a) There have never been any pictures of Lily.
 \therefore It is not true that Pete showed Roger a picture of Lily.
- (b) There have never been any pictures of Lily.
 \therefore It is not true that Pete owed Roger a picture of Lily.

2. Extensions

- Compositionality

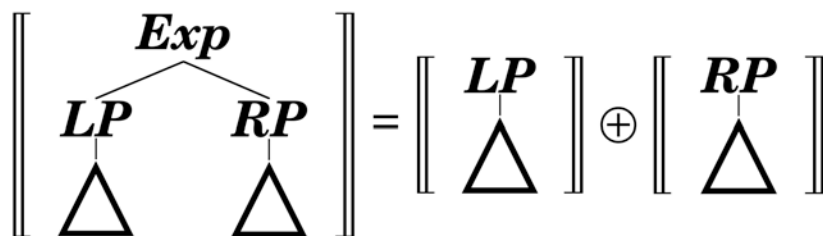
Substitution Principle

If two non-sentential expressions of the same category have the same meaning, either may replace the other in all positions within any sentence without thereby affecting the truth conditions of that sentence.

Principle of Compositionality

The meaning of a complex expression functionally depends on the meanings of its immediate parts and the way in which they are combined:

(8)



- Meaning as communicative function
- *Extension*: [contribution to] reference
- *Intension*: [contribution to] informational content
- ...

- Basic extensions

- (9a) $\llbracket \text{Austin} \rrbracket = \text{Austin}$ $\llbracket \text{proper name} \rrbracket = \text{bearer}$
 (b) $\llbracket \text{the capital of Texas} \rrbracket = \text{Austin}$ $\llbracket \text{definite description} \rrbracket = \text{descriptee}$
 (c) $\llbracket \text{city} \rrbracket = \{\text{London, Paris, Rome, Austin, Frankfurt, ...}\} = \{x \mid x \text{ is a city}\}$ $\llbracket \text{count noun} \rrbracket = \text{set of representatives}$
 (d) $\llbracket \text{snore} \rrbracket = \{x \mid x \text{ snores}\}$ $\llbracket \text{intransitive verb} \rrbracket = \text{set of satisfiers}$
 (e) $\llbracket \text{meet} \rrbracket = \{(x,y) \mid x \text{ meets } y\}$ $\llbracket \text{transitive verb} \rrbracket = \text{set of satisfier pairs}$
 (f) $\llbracket \text{show} \rrbracket = \{(x,y,z) \mid x \text{ shows } y \text{ to } z\}$ $\llbracket \text{ditransitive verb} \rrbracket = \text{set of satisfier triples}$
 (g) $\llbracket \text{shows Joe} \rrbracket = \{(x,y) \mid x \text{ shows } y \text{ to Joe}\}$ $\llbracket \text{2-place predicate} \rrbracket = \text{set of satisfier pairs}$
 (h) $\llbracket \text{shows Joe the Vatican} \rrbracket = \{(x) \mid x \text{ shows the Vatican to Joe}\}$
 $= \{x \mid x \text{ shows the Vatican to Joe}\}$ $\llbracket \text{1-place predicate} \rrbracket = \text{set of satisfiers}$

☞ *Parallelism between valency and type of extension*

Frege (1891)

The extension of an n -place predicate is a set of n -tuples.

E.g. $\llbracket \text{Benny shows Angie the Vatican} \rrbracket = \{() \mid \text{Benny shows the Vatican to Angie}\}$
 $= \text{the set of objects of the form ' () ' such that Benny shows the Vatican to Angie, i.e.:}$
 $\llbracket \text{Benny shows Angie the Vatican} \rrbracket = \begin{cases} \{() \}, & \text{if Benny does show the Vatican to Angie} \\ \emptyset & \text{otherwise} \end{cases}$

NB: $() = \emptyset = 0$; hence $\{()\} = \{\emptyset\} = \{0\} = 1$!

☞ *Frege's Generalization*

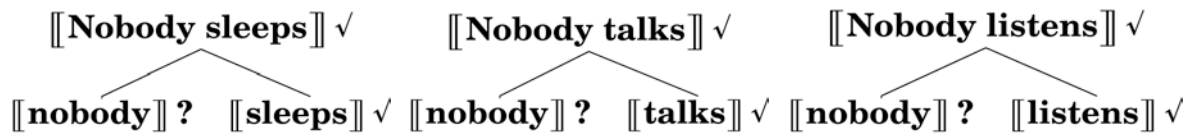
Frege (1892)

The extension of a sentence S is its truth value, i.e. 1 if S is true and 0 if S is false.

- Constructing contributing extensions

- (10a) *From:* $\llbracket \text{Exp} \rrbracket \checkmark$ *... to:* $\llbracket \text{Exp} \rrbracket \checkmark$
 $\llbracket \text{LP} \rrbracket ? \quad \llbracket \text{RP} \rrbracket \checkmark$ $\llbracket \text{LP} \rrbracket \checkmark \quad \llbracket \text{RP} \rrbracket \checkmark$
 (b) $\llbracket \text{LP} \rrbracket (\llbracket \text{RP} \rrbracket) = \llbracket \text{Exp} \rrbracket$
 (c) $\llbracket \text{LP} \rrbracket = \{(\llbracket \text{RP} \rrbracket , \llbracket \text{Exp} \rrbracket) \mid \text{Exp} = \text{LP} + \text{RP}\}$

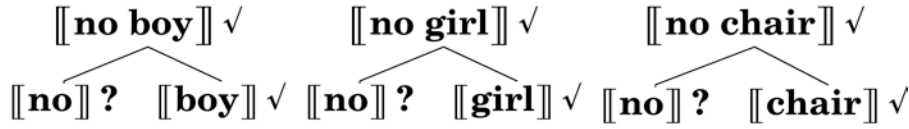
(11a)



- (b) $\llbracket \text{nobody} \rrbracket (\llbracket \text{sleeps} \rrbracket) = \llbracket \text{nobody sleeps} \rrbracket \Rightarrow \llbracket \text{nobody} \rrbracket (S) = 1$ S : sleepers
 $\llbracket \text{nobody} \rrbracket (\llbracket \text{talks} \rrbracket) = \llbracket \text{nobody talks} \rrbracket \Rightarrow \llbracket \text{nobody} \rrbracket (T) = 0$ T : talkers
 $\llbracket \text{nobody} \rrbracket (\llbracket \text{listens} \rrbracket) = \llbracket \text{nobody listens} \rrbracket \Rightarrow \llbracket \text{nobody} \rrbracket (L) = 1$ L : hearers
 (c) $\llbracket \text{nobody} \rrbracket = \{(S,1), (T,0), (L,1), \dots\}$
 $= \{(Y, \vdash \llbracket \text{person} \rrbracket \cap Y = \emptyset \neg) \mid Y \text{ is a (possible) predicate extension}\}$
 $= \lambda Y. \vdash \llbracket \text{person} \rrbracket \cap Y = \emptyset \neg$

NB: $\vdash \dots \neg :=$ the truth value that is 1 iff ...

(12a)



- (b) $\llbracket \text{no} \rrbracket (\llbracket \text{boy} \rrbracket) = \lambda Y. \vdash B \cap Y = \emptyset \dashv$ B : boys
 $\llbracket \text{no} \rrbracket (\llbracket \text{girl} \rrbracket) = \lambda Y. \vdash G \cap Y = \emptyset \dashv$ G : girls
 $\llbracket \text{no} \rrbracket (\llbracket \text{city} \rrbracket) = \lambda Y. \vdash C \cap Y = \emptyset \dashv$ C : cities
- (c) $\llbracket \text{no} \rrbracket = \lambda X. \lambda Y. \vdash X \cap Y = \emptyset \dashv$

