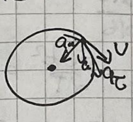
1.12

R, α, υ_0



$$a_\tau = \frac{d\upsilon}{dt}, \quad a_n = \frac{\upsilon^2}{R}, \quad \frac{a_\tau}{a_n} = \frac{R}{\upsilon^2} \cdot \frac{d\upsilon}{dt} = ctg\alpha$$

$$\int_{\upsilon_0}^{\upsilon} \frac{R}{\upsilon^2} d\upsilon = \int_0^t ctg\alpha \, dt \Rightarrow \frac{R}{\upsilon_0} - \frac{R}{\upsilon} = ctg\alpha t$$

$$\Rightarrow \upsilon = \frac{R}{\frac{R}{\upsilon_0} - ctg\alpha t} = \underline{\frac{R\upsilon_0}{R - ctg\alpha t \upsilon_0}}, \quad \underline{a_n = \frac{R\upsilon_0^2}{(R - ctg\alpha t \upsilon_0)^2}}$$

$$a_\tau = \frac{d \left(\frac{R\upsilon_0}{R - ctg\alpha t \upsilon_0} \right)}{dt} = \underline{\frac{R\, ctg\alpha\, \upsilon_0^2}{(R - \upsilon_0 ctg\alpha t)^2}}, \quad a = \sqrt{a_\tau^2 + a_n^2} = \underline{\frac{\upsilon_0 R \sqrt{1 + ctg^2\alpha}}{(R - \upsilon_0 ctg\alpha t)}}$$

1.34

$\omega = \omega_0 (a\varphi - b\varphi_0)$
$t_0 = 0$
$\varphi = \varphi_0$
$\varphi - ?\ \beta - ?$

$$\omega = \frac{d\varphi}{dt}, \quad \omega = \omega_0 (a\varphi - b\varphi_0) \Rightarrow \frac{d\varphi}{dt} = \omega_0 (a\varphi - b\varphi_0)$$

$$\frac{d\varphi}{a\varphi - b\varphi} = \omega_0 dt \Rightarrow \int_{\varphi_0}^{\varphi} \frac{d\varphi}{a\varphi - b\varphi_0} = \int_{t_0=0}^{t} \omega_0 dt$$

$$\int \frac{d\varphi}{a\varphi - b\varphi} = \left| \begin{matrix} a\varphi - b\varphi_0 = u \\ du = a\, d\varphi \Rightarrow d\varphi = \frac{du}{a} \end{matrix} \right| = \frac{1}{a} \int \frac{du}{a\varphi - b\varphi_0} = \frac{1}{a} \int \frac{du}{u} = \frac{1}{a} \ln|u|$$

$$= \frac{\ln|a\varphi - b\varphi_0|}{a} \Rightarrow \int_{\varphi_0}^{\varphi} \frac{d\varphi}{a\varphi - b\varphi_0} = \frac{1}{a} \cdot \ln|a\varphi - b\varphi_0| \Big|_{\varphi_0}^{\varphi = \frac{b}{a}}$$

$$= \frac{1}{a} \left(\ln|a\varphi - b\varphi_0| - \ln|a\varphi_0 - b\varphi_0| \right) = |a > b| =$$

$$= \frac{1}{a} \left(\ln(a\varphi - b\varphi_0) - \ln(a\varphi_0 - b\varphi_0) \right) = \frac{1}{a} \ln\left(\frac{a\varphi - b\varphi_0}{a\varphi_0 - b\varphi_0} \right)$$

$$\int_0^t \omega_0 dt = \omega_0 t \Big|_0^t = \omega_0 t \Rightarrow \frac{1}{a} \ln\left(\frac{a\varphi - b\varphi_0}{a\varphi_0 - b\varphi_0} \right) = \omega_0 t$$

$$\Rightarrow e^{\ln\left(\frac{a\varphi - b\varphi_0}{a\varphi_0 - b\varphi_0} \right)} = e^{a\omega_0 t} \Rightarrow \frac{a\varphi - b\varphi_0}{a\varphi_0 - b\varphi_0} = e^{a\omega_0 t}$$

$$\Rightarrow a\varphi - b\varphi_0 = e^{a\omega_0 t} (a\varphi_0 - b\varphi_0) \Rightarrow a\varphi = \varphi_0 \cdot e^{a\omega_0 t} (a - b) + b\varphi_0$$

$$\Rightarrow \underline{\varphi = \frac{\varphi_0}{a} \left(b(1 - e^{a\omega_0 t}) + a \cdot e^{a\omega_0 t} \right)}$$

$$\omega = \frac{d\varphi}{dt}, \quad \omega = \frac{d}{dt} \left(\frac{\varphi_0}{a} \left(b(1 - e^{a\omega_0 t}) + a e^{a\omega_0 t} \right) \right) =$$

