Домашня робота 3

4.13 $\int \sin^6 x\mathbf{d}x = \int (1-\cos^2 x)^3\mathbf{d}x = \int \left(1-3\cos^2 x+3\cos^4 x-\cos^6 x\right)\mathbf{d}x = x-3\int \left(\frac{1+\cos 2x}{2}\right)\mathbf{d}x+$ $3\int \left(\frac{1+\cos 2x}{2}\right)^2\mathbf{d}x - \int \cos^6 x$

4.14 $\int \sin^5 x\cos^5 x = \left|\begin{matrix} t=\sin x \\ \mathbf{d}t=\cos x\end{matrix}\right| = \int \cos^4 x\sin^5 x\mathbf{d}(\sin x) = \int t^5\cdot(1-\sin^2 x)\mathbf{d}(\sin x) =$
$= \int t^5\cdot(1-2t^2+t^4)\mathbf{d}t = \int t^5\mathbf{d}t - 2\int t^7\mathbf{d}t + \int t^9\mathbf{d}t = \frac{1}{6}t^6 - \frac{2}{8}t^8 + \frac{1}{10}t^{10} =$
$= \frac{1}{6}\sin^6 x - \frac{2}{8}\sin^8 x + \frac{1}{10}\sin^{10} x + c$

4.15 $\int \tan^5 x\mathbf{d}x = \left|\begin{matrix} t=\tan x \\ \mathbf{d}x=\frac{\mathbf{d}t}{1+t^2}\end{matrix}\right| = \int \frac{t^5}{1+t^2}\mathbf{d}t = \left|\begin{matrix} u=t^2 \\ \mathbf{d}u=2t\mathbf{d}t\end{matrix}\right| = \frac{1}{2}\int \frac{u^2}{u+1}\mathbf{d}u = \frac{1}{2}\int \left(u+\frac{1}{u+1}-1\right)\mathbf{d}u =$
$\frac{1}{2}\left(\frac{1}{2}u^2+\ln|u+1|-u\right) = \frac{1}{2}\left(\frac{1}{2}t^4+\ln|t^2+1|-t^2\right) = \frac{1}{2}\left(\frac{1}{2}\tan^4 x+\ln|\tan^2 x+1|-\tan^2 x\right)+c$

4.16 $\int \frac{\mathbf{d}x}{\sqrt{\tan x}}$

4.17 $\int \sin 5x\cos x\mathbf{d}x = \frac{1}{2}\int (\sin 6x+\sin 4x)\mathbf{d}x = \frac{1}{2}\left(-\frac{1}{6}\cos 6x-\frac{1}{4}\cos 4x\right)+c$

4.18 $\int \cos^2 ax\sin^2 bx\mathbf{d}x = \int \frac{1}{4}\left(\sin(b+a)x-\sin(b-a)x\right)^2\mathbf{d}x =$
$= \frac{1}{4}\int \left(\sin^2(b+a)x+2\sin(b+a)x\sin(b-a)x-\sin^2(b-a)x\right)\mathbf{d}x =$
$= \frac{1}{4}\left(\int \frac{1-\cos 2(b+a)x}{2}\mathbf{d}x + \int \left(-\cos(b+a+b-a)x+\cos(b+a-b+a)x\right)\mathbf{d}x + \int \frac{1-\cos 2(b-a)x}{2}\mathbf{d}x\right) =$
$= \frac{1}{4}\left(\frac{1}{2}\left(x-\frac{\sin 2(b+a)x}{2(b+a)}\right) - \frac{\sin 2bx}{2b} + \frac{\sin 2ax}{2x} + \frac{1}{2}\left(x-\frac{\sin 2(b-a)x}{2(b-a)}\right)\right)+c$

