

Домашня робота 11

$$9.9 \quad (a) \quad \int_0^1 \ln x \, dx = \lim_{a \rightarrow 0^+} \left(\int_a^1 \ln x \, dx \right) = \lim_{a \rightarrow 0^+} (-1 - \ln a \cdot a + a) = 1$$

$$(b) \quad \int_{-\infty}^{+\infty} \frac{dx}{1+x^2} = \int_{-\infty}^0 \frac{dx}{1+x^2} + \int_0^{+\infty} \frac{dx}{1+x^2} = \lim_{a \rightarrow -\infty} \left(\int_a^0 \frac{dx}{1+x^2} \right) + \lim_{a \rightarrow +\infty} \left(\int_0^a \frac{dx}{1+x^2} \right) =$$

$$= \lim_{a \rightarrow -\infty} (-\arctan a) + \lim_{a \rightarrow +\infty} (\arctan a) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$(c) \quad \int_{-1}^1 \frac{dx}{\sqrt{1-x^2}} = \int_{-1}^0 \frac{dx}{\sqrt{1-x^2}} + \int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{a \rightarrow -1^+} \left(\int_a^0 \frac{dx}{\sqrt{1-x^2}} \right) + \lim_{a \rightarrow +1^-} \left(\int_0^a \frac{dx}{\sqrt{1-x^2}} \right) =$$

$$= \lim_{a \rightarrow -1^+} (-\arcsin a) + \lim_{a \rightarrow +1^-} (\arcsin a) = \pi$$

$$9.10 \quad (a) \quad \int_1^{\infty} \frac{dx}{x^3 \sqrt{x^2+1}} - I$$

$$x \rightarrow \infty \quad f(x) = \frac{1}{x^3 \sqrt{x^2+1}} = x^{-\frac{5}{3}}, \quad p = \frac{5}{3} > 1 \Rightarrow \text{сходится}$$

(b)