Домашня робота 8

6.9 1. $I_1 = \int\limits_0^1 x^2 \mathbf{d}x,\ I_2 = \int\limits_0^1 x^3 \mathbf{d}x$

$$[0,\ 1]: \quad \begin{aligned} & x^3 \le x^2 \\ & I_2 \le I_1 \end{aligned}$$

2. $I_1 = \int\limits_1^2 x^2 \mathbf{d}x,\ I_2 = \int\limits_1^2 x^3 \mathbf{d}x$

$$[1,\ 2]: \quad \begin{aligned} & x^3 \ge x^2 \\ & I_2 \ge I_1 \end{aligned}$$

6.10 1. $I_1 = \int\limits_0^1 2^{x^2} \mathbf{d}x,\ I_2 = \int\limits_0^1 2^{x^3} \mathbf{d}x$

$$[0,\ 1]: \quad \begin{aligned} & x^3 \le x^2 \\ & 2^{x^3} \le 2^{x^2} \\ & I_2 \le I_1 \end{aligned}$$

2. $I_1 = \int\limits_1^2 x^2 \mathbf{d}x,\ I_2 = \int\limits_1^2 x^3 \mathbf{d}x$

$$[1,\ 2]: \quad \begin{aligned} & x^3 \ge x^2 \\ & 2^{x^3} \ge 2^{x^2} \\ & I_2 \ge I_1 \end{aligned}$$

3. $I_1 = \int\limits_1^2 \ln x \mathbf{d}x,\ I_2 = \int\limits_1^2 \ln^2 x \mathbf{d}x$

$$[1,\ 2]: \quad \begin{aligned} & \ln 1 = \ln^2 1 \\ & \ln 2 > \ln^2 2 \\ & \ln x \ge \ln^2 x \\ & I_2 \ge I_1 \end{aligned}$$

4. $I_1 = \int\limits_3^2 \ln x \mathbf{d}x,\ I_2 = \int\limits_3^2 \ln^2 x \mathbf{d}x$

$$[3,\ 4]: \quad \begin{aligned} & \ln 3 < \ln^2 3 \\ & \ln 4 < \ln^2 4 \\ & \ln x \le \ln^2 x \\ & I_2 \le I_1 \end{aligned}$$

6.11 1. $I = \int\limits_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 + \sin^2 x) \mathbf{d}x$

$$f(x) = 1 + \sin^2 x: \quad \begin{aligned} & 0 \le \sin|x| \le 1 \\ & 0 \le \sin^2 x \le 1 \\ & 1 \le 1 + \sin^2 x \le 2 \end{aligned} \qquad \begin{aligned} 1 \cdot \frac{\pi}{4} \le I \le 2 \cdot \frac{5\pi}{4} \\ \frac{\pi}{4} \le I \le \frac{5\pi}{2} \end{aligned}$$

2. $I = \int\limits_{\frac{1}{e}}^{e} x^2 e^{-x^2} \mathbf{d}x$

$$f(x) = x^2 e^{-x^2}$$

6.12 $I = \int\limits_0^1 \frac{x^n}{1+x}\mathbf{d}x$

$f(x) = \frac{x^n}{1+x}$ - неперервна на $[0,\ 1] \Rightarrow I = \frac{\xi^n}{1+\xi}(b-a)$

$\lim\limits_{n\to\infty} \frac{\xi^n}{1+\xi} = \frac{\infty}{1+x}$

6.13 $\frac{\mathbf{d}}{\mathbf{d}x} \int\limits_{\sin x}^{\cos x} \cos \pi t^2 \mathbf{d}t = (\cos x)'(\cos(\pi \cos x)) - (\sin x)'(\cos(\pi \sin x)) =$
$= -\sin x(\cos(\pi \cos x)) - \cos x(\cos(\pi \sin x))$

6.14 1.