4.19 $\int \frac{\sin^2 x}{\sin x+2\cos x}\mathbf{d}x = \left|\begin{matrix} t=\tan\frac{x}{2} \\ \mathbf{d}x=\frac{2}{t^2+1}\mathbf{d}t\end{matrix}\right| = \int \frac{\frac{4t^2}{(1+t^2)^2}}{\frac{2t}{1+t^2}+2\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\mathbf{d}t = \int \frac{\frac{4t^2}{1+t^2}}{2t+2-2t^2}\cdot\frac{2}{1+t^2}\mathbf{d}t =$
$= 4\int \frac{t^2}{(1+t^2)^2(1+t-t^2)}\mathbf{d}t \ominus$

$$\frac{t^2}{-(1+t^2)^2(1-t+t^2)} = \frac{At+B}{t^2+1} + \frac{Ct+D}{(t^2+1)^2} + \frac{Et+F}{t^2-t-1}$$
$t^2 = -(At+B)(t^2+1)(t^2-t-1) - (Ct+D)(t^2-t-1) - (Et+F)(t^2+1)^2$
$0\cdot t^5+0\cdot t^4+0\cdot t^3+t^2+0\cdot t+0 = -At^5+At^4+At^2+At-Bt^4+Bt^3+Bt+B-Ct^3+$
$+Ct^2+Ct-Dt^2+Dt+D-Et^5-2Et^3-Et-Ft^4-2Ft^2-F$
$0\cdot t^5+0\cdot t^4+0\cdot t^3+t^2+0\cdot t+0 = t^5(-A-E)+t^4(A-B-F)+t^3(B-C-2E)+t^2(A+$
$C-D-2F)+t(A+B+C+D-E)+(B+D-F)$
$$\begin{cases} t^5: & -A-E=0 & A=0 \\ t^4: & A-B-F=0 & B=\frac{1}{5} \\ t^3: & B-C-2F=0 & C=\frac{1}{5} \\ t^2: & A-C-D-2F=1 & D=-\frac{2}{5} \\ t^1: & A+B+C+D-E=0 & E=0 \\ t^0: & B+D-F=0 & F=-\frac{1}{5}\end{cases}$$
$\ominus -4\int \left(\frac{1}{5(t^2+1)}+\frac{t-2}{5(t^2+1)^2}-\frac{1}{5(t^2-t-1)}\right)\mathbf{d}t = -4\left(\frac{1}{5}\arctan t+\frac{1}{5}\int_i \left(\frac{t}{(t^2+1)^2}+\frac{2}{(t^2+1)^2}\right)\mathbf{d}t+\right.$
$\left.+\frac{1}{5}\int_{ii}\left(\frac{2}{\sqrt{5}(2t-\sqrt{5}-1)}-\frac{2}{\sqrt{5}(2t+\sqrt{5}-1)}\right)\mathbf{d}t\right)\ominus$

i. $\int \left(\frac{t}{(t^2+1)^2}+2\cdot\frac{1}{(t^2+1)^2}\right)\mathbf{d}t = \frac{1}{2}\int \frac{1}{(t^2+1)^2}\mathbf{d}(t^2+1)+\frac{t}{2(t^2+1)}+\frac{1}{2}\int \frac{1}{t^2+1}\mathbf{d}t =$
$= -\frac{1}{4(t^2+1)}+\frac{t}{2(t^2+1)}+\frac{1}{2}\arctan t+c$

ii. $\int \left(\frac{2}{\sqrt{5}(2t-\sqrt{5}-1)} - \frac{2}{\sqrt{5}(2t+\sqrt{5}-1)} \right) \mathbf{d}t =$

$= \frac{2}{\sqrt{5}} \left(\int \frac{1}{(2t-\sqrt{5}-1)} \mathbf{d}(2t - \sqrt{5} - 1) + \int \frac{1}{(2t+\sqrt{5}-1)} \mathbf{d}(2t + \sqrt{5} - 1) \right) =$

$= \frac{1}{\sqrt{5}} \left(\ln|2t - \sqrt{5} - 1| + \ln|2t + \sqrt{5} - 1| \right) + c$

$$\circeq - \frac{4}{5} \left(\arctan t + -\frac{1}{4(t^2+1)} + \frac{t}{2(t^2+1)} + \frac{1}{2} \arctan t + \frac{1}{\sqrt{5}} \ln|2t - \sqrt{5} - 1| + \ln|2t + \sqrt{5} - 1| \right) + c$$