РОЗРАХУНКОВА РОБОТА
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"
ВАРІАНТ №2

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1.

$$\sqrt[4]{\frac{-1+i\sqrt{3}}{2}}$$

$$\sqrt[4]{\frac{-1+i\sqrt{3}}{2}} = \sqrt[4]{\sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2}} \cdot \left(\cos\left(\frac{\arctan(-\sqrt{3})+\pi+2\pi k}{4}\right) + \right.$$

$$\left. + \, i\sin\left(\frac{\arctan(-\sqrt{3})+\pi+2\pi k}{4}\right)\right) = 1 \cdot \left(\cos\left(\frac{\pi}{6}+\frac{1}{2}\pi k\right) + i\sin\left(\frac{\pi}{6}+\frac{1}{2}\pi k\right)\right)$$

$$0 \le k \le 3$$

$$k=0: \quad \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$k=1: \quad \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$k=2: \quad \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$k=3: \quad \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

2.

$$\cos\left(\frac{\pi}{6}+2i\right)$$

$$\cos\left(\frac{\pi}{6}+2i\right) = \cos\frac{\pi}{6}\cos 2i - \sin\frac{\pi}{6}\sin 2i = \frac{\sqrt{3}}{2}\cos 2i - \frac{1}{2}\sin 2i = \frac{\sqrt{3}}{2}\operatorname{ch} 2 - \frac{1}{2}i\operatorname{sh} 2$$

3.

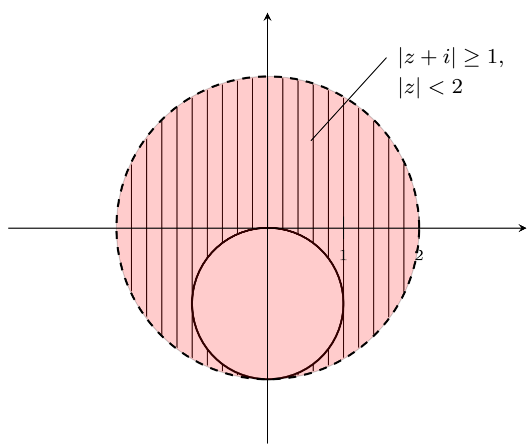
$$\operatorname{Arcsin} 4$$

$$\operatorname{Arcsin} 4 = -i \ \operatorname{Ln}\ \left(4i \pm \sqrt{1-16}\right) = -i \ \operatorname{Ln}\ \left(i\left(4\pm\sqrt{15}\right)\right) = \left\langle \begin{array}{c} z = 4\pm\sqrt{15}\ (x=0) \\ \arg z = \dfrac{\pi}{2} \end{array} \right\rangle =$$

$$= -i\left(\ln\left|4\pm\sqrt{15}\right| + i\left(\frac{\pi}{2}+2\pi k\right)\right) = -i\ln\left(4\pm\sqrt{15}\right) + \frac{\pi}{2} + 2\pi k, \quad k\in\mathbb{Z}$$

4.

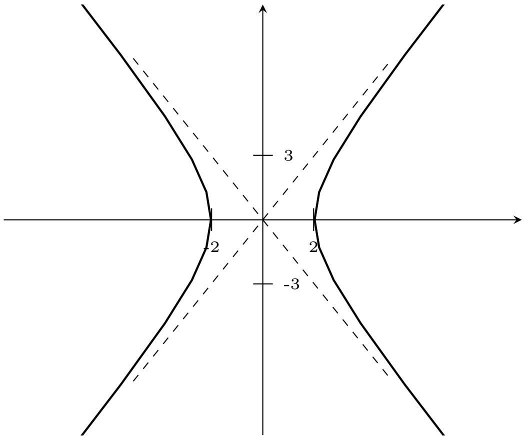
$$|z+i| \ge 1, \quad |z| < 2$$



5.

$$z = 2\sec t - 3i\,\mathrm{tg}\,t$$

$$\begin{cases} \dfrac{x}{2} = \dfrac{2}{\cos t}, \\ \dfrac{y}{3} = -\dfrac{\sin t}{\cos t} \end{cases} \Rightarrow \frac{x^2}{4} - \frac{y^2}{9} = -\frac{\sin^2 t}{\cos^2 t} + \frac{1}{\cos^2 t} = \frac{1-\sin^2 t}{\cos^2 t} = 1$$



6.

$$u = x^3 - 3xy^2 + 1, \quad f(0) = 1$$

$$u'_x = 3(x^2 - y^2), \quad u'_y = -6xy, \quad u'_x = v'_y, v'_x = -u'_y \Longrightarrow v = 3\int (x^2 - y^2)\ \mathrm{d}y =$$
$$= 3\left(x^2 y - \frac{1}{3}y^3\right), \quad v'_x = 6xy + C'(x), \quad v'_x = -u'_y \Longrightarrow 6xy + C'(x) = 6xy, \quad C'(x) = 0,$$
$$C(x) = C \in \mathbb{R}, \quad v = 3x^2 y - y^3 + C$$
$$f(z) = x^3 - 3xy^2 + 1 + 3x^2 yi - y^3 i + Ci, \quad f(0) = 1, \quad f(0) = 0 - 0 + 1 + 0i - 0i + Ci =$$
$$= Ci + 1 = 1 \Longrightarrow C = 0, \quad f(z)x^3 - 3xy^2 + 1 + 3x^2 yi - y^3 i = (x+iy)^3 + 1 = z^3 + 1$$

