Домашня контрольна робота

Варіант №12002

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Part I

ФІ-12

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1. $\left(L_1^R L_2^R\right)^+ = \left(\left(L_1^R\right)^+ \cup \left(L_2^R\right)^*\right)^+$

$$\begin{aligned}
L_1^R &= \left\{x \in \Sigma^* \mid x^R \in L_1\right\}\\
L_2^R &= \left\{y \in \Sigma^* \mid y^R \in L_1\right\}\\
L_1^R L_2^R &= \left\{xy \in \Sigma^* \mid x^R \in L_1,\ y^R \in L_2^R\right\}\\
\left(L_1^R L_2^R\right)^+ &= \left\{xy \in \Sigma^* \mid \exists n \in \mathbb{N}_1,\ x^R \in L_1^n,\ y^R \in L_2^n\right\}
\end{aligned}$$

$$\begin{aligned}
L_1^R &= \left\{x \in \Sigma^* \mid x^R \in L_1\right\}\\
L_2^R &= \left\{y \in \Sigma^* \mid y^R \in L_1\right\}\\
\left(L_1^R\right)^+ &= \left\{x \in \Sigma^* \mid \exists n \in \mathbb{N}_1,\ x^R \in L_1^n\right\}\\
\left(L_2^R\right)^* &= \left\{x \in \Sigma^* \mid \exists n \in \mathbb{N}_0,\ x^R \in L_2^n\right\}\\
\left(L_1^R\right)^+ \cup \left(L_2^R\right)^* &= \{z \in \Sigma^* \mid \exists n \in \mathbb{N}_1,\ z^R \in L_1^n \vee z^R \in L_2^n\}\\
\left(\left(L_1^R\right)^+ \cup \left(L_2^R\right)^*\right)^+ &= \{z \in \Sigma^* \mid \exists n \in \mathbb{N}_1,\ \ z^R \in L_1^n \vee z^R \in L_2^n\}
\end{aligned}$$

$\Rightarrow \left(L_1^R L_2^R\right)^+ \neq \left(\left(L_1^R\right)^+ \cup \left(L_2^R\right)^*\right)^+$
Let $\Sigma = \{\alpha_1,\ \alpha_2,\ \beta_1,\ \beta_2\},\ L_1 = \{\alpha_1,\ \alpha_1\alpha_2\},\ L_2 = \{\beta_1,\ \beta_1\beta_2\}$.

$$\begin{aligned}
L_1^R &= \{\alpha_1,\ \alpha_2\alpha_1\}\\
L_2^R &= \{\beta_1,\ \beta_2\beta_1\}\\
L_1^R L_2^R &= \{\alpha_1\beta_1,\ \alpha_1\beta_2\beta_1,\ \alpha_2\alpha_1\beta_1,\ \alpha_2\alpha_2\beta_2\beta_2\}\\
\left(L_1^R L_2^R\right)^+ &= \{\alpha_1\beta_1\alpha_1\beta_1,\ \alpha_1\beta_2\beta_1\alpha_1\beta_2\beta_1,\ \alpha_2\alpha_1\beta_1\alpha_2\alpha_1\beta_1 \ldots\}
\end{aligned}$$

$$\begin{aligned}
L_1^R &= \{\alpha_1,\ \alpha_2\alpha_1\}\\
L_2^R &= \{\beta_1,\ \beta_2\beta_1\}\\
\left(L_1^R\right)^+ &= \{\alpha_1,\ \alpha_2\alpha_1,\ \alpha_1\alpha_1,\ \alpha_2\alpha_1\alpha_2\alpha_1 \ldots\}\\
\left(L_2^R\right)^* &= \{\varepsilon,\ \beta_1,\ \beta_2\beta_1,\ \beta_1\beta_1,\ \beta_2\beta_1\beta_2\beta_1 \ldots\}\\
\left(L_1^R\right)^+ \cup \left(L_2^R\right)^* &= \{\varepsilon,\ \alpha_1,\ \alpha_2\alpha_1,\ \alpha_1\alpha_1,\ \alpha_2\alpha_1\alpha_2\alpha_1,\ \beta_1,\ \beta_2\beta_1,\ \beta_1\beta_1,\ \beta_2\beta_1\beta_2\beta_1 \ldots\}\\
\left(\left(L_1^R\right)^+ \cup \left(L_2^R\right)^*\right)^+ &= \{\alpha_1,\ \alpha_2\alpha_1,\ \alpha_1\alpha_1, \alpha_1\alpha_1\alpha_1\ \ \alpha_2\alpha_1\alpha_2\alpha_1,\ \beta_1,\ \beta_2\beta_1,\ \beta_1\beta_1,\ \beta_1\beta_1\beta_1\ \beta_2\beta_1\beta_2\beta_1 \ldots\}
\end{aligned}$$

