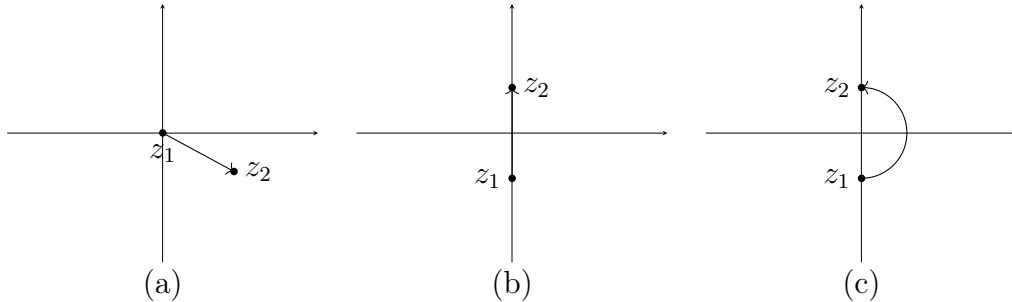

ДОМАШНЯ РОБОТА №6
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"
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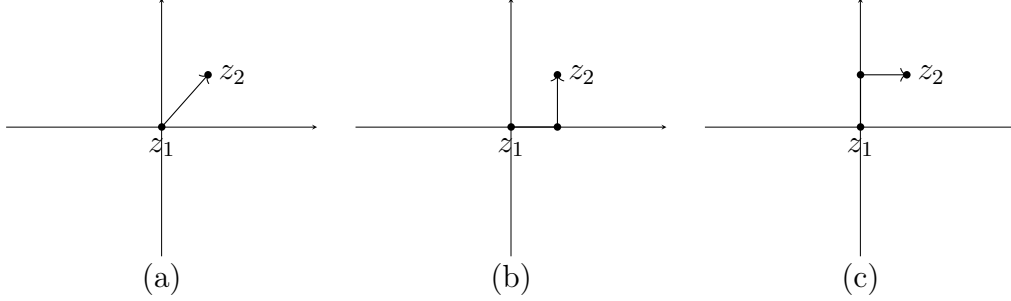
1. (a) $z_1 = 0, \quad z_2 = 2 - i$
 $y = -\frac{1}{2}x \Rightarrow z = -\frac{1}{2}ix + x = x \left(1 - \frac{1}{2}i\right), \quad x : 0 \rightarrow 2$
 $dz = \left(1 - \frac{1}{2}i\right) dx, \quad |z| = \left|x \left(1 - \frac{1}{2}i\right)\right| = \frac{\sqrt{5}}{2}|x|$
 $\int_C |z| dz = \int_0^2 \frac{\sqrt{5}}{2} \left(1 - \frac{1}{2}i\right) |x| dx = \frac{\sqrt{5}}{2} \left(1 - \frac{1}{2}i\right) \frac{x^2}{2} \Big|_0^2 = \sqrt{5} \left(1 - \frac{1}{2}i\right)$
- (b) $z_1 = -i, \quad z_2 = i$
 $x = 0, \quad y : -1 \rightarrow 1 \Rightarrow z = iy$
 $dz = i dy, \quad |z| = |y|$
 $\int_C |z| dz = i \int_{-1}^1 |y| dy = 2i \int_0^1 y dy = 2i \cdot \frac{y^2}{2} \Big|_0^1 = i$
- (c) $|z| = 1, \quad -\frac{\pi}{2} \leq \arg z \leq \frac{\pi}{2} \quad z = z_0 + R \cdot e^{i\varphi} = 0 + 1 \cdot e^{i\varphi} = e^{i\varphi}$
 $\varphi : -\frac{\pi}{2} \rightarrow \frac{\pi}{2}, \quad dz = ie^{i\varphi} d\varphi$
 $\int_C |z| dz = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ie^{i\varphi} d\varphi = e^{i\varphi} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = e^{i\frac{\pi}{2}} - e^{-i\frac{\pi}{2}} = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} - \cos -\frac{\pi}{2} - i \sin -\frac{\pi}{2} = 0 + i - 0 + i = 2i$



2. (a) $z_1 = 0, \quad z_2 = 1 + i$
 $y = x, \quad x : 0 \rightarrow 1, \quad z = x + ix = x(i + 1), \quad dz = (1 + i) dx$
 $\int_C (x - y + ix^2) dz = \int_0^1 (x - x + ix^2) \cdot (1 + i) dx = \int_0^1 x^2(i - 1) dx =$
 $= (i - 1) \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}(i - 1)$
- (b) $y = 0 : \quad x : 0 \rightarrow 1, \quad z = x, \quad dz = dx,$
 $x = 1 : \quad y : 0 \rightarrow 1, \quad z = 1 + iy, \quad dz = i dy,$
 $\int_C (x - y + ix^2) dz = \int_0^1 (x + ix^2) dx + \int_0^1 (1 - y + i)i dy = \frac{x^2}{2} \Big|_0^1 + i \frac{x^3}{3} \Big|_0^1 - \frac{iy^2}{2} \Big|_0^1 +$
 $+ (1 + i)iy \Big|_0^1 = \frac{1}{2} + i \frac{1}{3} + i - \frac{i}{2} - 1 = 1 + i \left(\frac{1}{3} + 1 - \frac{1}{2}\right) = -\frac{1}{2} + i \frac{5}{6}$
- (c) $x = 0 : \quad y : 0 \rightarrow 1, \quad z = iy, \quad dz = i dy,$
 $y = 1 : \quad x : 0 \rightarrow 1, \quad z = x + i, \quad dz = dx,$

