

Домашня контрольна робота

Варіант №12002

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Part I

ΦI-12

01.06.2022

$$1. (L_1^R L_2^R)^+ = \left((L_1^R)^+ \cup (L_2^R)^* \right)^+$$

$$\begin{aligned} L_1^R &= \{x \in \Sigma^* \mid x^R \in L_1\} \\ L_2^R &= \{y \in \Sigma^* \mid y^R \in L_1\} \\ L_1^R L_2^R &= \{xy \in \Sigma^* \mid x^R \in L_1, y^R \in L_2^R\} \\ (L_1^R L_2^R)^+ &= \{xy \in \Sigma^* \mid \exists n \in \mathbb{N}_1, x^R \in L_1^n, y^R \in L_2^n\} \end{aligned}$$

$$\begin{aligned} L_1^R &= \{x \in \Sigma^* \mid x^R \in L_1\} \\ L_2^R &= \{y \in \Sigma^* \mid y^R \in L_1\} \\ (L_1^R)^+ &= \{x \in \Sigma^* \mid \exists n \in \mathbb{N}_1, x^R \in L_1^n\} \\ (L_2^R)^* &= \{x \in \Sigma^* \mid \exists n \in \mathbb{N}_0, x^R \in L_2^n\} \\ (L_1^R)^+ \cup (L_2^R)^* &= \{z \in \Sigma^* \mid \exists n \in \mathbb{N}_1, z^R \in L_1^n \vee z^R \in L_2^n\} \\ \left((L_1^R)^+ \cup (L_2^R)^* \right)^+ &= \{z \in \Sigma^* \mid \exists n \in \mathbb{N}_1, z^R \in L_1^n \vee z^R \in L_2^n\} \end{aligned}$$

$$\Rightarrow (L_1^R L_2^R)^+ \neq \left((L_1^R)^+ \cup (L_2^R)^* \right)^+$$

Let $\Sigma = \{\alpha_1, \alpha_2, \beta_1, \beta_2\}$, $L_1 = \{\alpha_1, \alpha_1\alpha_2\}$, $L_2 = \{\beta_1, \beta_1\beta_2\}$.

$$\begin{aligned} L_1^R &= \{\alpha_1, \alpha_2\alpha_1\} \\ L_2^R &= \{\beta_1, \beta_2\beta_1\} \\ L_1^R L_2^R &= \{\alpha_1\beta_1, \alpha_1\beta_2\beta_1, \alpha_2\alpha_1\beta_1, \alpha_2\alpha_2\beta_2\beta_2\} \\ (L_1^R L_2^R)^+ &= \{\alpha_1\beta_1\alpha_1\beta_1, \alpha_1\beta_2\beta_1\alpha_1\beta_2\beta_1, \alpha_2\alpha_1\beta_1\alpha_2\alpha_1\beta_1 \dots\} \end{aligned}$$

$$\begin{aligned} L_1^R &= \{\alpha_1, \alpha_2\alpha_1\} \\ L_2^R &= \{\beta_1, \beta_2\beta_1\} \\ (L_1^R)^+ &= \{\alpha_1, \alpha_2\alpha_1, \alpha_1\alpha_1, \alpha_2\alpha_1\alpha_2\alpha_1 \dots\} \\ (L_2^R)^* &= \{\varepsilon, \beta_1, \beta_2\beta_1, \beta_1\beta_1, \beta_2\beta_1\beta_2\beta_1 \dots\} \\ (L_1^R)^+ \cup (L_2^R)^* &= \{\varepsilon, \alpha_1, \alpha_2\alpha_1, \alpha_1\alpha_1, \alpha_2\alpha_1\alpha_2\alpha_1, \beta_1, \beta_2\beta_1, \beta_1\beta_1, \beta_2\beta_1\beta_2\beta_1 \dots\} \\ \left((L_1^R)^+ \cup (L_2^R)^* \right)^+ &= \{\alpha_1, \alpha_2\alpha_1, \alpha_1\alpha_1, \alpha_1\alpha_1\alpha_1, \alpha_2\alpha_1\alpha_2\alpha_1, \beta_1, \beta_2\beta_1, \beta_1\beta_1, \beta_1\beta_1\beta_1, \beta_2\beta_1\beta_2\beta_1 \dots\} \end{aligned}$$

