Розрахункова Робота №4

ФІ-12 Бекешева Анастасія

June 7, 2022

1. $\int \arctan\sqrt{4x-1}\mathbf{d}x = \int \arctan\sqrt{4x-1}\mathbf{d}(4x-1) = x\arctan\sqrt{4x-1} -$
$- \frac{1}{2}\int \frac{\mathbf{d}(4x-1)}{\sqrt{4x-1}} = x\arctan\sqrt{4x-1} - \frac{1}{4}\sqrt{4x-1} + c$

2. $\int\limits_{-2}^{0} (x^2-4)\cos 3x\mathbf{d}x = \begin{vmatrix} u = x^2-4 & \mathbf{d}v = \cos 3x\mathbf{d}x \\ \mathbf{d}u = 2x\mathbf{d}x & v = \frac{1}{3}\sin 3x \end{vmatrix} = \frac{1}{3}(x^2-4)\sin 3x\Big|_{-2}^{0} -$
$- \frac{2}{3}\int\limits_{-2}^{0} x\sin 3x\mathbf{d}x = \begin{vmatrix} u = x & \mathbf{d}v = \sin 3x\mathbf{d}x \\ \mathbf{d}u = \mathbf{d}x & v = -\frac{1}{3}\cos 3x \end{vmatrix} = -\frac{2}{3}\left(-\frac{1}{3}x\cos 3x\Big|_{-2}^{0} + \frac{1}{3}\int\limits_{-2}^{0}\cos 3x\mathbf{d}x\right) =$
$= -\frac{2}{3}\left(-\frac{2}{3}\cos 6 + \frac{1}{9}\sin 3x\right) = \frac{4}{9}\cos 6 - \frac{2}{27}\sin 6$

3. $\int \frac{1+\ln x}{x}\mathbf{d}x = \int \frac{\mathbf{d}x}{x} + \int \ln x\mathbf{d}(\ln x) = \ln|x| + \frac{1}{2}\ln^2 x + c$

4. $\int\limits_{0}^{1} \frac{(x^2+1)\mathbf{d}x}{(x^3+3x+1)} = \begin{vmatrix} t = x^3+3x+1 \\ \mathbf{d}x = \frac{1}{3x^2+3}\mathbf{d}t \end{vmatrix} = \frac{1}{3}\int\limits_{0}^{1}\frac{\mathbf{d}t}{t^2} = -\frac{1}{3t} = -\frac{1}{3(x^3+3x+1)}\Big|_{0}^{1} = \frac{4}{15}$

5. $\int \frac{3x^3+1}{x^2-1}\mathbf{d}x = \int \left(3x + \frac{3x+1}{(x-1)(x+1)}\right)\mathbf{d}x \ominus$
$= \frac{A}{x-1} + \frac{B}{x+1}$
$A = \frac{3x+1}{x+1}\Big|_{x=1} = 2$
$B = \frac{3x+1}{x-1}\Big|_{x=-1} = 1$
$\ominus \int \left(3x + \frac{2}{x-1} + \frac{1}{x+1}\right)\mathbf{d}x = \frac{3}{2}x^2 + 2\ln|x-1| + \ln|x+1| + c$

6. $\int \frac{x^3+6x^2+13x+8}{x(x+2)^3}\mathbf{d}x = \ominus$
$= \frac{A}{x} + \frac{B}{(x+2)^3}$
$A = \frac{x^3+6x^2+13x+8}{(x+2)^2}\Big|_{x=0} = 1$
$B = \frac{x^3+6x^2+13x+8}{x}\Big|_{x=-2} = 1$
$\ominus \int \left(\frac{1}{x} + \frac{1}{(x+2)^3}\right)\mathbf{d}x = \ln|x| - \frac{1}{2(x+2)^2} + c$

8. $\int\limits_{0}^{\frac{\pi}{2}} \frac{\cos x\mathbf{d}x}{2+\cos x} = \int\limits_{0}^{\frac{\pi}{2}}\left(1 - \frac{2}{\cos x + 2}\right)\mathbf{d}x = x\Big|_{0}^{\frac{\pi}{2}} - 2\int \frac{1}{\cos^2\frac{x}{2}(\tan^2\frac{x}{2}) + 3}dx = \left| t = \frac{\tan\frac{x}{2}}{\sqrt{3}} \right| =$
$= x\Big|_{0}^{\frac{\pi}{2}} - \frac{4}{\sqrt{3}}\int \frac{1}{t^2+1}\mathbf{d}t = x\Big|_{0}^{\frac{\pi}{2}} - \frac{4}{\sqrt{3}}\arctan\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right)\Big|_{0}^{\frac{\pi}{2}} = \frac{\pi}{2} - \frac{2\pi}{\sqrt{3^3}}$