$$= \frac{\varphi_0}{a} \left(b(-e^{a\omega_0 t} a\omega_0) + a e^{a\omega_0 t} a\omega_0 \right) = \frac{\varphi_0}{a} \left(a^2 \omega_0 e^{a\omega_0 t} - \right.$$

$$\left. - ab\omega_0 e^{a\omega_0 t} \right) = \varphi_0 \left(a\omega_0 e^{a\omega_0 t} - b\omega_0 e^{a\omega_0 t} \right) =$$

$$= \underline{\varphi_0 \omega_0 e^{a\omega_0 t} (a - b)}$$

3.8

m, t_0
$F(t) = F_0(1 - \frac{t}{T})$
$x(t) - ?, t_n - ?, S - ?$

$F = m\frac{dv}{dt} \Rightarrow F_0(1 - \frac{t}{T}) = m\frac{dv}{dt}$

$\int F_0(1 - \frac{t}{T})dt = \int m\,dv \Rightarrow F_0 t - \frac{F_0 t^2}{2T} = mv$

$\Rightarrow v = \frac{2F_0 tT - F_0 t^2}{2Tm} = \frac{dS}{dt} \Rightarrow \int_0^T (2F_0 tT - F_0 t^2)dt =$

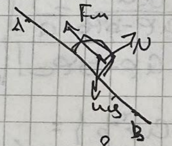
$= \int_0^S 2Tm\,dS \Rightarrow 2F_0 T\frac{T^2}{2} - F_0\frac{T^3}{3} = 2TmS \Rightarrow$

$\Rightarrow S = \frac{3F_0T^3 - F_0T^3}{3 \cdot 2Tm} = \frac{2F_0T^3}{6Tm} = \underline{\frac{F_0T^2}{3m}}$

$\Rightarrow x(t) = \underline{\frac{F_0t^2}{2m} - \frac{F_0t^3}{6Tm}}$

3.25

m, S, α
$\mu = \gamma\sqrt{x}$
L
$v(t) - ?$
$u - ?$



I – зісковзування

$N_y - mg\cos\alpha = 0 \Rightarrow N = N_y - mg\cos\alpha$

$A_{A\to B} = \int_A^B F_{тр}\,dS = \int_0^S \gamma mg\cos\alpha\sqrt{x}\,dx =$

$-\gamma mg\cos\alpha\int_0^S \sqrt{x}\,dx = -\gamma mg\cos\alpha\,\frac{2}{3}\sqrt{x^3}\Big|_0^S -$

$= -\frac{2}{3}\gamma mg\cos\alpha\sqrt{S^3} \Rightarrow E_B = E_A + A_{A\to B} = T_A + U_A + A_{A\to B}$

$\Rightarrow E_B = T_B + U_B = T_0 + A_x + A_{тр} \Rightarrow T_B = U_A - U_B + A_{A\to B} \Rightarrow$

$\frac{mv^2}{2} = mgS\sin\alpha - \frac{2}{3}\gamma mg\cos\alpha\sqrt{S^3} \Rightarrow v^2 = 2gS\sin\alpha -$

$- 4/3\,\gamma mg\cos\alpha\sqrt{S^3} = 2g(S\sin\alpha - 2/3\,\gamma\sqrt{S^3}\cos\alpha)$

II падіння

$v_x(0) = v\cdot\cos\alpha,\ v_y(0) = v\cdot\sin\alpha\ (t = 0),\ F_n = -kv$

$\begin{cases} m\dot v_x = -kv_x \\ m\dot v_y = mg - kv_y \end{cases} \Rightarrow \begin{cases} \dot v_x + k/m\ v_x = 0 \\ \dot v_y + k/m\ v_y = mg \end{cases}$

$\Rightarrow v_x = C_1 e^{-\frac{k}{m}t} \Rightarrow v_x(0) = C_1 = v\cos\alpha \Rightarrow v_x(t) = v_0\cos\alpha\cdot e^{-\frac{k}{m}t}$

$\begin{cases} v_y = C_2 e^{-\frac{k}{m}t} \\ v_y = C \Rightarrow k/m\ C = g \Rightarrow C \cdot mg \end{cases} \Rightarrow v_y = \frac{mg}{k} + C_2 e^{-\frac{k}{m}t}$

$v_y(0) = C_2 + \frac{mg}{k} = v\sin\alpha \Rightarrow C_2 = v\sin\alpha - \frac{mg}{k}$

$v_y(t) = mg/k + (v\cdot\sin\alpha - mg/k)e^{-\frac{k}{m}t} = \frac{mg}{k}(1 - e^{-\frac{k}{m}t})$
$+ v\sin\alpha e^{-\frac{k}{m}t}$.

