Домашня робота 9

1. $x = \dfrac{1}{y\sqrt{1+\ln y}}, x = 0, y = 1, y = e^3$

$$\int\limits_1^{e^3} \frac{1}{y\sqrt{1+\ln y}}\mathbf{d}y = \left| \begin{array}{ll} t = \ln y & \mathbf{d}t = \dfrac{1}{y}\mathbf{d}y \\ t_1 = 0 & t_2 = 3 \end{array} \right| = \int\limits_0^3 \frac{1}{\sqrt{1+t}}\mathbf{d}t = 2\sqrt{1+t}\Big|_0^3 = 2$$

2. $\left\{ \begin{array}{l} x = \sqrt{2}\cos t \\ y = 2\sqrt{2}\sin t \end{array} \right. , y = 2\ (y \geq 2)$

$2\sqrt{2}\sin t \geq 2 \Rightarrow t \in \left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$

$$S = \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\sqrt{2}\sin t(-\sqrt{2}\sin t)\mathbf{d}t - 2\cdot 2 = 2\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(1-\cos 2t)\mathbf{d}t - 4 = 2\left(1 - \frac{1}{2}\sin 2t\right)\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} =$$

$$= \pi - 2$$

3. $\left\{ \begin{array}{l} x = 2\cos t \\ y = 6\sin t \end{array} \right. , y = 3\ (y \geq 3)$

$6\sin t \geq 3 \Rightarrow t \in \left[\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right]$

$$S_1 = \int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 6\sin t\cdot 2(-\sin t)\mathbf{d}t = 12\int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} -\sin^2 t\mathbf{d}t = -6\int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(1-\cos 2t)\mathbf{d}t = -6\left(t - \frac{1}{2}\sin 2t\right)\Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} =$$

$$= 4\pi + 3\sqrt{3}. \qquad S = S_1 - 3(x_1 - x_2) = 4\pi + 3\sqrt{3} - 3\left(2\cos\frac{\pi}{6} - 2\cos\frac{5\pi}{6}\right) = 4\pi - 3\sqrt{3}$$

4. $\left\{ \begin{array}{l} x = 16\cos^3 t \\ y = \sin^3 t \end{array} \right. , x = 6\sqrt{3}\ (x \geq 6\sqrt{3})$

$16\cos^3 t \geq 6\sqrt{3} \Rightarrow t \in \left[0, \dfrac{\pi}{6}\right]$

$$S = 6\int\limits_{\frac{\pi}{6}}^{0} -\sin t\cdot 16\cos^2 t\cdot\sin^3 t\mathbf{d}t = 96\int\limits_0^{\frac{\pi}{6}} \sin^4\cos^2 t\mathbf{d}t = 12\int\limits_0^{\frac{\pi}{6}} \sin^2 t(1-\cos 2t)\mathbf{d}t =$$

$$= 6\int\limits_0^{\frac{\pi}{6}}(1-\cos 4t)(1-\cos 2t)\mathbf{d}t = 6\int\limits_0^{\frac{\pi}{6}}(1-\cos 4t-\cos 2t+\cos 4t\cos 2t)\mathbf{d}t =$$

$$= 6\left(t - \frac{1}{4}\sin 4t - \frac{1}{2}\sin 2t\right) + 3\int\limits_0^{\frac{\pi}{6}}(\cos 3t + \cos 6t)\mathbf{d}t = 6\left(t - \frac{1}{4}\sin 4t - \frac{1}{2}\sin 2t\right) +$$

$$+ 3\left(\frac{1}{2}\sin 2t + \frac{1}{6}\sin 6t\right)\Big|_0^{\frac{\pi}{6}} = \pi - \frac{3}{2}\sqrt{2}$$

5. $\left\{ \begin{array}{l} x = 32\cos^3 t \\ y = \sin^3 t \end{array} \right. , x = 4\ (x \geq 4)$

$32\cos^3 t \geq 4 \Rightarrow t \in \left[-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right]$

$$S = \int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 12\sin^3 t(1-\cos 2t)\mathbf{d}t = \int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = 12\left(\int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^2 t\mathbf{d}t + \int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^2 t\cos^2 t\mathbf{d}t\right) =$$

$$= \left|\begin{array}{ll} u = \sin 2t & \mathbf{d}u = 2\cos 2t\mathbf{d}t \\ t_1 = -\dfrac{\sqrt{3}}{2} & t_2 = \dfrac{\sqrt{3}}{2} \end{array}\right| = 6\int\limits_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1-\cos 4t)\mathbf{d}t - 6\int\limits_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} t^2\mathbf{d}t =$$

$$= 6\left(t - \frac{1}{4}\sin 4t\right)\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} - 2t^3\Big|_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} = 4\pi$$

6. $r = \cos 2\varphi$

$T = \pi, r = \cos 2\varphi \geq 0 \Rightarrow \varphi \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

$$S = 2\left(\frac{1}{2}\int\limits_0^{\frac{\pi}{4}} r^2\mathbf{d}\varphi\right) = \int\limits_0^{\frac{\pi}{4}} \cos^2 2\varphi\mathbf{d}\varphi = \frac{1}{2}\int\limits_0^{\frac{\pi}{4}} (1+\cos 4\varphi)\mathbf{d}\varphi = \frac{1}{2}\left(\varphi + \frac{1}{4}\sin 4\varphi\right)\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{8}$$

7. $\begin{cases} r = \sqrt{3}\cos\varphi \\ r = \sin\varphi \end{cases}, 0 \leq \varphi \leq \dfrac{\pi}{2}$

$\sqrt{3}\cos\varphi = \sin\varphi \Rightarrow \varphi = \dfrac{\pi}{3}$

$$S_1 = \frac{1}{2}\int\limits_0^{\frac{\pi}{3}} \sin^2\varphi\mathbf{d}\varphi = \frac{1}{4}\int\limits_0^{\frac{\pi}{3}} (1-\cos 2\varphi)\mathbf{d}\varphi = \frac{1}{4}\left(\varphi - \frac{1}{2}\sin^2\varphi\right)\Big|_0^{\frac{\pi}{3}} = \frac{1}{12}\pi - \frac{\sqrt{3}}{16}$$

$$S_2 = \frac{1}{2}\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\cos^2\varphi\mathbf{d}\varphi = \frac{3}{4}\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1+\cos 2\varphi)\mathbf{d}\varphi = \frac{3}{4}\left(\varphi + \frac{1}{2}\sin 2\varphi\right)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \frac{3}{24}\pi - \frac{3\sqrt{3}}{16}$$

$$\Rightarrow S = S_1 + S_2 = \frac{5}{24}\pi - \frac{\sqrt{3}}{4}$$