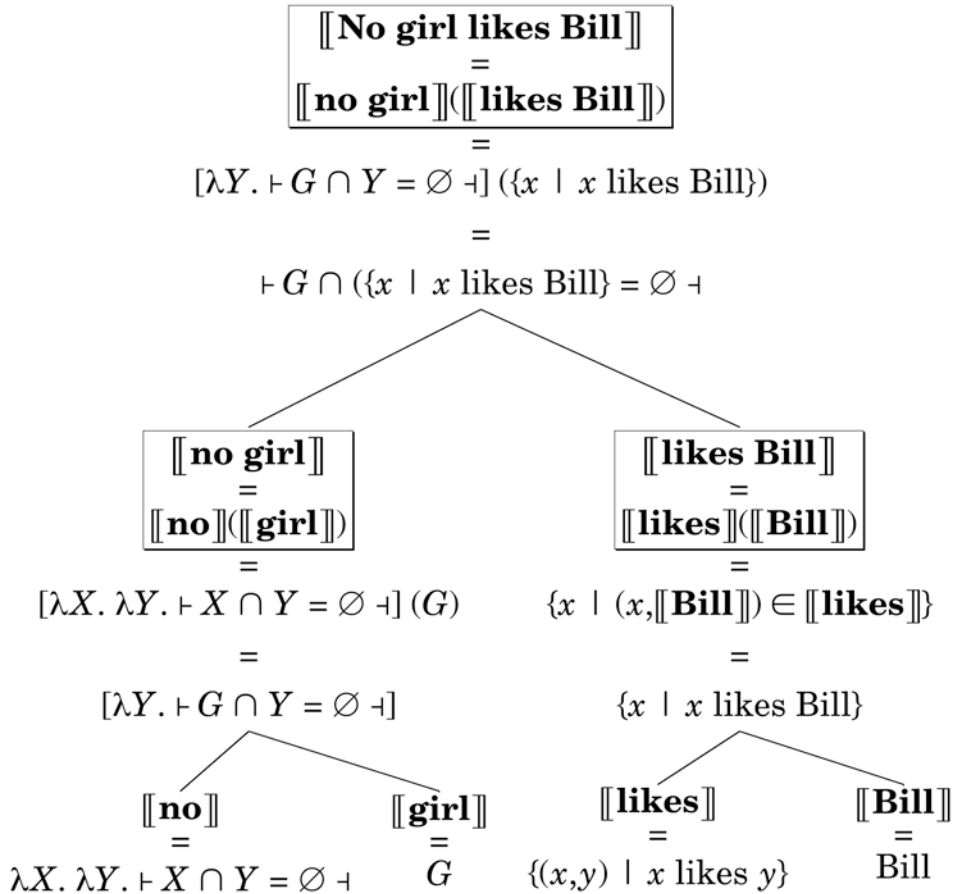
- (13) $\llbracket \text{every} \rrbracket = \lambda X. \lambda Y. \vdash X \subseteq Y \dashv$
 $\llbracket \text{some} \rrbracket = \lambda X. \lambda Y. \vdash X \cap Y \neq \emptyset \dashv$
 $\llbracket \text{one} \rrbracket = \lambda X. \lambda Y. \vdash |X \cap Y| = 1 \dashv$ $|Z|$: # of elements of Z (cardinality)
 $\llbracket \text{most} \rrbracket = \lambda X. \lambda Y. \vdash |X \cap Y| > |X \setminus Y| \dashv$

- (14a) $\llbracket \text{Bill}_M \rrbracket = \lambda X. \vdash \text{Bill} \in X \dashv$ *cf.* Montague (1970)

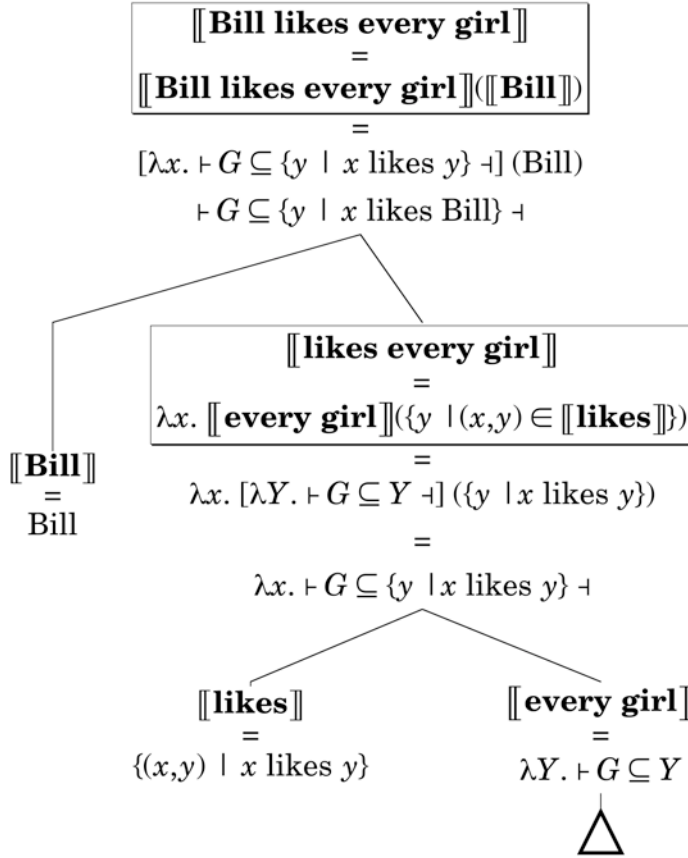
- (b) $\llbracket \text{the}_R \rrbracket = \lambda X. \lambda Y. \vdash |X| = 1 \ \& \ X \subseteq Y \dashv$ *cf.* Russell (1905)

- Extensional constructions

(15)



(16)



• Extensional types

U : domain of individuals

(17a) $A [\subseteq U] \approx \lambda x. \vdash x \in A \vdash$

characteristic function (of A rel. to U)

(b) $R [\subseteq U^2] \approx \lambda x. \lambda y. \vdash (x,y) \in R \vdash \approx \lambda y. \lambda x. \vdash (x,y) \in R \vdash$

(c) $R [\subseteq U^3] \approx \lambda z. \lambda y. \lambda x. \vdash (x,y,z) \in R \vdash$

(18) x is of type $e \Leftrightarrow x \in U$;

u is of type $t \Leftrightarrow u \in \{0,1\}$;

f is of type $(a,b) \Leftrightarrow f: \{x \mid x \text{ is of type } a\} \rightarrow \{y \mid y \text{ is of type } b\}$

(19) *Extensions and their types*

Category	Example	Extension	Type
Name	Austin	Austin $[\in U]$	<i>e</i>
Description	the capital of Texas	Austin $[\in U]$	<i>e</i>
Noun	city	$C [\subseteq U]$	<i>et</i>
1-place predicate	sleep	$S [\subseteq U]$	<i>et</i>
2-place predicate	eat	$\subseteq U \times U$	<i>et</i>
3-place predicate	give	$\subseteq U \times U \times U$	<i>e(et)</i>
Sentence	It's raining	$0 [\in \{0,1\}]$	<i>t</i>
Quantified NP	everybody	$\lambda Y. \vdash \llbracket \text{person} \rrbracket \subseteq Y \vdash$	<i>(et)t</i>
Determiner	no	$\lambda X. \lambda Y. \vdash X \cap Y = \emptyset \vdash$	<i>(et)((et)t)</i>

3. Intensions

- Logical Space as a model of content

- (20a) 4 fair coins are tossed.
(b) At least one of the 4 tossed coins lands heads up.
(c) At least one of the 4 tossed coins lands heads down.
(d) Exactly 2 of the 4 tossed coins land heads up.
(e) Exactly 2 of the 4 tossed coins land heads down.

☞ *Carnap's Content*

Carnap (1947)

The *proposition* expressed by a sentence is the set of possible cases of which that sentence is true.

- (21a) 4 coins were tossed when John coughed.
(b) 4 coins were tossed and no one coughed.

☞ *Wittgenstein's Paradise*

Wittgenstein (1921)

All (and only the) maximally specific cases (possible worlds) are members of a set **W** (aka *Logical Space*).

- From propositions to intensions

- (22) $p [\subseteq \mathbf{W}] \approx \lambda w. \vdash w \in p \dashv$

characteristic function (of p rel. to **W**)

- (23) The *intension* of an expression is its extension relative to Logical Space:
 $\llbracket \mathbf{E} \rrbracket : \mathbf{W} \rightarrow \{x \mid x \text{ is of the "appropriate" type}\}$

- Intensional types

☞ *Montagovian types*

Montague (1970)

x is of type **e** $\Leftrightarrow x \in U$;

u is of type **t** $\Leftrightarrow u \in \{0,1\}$;

f is of type (a,b) $\Leftrightarrow f: \{x \mid x \text{ is of type } a\} \rightarrow \{y \mid y \text{ is of type } b\}$

g is of type (\mathbf{s},c) $\Leftrightarrow g: \mathbf{W} \rightarrow \{y \mid y \text{ is of type } c\}$

☞ *Two-sorted types*

"Gallin (1975)"

x is of type **e** $\Leftrightarrow x \in U$;

u is of type **t** $\Leftrightarrow u \in \{0,1\}$;

w is of type **s** $\Leftrightarrow w \in \mathbf{W}$;

f is of type (a,b) $\Leftrightarrow f: \{x \mid x \text{ is of type } a\} \rightarrow \{y \mid y \text{ is of type } b\}$

- Notation

$$\llbracket \mathbf{Exp} \rrbracket^w = \llbracket \mathbf{Exp} \rrbracket(w)$$

4. Attitude reports

- Substitution failure

(24) Fritz thinks that Hamburg is larger than Cologne.

Hamburg is larger than Cologne.

Pfäffingen is larger than Breitenholz.

∴ Fritz thinks that Pfäffingen is larger than Breitenholz.