7.

$$\int\limits_{\mathcal{L}} (z+1)e^z\ \mathrm{d}z\ , \quad \mathcal{L} : \{|z| = 1, \quad \mathfrak{Re}\, z \geq 0\}$$

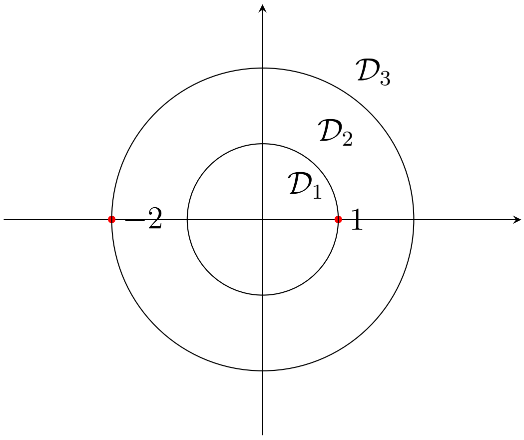
$$z = x + iy, \quad (z+1)e^z = (x + iy + 1)e^{x+iy} = (z + iy + 1)\cdot(\cos(x+iy) + i\sin(x+iy))$$

8.

$$f(z) = \frac{z-4}{z^4 + z^3 - 2z^2}$$

$$\left[\begin{array}{l} z_1 = 0 \\ z_2 = 1 \\ z_3 = -2 \end{array}\right. \left[\begin{array}{l} \mathcal{D}_1 :\ 0 < |z| < 1 \\ \mathcal{D}_2 :\ 1 < |z| < 2 \\ \mathcal{D}_3 :\ |z| > 2 \end{array}\right. f(z) = \frac{z-4}{z^2(z-1)(z+2)} = \frac{1}{z^2}\left(\frac{A}{z-1} + \frac{B}{z+2}\right) =$$
$$= \left\langle A = \left.\frac{z-4}{z+2}\right|_{z=1} = -\frac{3}{3} = -1, \quad B = \left.\frac{z-4}{z-1}\right|_{z=-2} = \frac{-6}{-3} = 2\right\rangle = \frac{1}{z^2}\left(\frac{2}{z+2} - \frac{1}{z-1}\right)$$
$$\frac{1}{z-1} = -\frac{1}{1-z} = -\sum_{n=0}^{\infty} z^n \in \mathcal{D}_1$$
$$\frac{1}{z-1} = \frac{1}{z}\cdot\frac{1}{1-\frac{1}{z}} = \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n = \sum_{n=0}^{\infty}\frac{1}{z^{n+1}} \in \mathcal{D}_2, \mathcal{D}_3$$
$$\frac{2}{z+2} = \frac{1}{1-\left(-\frac{z}{2}\right)} = \sum_{n=0}^{\infty}\left(-\frac{z}{2}\right)^n = \sum_{n=0}^{\infty}(-1)^n\frac{z^n}{2^n} \in \mathcal{D}_1, \mathcal{D}_2$$
$$\frac{2}{z+2} = \frac{2}{z}\cdot\frac{1}{1-\left(-\frac{2}{z}\right)} = \frac{2}{z}\sum_{n=0}^{\infty}\left(-\frac{2}{z}\right)^n = \sum_{n=0}^{\infty}(-1)^n\frac{2^{n+1}}{z^{n+1}} \in \mathcal{D}_3$$

$$\mathcal{D}_1: \quad f(z) = \frac{1}{z^2}\left(\sum_{n=0}^{\infty}(-1)^n\frac{z^n}{2^n} - \left(-\sum_{n=0}^{\infty} z^n\right)\right) = \sum_{n=0}^{\infty} z^{n-2}\left(\frac{(-1)^n}{2^n}+1\right)$$
$$\mathcal{D}_2: \quad f(z) = \frac{1}{z^2}\left(\sum_{n=0}^{\infty}(-1)^n\frac{z^n}{2^n} - \sum_{n=0}^{\infty}\frac{1}{z^{n+1}}\right) = \sum_{n=0}^{\infty}(-1)^n\frac{z^{n-2}}{2^n} - \sum_{n=0}^{\infty}\frac{1}{z^{n+3}}$$
$$\mathcal{D}_3: \quad f(z) = \frac{1}{z^2}\left(\sum_{n=0}^{\infty}(-1)^n\frac{2^{n+1}}{z^{n+1}} - \sum_{n=0}^{\infty}\frac{1}{z^{n+1}}\right) = \sum_{n=0}^{\infty}\frac{((-1)^n 2^{n+1}-1)}{z^{n+3}}$$



10.

$$f(z) = \sin\left(\frac{z}{z-1}\right), \qquad z_0 = 1$$