ДОМАШНЯ РОБОТА №7
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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1. (a) $\int\limits_{-2}^{-2+i} (z+2)^2 \ \mathrm{d}z = \int\limits_{-2}^{-2+i} (z+2)^2 \ \mathrm{d}(z+2) = \left.\frac{(z+2)^3}{3}\right|_{-2}^{-2+i} = \frac{(-2+i+2)^3}{3} - \frac{(-2+2)^3}{3} =$
$= \frac{1}{3}i^3 - \frac{1}{3}0^3 = \frac{i^3}{3} = \frac{-i}{3}$

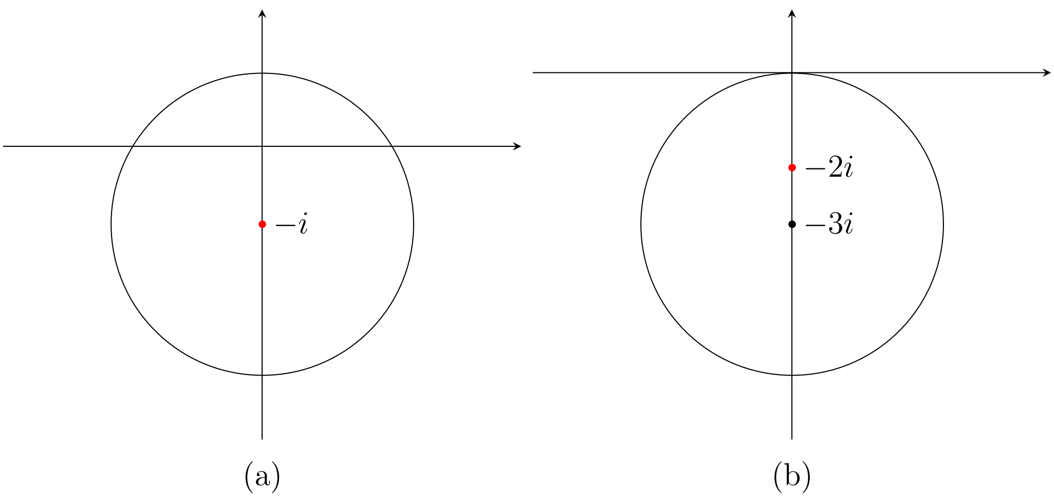
(b) $\int\limits_{0}^{\pi+2i} \cos\frac{z}{2} \ \mathrm{d}z = 2\int\limits_{0}^{\pi+2i} \cos\frac{z}{2} \ \mathrm{d}\frac{z}{2} = 2\sin\left.\frac{z}{2}\right|_{0}^{\pi+2i} = 2\sin\left(\frac{\pi}{2}+i\right) + 2\sin 0 = 2\cos i =$
$= 2\,\mathrm{ch}\,1$

(c) $\int\limits_{-i}^{i} (1+4iz^3) \ \mathrm{d}z = (z+iz^4)\big|_{-i}^{i} = (i + i\cdot i^4) - (-i + i\cdot i^4) = i + i - 0 = 2i$

(d) $\int\limits_{2e^{-\frac{\pi}{3}i}}^{2e^{\frac{\pi}{3}i}} (2-3z+z^2) \ \mathrm{d}z = \left.\frac{1}{3}z^3 - \frac{3}{2}z^2 + 2z\right|_{2e^{-\frac{\pi}{3}i}}^{2e^{\frac{\pi}{3}i}} = 4e^{i\frac{\pi}{3}} - 6e^{2i\frac{\pi}{3}} + \frac{8}{3}e^{i\pi} - 4e^{-i\frac{\pi}{3}} + 6e^{-2i\frac{\pi}{3}} -$
$- \frac{8}{3}e^{-i\pi} = 4\left(e^{i\frac{\pi}{3}} - e^{-i\frac{\pi}{3}}\right) - 6\left(e^{2i\frac{\pi}{3}} - e^{-2i\frac{\pi}{3}}\right) + \frac{8}{3}\left(e^{i\pi} - e^{-i\pi}\right) = 8\,\mathrm{sh}\,i\frac{\pi}{3} - 12\,\mathrm{sh}\,2i\frac{\pi}{3} +$
$+ \frac{16}{3}\,\mathrm{sh}\,i\pi = \frac{8}{i}\sin - \frac{12}{i}\sin -2\frac{\pi}{3} + \frac{16}{3i}\,\mathrm{sh}(-\pi) = \frac{2}{i}\sqrt{3}$