(25a)

$$\begin{aligned}
 & \llbracket \text{thinks that Hamburg is larger than Cologne} \rrbracket^{w^*} \\
 & \quad = \\
 & \llbracket \text{thinks} \rrbracket^{w^*} \oplus \llbracket \text{Hamburg is larger than Cologne} \rrbracket^{w^*} \\
 & \quad = \\
 & \boxed{\llbracket \text{thinks} \rrbracket^{w^*} \oplus 1} \\
 & \quad \swarrow \quad \searrow \\
 & \llbracket \text{thinks} \rrbracket^{w^*} \quad \llbracket \text{Hamburg is larger than Cologne} \rrbracket^{w^*} \\
 & \quad \quad \quad = \\
 & \quad \quad \quad 1
 \end{aligned}$$

(b)

$$\begin{aligned}
 & \llbracket \text{thinks that Pfäffingen is larger than Breitenholz} \rrbracket^{w^*} \\
 & \quad = \\
 & \llbracket \text{thinks} \rrbracket^{w^*} \oplus \llbracket \text{Pfäffingen is larger than Breitenholz} \rrbracket^{w^*} \\
 & \quad = \\
 & \boxed{\llbracket \text{thinks} \rrbracket^{w^*} \oplus 1} \\
 & \quad \swarrow \quad \searrow \\
 & \llbracket \text{thinks} \rrbracket^{w^*} \quad \llbracket \text{Pfäffingen is larger than Breitenholz} \rrbracket^{w^*} \\
 & \quad \quad \quad = \\
 & \quad \quad \quad 1
 \end{aligned}$$

- Intensional compositionality

(26a)

$$\begin{aligned}
 & \llbracket \text{thinks that Hamburg is larger than Cologne} \rrbracket^{w^* \vee} \\
 & \quad = \\
 & \llbracket \text{thinks} \rrbracket^{w^*} \oplus \llbracket \text{Hamburg is larger than Cologne} \rrbracket \\
 & \quad = \\
 & \boxed{\llbracket \text{thinks} \rrbracket^{w^*} \oplus p} \\
 & \quad \swarrow \quad \searrow \\
 & \llbracket \text{thinks} \rrbracket^{w^*} ? \quad \llbracket \text{Hamburg is larger than Cologne} \rrbracket \vee \\
 & \quad \quad \quad = \\
 & \quad \quad \quad p
 \end{aligned}$$

$$\begin{array}{c}
\llbracket \text{thinks that Pfäffingen is larger than Breitenholz} \rrbracket^{w*} \checkmark \\
= \\
\llbracket \text{thinks} \rrbracket^{w*} \oplus \llbracket \text{Pfäffingen is larger than Breitenholz} \rrbracket \\
= \\
\boxed{\llbracket \text{thinks} \rrbracket^{w*} \oplus q} \\
\swarrow \quad \searrow \\
\llbracket \text{thinks} \rrbracket^{w*} ? \quad \llbracket \text{Pfäffingen is larger than Breitenholz} \rrbracket \\
\quad \quad \quad = \\
\quad \quad \quad q \quad \checkmark
\end{array}$$

(28) *More expressions (of more types)*

👉 *Fregean Compositionality* Frege (1892)

$$\begin{array}{ccc} \left[\begin{array}{c} \textit{ExtExp} \\ \swarrow \quad \searrow \\ \textit{LP} \quad \textit{RP} \end{array} \right]^w & = [\textit{LP}]^w \oplus [\textit{RP}]^w & \text{or:} \quad \left[\begin{array}{c} \textit{IntExp} \\ \swarrow \quad \searrow \\ \textit{LP} \quad \textit{RP} \end{array} \right]^w = [\textit{LP}]^w \oplus [\textit{RP}]^w \quad [\text{or ...}] \end{array}$$

☞ *Intensional compositionality*

$$\llbracket \begin{array}{c} \textit{ArbExp} \\ \swarrow \quad \searrow \\ \textit{LP} \quad \textit{RP} \end{array} \rrbracket = \llbracket \textit{LP} \rrbracket \oplus \llbracket \textit{RP} \rrbracket$$

 Distinction between extensional and intensional constructions

$$\llbracket \mathbf{Exp}_i \rrbracket^w = \llbracket \mathbf{LP}_i \rrbracket^w \oplus_{\mathcal{F}} \llbracket \mathbf{RP}_i \rrbracket^w$$

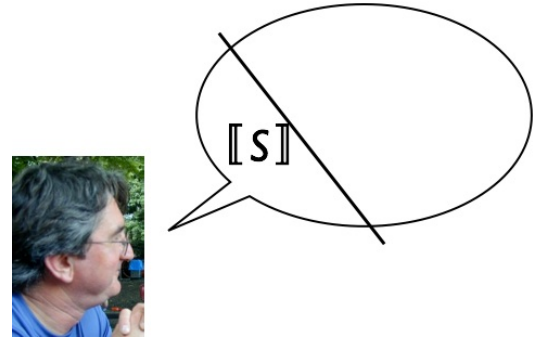
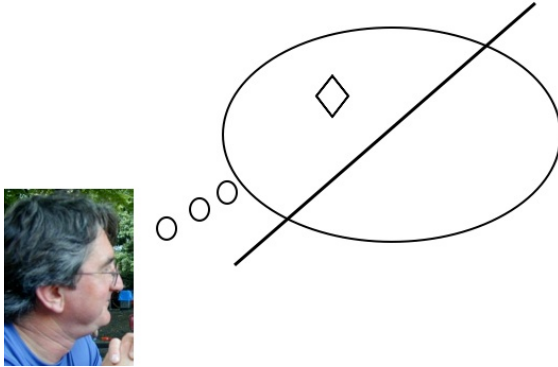
☞ *Pointwise calculation of intension*

$$\begin{aligned}
 & \boxed{\llbracket \text{John loves Mary} \rrbracket} \\
 &= \\
 & \lambda w. \llbracket \text{John loves Mary} \rrbracket^w \\
 &= \\
 & \lambda w. \llbracket \text{loves Mary} \rrbracket^w (\llbracket \text{John} \rrbracket^w) \\
 &= \dots \\
 & \lambda w. \lambda x^e. \vdash \text{in } w, \text{John loves Mary} \dashv \\
 & \quad \swarrow \quad \searrow \\
 & \begin{array}{cc}
 \llbracket \text{John} \rrbracket^w & \llbracket \text{loves Mary} \rrbracket^w \\
 = & = \\
 \text{John} & \llbracket \text{loves} \rrbracket^w (\llbracket \text{Mary} \rrbracket^w) \\
 & = \\
 & \lambda x^e. \vdash \text{in } w, x \text{ loves Mary} \dashv \\
 & \quad \swarrow \quad \searrow \\
 & \begin{array}{cc}
 \llbracket \text{loves} \rrbracket^w & \llbracket \text{Mary} \rrbracket^w \\
 = & = \\
 \lambda y^e. \lambda x^e. \vdash \text{in } w, x \text{ loves } y \dashv & \text{Mary}
 \end{array}
 \end{array}
 \end{aligned}$$

• Modelling cognitive states in Logical Space

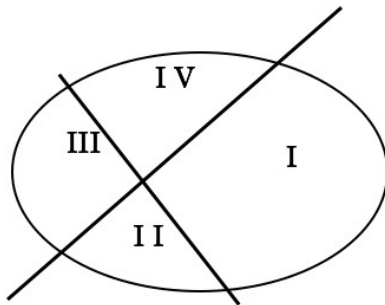
(29a) Fritz in w^* ...

(b)



S = Hamburg is larger than Cologne

(30)



I: $W \setminus (\diamond \cup \llbracket S \rrbracket)$; II: $\llbracket S \rrbracket \setminus \diamond$; III: $\diamond \cap \llbracket S \rrbracket$; IV: $\diamond \setminus \llbracket S \rrbracket$

(31) **$\llbracket \text{Fritz thinks that Hamburg is larger than Cologne} \rrbracket (w^*) = 1$**

$\Leftrightarrow \neg (\exists w \in \diamond) \llbracket S \rrbracket (w) = 0$

$\Leftrightarrow (\forall w \in \diamond) \llbracket S \rrbracket (w) = 1$

$\Leftrightarrow \text{IV} = \emptyset$

(32) \diamond depends on

... attitude subject (Fritz)

... world of evaluation: w^*

... lexical meaning of verb: **think**

$\Rightarrow \diamond = \mathbf{Dox}(\text{Fritz})(w^*) \subseteq W$

$\approx \mathbf{Dox}$ is of type $e(s(st))$

(dependent) accessibility relation

(33a) $\llbracket \mathbf{think} \rrbracket = \lambda w^*. \lambda p^{st}. \lambda x^e. \vdash (\forall w) \mathbf{Dox}(x)(w^*)(w) \leq p(w) \dashv$

