Домашня контрольна робота

Варіант №12002

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Part II

ФІ-12

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6. $(\forall x)(\neg Q(x) \land \neg P(x)) \lor (\forall x)(S(x) \to Q(x)) \models (\forall x)(S(x) \to \neg P(x))$

$$
\begin{aligned}
&(\forall x)(\neg Q(x) \land \neg P(x)) \lor (\forall x)(\neg S(x) \lor Q(x) \land \neg((\forall x)(\neg S(x) \lor \neg P(x))) \\
&(\forall x_1)(\neg Q(x_1) \land \neg P(x_1)) \lor (\forall x_2)(\neg S(x_2) \lor Q(x_2)) \land (\exists x_3)(S(x_3) \land P(x_3)) \\
&(\forall x_1)(\neg Q(x_1) \land \neg P(x_1)) \lor (\forall x_2)(\neg S(x_2) \lor Q(x_2)) \land (S(f(x_3)) \land P(f(x_3))) \\
&(\neg Q(x_1) \land \neg P(x_1)) \lor (\neg S(x_2) \lor Q(x_2)) \land (S(f(x_3)) \land P(f(x_3))) \\
&(\neg P(x_1) \lor P(f(x_3)) \lor S(f(x_3))) \land (\neg P(x_1) \lor Q(x_2) \lor \neg S(x_2)) \land \\
&\land (P(f(x_3)) \lor \neg Q(x_1) \lor S(f(x_3))) \land (\neg Q(x_1) \lor Q(x_2) \lor \neg S(x_2))
\end{aligned}
$$

(1) $\neg P(x_1) \lor P(f(x_3)) \lor S(f(x_3))$

(2) $\neg P(x_1) \lor Q(x_2) \lor \neg S(x_2)$

(3) $P(f(x_3)) \lor \neg Q(x_1) \lor S(f(x_3))$

(4) $\neg Q(x_1) \lor Q(x_2) \lor \neg S(x_2)$

(a)

$$S(f(x_3)) \qquad (1, \{f(x_3)/x_1\})$$

(b)

$$\neg S(x_2) \qquad (4, \{x_2/x_1\})$$

(c)

$$\# \qquad (a, b, \{f(x_3)/x_2\})$$

7. $(\forall x)\underbrace{(\neg Q(x) \land \neg P(x))}_{A} \lor (\forall x)\underbrace{(S(x) \to Q(x))}_{B}$

$Q(d_1) = \mathbb{F}, P(d_1) = \mathbb{F}, S(d_2) = \mathbb{T}, Q(d_2) = \mathbb{T} \Rightarrow A \lor B = \mathbb{T}$

$$
\begin{aligned}
A \lor B = \mathbb{T}: &\qquad Q(d_1) = \mathbb{F}, P(d_1) = \mathbb{F}, S(d_2) = \mathbb{T}, Q(d_2) = \mathbb{T} \\
A \lor B = \mathbb{F}: &\qquad Q(d_1) = \mathbb{T}, P(d_1) = \mathbb{T}, S(d_2) = \mathbb{T}, Q(d_2) = \mathbb{F}
\end{aligned}
$$

$\Rightarrow$ premise is neither a tautology, nor a contradiction

8. $\Sigma = \{8, d\}, w_1 = d8d8dd, w_2 = dd8dd8, S: \begin{cases} dd8 \to 8d \\ dd \to \cdot\varepsilon \\ 8 \to d8 \\ 8 \to dd \end{cases}$

$$
\begin{aligned}
w_1: &\qquad d8d8\underline{dd} \\
w_2: &\qquad \underline{dd8}dd8 \vdash 8d\underline{dd8} \vdash \underline{8}d8d \vdash d\underline{8}d8d \vdash \underline{dd}8d8d
\end{aligned}
$$

$$S: \begin{cases} dd8 \to 8d \\ 8 \to d8 \\ dd \to \cdot\varepsilon \\ 8 \to dd \end{cases}$$