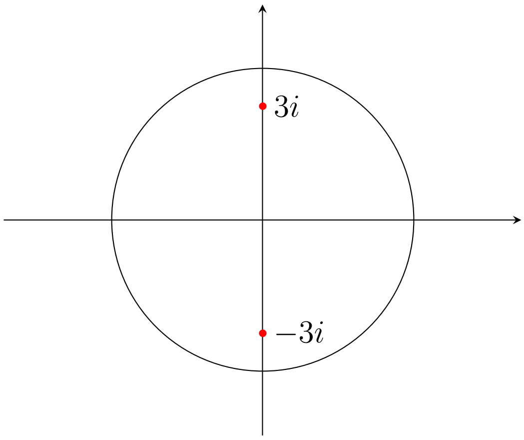
2. (a) $\int\limits_{|z+i|=3} \frac{\sin z}{z+i} \ \mathrm{d}z = \langle z \neq -i\rangle = 2\pi i \sin z\big|_{z=-i} = 2\pi i \sin -i = 2\pi\,\mathrm{sh}\,1$

(b) $C : x^2+y^2+6y = 0, \quad y = \pm\sqrt{9-x^2} - 3, \quad x = \pm 3 : y = -3, \quad x = 0 : y = 0, -6,$
$\int\limits_C \frac{\sin z \ \mathrm{d}z}{z^2+4} = \int\limits_C \frac{(z-2i)^{-1}\sin z \ \mathrm{d}z}{(z+2i)} = \langle z \neq \pm 2i\rangle = \left.\frac{2\pi i}{(z-2i)}\sin z\right|_{z=-2i} = -\frac{\pi}{2}\cdot$
$\cdot \sin(-2i) = \frac{\pi}{2}i\,\mathrm{sh}\,2$



3. $\int\limits_{|z|=4} \frac{\mathrm{d}z}{(z^2+9)(z+9)} = \langle z \neq \pm 3i, -9\rangle = \int\limits_{z\neq 3i} \frac{(z+9)^{-1} \ \mathrm{d}z}{(z^2+9)} = \int\limits_{|z|=4} \frac{\mathrm{d}z}{(z^2+9)(z+9)} +$
$+ \int\limits_{z\neq -3i} \frac{\mathrm{d}z}{(z^2+9)(z+9)} = (z+9)^{-1}(z+3i)^{-1}\Big|_{z=3i} + (z+9)^{-1}(z-3i)^{-1}\Big|_{z=-3i} =$

$$= \frac{2\pi i}{(3i+9)6i} - \frac{2\pi i}{(-3i+9)6i} = \frac{2\pi i}{6i} \cdot \frac{-i+3-i-3}{3(i+3)(-i+3)} = \frac{2\pi i}{6i}\frac{-2i}{(9-i^2)} = \frac{2\pi i}{9(9-(-1))} = -\frac{2\pi i}{90}$$

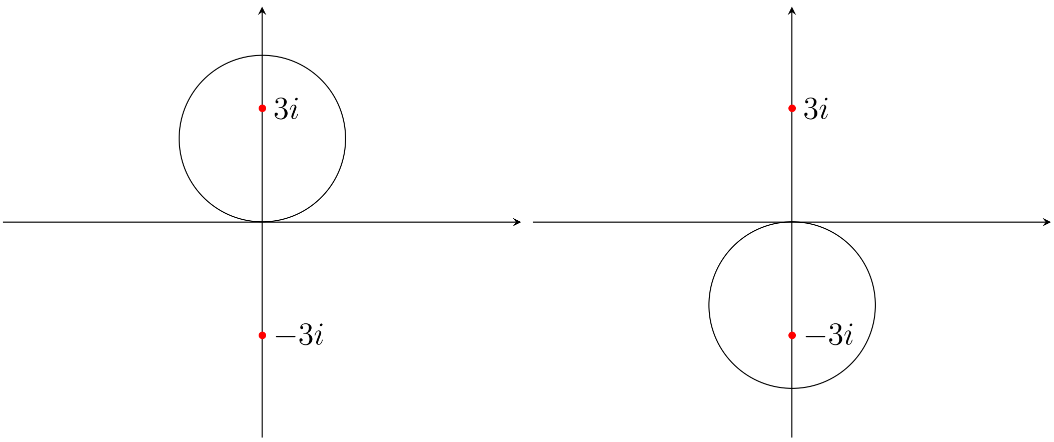


4. (a) $C : |z-2i| = 2, \quad \displaystyle\int\limits_C \frac{\mathrm{d}z}{z^2+9} = \langle z \neq \pm 3i \rangle = \int\limits_C \frac{(z+3i)^{-1}\ \mathrm{d}z}{z-3i} = \frac{2\pi i}{z+3i}\Big|_{z=3i} =$

$\displaystyle = \frac{2\pi i}{3i+3i} = \frac{\pi}{3}$

(b) $C : |z+2i| = 2, \quad \displaystyle\int\limits_C \frac{\mathrm{d}z}{z^2+9} = \langle z \neq \pm 3i \rangle = \int\limits_C \frac{(z-3i)^{-1}\ \mathrm{d}z}{z+3i} = \frac{2\pi i}{z-3i}\Big|_{z=-3i} =$