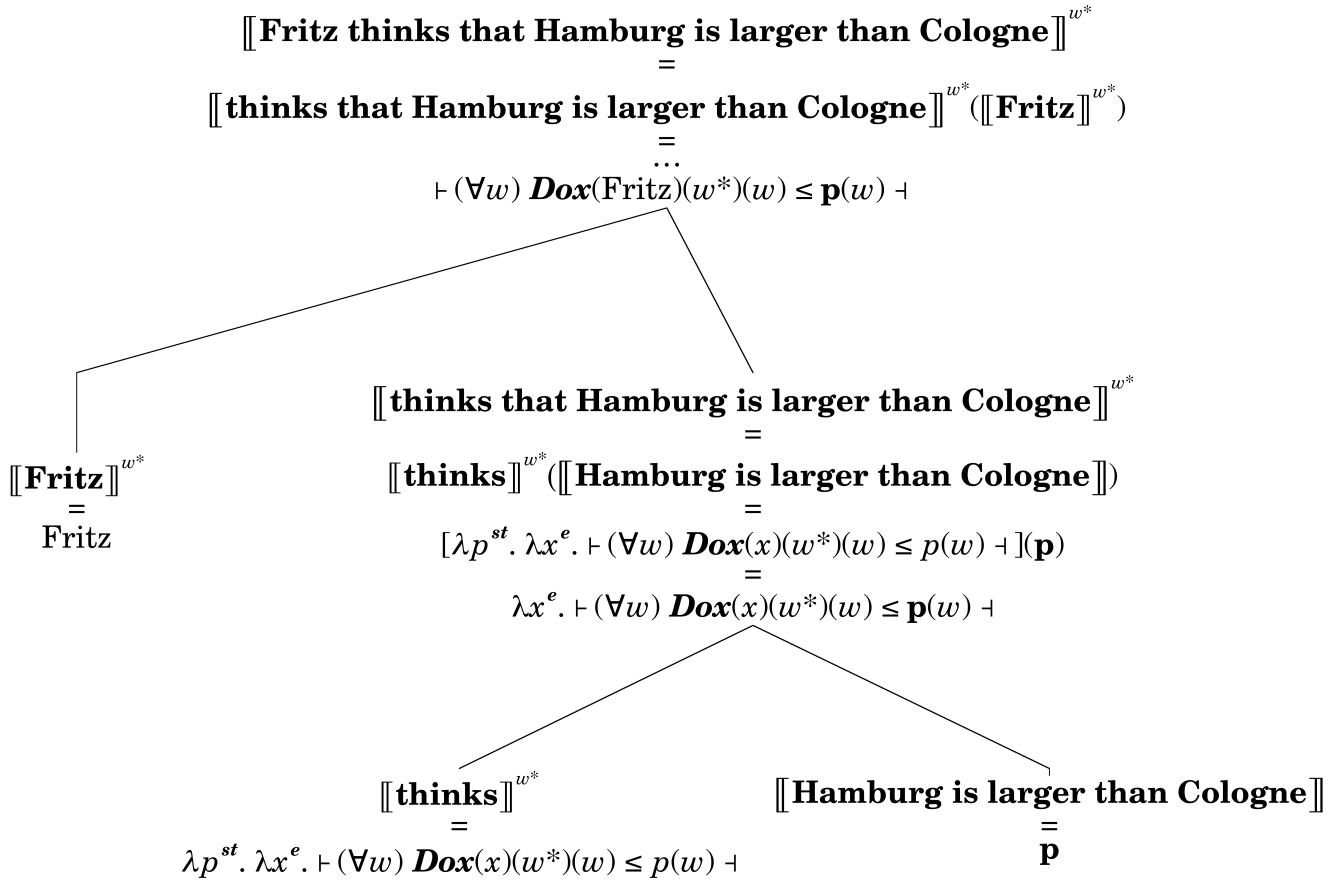
$\leq \approx \text{mat. impl.}$

(b) $\llbracket \mathbf{know} \rrbracket = \lambda w^*. \lambda p^{st}. \lambda x^e. \vdash (\forall w) \mathbf{Epi}(x)(w^*)(w) \leq p(w) \dashv$

(c) $\llbracket \mathbf{want} \rrbracket = \lambda w^*. \lambda p^{st}. \lambda x^e. \vdash (\forall w) \mathbf{Bou}(x)(w^*)(w) \leq p(w) \dashv$

... ..

(34)



(35a) [#] **Fritz knows that Breitenholz is larger than Pfäffingen.**

(b) $(\forall w^*) (\forall p^{st}) (\forall x^e) \llbracket \mathbf{know} \rrbracket (w^*)(p)(x) \leq p(w^*)$

(c) $(\forall w^*) (\forall x^e) \mathbf{Epi}(x)(w^*)(w^*) = 1$

(36a) [#] **Fritz knows that Rome is in Italy, but he doesn't think so.**

(b) $(\forall w^*) (\forall p^{st}) (\forall x^e) \llbracket \mathbf{know} \rrbracket (w^*)(p)(x) \leq \llbracket \mathbf{think} \rrbracket (w^*)(p)(x)$

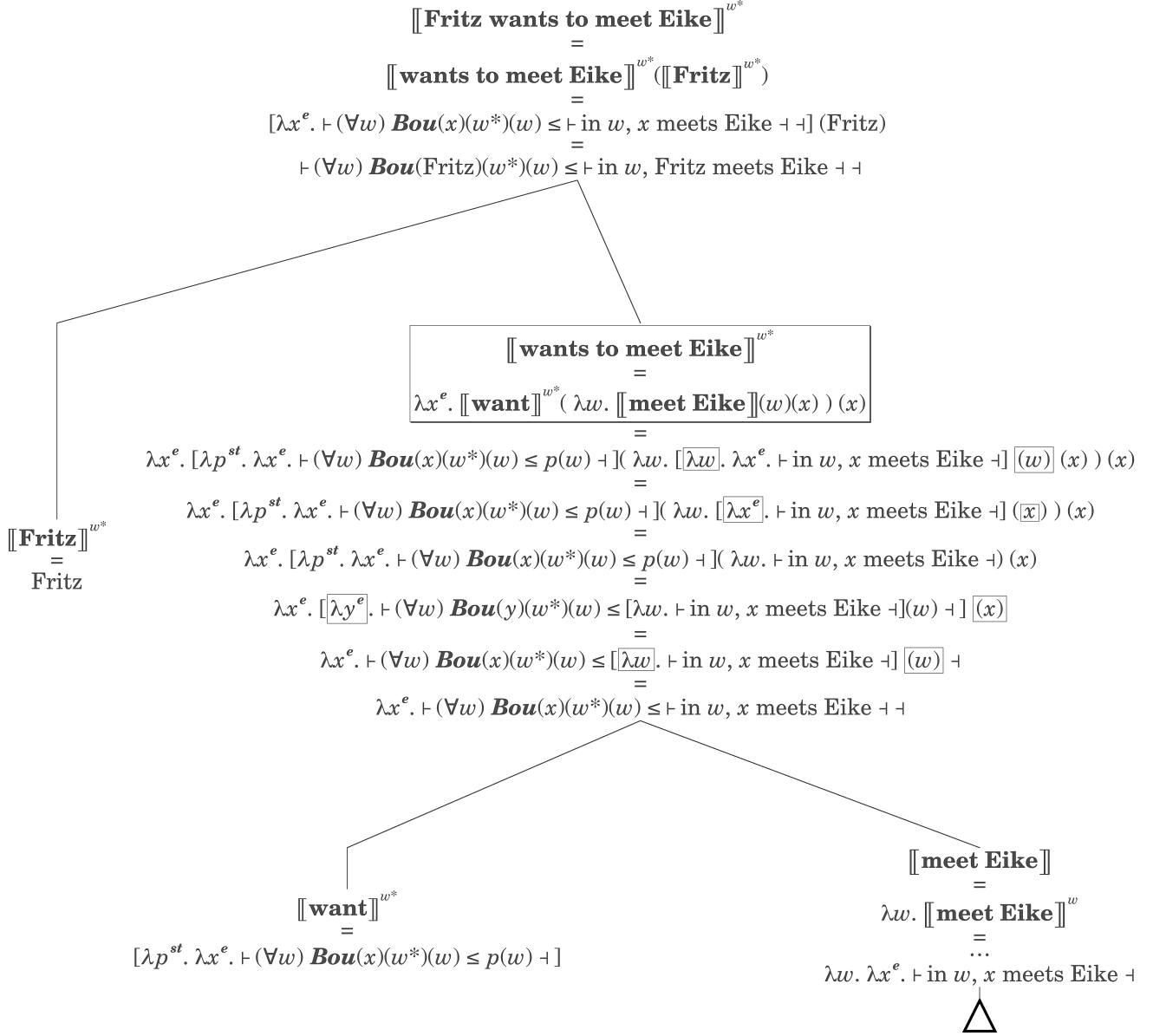
(c) $(\forall w^*) (\forall w) (\forall x^e) \mathbf{Dox}(x)(w^*)(w) \leq \mathbf{Epi}(x)(w^*)(w)$

(37a) * **Fritz wants that Fritz meets Eike.**

(b) **Fritz wants to meet Eike.**

(c) $\llbracket \text{want} \rrbracket = \lambda w^*. \lambda P^{s(et)}. \lambda x^e. \vdash (\forall w) \mathbf{Bou}(x)(w^*)(w) \leq P(w)(x) \dashv$

