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(Dated: May 24, 2018)

1. EXAMPLE: RUNNING A BNV OPERATOR FROM BELOW 10 TEV TO 1 GEV

In the Standard Model Effective Field Theory (SMEFT) there are four dimension six baryon number violating operators. In order to make predictions about proton decay rates or any other baryon number violating process, one must run the coefficients of these dimension six operators down to the scale where the physics process occurs, integrating out heavy particles as one passes through thresholds. We proceed with an example of this calculation below, utilizing two software packages: DSixTools[1], and RunDec[2]. DSixTools is used for running in the SMEFT, and RunDec is used for running in the Weak Effective Theory (WET).

Consider the BNV dimension 6 operator $Q^{duql}_{prst} = C^{duql}_{prst}(\bar{d}^{c\alpha}_p u^\beta_r)(\bar{q}^{ci\gamma}_s l^j_t)\epsilon_{\alpha\beta\gamma}\epsilon_{ij}$, where i,j,k are $SU(2)_L$ indices, α,β,γ are SU(3) indices, and p,r,s,t are flavor indices.

RG equations for wilson coefficients in the SMEFT can be found in the appendix of DSixTools[1], we will not list them here. We will run down $C^{duql}_{1113}, C^{duql}_{1123}, C^{duql}_{1133}$, as these three coefficients will contribute to a single matched wilson coefficient in the WET. We arbitrarily set the value of these coefficients at $\mu = 10$ TeV to be 5×10^{-5}.

We run these coefficients down to M_{Top} using DSixTools, the wilson coefficients each increase by ≈ 20 percent from $\mu = 10$ TeV to M_{Top}.

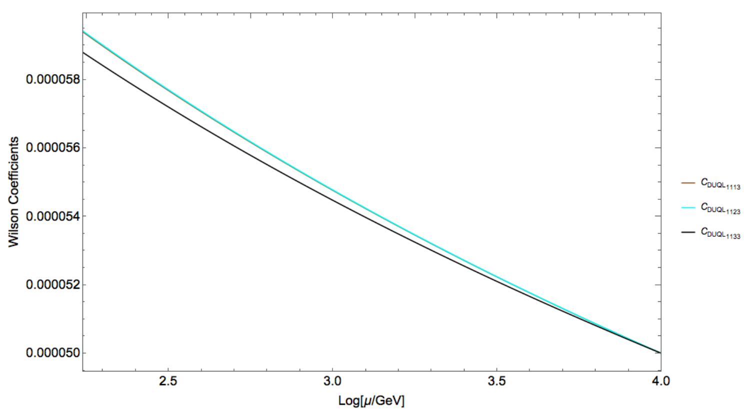

Figure 1: Running of the three SMEFT wilson coefficients: $C^{duql}_{1113}(\mu), C^{duql}_{1123}(\mu), C^{duql}_{1133}(\mu)$ from 10 TeV to M_{Top} using DSixTools.

Now we match the SMEFT to the WET where we have integrated out the top quark and broken electroweak symmetry. The SMEFT operators we considered above are matched to a single operator in the WET, $Q^{uds\nu_\tau}_{RL} = C^{uds\nu_\tau}_{RL}\epsilon_{\alpha\beta\gamma}(u^\alpha_R d^\beta_R)(s^\gamma_L \nu_\tau)$, with the matching condition

$$C^{uds\nu_\tau}_{RL}(M_{\text{Top}}) = -(V_{\text{CKM}})_{j2} C^{duql}_{11j3}(M_{\text{Top}}) \tag{1}$$

The above negative sign arises from contracting the SU(2) doublet index with the epsilon symbol. Using the above matching condition we can compute $C^{uds\nu_\tau}_{RL}(M_{\text{Top}}) = -7.361 \times 10^{-5}$. We then run down the wilson coefficient $C^{uds\nu_\tau}_{RL}(\mu)$ utilizing RunDec[2], from M_{Top} to 1 GeV. The two loop RG equation for the wilson coefficient $C(\mu)$ is given by

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$$\mu \frac{dC(\mu)}{d\mu} = -\left[4\frac{\alpha_s}{4\pi} + \left(\frac{14}{3} + \frac{4}{9}N_f - \frac{10}{3}\right)\frac{\alpha_s^2}{(4\pi)^2}\right] C(\mu) \tag{2}$$

Here we are only considering the QCD contribution to RG evolution in the WET because α_s is large. Using the two loop beta function for α_s, we can write the solution for $C(\mu)$ as

$$\frac{C(\mu)}{C(\mu_0)} = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{-\frac{2}{b_1}} \left[\frac{4\pi b_1 + b_2\alpha_s(\mu)}{4\pi b_1 + b_2\alpha_s(\mu_0)}\right]^{\frac{2}{b_1} - \frac{42+4N_f+9(-\frac{10}{3})}{18b_2}} \tag{3}$$

where $b_1 = -\frac{11N_c - 2N_f}{3}$ and $b_2 = -\frac{34}{3}N_c^2 + \frac{10}{3}N_c N_f + 2C_F N_f$. We can then calculate $C_{RL}^{uds\nu_\tau}(\mu)$ at any scale below M_{Top} which we show below.

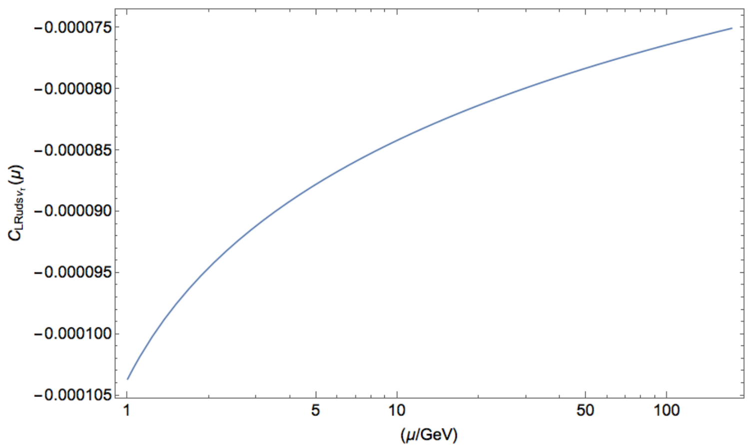

Figure 2: Running of the WET wilson coefficient $C_{RL}^{uds\nu_\tau}(\mu)$ from M_{Top} to 1 GeV

This gives $C_{RL}^{uds\nu_\tau}(\mu = 1\text{GeV}) = -1.037 \times 10^{-4}$, this is a 41 percent increase in magnitude from M_{Top} to 1 GeV.

[1] Celis, Alejandro et al., ”DSixTools: The Standard Model Effective Field Theory Toolkit”, arXiv:1704.04504 [hep-ph]
[2] Chetyrkin, K.G. et al., ”RunDec: a Mathematica package for running and decoupling of the strong coupling and quark masses”, arXiv:0004189 [hep-ph]