10. $\int\limits_{0}^{\pi} 2^4\sin^6 x\cos^2 x\mathbf{d}x = \int\limits_{0}^{\pi}\sin^2 x\left(2 - \frac{1-\cos 2x}{2}\right)\mathbf{d}x = \int\limits_{0}^{\pi}\sin^2 2x\left(1 - 2\cos 2x + \cos^2 x\right)\mathbf{d}x =$

$$= \int\limits_0^{\pi} \sin^2 2x\mathbf{d}x + 2\int\limits_0^{\pi} \sin^2 2x \cos x\mathbf{d}x + \int\limits_0^{\pi} \sin^2 2x \cos^2 x\mathbf{d}x = \int\limits_0^{\pi} \frac{1}{2}(1-\cos 4x)\mathbf{d}x -$$

$$-\int\limits_0^{\pi} \sin^2 2x\mathbf{d}(\sin 2x) + \int\limits_0^{\pi} \frac{1}{8}(1-\cos 8x)\mathbf{d}x = \frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + \frac{1}{3}\sin^3 x + \frac{1}{8}\left(x - \frac{1}{8}\sin 8x\right)\Bigg|_0^{\pi} =$$

$$= \frac{\pi}{2} + 0 + \frac{\pi}{8} = \frac{5\pi}{8}$$

11. $$\int\limits_1^{64} \frac{1-\sqrt[6]{x}+2\sqrt[3]{x}}{x+2\sqrt[3]{x}+\sqrt[3]{x^4}}\mathbf{d}x = \left| \begin{array}{ll} t = \sqrt[6]{x} & x = t^6 \\ \mathbf{d}x = 6t^5 & \mathbf{d}t \\ t_1 = 1 & t_2 = 2 \end{array} \right| = \int_1^2 \frac{1-t+2t^2}{t^6+2t^3+t^8} \cdot 6t^5\mathbf{d}t =$$

$$= 6\int_1^2 \frac{2t^2-t+1}{t(t+1)(2t^2-t+1)}\mathbf{d}t = 6\int_1^2 \frac{\mathbf{d}t}{t(t+1)} = 6\int\limits_1^2 \left(\frac{1}{t} - \frac{1}{t+1}\right)\mathbf{d}t = 6(\ln|t| - \ln|t+1|)\Bigg|_1^2 =$$

$$= 6(2\ln 2 - \ln 3) = 6\ln\frac{4}{3}$$

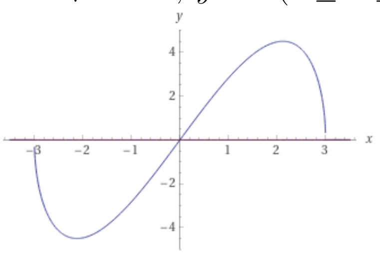
12. $$\int\limits_0^1 x^2\sqrt{1-x^2} = \left| \begin{array}{ll} x = \sin t & \\ \mathbf{d}x = \cos t & \mathbf{d}t \\ t_1 = 0 & t_2 = \dfrac{\pi}{2} \end{array} \right| = \int\limits_0^{\frac{\pi}{2}} \sin^2 t\sqrt{1-\sin^2 t}\cos t\mathbf{d}t = \int\limits_0^{\frac{\pi}{2}} \sin^2 t\cos^2 t\mathbf{d}t =$$

$$= \frac{1}{4}\int\limits_0^{\frac{\pi}{2}} \sin^2 t\mathbf{d}t = \frac{1}{8}\int\limits_0^{\frac{\pi}{2}} (1-\cos 4t)\,\mathbf{d}t = \frac{1}{8}\left(t - \frac{\sin 4t}{4}\right)\Bigg|_0^{\frac{\pi}{2}} = \frac{1}{8}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{16}$$

13. $$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x\sqrt[3]{x^2}}\mathbf{d}x = \int x^{-\frac{3}{2}}(x^{-\frac{1}{2}}+1)^{\frac{1}{3}}\mathbf{d}x = \left| \begin{array}{l} t = x^{-\frac{1}{2}}+1 \\ \mathbf{d}t = -\dfrac{1}{2}x^{-\frac{3}{2}}\mathbf{d}x \end{array} \right| = -2\int t^{\frac{1}{3}}dt = -\frac{3}{2}t^{\frac{4}{3}} =$$

$$= -\frac{3}{2}\sqrt{\left(1+\frac{1}{\sqrt{x}}\right)} + c$$

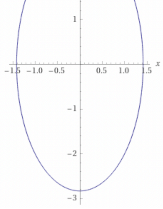
14. $y = x\sqrt{9-x^2},\ y = 0\ (0 \le x \le 3)$



$$S = \int\limits_0^3 y\mathbf{d}x = \int\limits_0^3 x\sqrt{9-x^2} = \left| \begin{array}{ll} t = 9-x^2 & \mathbf{d}t = -2x\mathbf{d}x \\ t_1 = 9 & t_2 = 0 \end{array} \right| =$$

$$= -\int\limits_9^0 \frac{\sqrt{t}}{2}\mathbf{d}x = -\frac{1}{6}t^{\frac{3}{2}}\Bigg|_9^0 = \frac{27}{3} - 0 = 9$$

15. $$\left\{ \begin{array}{l} x = \sqrt{2}\cos t \\ y = 2\sqrt{2}\sin t \end{array} \right. ,y = 2\ (y \ge 2)$$