$$2. A = P \wedge R \rightarrow \neg(Q \leftrightarrow \neg P \wedge R)$$

$$\begin{aligned} \text{cnf}(A) &= \neg(P \wedge R) \vee \neg(Q \leftrightarrow \neg P \wedge R) = \\ &= \neg(P \wedge R) \vee \neg((Q \vee \neg(\neg P \wedge R)) \wedge (\neg Q \vee (\neg P \wedge R))) = \\ &= \neg P \vee \neg R \vee ((\neg Q \wedge \neg P \wedge R) \vee (Q \wedge (P \vee \neg R))) = \\ &= (\neg P \vee \neg R \vee Q) \wedge (\neg P \vee \neg R \vee P \vee \neg R) \vee (\neg Q \wedge \neg P \wedge R) = \\ &= (\neg P \vee \neg R \vee Q) \vee (\neg Q \wedge \neg P \wedge R) = \\ &= ((\neg P \vee \neg R \vee Q) \vee \neg Q) \wedge ((\neg P \vee \neg R \vee Q) \vee (\neg P \vee R)) = \\ &= \neg P \vee \neg R \vee Q \end{aligned}$$

Tseitin:

$$\mathcal{T}(a) \setminus \mathcal{T}_0(A) = \{(\neg P), (P \wedge R), (\neg P \wedge R), (Q \leftrightarrow \neg P \wedge R), \\ (\neg(Q \leftrightarrow \neg P \wedge R)), (P \wedge R \rightarrow \neg(Q \leftrightarrow \neg P \wedge R))\}$$

t

substitute $\neg P$ with S_1

$P \wedge R$ with S_2

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R)$$

$$A_1 = S_2 \rightarrow \neg(Q \leftrightarrow S_1 \wedge R)$$

substitute $S_1 \wedge R$ with S_3

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R)$$

$$A_2 = S_2 \rightarrow \neg(Q \leftrightarrow S_3)$$

substitute $Q \leftrightarrow S_3$ with S_4

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\ \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3))$$

$$A_3 = S_2 \rightarrow \neg(S_4)$$

substitute $\neg S_4$ with S_5

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\ \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3)) \wedge (S_5 \leftrightarrow \neg S_4)$$

$$A_4 = S_2 \rightarrow S_5$$

substitute $S_2 \rightarrow S_5$ with S_6

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\ \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3)) \wedge (S_5 \leftrightarrow \neg S_4) \wedge (S_6 \leftrightarrow (S_2 \rightarrow S_5)) \wedge S_6$$

$$A_5 = S_6$$

$$\text{cnf}(\tilde{\varphi}_T(A)) = (P \vee S_1) \wedge (\neg S_1 \vee \neg P) \wedge (\neg P \vee \neg R \vee S_2) \wedge \\ \wedge (\neg S_2 \vee P) \wedge (\neg S_2 \vee R) \wedge (\neg S_1 \vee \neg R \vee S_3) \wedge (\neg S_3 \vee S_1) \wedge \\ \wedge (\neg S_3 \vee R) \wedge (\neg Q \vee \neg S_3 \vee S_4) \wedge (Q \vee S_3 \vee S_4) \wedge (\neg S_3 \vee Q \vee \neg S_4) \wedge \\ \wedge (\neg Q \vee S_3 \vee \neg S_4) \wedge (S_4 \vee S_5) \wedge (\neg S_5 \vee \neg S_4) \wedge (\neg S_2 \vee S_5) \wedge S_6 = \\ = (\neg P \vee \neg R \vee S_2) \wedge (\neg P \vee \neg S_1) \wedge (P \vee \neg Q \vee \neg R \vee \neg S_5) \wedge \\ \wedge (P \vee S_1) \wedge (\neg Q \vee S_3 \vee \neg S_4) \wedge (Q \vee \neg R \vee \neg S_4) \wedge (Q \vee S_3 \vee \neg S_5) \wedge \\ \wedge (R \vee \neg S_2) \wedge (R \vee \neg S_3) \wedge (\neg S_2 \vee \neg S_3) \wedge (S_4 \vee S_5) \wedge S_6$$

$$\text{Rank} = |\{P, Q, R, S_1, S_2, S_3, S_4, S_5, S_6\}| = 9, \text{Complexity} = \sum_{\Lambda \in \text{cnf}(\tilde{\varphi}_T(A))} 1 = 11$$

$$3. W = \{P_2 \vee P_4 \vee \neg P_5, P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_2 \vee \neg P_5 \vee \neg P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