$$\int_C (x - y + ix^2) dz = \int_0^1 (-iy) dy + \int_0^1 (x - 1 + ix^2) dx = -i \frac{y^2}{2} \Big|_0^1 + \frac{x^2}{2} \Big|_0^1 - x \Big|_0^1 +$$

$$-\frac{1}{2} - \frac{i}{6}$$



3. (a) $z_1 = -2, \quad z_2 = 2,$
 $z = 2e^{i\varphi}, \quad \varphi : \pi \rightarrow 2\pi, \quad dz = 2ie^{i\varphi} d\varphi, \quad |z| = 2,$

$$\int_L z|z| dz = \int_{\pi}^{2\pi} 2 \cdot 2e^{i\varphi} \cdot 2ie^{i\varphi} d\varphi = 8i \int_{\pi}^{2\pi} e^{2i\varphi} d\varphi = -\frac{i}{2} \cdot 8i \int_{\pi}^{2\pi} e^{2i\varphi} d(2i\varphi) =$$

$$= 4e^{2i\varphi} \Big|_{\pi}^{2\pi} = 4(e^{4\pi i} - e^{2\pi i}) = 4e^{2\pi i}(e^{2\pi i} - 1) = 4 \cdot (\cos 2\pi + i \sin 2\pi) \cdot (\cos 2\pi + i \sin 2\pi - 1)$$

$$= 4 \cdot (1 + i \cdot 0) \cdot (1 + i \cdot 0 - 1) = 4 \cdot 1 \cdot 0 = 0$$
- (c) $z_1 = 0, \quad z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2},$
 $y = \sqrt{3}x, \quad x : 0 \rightarrow \frac{1}{2}, \quad z = x + i\sqrt{3}x, \quad dz = (1 + i\sqrt{3}) dx,$

$$\int_L e^{|z|^2} \Re z dz = \int_0^{\frac{1}{2}} x(1 + i\sqrt{3})e^{\sqrt{x^2 + 3x^2}} dx = (1 + i\sqrt{3}) \int_0^{\frac{1}{2}} e^{4x^2} dx = \frac{(1 + i\sqrt{3})}{2} \cdot$$

$$\int_0^{\frac{1}{2}} \frac{1}{x} e^{x^2} d(4x^2) = \frac{(1 + i\sqrt{3})}{2} \cdot \frac{e^{4x^2}}{4} \Big|_0^{\frac{1}{2}} = \frac{(1 + i\sqrt{3})}{8} (e - 1)$$
- (d) $|z - 1| = 1, \quad z = e^{i\varphi}, \quad \varphi : 0 \rightarrow 2\pi, \quad dz = ie^{i\varphi} d\varphi,$
 $\Re(z) = \Re(e^{i\varphi}) = \Re(\cos \varphi + i \sin \varphi) = \cos \varphi$

$$\oint_{|z-1|=1} \Re(z) dz = \int_0^{2\pi} \cos \varphi \cdot ie^{i\varphi} d\varphi = -i \cdot i \int_0^{2\pi} e^{i\varphi} \operatorname{ch} i\varphi d(i\varphi) = \int_0^{2\pi} \frac{1}{4} \cdot 2e^{2i\varphi} \cdot$$

$$e^{-2i\varphi} (e^{2i\varphi} + 1) d(i\varphi) = \left\langle t = e^{2i\varphi}, \quad d(i\varphi) = \frac{1}{2} e^{-2i\varphi} dt \right\rangle = \frac{1}{4} \int_0^{2\pi} \frac{t+1}{t} dt =$$

$$= \frac{1}{4} \int_0^{2\pi} \left(\frac{1}{t} + 1 \right) dt = \frac{1}{4} \ln t + \frac{1}{4} t \Big|_0^{2\pi} = \frac{1}{2} i\varphi + \frac{1}{4} e^{2i\varphi} \Big|_0^{2\pi} = i\pi + \frac{1}{4} e^{4i\pi} - 0 - \frac{1}{4} e^0 =$$

$$= i\pi - \frac{1}{4} + \frac{1}{4} (\cos 4\pi + i \sin 4\pi) = i\pi - \frac{1}{4} + \frac{1}{4} = i\pi$$

