Розрахункова Робота №6

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1. $$\sum_{n=9}^{\infty}\frac{18}{n^2-13n+40}=\sum_{n=9}^{\infty}\left(-\frac{6}{n-5}+\frac{6}{n-8}\right)=\left(6-\frac{3}{2}\right)+\left(3-\frac{6}{5}\right)+(2-1)+$$
$$+\left(\frac{3}{2}-\frac{6}{7}\right)+\left(\frac{6}{5}-\frac{3}{4}\right)+\left(1-\frac{2}{3}\right)+\cdots=11$$

2. $$\sum_{n=1}^{\infty}\frac{\arctan^n}{n^3}$$
$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{\frac{\arctan^2 n}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\arctan^2 n=\frac{\pi^2}{4}\Rightarrow \text{ряд збіжний}$$

3. $$\sum_{n=1}^{\infty}\frac{1}{n+4}\tan\frac{1}{\sqrt{n}},\ \sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}\ \text{збіжний при } p=\frac{3}{2}\geq 0$$
$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{\frac{1}{n+4}\tan\frac{1}{\sqrt{n}}}{\frac{1}{n^{\frac{3}{2}}}}=\lim_{n\to\infty}\frac{n^{\frac{3}{2}}}{\sqrt{n}(n+4)}=1\neq 0\Rightarrow \text{ряд збіжний}$$

4. $$\sum_{n=1}^{\infty}\frac{4^n}{(n!)^2}$$
$$\lim_{n\to\infty}\frac{\frac{4^{n+1}}{((n+1)!)^2}}{\frac{4^n}{(n!)^2}}=4\lim_{n\to\infty}\left(\frac{n!}{(n+1)1}\right)^2=0\Rightarrow \text{ряд збіжний}$$

5. $$\sum_{n=1}^{\infty}n^4\left(\frac{2n}{3n+5}\right)^n$$
$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}n^{\frac{4}{n}}\frac{2n}{3n+5}=\lim_{n\to\infty}n^{\frac{4}{v}}\frac{2}{3+\frac{5}{n}}=\frac{2}{3}\Rightarrow \text{ряд збіжний}$$

6. $$\sum_{n=1}^{\infty}\frac{1}{n\ln^2(2n+1)}$$
$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{\frac{1}{n\ln^2(2n+1)}}{\frac{1}{(2n+1)\ln^2(2n+1)}}=\lim_{n\to\infty}\frac{2n+1}{4}=2\Rightarrow \text{ряд збіжний}$$

7. $$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{2n+1}\right)^n$$
$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}(-1)^{\frac{n+1}{n}}\frac{n}{2n+1}=\frac{1}{2}\Rightarrow \text{ряд збіжний}$$

8. $$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!},\ \alpha=0.01$$
$$\lim_{n\to\infty}\frac{1}{n!}=0,\ 1>0.01,\frac{1}{2}>0.01,\frac{1}{6}>0.01,\frac{1}{24}>0.01,\frac{1}{120}<0.01$$
$$\Rightarrow\frac{1}{120}-\frac{1}{720}+\frac{1}{5040}-\cdots<\frac{1}{120}<0.01\Rightarrow S\approx 1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24}=\frac{15}{24}\approx 0.62$$

10. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{2n+3}(x+3)^{2n}$

$\lim\limits_{n\to\infty}\left|\dfrac{(x-3)^{n+1}(n+1)5^n}{(n+2)5^{n+1}(x-3)^n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{(x-3)(n+1)}{(n+2)}\right| = \left|\dfrac{n-3}{5}\right| < 1$

$|x-3| < 5 \Rightarrow \left[\begin{array}{l} x < 8 \\ x > -2 \end{array}\right. \Rightarrow x \in (-2, 8)$

$x = -2 : a_n = \dfrac{(-1)^n(-5)^n}{(n+1)5^n} = \dfrac{1}{n+1}$

$\lim\limits_{n\to\infty}\dfrac{a_n}{b_n} = \lim\limits_{n\to\infty}\dfrac{\dfrac{1}{n+1}}{\dfrac{1}{n}} = \lim\limits_{n\to\infty}\dfrac{n}{n+1} = 1 \Rightarrow$ ряд розбіжний

$x = 8 : a_n = \dfrac{(-1)^n}{n+1},\ b_n = \dfrac{1}{n}, \lim\limits_{n\to\infty}\dfrac{a_n}{b_n} \Rightarrow$ ряд розбіжний

Область збіжності: $x \in (-2, 8)$

14. $\dfrac{x^2}{\sqrt{4-5x}} = \dfrac{1}{2}\dfrac{x^2}{\sqrt{1-\frac{5}{4}x}}$

$(1+t)^\alpha = 1 + \alpha t + \dfrac{\alpha^2-\alpha}{2}t^2 + \dfrac{(\alpha^2-\alpha)(\alpha-2)}{6}t^3 + \ldots$

$\alpha = -\dfrac{1}{2} : (1+t)^{-\frac{1}{2}} = \dfrac{1}{\sqrt{1+t}} = 1 - \dfrac{1}{2}t + \dfrac{3}{8}t^2 - \dfrac{15}{48}t^3 + \ldots$

$\dfrac{1}{\sqrt{1-\frac{5}{4}x}} = 1 + \dfrac{5}{8}x + \dfrac{75}{128}x^2 + \dfrac{625}{3072}x^3 + \ldots$

$f(x) = \dfrac{x^2}{2}\left(\dfrac{1}{\sqrt{1-\frac{5}{4}x}}\right) = \dfrac{x^2}{2} + \dfrac{5x^3}{16} + \dfrac{75x^4}{256} + \dfrac{625x^5}{6144} + \ldots$