Домашня робота 1

1.7 a) $\int \left(\frac{1-x}{x}\right)^2 dx = \int \frac{1-2x-x^2}{x^2}dx = \int(1 - \frac{2}{x} + \frac{1}{x^2})dx = x - 2\ln|x| - \frac{1}{x} + c$

b) $\int \frac{\sqrt{x}-2\sqrt[3]{x^2}+1}{\sqrt[4]{x}}dx = \int(\sqrt[4]{x} - 2x^{\frac{5}{4}} + \frac{1}{\sqrt[4]{x}})dx = -\frac{8x^{\frac{9}{4}}}{9} + \frac{4x^{\frac{5}{4}}}{5} + \frac{4x^{\frac{3}{4}}}{3} + c$

c) $\int \frac{x^2+3}{x^2-1}dx = \int(-\frac{2}{x+1} + \frac{2}{x-1} + 1)dx = -2\ln|x+1| + 2\ln|x-1| + x + c$

1.8 a) $\int \cot^2 x dx = \int \frac{\cos^x}{\sin^2 x}dx = \int \frac{1-\sin^2 x}{\sin^2 x}dx = \int(\frac{1}{\sin^2 x} - 1)dx = -\cot x - x + c$

b) $\int \sqrt[3]{1-3x}dx = \int(1-3x)^{\frac{1}{3}} \cdot (-\frac{1}{3})d(1-3x) = - \cdot \frac{1}{4}(1-3x)^{\frac{4}{3}} + c$

c) $\int \frac{dx}{2-3x^2} = \frac{1}{2}\int \frac{dx}{1-\frac{3}{2}x^2} = \left|d(\sqrt{\frac{3}{2}}x) = \sqrt{\frac{3}{2}}dx\right| = \frac{1}{\sqrt{6}}\int \frac{d(\sqrt{\frac{3}{2}}x)}{1-\frac{3}{2}x^2} = \frac{1}{2}\ln\left|\frac{1+\sqrt{\frac{3}{2}}x}{1-\sqrt{\frac{3}{2}}x}\right| + c$

1.9 a) $\int \frac{dx}{1+\sin x} = \left|t = \tan\frac{x}{2} \Rightarrow \sin x = \frac{2t}{1+t^2}\right| = \int \frac{2dt}{(t^2+1)\left(\frac{2t}{1+t^2}\right)} = \int \frac{2dt}{t^2+2t+1} = 2\int \frac{d(t+1)^2}{(t+1)^2} =$
$= -\frac{2}{t+1} + c = -\frac{2}{\tan\frac{x}{2}+1} + c$

b) $\int \frac{\sin\frac{1}{x}dx}{x^2} = \left|d(\frac{1}{x}) = -\frac{1}{x}^2 dx\right| - \int \sin(\frac{1}{x})d(\frac{1}{x}) = \cos(\frac{1}{x}) + c$

1.10 a) $\int x^2\sqrt[3]{1+x^3}dx = \left|d(1+x^3) = 3x^2dx\right| = \frac{1}{3}\int \sqrt[3]{1+x^3}d(1+x^3) = \frac{\sqrt[3]{(1+x^3)^4}}{4} + c$

b) $\int \frac{xdx}{4+x^4} = \left|d(x^2) = 2xdx\right| = \frac{1}{8}\int \frac{d(x^2)}{\frac{x^4}{4}+1} = \frac{1}{4}\int \frac{d(\frac{x^2}{2})}{\frac{x^4}{4}+1} = \frac{1}{4}\arctan\frac{x^2}{2} + c$

c) $\int \frac{dx}{e^x-e^{-x}} = \int \frac{e^x dx}{e^{2x}-1} = \int \frac{d(e^x)}{e^{2x}-1} = \frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right| + c$

1.11 a) $\int \frac{\sin x dx}{\sqrt{\cos^3 x}} = -\int \frac{d(\cos x)}{\sqrt{\cos^3 x}} = \frac{2}{\sqrt{\cos x}} + c$

b) $\int \frac{\sin x+\cos x}{\sqrt[3]{\sin x-\cos x}}dx = \int \frac{d(\sin x-\cos x)}{\sqrt[3]{\sin x-\cos x}} = \frac{3}{2}\sqrt[3]{(\sin x-\cos x)^2} + c$

1.12 a) $\int \frac{dx}{\mathrm{ch}x} = \int \frac{\mathrm{ch}x\,dx}{\mathrm{ch}^2 x} = \int \frac{d(\mathrm{sh}x)}{\mathrm{sh}^2 x+1} = \arctan(\mathrm{sh}x) + c$

b) $\int \frac{(1+e^x)^2 dx}{e^{2x}+1} =$