2. $A = P \wedge R \to \neg(Q \leftrightarrow \neg P \wedge R)$

$$\begin{aligned}
\operatorname{cnf}(A) &= \neg(P \wedge R) \vee \neg(Q \leftrightarrow \neg P \wedge R) =\\
&= \neg(P \wedge R) \vee \neg((Q \vee \neg(\neg P \wedge R)) \wedge (\neg Q \vee (\neg P \wedge R))) =\\
&= \neg P \vee \neg R \vee ((\neg Q \wedge \neg P \wedge R) \vee (Q \wedge (P \vee \neg R))) =\\
&= (\neg P \vee \neg R \vee Q) \wedge (\neg P \vee \neg R \vee P \vee \neg R) \vee (\neg Q \wedge \neg P \wedge R) =\\
&= (\neg P \vee \neg R \vee Q) \vee (\neg Q \wedge \neg P \wedge R) =\\
&= ((\neg P \vee \neg R \vee Q) \vee \neg Q) \wedge ((\neg P \vee \neg R \vee Q) \vee (\neg P \vee R)) =\\
&= \neg P \vee \neg R \vee Q
\end{aligned}$$

Tseitin:

$$\mathcal{T}(a)\backslash\mathcal{T}_0(A) = \{(\neg P), (P \wedge R), (\neg P \wedge R), (Q \leftrightarrow \neg P \wedge R), \\ (\neg(Q \leftrightarrow \neg P \wedge R)), (P \wedge R \rightarrow \neg(Q \leftrightarrow \neg P \wedge R))\}$$

$$\begin{aligned}
\text{substitute } \neg P \text{ with } S_1 & \\
P \wedge R \text{ with } S_2 & \\
& \tilde{\varphi_T}(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \\
& A_1 = S_2 \rightarrow \neg(Q \leftrightarrow S_1 \wedge R) \\
\text{substitute } S_1 \wedge R \text{ with } S_3 & \\
& \tilde{\varphi_T}(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \\
& A_2 = S_2 \rightarrow \neg(Q \leftrightarrow S_3) \\
\text{substitute } Q \leftrightarrow S_3 \text{ with } S_4 & \\
& \tilde{\varphi_T}(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\
& \quad \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3)) \\
& A_3 = S_2 \rightarrow \neg(S_4) \\
\text{substitute } \neg S_4 \text{ with } S_5 & \\
& \tilde{\varphi_T}(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\
& \quad \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3)) \wedge (S_5 \leftrightarrow \neg S_4) \\
& A_4 = S_2 \rightarrow S_5 \\
\text{substitute } S_2 \rightarrow S_5 \text{ with } S_6 & \\
& \tilde{\varphi_T}(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\
& \quad \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3)) \wedge (S_5 \leftrightarrow \neg S_4) \wedge (S_6 \leftrightarrow (S_2 \rightarrow S_5)) \wedge S_6 \\
& A_5 = S_6
\end{aligned}$$

$$\begin{aligned}
\text{cnf}(\tilde{\varphi_T}(A)) = & (P \vee S_1) \wedge (\neg S_1 \vee \neg P) \wedge (\neg P \vee \neg R \vee S_2) \wedge \\
& \wedge (\neg S_2 \vee P) \wedge (\neg S_2 \vee R) \wedge (\neg S_1 \vee \neg R \vee S_3) \wedge (\neg S_3 \vee S_1) \wedge \\
& \wedge (\neg S_3 \vee R) \wedge (\neg Q \vee \neg S_3 \vee S_4) \wedge (Q \vee S_3 \vee S_4) \wedge (\neg S_3 \vee Q \vee \neg S_4) \wedge \\
& \wedge (\neg Q \vee S_3 \vee \neg S_4) \wedge (S_4 \vee S_5) \wedge (\neg S_5 \vee \neg S_4) \wedge (\neg S_2 \vee S5_) \wedge S_6
\end{aligned}$$

$$\text{Rank} = |\{P, Q, R, S_1, S_2, S_3, S_4, S_5, S_6\}| = 9, \text{ Complexity} = \sum_{\wedge \in \text{cnf}(\tilde{\varphi_T}(A))} 1 = 14$$