DPLL:

MULT $(\neg P_2 \vee \neg P_5 \vee \neg P_2)$

$$W_1 = \{P_2 \vee P_4 \vee \neg P_5, P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_5 \vee \neg P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

SUS ($\neg P_5 \vee P_2$)

$$W_1 = \{P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_5 \vee P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

SPLIT (P_1)

$$W_{31} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{32} = \{\neg P_3 \vee P_5, \neg P_4 \vee P_5, \neg P_5 \vee P_2, P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

UNIT ($P_3 \in W_{32}$)

$$W_{41} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{42} = \{P_5, \neg P_4 \vee P_5, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

UNIT ($P_5 \in W_{42}$)

$$W_{41} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{42} = \{\neg P_4, P_2, P_2 \vee \neg P_4, \neg P_2\}$$

UNIT ($P_2 \in W_{42}$)

$$W_{51} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{52} = \{\neg P_4, \neg P_4\}$$

SAME ($\neg P_4 \in W_{52}$)

$$W_{61} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{62} = \{\neg P_4\}$$

UNIT ($\neg P_4 \in W_{62}$)

$$W_{71} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{72} = \emptyset$$

PURE ($P_3 \in W_{71}$)

$$W_{81} = \{\neg P_2 \vee \neg P_4, \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{72} = \emptyset$$

UNIT ($\neg P_4 \in W_{81}$)

$$W_{91} = \{\neg P_2, \neg P_5 \vee P_2, P_2, \neg P_2 \vee P_5\}$$

$$W_{72} = \emptyset$$

UNIT ($P_2 \in W_{91}$)

$$W_{101} = \{\neg P_5, P_5\}$$

$$W_{72} = \emptyset$$

UNIT ($P_5 \in W_{101}$)

$$W_{111} = \emptyset$$

$$W_{72} = \emptyset$$

W - unsatisfiable

Resolution:

$$(1) P_2 \vee P_4 \vee \neg P_5$$

$$(2) P_1 \vee \neg P_2 \vee \neg P_4$$

$$(3) \neg P_1 \vee \neg P_3 \vee P_5$$

(a)

$$(4) P_1 \vee P_3 \vee \neg P_4$$

$$(5) \neg P_1 \vee \neg P_4 \vee P_5$$

$$(6) \neg P_2 \vee \neg P_5 \vee \neg P_2$$

(b)

$$(7) \neg P_1 \vee P_3$$

$$(8) P_2 \vee \neg P_4$$

$$(9) \neg P_2 \vee P_5$$

$$P_2 \vee \neg P_5$$

$$(1, 8, P_4)$$

3

$$P_3 \vee P_4$$

$$(4, 7, P_1)$$

(c)		(g)
	$P_2 \vee P_3$	$(8, b, P_4)$
		$\neg P_4$
(d)		$(8, d, P_2)$
	$\neg P_2$	(6, 9, P_5)
(e)		
	$\neg P_5$	(a, d, P_2)
(f)		(i)
	P_3	(c, d, P_3)
		$\neg P_1$
		(e, h, P_5)

Resolution:

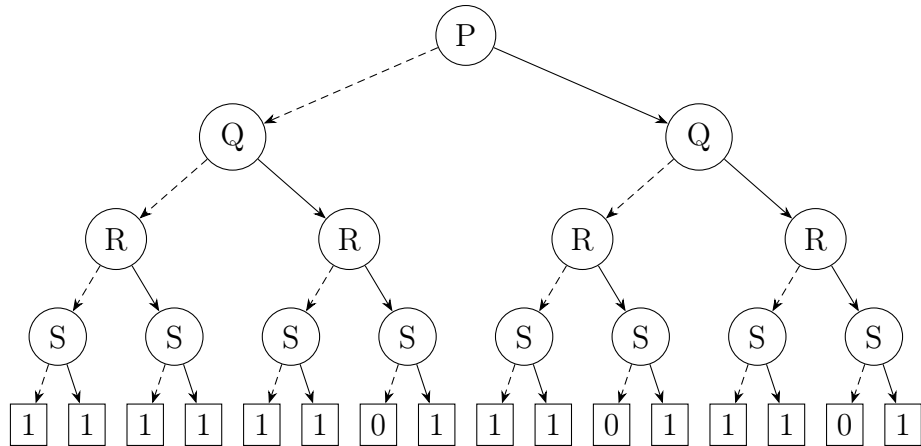
(1)	$P_2 \vee P_4 \vee \neg P_5$	(4)	$P_1 \vee P_3 \vee \neg P_4$	(7)	$\neg P_1 \vee P_3$
(2)	$P_1 \vee \neg P_2 \vee \neg P_4$	(5)	$\neg P_1 \vee \neg P_4 \vee P_5$	(8)	$P_2 \vee \neg P_4$
(3)	$\neg P_1 \vee \neg P_3 \vee P_5$	(6)	$\neg P_2 \vee \neg P_5 \vee \neg P_2$	(9)	$\neg P_2 \vee P_5$
(a)		(k)			
	$\neg P_4 \vee \neg P_2$	(8, 9, P_2)		$\neg P_1 \vee \neg P_4$	$(8, j, P_2)$
(b)		(l)			
	$P_1 \vee \neg P_2 \vee P_3$	(4, a, P_4)		$\neg P_1 \vee P_2 \vee \neg P_5$	$(1, k, P_4)$
(c)		(m)			
	$P_1 \vee P_4 \vee P_3 \vee \neg P_5$	(1, b, P_2)		$\neg P_1 \vee P_2 \vee \neg P_4$	$(5, j, P_5)$
(d)		(n)			
	$P_4 \vee P_3 \vee \neg P_5$	(7, c, P_1)		$P_2 \vee P_3 \vee \neg P_4$	$(4, m, P_1)$
(e)		(o)			
	$\neg P_2 \vee P_3 \vee P_4$	(9, d, P_5)		$\neg P_1 \vee \neg P_4 \vee P_5$	$(3, o, P_3)$
(f)		(p)			
	$\neg P_1 \vee \neg P_2 \vee P_3 \vee P_5$	(5, e, P_4)		$\neg P_1 \vee \neg P_2 \vee \neg P_4$	$(6, p, P_5)$
(g)		(q)			
	$\neg P_2 \vee P_3 \vee \neg P_4 \vee P_5$	(2, e, P_1)		$\neg P_2 \vee P_3 \vee \neg P_4$	$(4, p, P_1)$
(h)		(r)			
	$\neg P_1 \vee \neg P_2 \vee \neg P_4 \vee P_5$	(3, h, P_3)		$P_3 \vee \neg P_4$	$(8, q, P_2)$
(i)		(s)			
	$\neg P_1 \vee \neg P_4 \vee P_5$	(9, h, P_2)		$\neg P_1 \vee \neg P_4 \vee P_5$	$(3, s, P_3)$
(j)		(t)			
	$\neg P_1 \vee \neg P_2 \vee \neg P_4$	(6, i, P_5)		$\neg P_1 \vee \neg P_2 \vee \neg P_4$	$(6, t, P_5)$

$$4. A = ((P \rightarrow \neg R) \leftrightarrow (Q \wedge \neg P \wedge R)) \rightarrow S. \quad P < Q < R < S$$