$\vec v(t) = v_x(t)\vec i + v_y\vec j = \underline{v\cos\alpha e^{-\frac{k}{m}t}\cdot\vec i + [\frac{mg}{k}(1 - e^{-\frac{k}{m}t})} +$
$\underline{+ v\cdot\sin\alpha e^{-\frac{k}{m}t}]\vec j}$

$x(0) = 0 \Rightarrow x(t) = \int^t v_x(t')dt' + x(0) = \int_0^t v\cos\alpha e^{-\frac{k}{m}t}dt$

$= v\cos\alpha(-\frac{m}{k}e^{-\frac{k}{m}t})\Big|_0^t = v\cdot\cos\alpha(1 - e^{-\frac{k}{m}t})\frac{m}{k} =$

$= \frac{mv}{k}\cos\alpha(1 - e^{-\frac{k}{m}t}) \Rightarrow x(\tau) = L \Rightarrow \frac{mv}{k}\cos\alpha(1 - e^{-\frac{k}{m}\tau}) = L$

$(y(0)=0)$ $y(t)=\int_0^t v_y(t')dt' + y(0) = \int_0^t \frac{mg}{k}dt' - \frac{mg}{k}\int e^{-\frac{k}{m}t'}dt'$

$= \frac{mgt}{k} + \frac{m^2g}{k^2}e^{-\frac{k}{m}t'}\Big|_0^t - \frac{mv}{k}\sin\alpha\, e^{-\frac{k}{m}t'}\Big|_0^t = \frac{mgt}{k} + \left(\frac{mv}{k}\sin\alpha - \frac{m^2g}{k^2}\right)\left(1-e^{-\frac{k}{m}t}\right)$

$L = y(\tau) \Rightarrow \left(1-e^{-\frac{k}{m}\tau}\right) = \frac{Lk}{mv\cos\alpha}$, $e^{-\frac{k}{m}\tau} = \frac{mv\cos\alpha - kL}{mv\cos\alpha}$

$\Rightarrow \frac{k}{m}\tau = \ln\left(\frac{mv\cos\alpha - kL}{mv\cos\alpha}\right) \Rightarrow \tau = \frac{m}{k}\ln\left(\frac{mv\cos\alpha}{mv\cos\alpha - Lk}\right)$

$\Rightarrow h = \frac{m^2g}{k^2}\ln\left(\frac{mv\cos\alpha}{mv\cos\alpha - Lk}\right) + \left(\frac{mv}{k}\sin\alpha - \frac{m^2g}{k^2}\right)\frac{L}{mv\cos\alpha} =$

$= \frac{m^2g}{k^2}\ln\left(\frac{mv\cos\alpha}{mv\cos\alpha - Lk}\right) + L\,\mathrm{tg}\,\alpha - \frac{mgL}{kv\cos\alpha}$

4.15

m_1, m_2
$v_1 = (3i + 4j)$
$v_2(-2i + 3j)$
$v - ?$

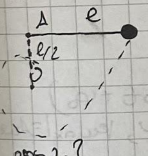
$p = m_1v_1 + m_2v_2$, ~~$v = p_j - (m_1 - m_2)v$~~

$m_1v_1 + m_2v_2 = (m_1 + m_2)v$

$\Rightarrow v = \frac{m_1}{m_1+m_2}\bar{v}_1 + \frac{m_2}{m_1+m_2}\bar{v}_2$

$\Rightarrow v_x = \frac{3m_1 - m_2}{m_1+m_2}$, $v_y = \frac{4m_1 + 3m_2}{m_1+m_2}$

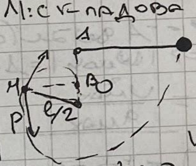
4.35



$\cos\alpha - ?$
$y_{max} - ?$
$y - ?$

I ~~...~~ кулька летить чверть кола $r = l$ → II цвях задіває нитку, коло $r = l/2$ → $T_M = 0$ у т. M

M: складова ваги $= P\cos\alpha$, $v_M^2 = 2gH$, $H = AB =$



$= AO - BO = \frac{1}{2}l(1-\cos\alpha)$

$\Rightarrow \frac{mv^2}{R} = \frac{2\cdot 2ml(1-\cos\alpha)g}{2l} = 2P(1-\cos\alpha)$

$\Rightarrow P\cos\alpha = 2P(1-\cos\alpha) \Rightarrow \boxed{\cos\alpha = \frac{2}{3}}$

кулька летить як тіло ~~брошене~~ кинуте під кутом α до горизонту $\Rightarrow v = \sqrt{\frac{1}{3}gl}$

$h = \frac{v^2\sin^2\alpha}{2g} = \frac{5}{54}l \Rightarrow y_{max} = \frac{l}{3} + h = \frac{23}{54}l \Rightarrow$