(38)



5. Unspecific Objects

• Paraphrases

Quine (1956)

(39a) **John is looking for a sweater.**

(b) **John wants to find a sweater.**

(40a) **Mary owes me a horse.**

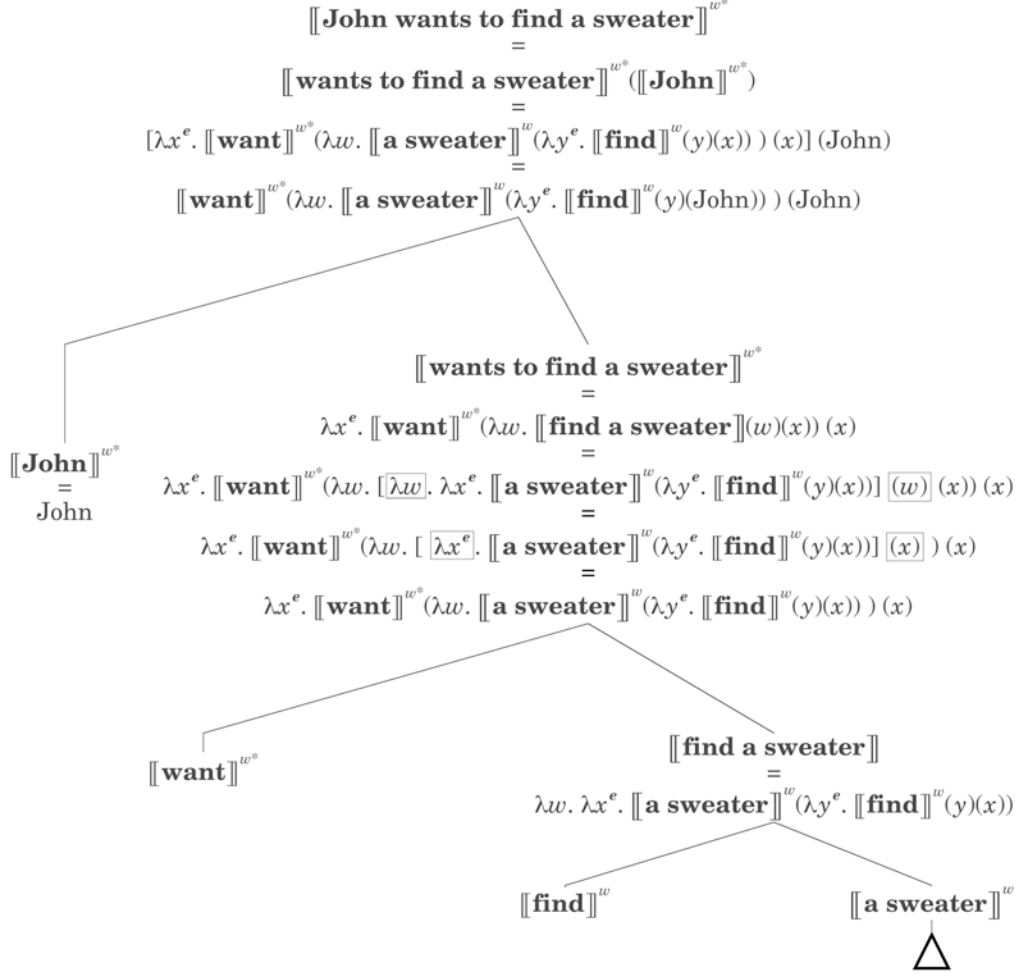
(b) **Mary is obliged to give me a horse.**

(41a) **This horse resembles a unicorn.**

(b) **This horse could (almost) be a unicorn.**

- Relational analyses

(42a) *Analysis of paraphrase*

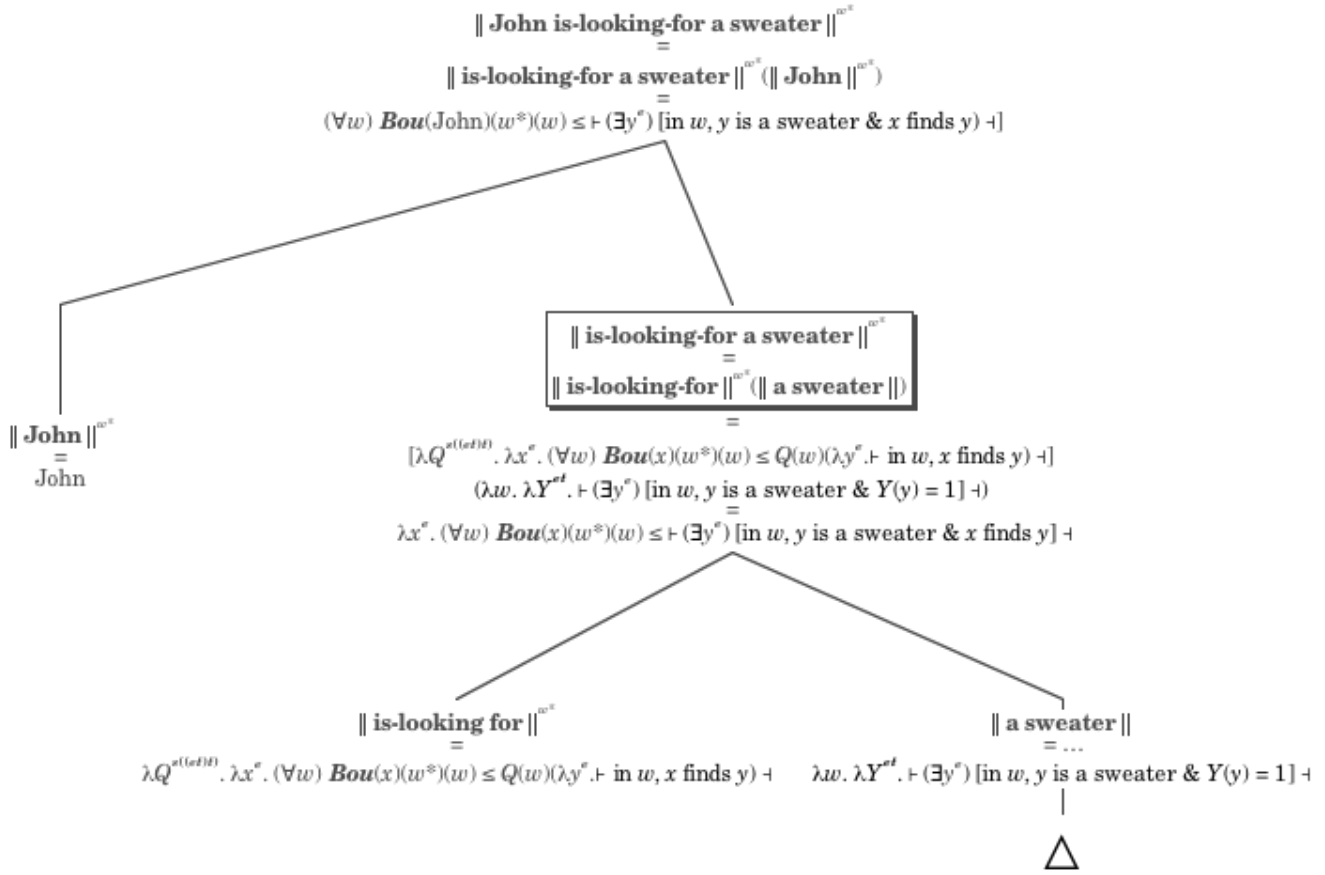


(b) *Dissection*

$$\begin{aligned}
& \lambda x^e. \llbracket \text{want} \rrbracket^{w^*} (\lambda w. \llbracket \text{a sweater} \rrbracket^w (\lambda y^e. \llbracket \text{find} \rrbracket^w (y)(x))) (x) \\
& = \lambda x^e. \mathbf{W}(\lambda w. \mathbf{S}(w) \lambda y^e. \mathbf{F}(w)(y)(x))(x) \\
& = [\lambda Q^{s((et)t)}. \lambda x^e. \mathbf{W}(\lambda w. Q(w) (\lambda y^e. \mathbf{F}(y)(x)))(x)] (\mathbf{S})
\end{aligned}$$