3. $W = \{P_2 \vee P_4 \vee \neg P_5, P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_2 \vee \neg P_5 \vee \neg P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$

***DPLL*:**

MULT $(\neg P_2 \vee \neg P_5 \vee \neg P_2)$
$W_1 = \{P_2 \vee P_4 \vee \neg P_5, P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_5 \vee P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$

SUS $(\neg P_5 \vee P_2)$
$W_1 = \{P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_5 \vee P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$

SPLIT (P_1)
$W_{31} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$
$W_{32} = \{\neg P_3 \vee P_5, \neg P_4 \vee P_5, \neg P_5 \vee P_2, P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$

UNIT $(P_3 \in W_{32})$
$W_{41} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$
$W_{42} = \{P_5, \neg P_4 \vee P_5, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$

UNIT $(P_5 \in W_{42})$
$W_{41} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$
$W_{42} = \{\neg P_4, P_2, P_2 \vee \neg P_4, \neg P_2\}$

UNIT $(P_2 \in W_{42})$
$W_{51} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$
$W_{52} = \{\neg P_4, \neg P_4\}$

SAME $(\neg P_4 \in W_{52})$
$W_{61} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$
$W_{62} = \{\neg P_4\}$

UNIT $(\neg P_4 \in W_{62})$
$W_{71} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$
$W_{72} = \emptyset$

PURE $(P_3 \in W_{71})$
$W_{81} = \{\neg P_2 \vee \neg P_4, \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$
$W_{72} = \emptyset$

UNIT $(\neg P_4 \in W_{81})$
$W_{91} = \{\neg P_2, \neg P_5 \vee P_2, P_2, \neg P_2 \vee P_5\}$
$W_{72} = \emptyset$

UNIT $(P_2 \in W_{91})$
$W_{101} = \{\neg P_5, P_5\}$
$W_{72} = \emptyset$

UNIT $(P_5 \in W_{101})$
$W_{111} = \emptyset$
$W_{72} = \emptyset$

W - unsatisfiable

Resolution:

(1) $P_2 \vee P_4 \vee \neg P_5$
(2) $P_1 \vee \neg P_2 \vee \neg P_4$
(3) $\neg P_1 \vee \neg P_3 \vee P_5$
(4) $P_1 \vee P_3 \vee \neg P_4$
(5) $\neg P_1 \vee \neg P_4 \vee P_5$
(6) $\neg P_2 \vee \neg P_5 \vee \neg P_2$
(7) $\neg P_1 \vee P_3$
(8) $P_2 \vee \neg P_4$
(9) $\neg P_2 \vee P_5$

(a)

$P_2 \vee \neg P_5 \qquad (1, 8, P_4)$

(b)

$P_3 \vee P_4 \qquad (4, 7, P_1)$

(c)

$P_2 \vee P_3 \qquad (8, b, P_4)$

(d)

$\neg P_2 \qquad (6, 9, P_5)$

(e)

$$\neg P_5 \qquad (a, d, P_2)$$

(f)

$$P_3 \qquad (c, d, P_3)$$

(g)

$$\neg P_4 \qquad (8, d, P_2)$$

(h)

$$\neg P_1 \vee P_5 \qquad (3, f, P_3)$$

(i)

$$\neg P_1 \qquad (e, h, P_5)$$

Resolution:

(1) $P_2 \vee P_4 \vee \neg P_5$
(2) $P_1 \vee \neg P_2 \vee \neg P_4$
(3) $\neg P_1 \vee \neg P_3 \vee P_5$
(4) $P_1 \vee P_3 \vee \neg P_4$
(5) $\neg P_1 \vee \neg P_4 \vee P_5$
(6) $\neg P_2 \vee \neg P_5 \vee \neg P_2$
(7) $\neg P_1 \vee P_3$
(8) $P_2 \vee \neg P_4$
(9) $\neg P_2 \vee P_5$