$$
\begin{aligned}
w_1: &\qquad d\underline{8}d8dd \vdash \underline{dd8}d8dd \vdash 8\underline{dd8}dd \vdash \underline{8}8ddd \vdash d\underline{8}8ddd \vdash \underline{dd8}8ddd \vdash \\
&\qquad \vdash \underline{8}d8ddd \vdash d\underline{8}d8ddd \vdash \underline{dd8}d8ddd \vdash 8\underline{dd8}ddd \vdash 88dddd \qquad \Rightarrow \text{loop} \\
w_2: &\qquad \underline{dd8}dd8 \vdash 8d\underline{dd8} \vdash \underline{8}d8d \vdash d\underline{8}d8d \vdash \underline{dd8}d8d \vdash 8\underline{dd8}d \vdash \underline{8}8dd \vdash \\
&\qquad \vdash d\underline{8}8dd \vdash \underline{dd8}8dd \vdash \underline{8}d8dd \vdash d\underline{8}d8dd \vdash \underline{dd8}d8dd \vdash 8\underline{dd8}dd \qquad \Rightarrow \text{loop}
\end{aligned}
$$

$$S: \begin{cases} dd8 \to 8d \\ dd \to \varepsilon \\ 8 \to d8 \\ 8 \to dd \end{cases}$$

$$\begin{aligned}
w_1: \quad & d8d8\underline{dd} \vdash d\underline{8}d8 \vdash \underline{dd8}d8 \vdash 8\underline{dd}8 \vdash \underline{8}8 \vdash d\underline{8}8 \vdash \underline{dd8}8 \vdash \\
& \vdash \underline{8}d8 \vdash d\underline{8}d8 && \Rightarrow \text{loop} \\
w_2: \quad & \underline{dd8}dd8 \vdash 8d\underline{dd8} \vdash \underline{8}d8d \vdash d\underline{8}d8d \vdash \underline{dd8}d8d \vdash 8\underline{dd8}d \vdash \\
& \vdash 88\underline{dd} \vdash \underline{8}8 \vdash d\underline{8}8 \vdash \underline{dd8}8 \vdash \underline{8}d8 \vdash d\underline{8}d8 \vdash \underline{dd8}d8 \vdash \\
& \vdash 8\underline{dd8} \vdash \underline{8}8d \vdash d\underline{8}8d && \Rightarrow \text{loop}
\end{aligned}$$

$w = 88$

$$\begin{aligned}
w: \quad & \underline{8}8 \vdash d\underline{8}8 \vdash \underline{dd8}8 \vdash \underline{8}d8 \vdash d\underline{8}d8 \vdash \underline{dd8}d8 \vdash 8\underline{dd8} \vdash \\
& \vdash \underline{8}8d \vdash d\underline{8}8d \vdash \underline{dd8}8d \vdash \underline{8}d8d \vdash d\underline{8}d8d \vdash \underline{dd8}d8d && \Rightarrow \text{loop}
\end{aligned}$$

9. $w_1 = 11100, w_2 = 00111, C_1 = 1q01011, C_2 = 11q110\#11, \Pi = \{q_0 0 q_1 1 R, q_0 1 q_2 0 R, q_0 \# q_1 0 S, q_1 0 q_0 1 L, q_1 1 q_2 1 S, q_2 0 q_0 1 R, q_2 1 q_4 1 S, q_3 0 q_0 1 S, q_4 0 q_1 0 S, q_4 1 q_0 \# R\}$

$$\begin{aligned}
w_1: \quad & q_0 11100 \vdash 0q_2 1100 \vdash 0q_4 1100 \vdash 0\#q_0 100 \vdash 0\#0q_2 00 \vdash \\
& \vdash 0\#01q_0 0 \vdash 0\#011q_1\# \\
w_2: \quad & q_0 00111 \vdash 1q_1 0111 \vdash\vdash q_0 11111 \vdash 0q_2 1111 \vdash 0q_4 1111 \vdash \\
& \vdash 0\#q_0 111 \vdash 0\#0q_2 11 \vdash 0\#0q_4 11 \vdash 0\#0\#q_0 1 \vdash 0\#0\#0q_2\# \\
C_1: \quad & 1q_0 1011 \vdash 10q_2 011 \vdash 101q_0 11 \vdash 1010q_2 1 \vdash 1010q_4 1 \vdash \\
& \vdash 1010\#q_0\# \vdash 1010\#q_1 0 \vdash 1010q_0\#1 \vdash 1010q_1 01 \vdash \\
& \vdash 101q_0 011 \vdash 1011q_1 11 \vdash 1011q_2 11 \vdash 1011q_4 11 \vdash 101\#q_0 1 \vdash \\
& \vdash 1011\#0q_2\# \\
C_2: \quad & 11q_1 10\#11 \vdash 11q_2 10\#11 \vdash 11q_4 10\#11 \vdash 11\#q_0 10\#11 \vdash \\
& \vdash 11\#0q_2 0\# \vdash \underline{11\#01q_0\#11} \vdash 11\#01q_1 011 \vdash 11\#0q_1 1111 \vdash \\
& \vdash 11\#0q_2 1111 \vdash 11\#0q_4 1111 \vdash \underline{11\#01q_0\#11} && \Rightarrow \text{loop} \\
& C_1 \models C_F, C_2 \not\models C_F \\
w = 1: \quad & q_0 1 \vdash 0q_2\#
\end{aligned}$$