(c) *Simplification*

$$\begin{aligned}
& \llbracket \text{look-for} \rrbracket (w^*) \\
& = \lambda Q^{s((et)t)}. \lambda x^e. \mathbf{W}(\lambda w. Q(w) (\lambda y^e. \mathbf{F}(y)(x)))(x) \\
& = \lambda Q^{s((et)t)}. \lambda x^e. \llbracket \text{want} \rrbracket (w^*)(\lambda w. Q(w) (\lambda y^e. \llbracket \text{find} \rrbracket (w)(y)(x)))(x) \\
& = \lambda Q^{s((et)t)}. \lambda x^e. [\lambda p^{st}. \lambda x^e. \vdash (\forall w) \mathbf{Bou}(x)(w^*)(w) \leq p(w) \dashv] \\
& \quad (\lambda w. Q(w) (\lambda y^e. [\lambda w. \lambda y^e. \lambda x^e. \vdash \text{in } w, x \text{ finds } y \dashv](w)(y)(x)))(x) \\
& = \lambda Q^{s((et)t)}. \lambda x^e. [\lambda p^{st}. \lambda x^e. \vdash (\forall w) \mathbf{Bou}(x)(w^*)(w) \leq p(w) \dashv] \\
& \quad (\lambda w. Q(w) (\lambda y^e. \vdash \text{in } w, x \text{ finds } y \dashv))(x) \\
& = \lambda Q^{s((et)t)}. \lambda x^e. [\lambda p^{st}. \lambda z^e. \vdash (\forall w) \mathbf{Bou}(z)(w^*)(w) \leq p(w) \dashv] \\
& \quad (\lambda w. Q(w) (\lambda y^e. \vdash \text{in } w, x \text{ finds } y \dashv))(x) \\
& = \lambda Q^{s((et)t)}. \lambda x^e. \vdash (\forall w) \mathbf{Bou}(x)(w^*)(w) \leq Q(w) (\lambda y^e. \vdash \text{in } w, x \text{ finds } y \dashv) \dashv
\end{aligned}$$

(43a) **John is looking for most unicorns.**(b) $(\forall w) \text{Bou}(x)(w^*)(w) \leq \vdash \text{in } w, \#(\text{unicorns } x \text{ finds}) > \#(\text{unicorns } x \text{ doesn't find}) \vdash$ (c) **John wants to find most unicorns.**(44a) **John is looking for each unicorn.**(b) $(\forall w) \text{Bou}(x)(w^*)(w) \leq \vdash \text{in } w, \text{John finds each unicorn} \vdash$ (c) **John wants to find each unicorn.**(45a) **John is looking for no unicorn.**(b) $(\forall w) \text{Bou}(x)(w^*)(w) \leq \vdash \text{in } w, \text{John doesn't find a unicorn} \vdash$ (c) **John wants to find no unicorn.**(46a) An intension Q of type $s((et)t)$ is *existential* iff

$$Q = \lambda w. \lambda Y^{et}. \vdash (\exists x) [P(w)(x) = Y(x) = 1] \vdash$$

for some intension P of ('property') type $s(et)$.(b) $\lambda P^{s(et)}. \lambda w. \lambda Y^e. \vdash (\exists x) [P(w)(x) = Y(x) = 1] \vdash$

Partee (1987)

is a one-one mapping (called A) whose inverse (called BE) is:

$$\lambda Q^{s((et)t)}. \lambda w. \lambda x^e. Q(\lambda y^e. \vdash x = y \vdash).$$

(47) $\llbracket \text{look-for} \rrbracket (w^*)$

Zimmermann (1993)

$$= \lambda P^{s(et)}. \lambda x^e. \vdash (\forall w) \text{Bou}(x)(w^*)(w) \leq \vdash (\exists y^e) \text{in } w, P(y) = 1 \& x \text{ finds } y \vdash$$

- Relational readings

(48) **I owe you a horse.**

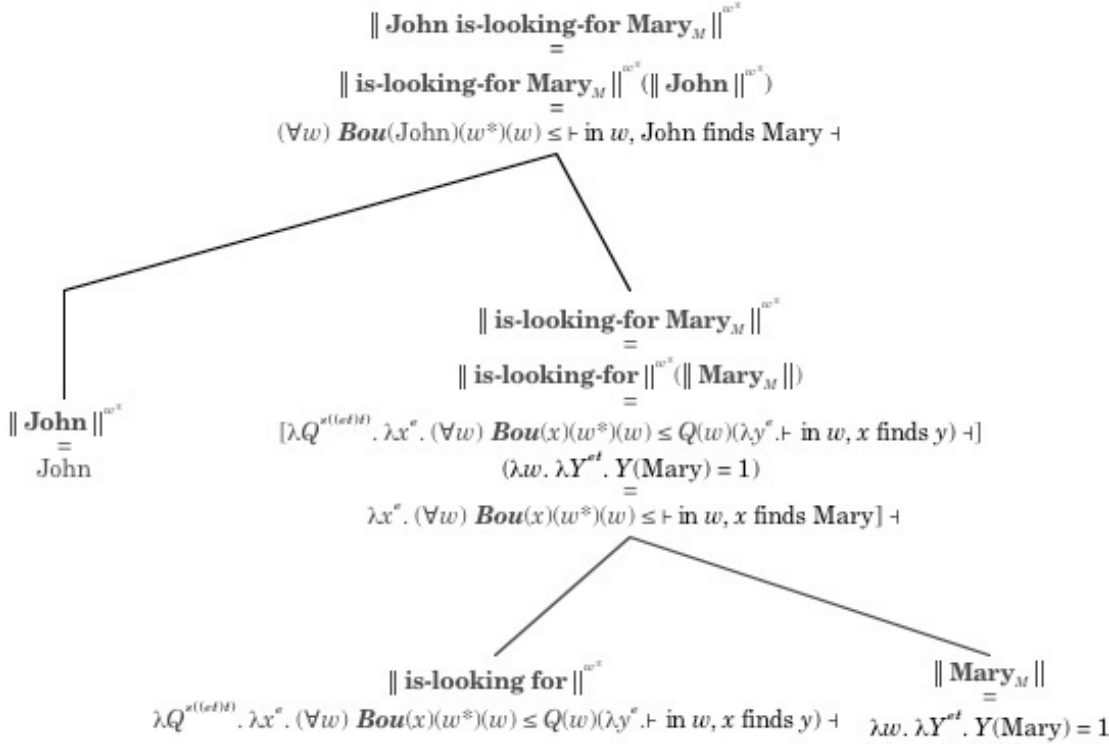
Buridanus (1350)

(49) **John is looking for Mary.**

Mary is a Swiss student.

∴ **John is looking for a Swiss student.**

(50a)

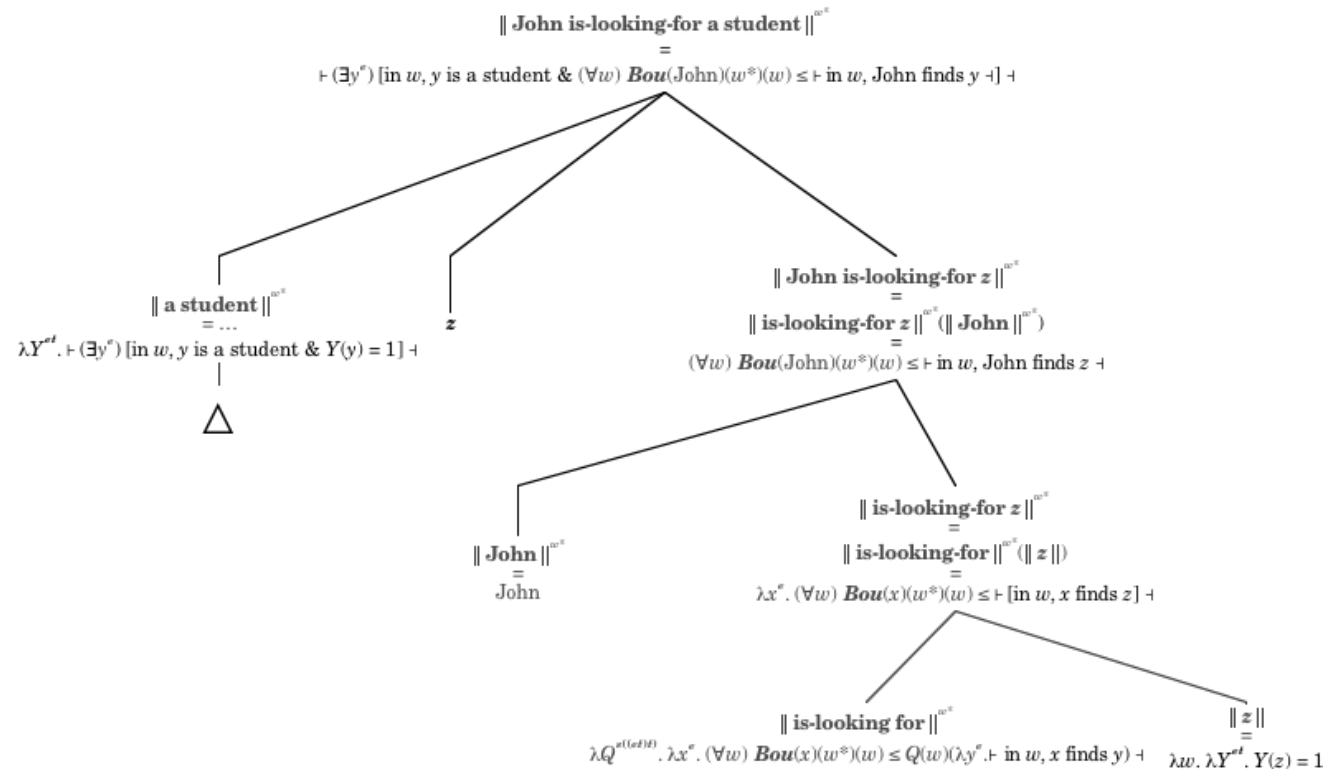


(a') $(\exists m^{s(e)}) [m \text{ is a mode of presentation \& } m(w^*)(\text{John}) = \text{Mary} \& (\forall w) \text{Bou}(x)(w^*)(w) \leq \vdash \text{ in } w, \text{John finds } m(w)(\text{John})]$

de re

Kaplan (1969)