$y = (M_y - O_y) = \frac{l}{3}$

9.14

$r = r(\varphi)$
$U = -\alpha/r^2$
$E = 0$
M
$r = ?$

$M = m[\bar{r}\cdot\bar{v}] = mr^2\dot\varphi$

$\frac{m\dot r^2}{2} + \frac{M^2}{2mr^2} = \frac{\alpha}{r^2} \Rightarrow m^2\dot r^2 r^2 + M^2 = 2\alpha m \Rightarrow$

$\Rightarrow dt = \frac{mr\,dr}{\sqrt{2m\alpha - M^2}}$, $d\varphi = \frac{M}{r^2}\cdot\frac{mr\,dr}{\sqrt{2m\alpha - M^2}}$

$d\varphi = \frac{M}{r}\cdot\frac{dr}{\sqrt{2m\alpha - M^2}}$, $\frac{1}{r}dr = \frac{\sqrt{2m\alpha - M^2}}{M}d\varphi \Rightarrow$

$\ln r = \frac{\varphi\sqrt{2m\alpha - M^2}}{M} \Rightarrow r = e^{\frac{\varphi}{M}\sqrt{2m\alpha - M^2}}$

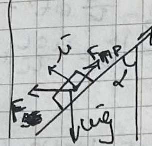
10.18

$m, \angle\alpha$

a_0

$\mu - ?$

$t - ?$



$Ox: F_{тр} - mg \cdot \cos\alpha = F_{зб} \cdot \cos\alpha + F_{зб}\mu \cdot \sin\alpha - N \sin\alpha = 0$

$\mu\, mg\cos\alpha - a_0^2 k \sin\alpha\,\mu - a_0^2 k \cos\alpha - mg \sin\alpha = 0$

$\mu(g\cos\alpha - a_0^2 k \sin\alpha) - a_0^2 k \cos\alpha - mg\sin\alpha = 0$

$m\bar{a} = m\bar{g} + \bar{N} + \bar{F}_{тр} \Rightarrow$

$\begin{cases} ma = mg\sin\alpha - \mu N \\ 0 = N - mg\cos\alpha \end{cases} \Rightarrow a = g(\sin\alpha - \mu\cos\alpha)$

$a_x = g(\sin\alpha - \mu\cos\alpha)$

$A = a + a_x a_0$

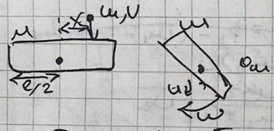
$v_{0x} = 0 \Rightarrow l = \frac{at^2}{2} \Rightarrow t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2l}{g(\sin\alpha - \mu\cos\alpha + a_0(\sin\alpha + \mu\cos\alpha))}}$

11.16

M, l

m

$x - ?$



$\frac{mv^2}{2} = \frac{I\omega^2}{2} + \frac{Mu^2}{2}$

$\Rightarrow I = \frac{mL^2}{12}$

$\Rightarrow mvx = I\omega \Rightarrow \omega = mvx/I$

$\Rightarrow mv = Mu \Rightarrow u = \frac{mv}{M} \Rightarrow mv^2 = I\frac{m^2v^2x^2}{I^2} + M\cdot\frac{m^2v^2}{M^2} \Rightarrow 1 = \frac{mx^2}{I} + \frac{m}{M}$

$\Rightarrow x^2 = \frac{l^2}{12}\left(\frac{M}{m} - 1\right) \Rightarrow x = \sqrt{\frac{x^2}{12}\left(\frac{M}{m} - 1\right)}$

12.26

v_{max}

a_{max}

$\omega - ?$

$A - ?$

$v - ?$

$x(t) = A\cos(\omega t + \varphi_0) \quad v = \dot{x} = -A\omega\sin(\omega t + \varphi_0)$

$a = \ddot{x} = -A\omega^2\cos(\omega t + \varphi_0) \Rightarrow |v_{max}| = A\omega, |a_{max}| = A\omega^2$

$\Rightarrow \frac{a_{max}}{v_{max}} = \omega, \quad A = \frac{v_{max}}{\omega} = \frac{v_{max}^2}{a_{max}}$

$x(t) = \pm\frac{A}{2} \Rightarrow \cos(\omega t + \varphi_0) = \pm\frac{1}{2} \Rightarrow |\cos(\omega t + \varphi_0)| = \frac{1}{2}$

$\Rightarrow |\sin(\omega t + \varphi_0)| = \sqrt{\sin^2(\omega t + \varphi_0)} = \sqrt{1 - \cos^2(\omega t + \varphi_0)} = \frac{\sqrt{3}}{2}$

$v = |v(t)| = A\omega\sin(\omega t + \varphi_0) = A\omega\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}v_{max}$