ДОМАШНЯ РОБОТА №16
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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1. (a) $f(t) = 3t^2 - 2 - 5\sin t + 2e^{2t} \longleftrightarrow 3 \cdot \dfrac{2}{p^3} - \dfrac{2}{p} - 5 \cdot \dfrac{1}{p^2-1} + 2 \cdot \dfrac{1}{p-2}$

(b) $f(t) = te^t + t^2 \sin 3t \longleftrightarrow \dfrac{1}{(p-1)^2} + \dfrac{\mathrm{d}^2}{\mathrm{d}p^2}\left(\dfrac{3}{p^2+9}\right) = \dfrac{1}{(p-1)^2} + 18 \cdot \dfrac{p^2-3}{(p^2+9)^3}$

(c) $f(t) = (t+2)\cos t \longleftrightarrow \dfrac{p^2-1}{(p^2+1)^2} + 2 \cdot \dfrac{p}{p^2+1}$

(d) $f(t) = e^{2t}\operatorname{ch} t - sh2t = \dfrac{1}{2}\left(e^{3t} + e^t\right) - \operatorname{sh} 2t \longleftrightarrow \dfrac{1}{2}\left(\dfrac{1}{p-3} + \dfrac{1}{p-1}\right) - \dfrac{2}{p^2-4}$

2. (a) $F(p) = \dfrac{6}{p^3-8} = \dfrac{6}{(p-2)(p^2+2p+4)} = \dfrac{1}{2}\left(-\dfrac{p+1+3}{(p+1)^2+3} + \dfrac{1}{p-2}\right) \longleftrightarrow$
$\longleftrightarrow \dfrac{1}{2}\left(-e^{-t}\cos\sqrt{3}t - \sqrt{3}e^{-t}\sin\sqrt{3}t + e^2 t\right)$

(b) $F(p) = \dfrac{p}{(p+1)(p^2+p+1)} = \dfrac{p+1}{p^2+p+1} - \dfrac{1}{p+1} = \dfrac{p+\frac{1}{2}+\frac{1}{2}}{\left(p+\frac{1}{2}\right)^2 + \dfrac{3}{4}} - \dfrac{1}{p+1} \longleftrightarrow$
$\longleftrightarrow -e^t + e^{-\frac{t}{2}}\left(\cos\dfrac{\sqrt{3}}{2}t + \dfrac{\sqrt{3}}{3}\sin\dfrac{\sqrt{3}}{2}t\right)$

(c) $F(p) = \dfrac{3p+2}{(p+1)(p^2+4p+5)} = \dfrac{1}{2}\left(\dfrac{p+2+7}{(p+2)^2+1} - \dfrac{1}{p+1}\right) \longleftrightarrow \dfrac{1}{2}\left(e^{-2t}\cos t + \right.$
$\left. +7e^{-2t}\sin t - e^{-t}\right)$

3. $y'' + y' = t^2 + 2t, \quad y(0) = 0, \quad y'(0) = -2$
$y' \longleftrightarrow pY(p) - y(0) = pY(p), \quad y'' \longleftrightarrow p^2Y(p) - py(0) - p'(0) = p^2Y(p) + 2,$
$t^2 + 2t \longleftrightarrow \dfrac{2}{p^3} + \dfrac{2}{p^2}, \quad p^2Y(p) + 2 + pY(p) = \dfrac{2}{p^3} + \dfrac{2}{p^2}, \quad Y(p) = \left(\dfrac{2}{p^3} + \dfrac{2}{p^2} - 2\right) \cdot \dfrac{1}{p^2+p}$
$Y(p) = -2\left(\dfrac{1}{p^2} - \dfrac{1}{p^3} - \dfrac{1}{p+1} + \dfrac{1}{p} - \dfrac{1}{p^2} + \dfrac{1}{p^3} - \dfrac{1}{p^4}\right) = 2\left(\dfrac{1}{p^4} + \dfrac{1}{p+1} - \dfrac{1}{p}\right) \longleftrightarrow y(t) =$
$= 2\left(e^{-t} - 1 + \dfrac{t^3}{6}\right)$

4. $\begin{cases} \dot{x} + y = 0 \\ \dot{y} + x = 0 \end{cases}, \quad x(0) = 1, \quad y(0) = -1$
$x(t) \leftrightarrow X(p) \qquad \dot{x}(t) \leftrightarrow pX(p) - x(0) = pX(p) - 1 \qquad Y(p) = 1 - pX(p)$
$y(t) \leftrightarrow Y(p) \qquad \dot{y}(t) \leftrightarrow pY(p) - y(0) = pY(p) + 1 \qquad X(p) = -1 - pY(p)$
$Y(p) = 1 - pX(p), \quad p - p^2X(p) + X(p) + 1 = 0, \quad X(p)(1-p^2) = -p - 1,$
$X(p) = \dfrac{p+1}{p^2-1} = \dfrac{1}{p-1} \longleftrightarrow e^t, \quad Y(p) = -\dfrac{1}{p-1} \longleftrightarrow -e^t$

5. $y(x) = x - \displaystyle\int_0^x e^{x-t}y(t)\ \mathrm{d}t$
$f(x-t) = e^{x-t}, \quad f(x) = e^x, \quad y(x) \leftrightarrow Y(p) = \dfrac{1}{p^2} - \dfrac{Y(p)}{p-1}, \quad Y(p)\left(1 - \dfrac{1}{p+1}\right) = \dfrac{1}{p^2},$
$Y(p) = \dfrac{p-1}{p^3} = \dfrac{1}{p^2} - \dfrac{1}{p^3} \longleftrightarrow x - \dfrac{1}{2}x^2$