

Домашня робота 3

$$4.13 \quad \int \sin^6 x \, dx = \int (1 - \cos^2 x)^3 \, dx = \int (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x) \, dx = x - 3 \int \left(\frac{1+\cos 2x}{2}\right) \, dx + 3 \int \left(\frac{1+\cos 2x}{2}\right)^2 \, dx - \int \cos^6 x$$

$$4.14 \quad \int \sin^5 x \cos^5 x = \left| \begin{array}{l} t = \sin x \\ dt = \cos x \end{array} \right| = \int \cos^4 x \sin^5 x \, d(\sin x) = \int t^5 \cdot (1 - \sin^2 x) \, d(\sin x) = \\ = \int t^5 \cdot (1 - t^2 + t^4) \, dt = \int t^5 \, dt - 2 \int t^7 \, dt + \int t^9 \, dt = \frac{1}{6}t^6 - \frac{2}{8}t^8 + \frac{1}{10}t^{10} = \\ = \frac{1}{6}\sin^6 x - \frac{2}{8}\sin^8 x + \frac{1}{10}\sin^{10} x + c$$

$$4.15 \quad \int \tan^5 x \, dx = \left| \begin{array}{l} t = \tan x \\ dx = \frac{dt}{1+t^2} \end{array} \right| = \int \frac{t^5}{1+t^2} \, dt = \left| \begin{array}{l} u = t^2 \\ du = 2t \, dt \end{array} \right| = \frac{1}{2} \int \frac{u^2}{u+1} \, du = \frac{1}{2} \int \left(u + \frac{1}{u+1} - 1\right) \, du = \\ = \frac{1}{2} \left(\frac{1}{2}u^2 + \ln|u+1| - u\right) = \frac{1}{2} \left(\frac{1}{2}t^4 + \ln|t^2+1| - t^2\right) = \frac{1}{2} \left(\frac{1}{2}\tan^4 x + \ln|\tan^2 x + 1| - \tan^2 x\right) + c$$

$$4.16 \quad \int \frac{dx}{\sqrt{\tan x}}$$

$$4.17 \quad \int \sin 5x \cos x \, dx = \frac{1}{2} \int (\sin 6x + \sin 4x) \, dx = \frac{1}{2} \left(-\frac{1}{6}\cos 6x - \frac{1}{4}\cos 4x\right) + c$$

$$4.18 \quad \int \cos^2 ax \sin^2 bx \, dx = \int \frac{1}{4} (\sin(b+a)x - \sin(b-a)x)^2 \, dx = \\ = \frac{1}{4} \int (\sin^2(b+a)x + 2\sin(b+a)x \sin(b-a)x - \sin^2(b-a)x) \, dx = \\ = \frac{1}{4} \left(\int \frac{1-\cos 2(b+a)x}{2} \, dx + \int (-\cos(b+a+b-a)x + \cos(b+a-b+a)x) \, dx + \int \frac{1-\cos 2(b-a)x}{2} \, dx \right) = \\ = \frac{1}{4} \left(\frac{1}{2} \left(x - \frac{\sin 2(b+a)x}{2(b+a)}\right) - \frac{\sin 2bx}{2b} + \frac{\sin 2ax}{2a} + \frac{1}{2} \left(x - \frac{\sin 2(b-a)x}{2(b-a)}\right) \right) + c$$

$$4.19 \quad \int \frac{\sin^2 x}{\sin x + 2 \cos x} \, dx = \left| \begin{array}{l} t = \tan \frac{x}{2} \\ dx = \frac{2}{t^2+1} \, dt \end{array} \right| = \int \frac{\frac{4t^2}{(1+t^2)^2}}{\frac{2t}{1+t^2} + 2\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2} \, dt = \int \frac{\frac{4t^2}{1+t^2}}{2t+2-2t^2} \cdot \frac{2}{1+t^2} \, dt = \\ = 4 \int \frac{t^2}{(1+t^2)^2(1+t-t^2)} \, dt \ominus$$

$$\frac{t^2}{-(1+t^2)^2(1-t+t^2)} = \frac{At+B}{t^2+1} + \frac{Ct+D}{(t^2+1)^2} + \frac{Et+F}{t^2-t-1}$$

$$t^2 = -(At+B)(t^2+1)(t^2-t-1) - (Ct+D)(t^2-t-1) - (Et+F)(t^2+1)^2$$

$$0 \cdot t^5 + 0 \cdot t^4 + 0 \cdot t^3 + t^2 + 0 \cdot t + 0 = -At^5 + At^4 + At^2 + At - Bt^4 + Bt^3 + Bt + B - Ct^3 +$$

$$+ Ct^2 + Ct - Dt^2 + Dt + D - Et^5 - 2Et^3 - Et - Ft^4 - 2Ft^2 - F$$

$$0 \cdot t^5 + 0 \cdot t^4 + 0 \cdot t^3 + t^2 + 0 \cdot t + 0 = t^5(-A-E) + t^4(A-B-F) + t^3(B-C-2E) + t^2(A+C-D-2F) + t(A+B+C+D-E) + (B+D-F)$$

$$\left\{ \begin{array}{ll} t^5: & -A-E=0 \quad A=0 \\ t^4: & A-B-F=0 \quad B=\frac{1}{5} \\ t^3: & B-C-2F=0 \quad C=\frac{1}{5} \\ t^2: & A-C-D-2F=1 \quad D=-\frac{2}{5} \\ t^1: & A+B+C+D-E=0 \quad E=0 \\ t^0: & B+D-F=0 \quad F=-\frac{1}{5} \end{array} \right.$$

$$\ominus 4 \int \left(\frac{1}{5(t^2+1)} + \frac{t-2}{5(t^2+1)^2} - \frac{1}{5(t^2-t-1)} \right) \, dt = -4 \left(\frac{1}{5} \arctan t + \frac{1}{5} \int_i \left(\frac{t}{(t^2+1)^2} + \frac{2}{(t^2+1)^2} \right) \, dt + \frac{1}{5} \int_{ii} \left(\frac{t}{(t^2+1)^2} + \frac{2}{(t^2+1)^2} \right) \, dt \right)$$