10. $\mathcal{M} = (\{\#, 0, x, y, z\}, \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_{\text{exit}}, q_{\text{end}}, q_{\text{blank}}, q_{\text{check}}, q_{\text{backc}}, q_{\text{getback}}, q_{\text{gbagain}}, q_{\text{addzero}}, q_{\text{fx}}, q_{\text{fy}}, q_{\text{fz}}, q_{\text{x}}, q_{\text{y}}, q_{\text{z}}, q_{\text{delx}}\}, \Pi)$

Algorithm(steps):

Input: $\omega \in \{x, y, z\}^*$

(1) Find and mark two z entries, or else end

(2) Compare amount of z and y entries:

 (i) Find and mark y entry, then z entry, or else end

 (ii) Find and mark z entry, then y entry, or else end

(3) Get back to initial word(clear marking)

(4) Find first x entry, or else end

(5) Find second x entry, or else end

(6) Delete second x entry:

(i) Mark(set) second x entry as 0

(ii) Move each letter(left of 0) one cell right

Output: $\omega' \in \{x, y, z\}^*$

$$
\begin{aligned}
\Pi = \{&q_0xq_0xR, q_0yq_0yR, q_0zq_1ZR, q_1xq_1xR, q_1yq_1yR, q_1zq_{\text{check}}ZL, q_1\#q_{\text{exit}}\#L, q_{\text{exit}}xq_{\text{exit}}xL,\\
&q_{\text{exit}}yq_{\text{exit}}yL, q_{\text{exit}}zq_{\text{exit}}zL, q_{\text{exit}}Zq_{\text{exit}}zL, q_{\text{exit}}Yq_{\text{exit}}yL, q_{\text{exit}}\#q_{\text{end}}\#S, q_{\text{b}}xq_{\text{b}}x, q_{\text{b}}yq_{\text{b}}yL,\\
&q_{\text{b}}zq_{\text{b}}z, q_{\text{b}}Xq_{\text{b}}X, q_{\text{b}}Yq_{\text{b}}Y, q_{\text{b}}Zq_{\text{b}}ZL, q_{\text{b}}\#q_{\text{blank}}\#R, q_{\text{blank}}\#q_{\text{blank}}\#L, q_{\text{blank}}xq_2xR,\\
&q_{\text{blank}}yq_2yR, q_{\text{blank}}Yq_2YR, q_{\text{blank}}Zq_2ZR, q_{\text{blank}}zq_2zR, q_2\#q_{\text{check}}\#L, q_2xq_2xR, q_2Zq_2ZR,\\
&q_2Yq_2YR, q_2yq_{\text{fz}}YR, q_2zq_{\text{fy}}ZR, q_{\text{fz}}xq_{\text{fz}}xR, q_{\text{fz}}yq_{\text{fz}}yR, q_{\text{fz}}Yq_{\text{fz}}YR, q_{\text{fz}}Zq_{\text{fz}}ZR, q_{\text{fz}}zq_{\text{check}}ZR,\\
&q_{\text{fz}}\#q_{\text{exit}}\#L, q_{\text{fy}}xq_{\text{fy}}xR, q_{\text{fy}}zq_{\text{fy}}zR, q_{\text{fy}}Zq_{\text{fy}}ZR, q_{\text{fy}}Yq_{\text{fy}}YR, q_{\text{fy}}yq_{\text{check}}YR, q_{\text{fy}}\#q_{\text{exit}}\#L,\\
&q_{\text{check}}yq_{\text{backc}}yL, q_{\text{check}}zq_{\text{backc}}zL, q_{\text{check}}xq_{\text{check}}xR, q_{\text{check}}Yq_{\text{check}}YR, q_{\text{check}}Zq_{\text{check}}ZR,\\