(b)



- More paraphrases

(51a) **John is looking for a sweater.**

(b) **John wants to find a sweater.**

(c) **John is looking for an intentional sweater.**

(52a) **Mary owes me a horse.**

(b) **Mary is obliged to give me a horse.**

(c) **Mary owes me an arbitrary horse.**

(53a) **This horse resembles a unicorn.**

(b) **This horse could (almost) be a unicorn.**

(c) **This horse resembles a generic unicorn.**

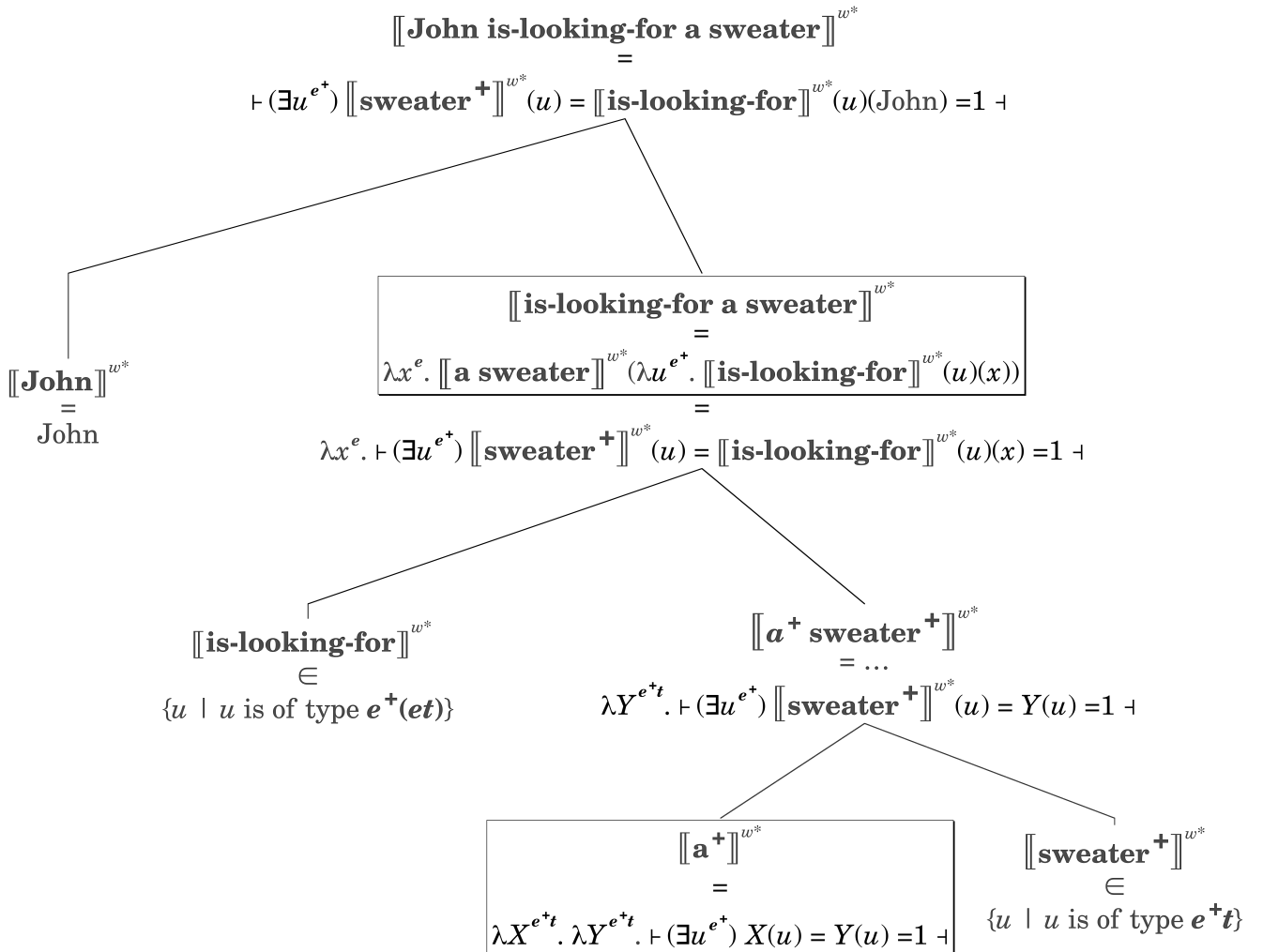
(53a) **Jones hired an assistant.**

(b) **Jones saw to it that someone would become an/his assistant.**

(c) **Jones hired a would-be assistant.**

- Quantificational analyses

(54)



(55a) $e^+ = s(et)$

Condoravdi *et al.* (2001)

(b) $\llbracket \text{sweater}^+ \rrbracket (w^*) = \lambda P^{s(et)}. \vdash (\forall w) (\forall x^e) P \sqsubseteq \llbracket \text{sweater} \rrbracket \rightarrow$

(c) $\llbracket \text{look-for} \rrbracket (w^*)$ Zimmermann (2006): ‘exact match’

$= \lambda P^{s(et)}. \lambda x^e. \vdash (\forall w) [\mathbf{Bou}(x)(w^*)(w) \leftrightarrow (\exists y^e) \text{ in } w, P(y) = 1 \ \& \ x \text{ finds } y] \rightarrow$

Notation: $P \sqsubseteq Q :\Leftrightarrow (\forall w) (\forall x^e) P(w)(x) \leq Q(w)(x)$

sub-concepthood

- Monotonicity

(56a) **John is a looking for a red sweater.**

\therefore **John is looking for a sweater.**

(b) **John is looking for a sweater.**

Mary is looking for a book.

\therefore **John is looking for something Mary is looking for.**

Intersective construal (for simplicity): $\llbracket \text{red sweater} \rrbracket = \llbracket \text{sweater} \rrbracket \sqcap \llbracket \text{red} \rrbracket$

Notation: $P \sqcap Q := \lambda w. \lambda x^e. P(w)(x) = Q(w)(x) = 1$

(57) *Relational analyses (with lexical decomposition)*

(a) $(\forall w) \mathbf{Bou}(\text{John})(w^*)(w) \leq \vdash (\exists y^e) [\text{in } w, y \text{ is a sweater} \ \& \ y \text{ is red} \ \& \ \text{John finds } y] \rightarrow$

$\Rightarrow (\forall w) \mathbf{Bou}(\text{John})(w^*)(w) \leq \vdash (\exists y^e) [\text{in } w, y \text{ is a sweater} \ \& \ \text{John finds } y] \rightarrow$

(b) $[(\forall w) \mathbf{Bou}(\text{John})(w^*)(w) \leq \vdash (\exists y^e) [\text{in } w, y \text{ is a sweater} \ \& \ \text{John finds } y] \rightarrow$

$\& (\forall w) \mathbf{Bou}(\text{Mary})(w^*)(w) \leq \vdash (\exists y^e) [\text{in } w, y \text{ is a book} \ \& \ \text{Mary finds } y] \rightarrow] \dots$

– quantifier analysis – e.g. $Q \equiv \lambda w. \lambda P. P=P$:

$\dots \Rightarrow (\exists Q^{s(et)t}) [\llbracket \text{look-for} \rrbracket (w^*)(Q)(\text{Mary}) \ \& \ \llbracket \text{look-for} \rrbracket (w^*)(Q)(\text{John})]$

– property analysis – e.g. $Q \equiv \lambda w. \lambda P. P=P$:

$\dots \Rightarrow (\exists P^{s(et)}) [\llbracket \text{look-for} \rrbracket (w^*)(P)(\text{Mary}) \ \& \ \llbracket \text{look-for} \rrbracket (w^*)(P)(\text{John})]$

(58) *Quantificational analysis (with exact match)*

(a) $(\exists P^{s(et)}) \sqsubseteq \llbracket \text{sweater} \rrbracket \sqcap \llbracket \text{red} \rrbracket (\forall w) [\mathbf{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \text{ in } w, P(y)=1 \ \& \ j \text{ finds } y]$

$\Rightarrow (\exists P^{s(et)}) \sqsubseteq \llbracket \text{sweater} \rrbracket (\forall w) [\mathbf{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \text{ in } w, P(y)=1 \ \& \ j \text{ finds } y]$

(b) $[(\exists P^{s(et)}) \sqsubseteq \llbracket \text{sweater} \rrbracket (\forall w) [\mathbf{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \text{ in } w, P(y)=1 \ \& \ j \text{ finds } y]$

$\& (\exists P^{s(et)}) \sqsubseteq \llbracket \text{book} \rrbracket (\forall w) [\mathbf{Bou}(m)(w^*)(w) \leftrightarrow (\exists y^e) \text{ in } w, P(y)=1 \ \& \ m \text{ finds } y]]$

$\neq > (\exists P^{s(et)}) (\forall w) [\mathbf{Bou}(m)(w^*)(w) \leftrightarrow (\exists y^e) \text{ in } w, P(y)=1 \ \& \ m \text{ finds } y]$

$\dots \& [\mathbf{Bou}(j)(w^*)(w) \leftrightarrow (\exists y^e) \text{ in } w, P(y)=1 \ \& \ j \text{ finds } y]$

$\equiv (\exists P^{s(et)}) [\llbracket \text{look-for} \rrbracket (w^*)(P)(\text{Mary}) \ \& \ \llbracket \text{look-for} \rrbracket (w^*)(P)(\text{John})]$

- Unspecificity \Rightarrow Intensionality?