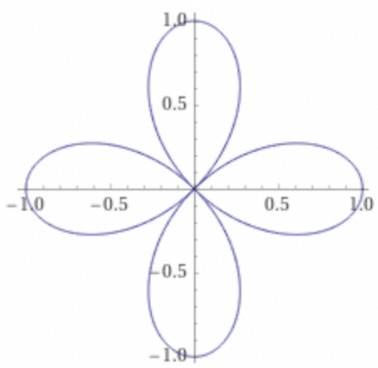


$$2\sqrt{2}\sin t \ge 2 \Rightarrow t \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

$$S = \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\sqrt{2}\sin t(-\sqrt{2}\sin t)\mathbf{d}t - 2\cdot 2 = 2\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1-\cos 2t)\mathbf{d}t - 4 =$$

$$= 2\left(1 - \frac{1}{2}\sin 2t\right)\Bigg|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \pi - 2$$

16. $r = \cos 2\varphi$

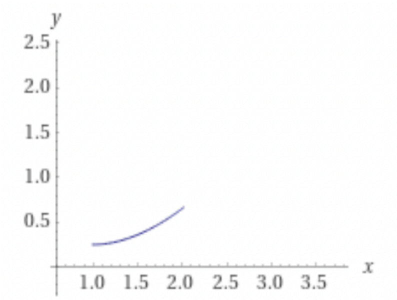


$$T = \pi, r = \cos 2\varphi \geq 0 \Rightarrow \varphi \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

$$S = 2\left(\frac{1}{2}\int\limits_0^{\frac{\pi}{4}} r^2 \mathbf{d}\varphi\right) = \int\limits_0^{\frac{\pi}{4}} \cos^2 2\varphi \mathbf{d}\varphi = \frac{1}{2}\int\limits_0^{\frac{\pi}{4}} (1 + \cos 4\varphi)\mathbf{d}\varphi =$$

$$= \frac{1}{2}\left(\varphi + \frac{1}{4}\sin 4\varphi\right)\Bigg|_0^{\frac{\pi}{4}} = \frac{\pi}{8}$$

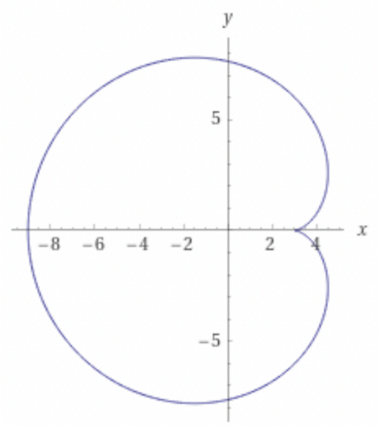
17. $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}, 1 \leq x \leq 2$



$$l = \int\limits_1^2 \sqrt{1 + \frac{(x^2-1)^2}{4x^2}}\mathbf{d}x = \int\limits_1^2 \frac{x^2+1}{2x}\mathbf{d}x = \frac{1}{2}\int\limits_1^2 \left(x + \frac{1}{x}\right)\mathbf{d}x =$$

$$= \frac{1}{2}\left(\frac{x^2}{2} + \ln|x|\right)\Bigg|_1^2 = \frac{1}{2}\left(\frac{3}{2} + \ln 2\right)$$

18. $\begin{cases} x = 3(2\cos t - \cos 2t) \\ y = 3(2\sin t - \sin 2t) \end{cases}, 0 \leq t \leq 2\pi$

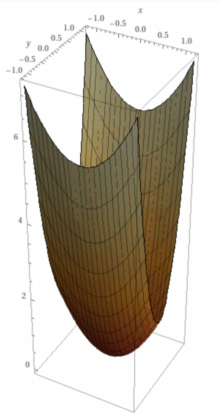


$$l = \int\limits_0^{\frac{\pi}{2}} \sqrt{9(-2\sin t + 2\sin t)^2 + 9(2\cos t - \cos 2t)^2}\mathbf{d}t =$$

$$= 6\int\limits_0^{\frac{\pi}{2}} \sqrt{2 - 2(\sin t \sin 2t + \cos t \cos 2t}\mathbf{d}t = 6\int\limits_0^{\frac{\pi}{2}} \sqrt{2 - 2\cos t}\mathbf{d}t =$$

$$= 6\int\limits_0^{\frac{\pi}{2}} \sqrt{4\sin^2\frac{t}{2}}dt = 12\int\limits_0^{\frac{\pi}{2}} \sin\frac{t}{2}\mathbf{d}t = -24\cos\frac{t}{2}\Bigg|_0^{\frac{\pi}{2}} = 48$$

20. $z = x^2 + 4y^2, z = 2$



$$\frac{x^2}{z} + \frac{4y^2}{z} = 1 \Rightarrow a = \sqrt{z},\ b = \frac{\sqrt{z}}{2} \Rightarrow S = \pi ab = \frac{\pi}{2}z$$

$$V = \int\limits_0^2 S(z)\mathbf{d}z = \frac{\pi}{2}\int\limits_0^2 Sz\mathbf{d}z = \frac{\pi}{2}\left(\frac{z^2}{2}\right)\Bigg|_0^2 = \pi$$