AI-512 MATHEMATICS FOR MACHINE LEARNING

Name: Yugaam Mangesh Hagargi
Roll No: IMT2020536

Q.1.a) To minimize $\|z\|^2$ [z : n-vector of modifications]

We minimize w.r.t $\underbrace{D(x+z) = \alpha S}_{\text{equality constraint}}$

Assumption: D has linearly independent rows.

$$D(x+z) = \alpha S$$
$$Dx + Dz = \alpha S$$

Lagrangian function $L(z, \lambda)$

$$L(z, \underset{k\times 1}{\lambda}) = z^T z + \lambda^T (Dz - \alpha S + Dx)$$

$$\frac{\partial L}{\partial z_i}(\hat{z}, \hat{\lambda}) = 0 \quad \forall i \in [1 \ldots n]$$

$$\frac{\partial L}{\partial \lambda_i}(\hat{z}, \hat{\lambda}) = 0 \quad \forall i \in [1 \ldots k]$$

$$\frac{\partial L}{\partial \lambda_i}(\hat{z}, \hat{\lambda}) = D_i(x+z) - \alpha S_i = 0$$

$$\frac{\partial L}{\partial z_i}(\hat{z}, \hat{\lambda}) = 2\sum_{j=1}^{n}(1)(\hat{z}_j) + \sum_{j=1}^{k}\hat{\lambda}_j (D_j)_i = 0$$

Writing in compact matrix form

$$\nabla_z L = 2\hat{z} + D^T\hat{\lambda} = 0$$

$$z = \frac{-D^T\lambda}{2}$$

$$\nabla_\lambda L = D(x+z) - \alpha S = 0$$

Substituting z in above equation

$$D\left(x - \frac{D^T\lambda}{2}\right) - \alpha S = 0$$

$$2(Dx - \alpha S) = DD^T\lambda$$

$$\lambda = 2(DD^T)^{-1}(Dx - \alpha S)$$

Substituting λ in $\nabla_z L$

$$2\hat{z} + D^T\left(2(DD^T)^{-1}\right)(Dx - \alpha S) = 0$$

$$\hat{z} = D^T(DD^T)^{-1}(\alpha S - Dx)$$

Pseudo inverse of $D^{+} = D^{T}(DD^{T})^{-1}$

$$\boxed{\hat{z} = D^{+}(\alpha S - Dx)}$$