$\displaystyle = \frac{2\pi i}{-3i-3i} = -\frac{\pi}{3}$



(a) (b)

5. (a) $L : |z-1| = \frac{1}{3}, \quad \displaystyle\int\limits_L \frac{\operatorname{sh} z\pi\ \mathrm{d}z}{(z-1)(z^2+4)^2} = \langle z \neq 1, \pm 2i \rangle = \int\limits_L \frac{(z^2+4)^{-2} \operatorname{sh} z\pi\ \mathrm{d}z}{z-1} =$

$\displaystyle = 2\pi i (z^2+4)^{-2} \operatorname{sh} z\pi\Big|_{z=1} = 2\pi \frac{\operatorname{sh} z\pi}{25}$

(b) $L : |z+2i| = \frac{1}{3}, \quad \displaystyle\int\limits_L \frac{\operatorname{sh} z\pi\ \mathrm{d}z}{(z-1)(z^2+4)^2} = \langle z \neq 1, \pm 2i \rangle = \int\limits_L \frac{\operatorname{sh} z\pi (z+2i)^{-2}}{(z-1)(z-2i)^2} =$

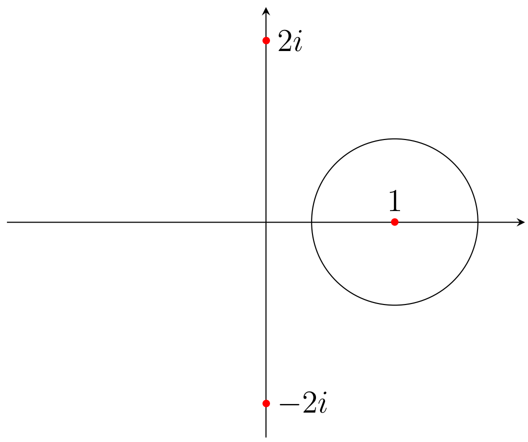
$\displaystyle = 2\pi i \frac{\mathrm{d}}{\mathrm{d}z}\left(\operatorname{sh} z\pi (z-1)^{-1}(z-2i)^{-2}\right)_{3}\Big|_{z=-2i} =$

$$= 2\pi i \left(\frac{\pi \operatorname{ch}(\pi z)(z-1)(z-2i)^2 - \operatorname{sh}(\pi z)(2(z-1)(z-2i)+(z-2i)^2)}{(z-1)^2(z-2i)^4} \right) =$$

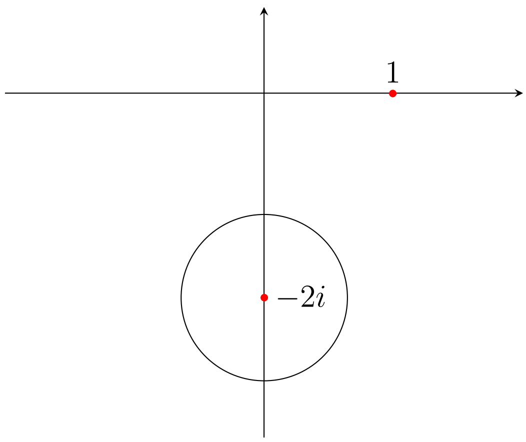
$$= 2\pi i \frac{\pi \operatorname{ch}(-2\pi i)(-16)(-2i-1) + \operatorname{sh}(2\pi i)(-8(-2i-1)-16)}{(-16)^2(-3+4i)} = \frac{2\pi^2 i(16+32i)+0(16i-8)}{(16+32i)^2} =$$

$$= \frac{\pi^2 i}{8+16i} = \frac{\pi^2 i \cdot 8 - \pi^2 i \cdot 16i}{320} = \frac{\pi^2(i+2)}{40}$$

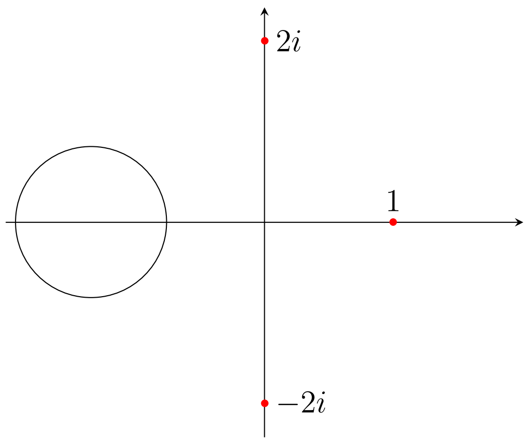
(d) $L : |z+2| = 1, \quad \int\limits_L \frac{\operatorname{sh} z \ \mathrm{d}z}{(z-1)(z^2+4)^2} = \langle z \neq 1, \pm 2i \rangle = 0 (z \notin D)$



(a)



(b)



(a)