Домашня робота 6

5.11 $\displaystyle\int \frac{x\sqrt[3]{2+x}}{x+\sqrt[3]{2+x}}\mathbf{d}x = \left| \begin{matrix} 2+x=t^3, & x=t^3-2 \\ \dfrac{\mathbf{d}x}{\mathbf{d}t}=3t^2, & \mathbf{d}x=3t^2\mathbf{d}t \end{matrix} \right| = \int \frac{3t^3(t^3-2)}{t^3+t-2}\mathbf{d}t = 3\int \frac{t^6-2t^3}{t^3+t-2}\mathbf{d}t =$

$\displaystyle= 3\int \left(t^3-t+1+\frac{t-2}{t^2+t+2}\right)\mathbf{d}t = 3\left(\frac{t^4}{4}-\frac{t^2}{2}+t+\frac{1}{2}\ln|t^2+t+2-\right.$

$\displaystyle\left.-\frac{5}{\sqrt{7}}\arctan\left(\frac{2t+1}{\sqrt{7}}\right)\right)+c$

5.12 $\displaystyle\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\mathbf{d}x =$

$$\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} = \frac{(\sqrt{x+1}-\sqrt{x-1})^2}{x+1-x+1} = \frac{2\left(x-\sqrt{x^2-1}\right)}{2} = x-\sqrt{x^2-1}$$

$\displaystyle= \int \left(x-\sqrt{x^2-1}\right)\mathbf{d}x = \frac{x^2}{2}-\int \sqrt{\operatorname{ch}^2 t-1}\operatorname{sh} t\mathbf{d}t = \frac{x^2}{2}-\int \operatorname{sh}^2 t\mathbf{d}t = \frac{x^2}{2}-\int \frac{\operatorname{ch}2t-1}{2}\mathbf{d}t =$

$\displaystyle= \frac{1}{4}\left(2x^2-\operatorname{sh}2t-t\right) = \frac{1}{4}\left(2x^2-\operatorname{sh}(2\operatorname{arcch}x)-\operatorname{arcch}x\right)+c$

5.13 $\displaystyle\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}\mathbf{d}x = \left| \begin{matrix} x+1=t^6 \\ \mathbf{d}x=6t^5\mathbf{d}t \end{matrix} \right| = \int \frac{(1-t^3)6t^5}{1+t^2}\mathbf{d}t = -6\int \left(\frac{2}{t+1}+t^7-t^6-2t^4+\right.$

$\displaystyle\left.+2t^3-2t^2+2t-2\right)\mathbf{d}t = -6\left(2\ln|t+1|+\frac{t^8}{8}-\frac{t^7}{7}-\frac{2t^5}{5}+\frac{t^4}{2}-\frac{2t^3}{3}+t^2-2t\right)+c=$

$\displaystyle= -6\left(2\ln|\sqrt[6]{x+1}+1|+\frac{(\sqrt[6]{x+1})^8}{8}-\frac{(\sqrt[6]{x+1})^7}{7}-\frac{2(\sqrt[6]{x+1})^5}{5}+\frac{(\sqrt[6]{x+1})^4}{2}-\right.$

$\displaystyle\left.-\frac{2(\sqrt[6]{x+1})^3}{3}+(\sqrt[6]{x+1})^2-2\sqrt[6]{x+1}\right)+c$

5.16 $\displaystyle\int \mathbf{d}x = \left| \begin{matrix} \sqrt{x^2-2x+2}=t+x \\ x=-\dfrac{t^2-2}{2+2t} \\ \mathbf{d}x=\dfrac{2t^2+4t+4}{(2+2t)^2}\mathbf{d}t \end{matrix} \right| = \int -\frac{2t^6+6t^5+8t^4-16t^2-24t-16}{(2+2t)^4}\mathbf{d}t$