(a)

$$\neg P_4 \vee \neg P_2 \qquad (8, 9, P_2)$$

(b)

$$P_1 \vee \neg P_2 \vee P_3 \qquad (4, a, P_4)$$

(c)

$$P_1 \vee P_4 \vee P_3 \vee \neg P_5 \qquad (1, b, P_2)$$

(d)

$$P_4 \vee P_3 \vee \neg P_5 \qquad (7, c, P_1)$$

(e)

$$\neg P_2 \vee P_3 \vee P_4 \qquad (9, d, P_5)$$

(f)

$$\neg P_1 \vee \neg P_2 \vee P_3 \vee P_5 \qquad (5, e, P_4)$$

(g)

$$\neg P_2 \vee P_3 \vee \neg P_4 \vee P_5 \qquad (2, e, P_1)$$

(h)

$$\neg P_1 \vee \neg P_2 \vee \neg P_4 \vee P_5 \qquad (3, h, P_3)$$

(i)

$$\neg P_1 \vee \neg P_4 \vee P_5 \qquad (9, h, P_2)$$

(j)

$$\neg P_1 \vee \neg P_2 \vee \neg P_4 \qquad (6, i, P_5)$$

(k)

$$\neg P_1 \vee \neg P_4 \qquad (8, j, P_2)$$

(l)

$$\neg P_1 \vee P_2 \vee \neg P_5 \qquad (1, k, P_4)$$

(m)

$$\neg P_1 \vee P_2 \vee \neg P_4 \qquad (5, j, P_5)$$

(n)

$$P_2 \vee P_3 \vee \neg P_4 \qquad (4, m, P_1)$$

(o)

$$\neg P_1 \vee \neg P_4 \vee P_5 \qquad (3, o, P_3)$$

(p)

$$\neg P_1 \vee \neg P_2 \vee \neg P_4 \qquad (6, p, P_5)$$

(q)

$$\neg P_2 \vee P_3 \vee \neg P_4 \qquad (4, p, P_1)$$

(r)

$$P_3 \vee \neg P_4 \qquad (8, q, P_2)$$

(s)

$$\neg P_1 \vee \neg P_4 \vee P_5 \qquad (3, s, P_3)$$

(t)

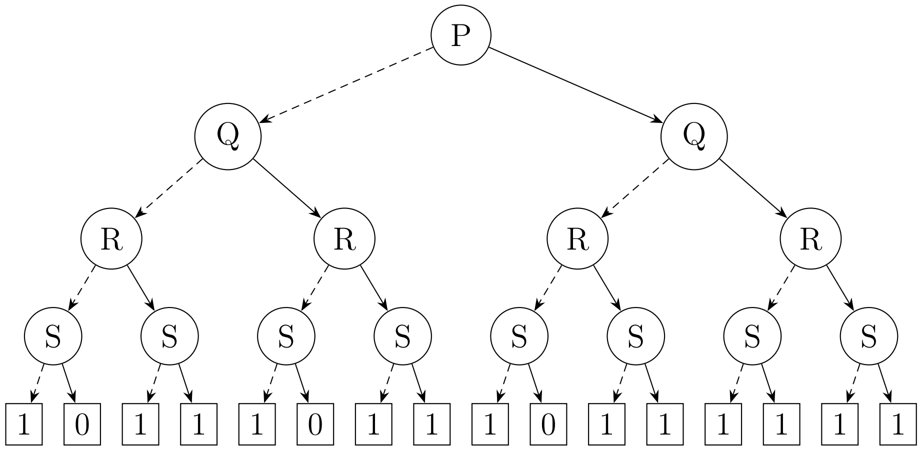
$$\neg P_1 \vee \neg P_2 \vee \neg P_4 \qquad (6, t, P_5)$$

4. $A = ((P \to \neg R) \leftrightarrow (Q \wedge \neg P \wedge R)) \to S. \qquad P < Q < R < S$

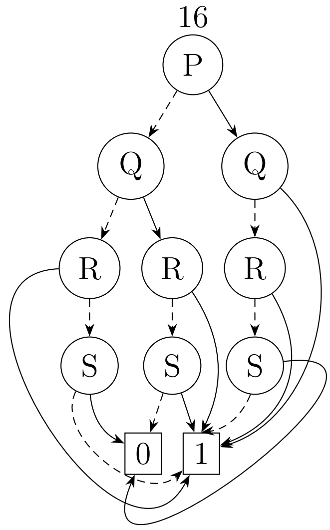
Shannon:

$$
\begin{aligned}
A[1/P] &= ((1 \to \neg R) \leftrightarrow (Q \wedge \neg 1 \wedge R)) \to S = ((1 \to \neg R) \leftrightarrow 0) \to S \\
A[0/P] &= ((0 \to \neg R) \leftrightarrow (Q \wedge \neg 0 \wedge R)) \to S = (1 \leftrightarrow (Q \wedge R)) \to S \\
A &= (P \Rightarrow A[1/P], A[0/P]) \\
A_0[1/Q] &= ((1 \to \neg R) \leftrightarrow (1 \wedge \neg 1 \wedge R)) \to S = ((1 \to \neg R) \leftrightarrow 0) \to S \\
A_0[0/Q] &= ((1 \to \neg R) \leftrightarrow (0 \wedge \neg 1 \wedge R)) \to S = ((1 \to \neg R) \leftrightarrow 0) \to S \\
A_0 &= (Q \Rightarrow A[1/Q], A[0/Q]) \\
A_1[1/Q] &= ((0 \to \neg R) \leftrightarrow (1 \wedge \neg 0 \wedge R)) \to S = (1 \leftrightarrow R) \to S \\
A_1[0/Q] &= ((0 \to \neg R) \leftrightarrow (0 \wedge \neg 0 \wedge R)) \to S = ((0 \to \neg R) \leftrightarrow 0) \to S = 1 \\
A_1 &= (Q \Rightarrow A[1/Q], A[0/Q]) \\
A_{00}[1/R] &= ((1 \to \neg 1) \leftrightarrow (1 \wedge \neg 1 \wedge 1)) \to S = S \\
A_{00}[0/R] &= ((1 \to \neg 0) \leftrightarrow (1 \wedge \neg 1 \wedge 0)) \to S = 1 \\
A_{01}[1/R] &= ((1 \to \neg 1) \leftrightarrow (0 \wedge \neg 1 \wedge 1)) \to S = S \\
A_{01}[0/R] &= ((1 \to \neg 0) \leftrightarrow (0 \wedge \neg 1 \wedge 0)) \to S = 1 \\
A_{00} &= (R \Rightarrow A[1/R], A[0/R]) \\
A_{01} &= (R \Rightarrow A[1/R], A[0/R]) \\
A_{10}[1/R] &= ((0 \to \neg 1) \leftrightarrow (1 \wedge \neg 0 \wedge 1)) \to S = S \\
A_{10}[0/R] &= ((0 \to \neg 0) \leftrightarrow (1 \wedge \neg 0 \wedge 0)) \to S = 1 \\
A_{11}[1/R] &= ((0 \to \neg 1) \leftrightarrow (0 \wedge \neg 0 \wedge 1)) \to S = 1 \\
A_{11}[0/R] &= ((0 \to \neg 0) \leftrightarrow (0 \wedge \neg 0 \wedge 0)) \to S = 1 \\
A_{10} &= (R \Rightarrow A[1/R], A[0/R]) \\
A_{11} &= (R \Rightarrow A[1/R], A[0/R]) \\
A_{000}[1/S] &= 1,\ A_{000}[0/S] = 0,\ A_{001}[1/S] = 1,\ A_{001}[0/S] = 1 \\
A_{010}[1/S] &= 1,\ A_{010}[0/S] = 0,\ A_{011}[1/S] = 1,\ A_{011}[0/S] = 1 \\
A_{100}[1/S] &= 1,\ A_{100}[0/S] = 0,\ A_{101}[1/S] = 1,\ A_{101}[0/S] = 1 \\
A_{110}[1/S] &= 1,\ A_{110}[0/S] = 1,\ A_{111}[1/S] = 1,\ A_{111}[0/S] = 1
\end{aligned}
$$

BDT:



ORBDD:



$$\mathcal{T}(A) = \{P, Q, R, S\},\ |\mathcal{T}(\mathcal{A})| = 4$$
$$A : \{\neg P, \neg Q, R, S\}, \{\neg P, Q, R, S\},\ \{P, \neg Q, R, S\},\ \{P, Q, R, S\}$$
$$(P \Rightarrow (Q \Rightarrow 1, (R \Rightarrow 1, (S \Rightarrow 1, 0))), (Q \Rightarrow (R \Rightarrow 1, (S \Rightarrow 1, 0)), (R \Rightarrow 1, (S \Rightarrow 1, 0))))$$

$=(P \Rightarrow (Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0))))$ - ordered

$=(P \Rightarrow (Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0), (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0))),$
$(Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0), (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0))))$ - full unordered

$=(P \Rightarrow (Q \Rightarrow 1, (R \Rightarrow 1, (S \Rightarrow 1, 0))))$ - full ordered

5.