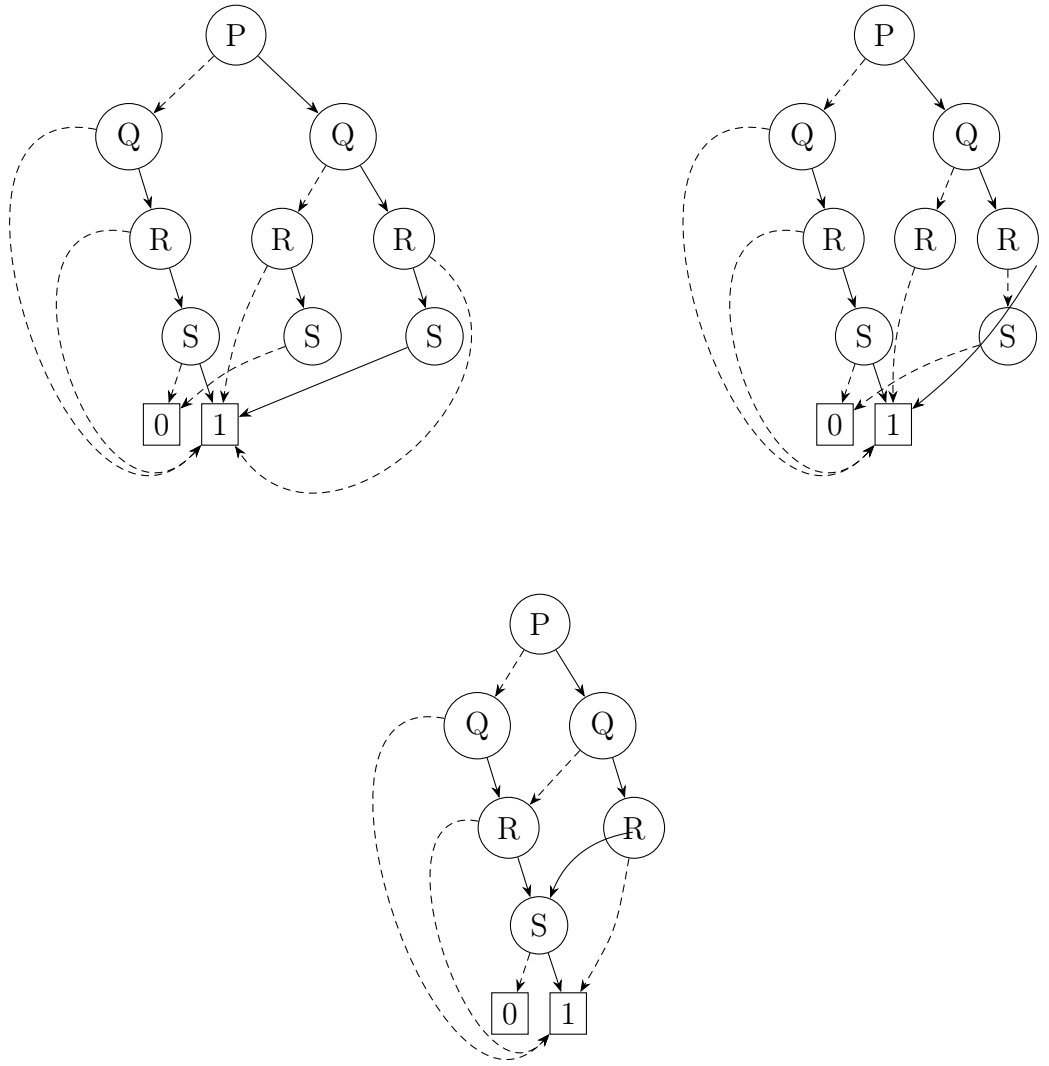
Shannon:

$$\begin{aligned}
A[1/P] &= ((1 \rightarrow \neg R) \leftrightarrow (Q \wedge \neg 1 \wedge R)) \rightarrow S = ((1 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S \\
A[0/P] &= ((0 \rightarrow \neg R) \leftrightarrow (Q \wedge \neg 0 \wedge R)) \rightarrow S = (1 \leftrightarrow (Q \wedge R)) \rightarrow S \\
A &= (P \Rightarrow A[1/P], A[0/P]) \\
A_1[1/Q] &= ((1 \rightarrow \neg R) \leftrightarrow (1 \wedge \neg 1 \wedge R)) \rightarrow S = ((1 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S \\
A_1[0/Q] &= ((1 \rightarrow \neg R) \leftrightarrow (0 \wedge \neg 1 \wedge R)) \rightarrow S = ((1 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S \\
A_1 &= (Q \Rightarrow A_1[1/Q], A_1[0/Q]) \\
A_0[1/Q] &= ((0 \rightarrow \neg R) \leftrightarrow (1 \wedge \neg 0 \wedge R)) \rightarrow S = (1 \leftrightarrow R) \rightarrow S \\
A_0[0/Q] &= ((0 \rightarrow \neg R) \leftrightarrow (0 \wedge \neg 0 \wedge R)) \rightarrow S = ((0 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S = 1 \\
A_1 &= (Q \Rightarrow A_1[1/Q], A_1[0/Q]) \\
A_{11}[1/R] &= ((1 \rightarrow \neg 1) \leftrightarrow (1 \wedge \neg 1 \wedge 1)) \rightarrow S = S \\
A_{11}[0/R] &= ((1 \rightarrow \neg 0) \leftrightarrow (1 \wedge \neg 1 \wedge 0)) \rightarrow S = 1 \\
A_{10}[1/R] &= ((1 \rightarrow \neg 1) \leftrightarrow (0 \wedge \neg 1 \wedge 1)) \rightarrow S = S \\
A_{10}[0/R] &= ((1 \rightarrow \neg 0) \leftrightarrow (0 \wedge \neg 1 \wedge 0)) \rightarrow S = 1 \\
A_{11} &= (R \Rightarrow A_{11}[1/R], A_{11}[0/R]) \\
A_{10} &= (R \Rightarrow A_{10}[1/R], A_{10}[0/R]) \\
A_{01}[1/R] &= ((0 \rightarrow \neg 1) \leftrightarrow (1 \wedge \neg 0 \wedge 1)) \rightarrow S = S \\
A_{01}[0/R] &= ((0 \rightarrow \neg 0) \leftrightarrow (1 \wedge \neg 0 \wedge 0)) \rightarrow S = 1 \\
A_{00}[1/R] &= ((0 \rightarrow \neg 1) \leftrightarrow (0 \wedge \neg 0 \wedge 1)) \rightarrow S = 1 \\
A_{00}[0/R] &= ((0 \rightarrow \neg 0) \leftrightarrow (0 \wedge \neg 0 \wedge 0)) \rightarrow S = 1 \\
A_{01} &= (R \Rightarrow A_{01}[1/R], A_{01}[0/R]) \\
A_{00} &= (R \Rightarrow A_{00}[1/R], A_{00}[0/R]) \\
A_{111}[1/S] &= 1, A_{111}[0/S] = 0, A_{110}[1/S] = 1, A_{110}[0/S] = 1 \\
A_{101}[1/S] &= 1, A_{101}[0/S] = 0, A_{100}[1/S] = 1, A_{100}[0/S] = 1 \\
A_{011}[1/S] &= 1, A_{011}[0/S] = 0, A_{010}[1/S] = 1, A_{010}[0/S] = 1 \\
A_{001}[1/S] &= 1, A_{001}[0/S] = 1, A_{000}[1/S] = 1, A_{000}[0/S] = 1
\end{aligned}$$

BDT:



ROBDD:



$$\mathcal{T}(A) = \{P, Q, R, S\}, |\mathcal{T}(A)| = 4$$

$$A : \{\neg P, \neg Q, R, S\}, \{\neg P, Q, R, S\}, \{P, \neg Q, R, S\}, \{P, Q, R, S\}$$

$$(P \Rightarrow (Q \Rightarrow 1, (R \Rightarrow 1, (S \Rightarrow 1, 0))), (Q \Rightarrow (R \Rightarrow 1, (S \Rightarrow 1, 0)), (R \Rightarrow 1, (S \Rightarrow 1, 0))))$$

$$=(P \Rightarrow (Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0))))$$

- ordered

$$=(P \Rightarrow (Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0), (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0))),$$

$$(Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0), (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0))))$$

- full unordered

$$=(P \Rightarrow (Q \Rightarrow 1, (R \Rightarrow 1, (S \Rightarrow 1, 0))))$$

- full ordered

5. A - Serhiy and Boris are about the same age, B - Serhiy is older than Boris, C - Nadiya and Boris are of different age, D - Boris is older than Fedor

$$\begin{aligned} T &= (A \vee B) \wedge (A \rightarrow C) \wedge (B \rightarrow D) \wedge (C \rightarrow D) = \\ &= (A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D) = \\ &= (\neg A \vee C) \wedge (A \vee B) \wedge D \end{aligned}$$

$$W = \{A \vee B, \neg A \vee C, \neg B \vee D, \neg C \vee D\}$$

DPLL:

SPLIT (A)

$$W_{11} = \{B, \neg B \vee D, \neg C \vee D\}$$

$$W_{12} = \{C, \neg B \vee D, \neg C \vee D\}$$

UNIT ($B \in W_{11}, C \in W_{12}$)

$$W_{21} = \{D, \neg C \vee D\}$$

$$W_{22} = \{\neg B \vee D, D\}$$

UNIT ($D \in W_{21}, D \in W_{22}$)

$$W_{31} = \{\neg C\}$$

$$W_{32} = \{\neg B\}$$

UNIT ($C \in W_{21}, B \in W_{22}$)

$$W_{41} = \emptyset$$

$$W_{42} = \emptyset$$

W - unsatisfiable

Resolution:

$$(1) \ A \vee B$$

$$(2) \ \neg A \vee C$$

$$(3) \ \neg B \vee D$$

$$(4) \ \neg C \vee D$$

(a)