&q_{\text{check}}\#q_{\text{backc}}\#R, q_{\text{getback}}xq_{\text{getback}}xL, q_{\text{getback}}yq_{\text{getback}}yL, q_{\text{getback}}Yq_{\text{getback}}YL,\\
&q_{\text{getback}}zq_{\text{getback}}zL, q_{\text{getback}}Zq_{\text{getback}}ZL, q_{\text{getback}}\#q_{\text{gbagain}}\#R, q_{\text{gbagain}}q_4\#S,\\
&q_{\text{gbagain}}xq_{\text{check}}xS, q_{\text{gbagain}}yq_{\text{check}}yS, q_{\text{gbagain}}Yq_{\text{check}}YS, q_{\text{gbagain}}zq_{\text{check}}zS,\\
&q_{\text{gbagain}}Zq_{\text{check}}ZS, q_{\text{backc}}xq_{\text{backc}}x, q_{\text{backc}}yq_{\text{backc}}yL, q_{\text{backc}}zq_{\text{backc}}z, q_{\text{backc}}Xq_{\text{backc}}X,\\
&q_{\text{backc}}Yq_{\text{backc}}Y, q_{\text{backc}}Zq_{\text{backc}}ZL, q_{\text{backc}}\#q_{\text{blankc}}\#R, q_{\text{blankc}}\#q_{\text{blankc}}\#Lzq_{\text{blankc}}xq_3xR,\\
&q_{\text{blankc}}yq_3yR, q_{\text{blankc}}Yq_3YR, q_{\text{blankc}}Zq_3ZR, q_{\text{blankc}}zq_3zR, q_3xq_2xS, q_3yq_2yS, q_3zq_2zS,\\
&q_3Yq_2YS, q_3Zq_2ZS, q_3\#q_2\#L, q_4\#q_4\#L, q_4xq_5xS, q_4Yq_5YS, q_4Zq_5ZS, q_5xq_5xL,\\
&q_5Yq_5yL, q_5Zq_5zL, q_5\#q_{\text{fx}}\#R, q_{\text{fx}}yq_{\text{fx}}yR, q_{\text{fx}}zq_{\text{fx}}zR, q_{\text{fx}}xq_{\text{delx}}xR, q_{\text{delx}}yq_{\text{delx}}yR,\\
&q_{\text{delx}}zq_{\text{delx}}zR, q_{\text{delx}}\#q_{\text{end}}\#S, q_{\text{delx}}xq_60L, q_6xq_6xL, q_6yq_6yL, q_6zq_6zL, q_6\#q_7\#S, q_7\#q_7\#R,\\
&q_7xq_{\text{x}}xR, q_7yq_{\text{y}}yR, q_7zq_{\text{z}}zR, q_{\text{x}}0q_{\text{addzero}}xL, q_{\text{x}}yq_{\text{y}}yR, q_{\text{x}}zq_{\text{z}}zR, q_{\text{y}}zq_{\text{z}}zR, q_{\text{z}}yq_{\text{y}}yR,\\
&q_{\text{y}}yq_{\text{y}}yR, q_{\text{x}}xq_{\text{x}}xR, q_{\text{z}}zq_{\text{z}}zR, q_{\text{z}}xq_{\text{x}}xR, q_{\text{y}}0q_{\text{addzero}}yL, q_{\text{z}}0q_{\text{addzero}}zL, q_{\text{addzero}}zq_60L,\\
&q_{\text{addzero}}xq_60L, q_{\text{addzero}}yq_60L, q_{\text{x}}\#q_{\text{end}}\#S, q_{\text{y}}\#q_{\text{end}}\#S, q_{\text{z}}\#q_{\text{end}}\#S, q_70q_{\text{end}}\#S\}
\end{aligned}
$$

11. $f_1(x) = x^2 + 3, f_2(x) = 2x + 2, f_3(x, y, z) = yz + 3(x + z), f_4(x, y) = |2x - 3y + 5| + 2y$

$$
\begin{aligned}
g(x) &= S^1(f_1(x)) = x^2 + 4\\
h(x) &= f_1(S^1(x)) = (x + 1)^2 + 3\\
y(x) &= S^1(f_2(x)) = 2x + 3\\
&f_2, f_3\\
&I_3^3(x, y, z)\\
g^2(0, x) &= 2x + 2\\
g^2(y + 1, x) &= I^3(y, g(y, x), x) = y
\end{aligned}
$$