Zimmermann (1983; 2001)

(59) **Arnim owns a bottle of 1981 Riesling-Sylvaner.**

Rooth (p.c., anno 1991)

Riesling-Sylvaner is Müller-Thurgau.

Arnim owns a bottle of 1981 Müller-Thurgau.

(60) **Arnim owns the bottle that Franzis does not own.**

(a) **[[the] (w^*)([bottle Franzis doesn't own])(w^*)**

$(\lambda y^e. \text{[own]} (w^*) (\lambda Y^{et}. Y(y))(\text{Arnim}))$

$\leq \vdash (\exists y^e) [\text{[bottle]} (w^*) (y) = \text{[own]} (w^*) (\lambda Y^{et}. Y(y))(\text{Arnim}) = 1] \neg$

(b) **[[own] (w^*)([the] (w^*)([bottle Franzis doesn't own])(w^*))(\text{Arnim})**

$\leq \text{[own]} (w^*) (\text{[the]} (w^*) (\text{[unicorn]}) (w^*)) (\text{Arnim})$

(in given scenario)

- Landscape of intensional verbs

(61)

VERBS OF ...	EXAMPLES
<i>Absence</i>	avoid, lack, omit
<i>Anticipation</i>	allow* (for), anticipate, expect, fear, foresee, plan, wait* (for)
<i>Calculation</i>	calculate, compute, derive
<i>Creation</i>	assemble, bake, build, construct, fabricate, make (these verbs in progressive aspect only)
<i>Depiction</i>	caricature, draw, imagine, portray, sculpt, show, visualize, write* (about)
<i>Desire</i>	hope* (for), hunger* (for), lust* (after), prefer, want
<i>Evaluation</i>	admire, disdain, fear, respect, scorn, worship (verbs whose corresponding noun can fill the gap in the evaluation 'worthy of _' or 'merits_')
<i>Requirement</i>	cry out* (for), demand, deserve, merit, need, require
<i>Search</i>	hunt* (for), look* (for), rummage about* (for), scan* (for), seek
<i>Similarity</i>	imitate, be reminiscent* (of), resemble, simulate
<i>Transaction</i>	buy, order, owe, own, reserve, sell, wager

Forbes (2006: 50)

(62a) **Matt needed some change before the conference.**

Schwarz (2006)

(b) **Matt was looking for some change before the conference.**

(63a) **Matt needs most of the small bills that were in the cash-box.**

(b) **Matt is looking for most of the small bills that were in the cash-box.**

(64)

Zimmermann (2001: 526)

Existential Impact⁵

From x Rs an N infer: *There is at least one N .*

Extensionality⁶

From x Rs an N , *Every N is an M* , and *Every M is an N* infer: x Rs an M .

Specificity

From x Rs an N infer: *Some (specific) individual is Red by x .*

5. General topics

• Propositionalism

Forbes (2000; 2006); M. Montague (2007)

(P) All (linguistic, mental, perceptual, pictorial,...) content is propositional.

(Q) All intensional contexts are parts of embedded clauses.

Quine (1956)

(65a) **[[Hesperus is a planet]]** \neq **[[Phosphorus is a planet]]**

Frege (1892)

\Rightarrow **[[Hesperus]]** \neq **[[Phosphorus]]**

non-propositional content

(b) **The thirsty man wants beer.**

Meinong (1904): intentional object

(c) **Jones worships a Greek goddess.**

R. Montague (1969) [crediting H. Kamp]

(d) **Lex Luthor fears Superman (but not Clark Kent).**

Forbes (2000)

(e) **Horatio believes that things Horatio doesn't believe in exist.**

Szabó (2003): coherent belief

(e) **John likes chocolate.**

... (partly) explains why ...

John wants to have chocolate.

M. Montague (2007)

• Russellian analysis

Russell (1905); Whitehead & Russell (1910); Cresswell (1973)

(66) *Denotations and their types*

Category	Example	Type
Name	Ljubljana	e
Description	the capital of Slovenia	$(e(st))(st)$
Noun	city	$e(st)$
1-place predicate	sleep	$e(st)$
2-place predicate	eat	$e(e(st))$
3-place predicate	give	$e(e(e(st)))$
Sentence	It's raining	st
Quantified NP	everybody	$(e(st))(st)$
Determiner	no	$(e(st))((e(st))(st))$
Attitude verb	think	$(st)(et)$
Connective	or	$(st)((st)(st))$

(67) *How to Russell a Frege-Church*

Kaplan (1975)

(a) R (**[[the capital of Slovenia is larger than Breitenholz]]**)

= R (**[[is larger than]]**) R (**[[Breitenholz]]**) (R (**[[the capital of Slovenia]]**))

- (b) $R(\llbracket \text{the capital of Slovenia} \rrbracket) = \lambda x^e. \lambda w. x = \llbracket \text{the capital of Slovenia} \rrbracket (w)$
- (c) $R(\llbracket \text{Breitenholz} \rrbracket) = \lambda x^e. \lambda w. x = \llbracket \text{Breitenholz} \rrbracket (w) \quad [= \lambda x^e. \lambda w. x = \text{Breitenholz}]$
- (d) $R(\llbracket \text{is larger than} \rrbracket)$
- $= \lambda P^e. \lambda Q^e. \lambda w. \vdash (\forall x) (\forall y) P(x)(w) \times Q(x)(w) \leq \llbracket \text{is larger than} \rrbracket (w)(x)(y)$

• Relativity of Reference

(68a) $\llbracket A \rrbracket = \lambda w. \llbracket A \rrbracket$, for lexical A Lewis (1974)

(b) $\llbracket A B \rrbracket = \lambda w. \llbracket A \rrbracket(w) \oplus \llbracket B \rrbracket(w)$, if $\llbracket A B \rrbracket = \llbracket A \rrbracket \oplus \llbracket B \rrbracket$

(69a) $\llbracket \text{John thinks it's raining} \rrbracket$

$= \text{APP}^{ext}(\text{APP}^{int}(\llbracket \text{thinks} \rrbracket, \llbracket \text{it's raining} \rrbracket), \llbracket \text{John} \rrbracket)$

NB: $\text{APP}^{ext}(A,B) = \lambda w. A(w)(B(w))$; $\text{APP}^{int}(A,B) = \lambda w. A(w)(B)$

(b) $\llbracket \text{John thinks it's raining} \rrbracket (w)$

$= \text{APP}^{ext}(\llbracket \text{thinks it's raining} \rrbracket(w), \llbracket \text{John} \rrbracket(w))$

$= \text{APP}^{ext}(\text{APP}^{int}(\llbracket \text{thinks} \rrbracket(w), \llbracket \text{it's raining} \rrbracket(w)), \llbracket \text{John} \rrbracket(w))$

$= \text{APP}^{ext}(\text{APP}^{int}(\llbracket \text{thinks} \rrbracket, \llbracket \text{it's raining} \rrbracket), \llbracket \text{John} \rrbracket)$

$= \llbracket \text{John thinks it's raining} \rrbracket$

(70) $\llbracket A \rrbracket = \pi(\llbracket A \rrbracket)$, for lexical A

Putnam (1980)

(b) $\llbracket A B \rrbracket = \llbracket A \rrbracket \oplus \llbracket B \rrbracket$, if $\llbracket A B \rrbracket = \llbracket A \rrbracket \oplus \llbracket B \rrbracket$

(c) $\pi_e: U \rightarrow U$ is a (non-trivial) bijection; π_s and π_t are identities on W and $\{0,1\}$;

π_{ab} maps any f of type ab to $\{(\pi x, \pi y) \mid f(x) = y\}$

(d) $\llbracket S \rrbracket = \llbracket S \rrbracket$, for any expression S

... provided that all compositions \oplus are invariant

NB: \oplus is invariant iff $\pi(\oplus) = \oplus$ for all permutations π

• Further topics

- Externalism
- Attitudes *de se*
- Granularity

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