(c)

$$B \vee C \quad (1, 2, A)$$

$$A \vee D \quad (1, 3, B)$$

(b)

(d)

$$\neg A \vee D \quad (2, 4, C)$$

$$D \quad (b, c, A)$$

Shannon:

$$T = (A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D), \quad A < B < C < D$$

$$T[1/A] = (1 \vee B) \wedge (\neg 1 \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D)$$

$$T[0/A] = (0 \vee B) \wedge (\neg 0 \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D)$$

$$T = (A \Rightarrow T[1/A], T[0/A])$$

$$T_0[1/B] = (1 \vee 1) \wedge (\neg 1 \vee C) \wedge (\neg 1 \vee D) \wedge (\neg C \vee D)$$

$$T_0[0/B] = (1 \vee 0) \wedge (\neg 1 \vee C) \wedge (\neg 0 \vee D) \wedge (\neg C \vee D)$$

$$T_0 = (B \Rightarrow T_0[1/B], T_0[0/B])$$

$$T_1[1/B] = (0 \vee 1) \wedge (\neg 0 \vee C) \wedge (\neg 1 \vee D) \wedge (\neg C \vee D)$$

$$T_1[0/B] = (0 \vee 0) \wedge (\neg 0 \vee C) \wedge (\neg 0 \vee D) \wedge (\neg C \vee D) = 0$$

$$T_1 = (B \Rightarrow T_1[1/B], T_1[0/B])$$

$$T_{00}[1/C] = (1 \vee 1) \wedge (\neg 1 \vee 1) \wedge (\neg 1 \vee D) \wedge (\neg 1 \vee D) = D$$

$$T_{00}[0/C] = (1 \vee 1) \wedge (\neg 1 \vee 0) \wedge (\neg 1 \vee D) \wedge (\neg 0 \vee D) = 0$$

$$T_{01}[1/C] = (1 \vee 0) \wedge (\neg 1 \vee 1) \wedge (\neg 0 \vee D) \wedge (\neg 1 \vee D) = D$$

$$T_{01}[0/C] = (1 \vee 0) \wedge (\neg 1 \vee 0) \wedge (\neg 0 \vee D) \wedge (\neg 0 \vee D) = 0$$

$$T_{00} = (C \Rightarrow T_{00}[1/C], T_{00}[0/C])$$

$$T_{01} = (C \Rightarrow T_{01}[1/C], T_{01}[0/C])$$

$$T_{10}[1/C] = (0 \vee 1) \wedge (\neg 0 \vee 1) \wedge (\neg 1 \vee D) \wedge (\neg 1 \vee D) = D$$

$$T_{10}[0/C] = (0 \vee 1) \wedge (\neg 0 \vee 0) \wedge (\neg 1 \vee D) \wedge (\neg 0 \vee D) = D$$

$$T_{11}[1/C] = (0 \vee 0) \wedge (\neg 0 \vee 1) \wedge (\neg 0 \vee D) \wedge (\neg 1 \vee D) = 0$$

$$T_{11}[0/C] = (0 \vee 0) \wedge (\neg 0 \vee 0) \wedge (\neg 0 \vee D) \wedge (\neg 0 \vee D) = 0$$

$$T_{10} = (C \Rightarrow T_{10}[1/C], T_{10}[0/C])$$

$$T_{11} = (C \Rightarrow T_{11}[1/C], T_{11}[0/C])$$

$$T_{000}[1/D] = 1, \quad T_{000}[0/D] = 0, \quad T_{001}[1/D] = 0, \quad T_{001}[0/D] = 0$$

$$T_{010}[1/D] = 1, \quad T_{010}[0/D] = 0, \quad T_{011}[1/D] = 0, \quad T_{011}[0/D] = 0$$

$$T_{100}[1/D] = 1, \quad T_{100}[0/D] = 0, \quad T_{101}[1/D] = 1, \quad T_{101}[0/D] = 0$$

$$T_{101}[1/D] = 0, \quad T_{101}[0/D] = 0, \quad T_{111}[1/D] = 0, \quad T_{111}[0/D] = 0$$