Домашня контрольна робота

Варіант №23

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Дискретна математика 2

ФІ-12

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1.
$$494x + 627y = 133$$

$\gcd(494, 627):$
$627 = 494 \cdot 1 + 133$
$494 = 133 \cdot 3 + 95$
$133 = 95 \cdot 1 + 38$
$95 = 38 \cdot 2 + 19$
$38 = 19 \cdot 2 + 0 \Rightarrow 494 \cdot 14 - 11 \cdot 627 =$ $\gcd(494, 627) = 1919$

q_i		1	3	1	2	
u_i	0	1	-1	4	-5	14
v_i	1	0	1	-3	4	-11

$26x + 33y = 7$

$\gcd(33, 26):$
$33 = 26 \cdot 1 + 7$
$26 = 7 \cdot 3 + 5$
$7 = 5 \cdot 1 + 2$
$5 = 2 \cdot 2 + 1$
$2 = 1 \cdot 2 + 0 \Rightarrow 26 \cdot 14 - 11 \cdot 33 = 1$ $\gcd(33, 26) = 1$

q_i		1	3	1	2	
u_i	0	1	-1	4	-5	14
v_i	1	0	1	-3	4	-11

$(x_0, y_0) : (98, -77)$

$$\begin{cases} x = 98 + 33k \\ y = -77 - 26k \end{cases} \Rightarrow 494 \cdot 98 - 77 \cdot 627 = 133. \ 26 \cdot 98 - 77 \cdot 33 = 7$$

2.
$$\begin{cases} x \equiv 12 \ (\mod 29) \\ x \equiv 6 \ (\mod 43) \\ x \equiv 76 \ (\mod 78) \end{cases}$$

$N = 97266, \ N_1 = 3354, \ N_2 = 2262, \ N_3 = 1247$

$$\gcd(29, 3354):$$
$$3354 = 29 \cdot 115 + 19$$
$$29 = 19 \cdot 1 + 10$$
$$19 = 10 \cdot 1 + 9$$
$$10 = 9 \cdot 1 + 1$$
$$9 = 1 \cdot 9 + 0$$

$$\gcd(43, 2262):$$
$$2262 = 43 \cdot 52 + 26$$
$$43 = 26 \cdot 1 + 17$$
$$26 = 17 \cdot 1 + 9$$
$$17 = 9 \cdot 1 + 8$$
$$9 = 8 \cdot 1 + 1$$
$$8 = 1 \cdot 8 + 0$$

$$\gcd(78, 1247):$$
$$1247 = 78 \cdot 15 + 77$$
$$78 = 77 \cdot 1 + 1$$
$$77 = 1 \cdot 77 + 0$$

q_i		115	1	1	1	
u_i	0	1	-115	116	-231	347
v_i	1	0	1	-1	2	-3

q_i		15	1	
u_i	0	1	-15	16
v_i	1	0	1	-1

q_i		52	1	1	1	1	
u_i	0	1	-52	53	-105	158	-263
v_i	1	0	1	-1	2	-3	5

n_i	29	43	78
N_i	3354	2262	1247
M_i	-3	5	-1

$$x_0 = (12 \cdot 3354 \cdot (-3) + 6 \cdot 2262 \cdot 5 + 76 \cdot 1247 \cdot (-1))(\mod 97266) \equiv -147656(\mod 97266) \equiv$$
$$\equiv 46876$$

$$\begin{cases} 46876 \equiv 12 \ (\mod 29) \\ 46876 \equiv 6 \ (\mod 43) \\ 46876 \equiv 76 \ (\mod 78) \end{cases}$$

3.
$$387^{1202} \mod 775$$

$$\gcd(775, 387):$$
$$775 = 387 \cdot 2 + 1$$
$$387 = 1 \cdot 387 + 0 \Rightarrow -2 \cdot 387 + 1 \cdot 775 = 1 \qquad\qquad \gcd(387, 775) = 1$$

q_i		2	
u_i	0	1	-2
v_i	1	0	1

$$\varphi(775) = \varphi(5 \cdot 5 \cdot 31) = 775 \cdot \left(1 - \frac{1}{5}\right) \cdot \left(1 - \frac{1}{31}\right) = 600$$
$$387^{1202} \equiv 387^{600 \cdot 2} \cdot 387^2(\mod 775) \equiv 194$$

4. (a)

$$256 \equiv 13 \ (\mod 540)$$

gcd(256, 540) :
$540 = 256 \cdot 2 + 28$
$256 = 28 \cdot 9 + 4$
$28 = 4 \cdot 7 + 0$ $\hspace{4cm}$ gcd(256, 540) = 4

$\Rightarrow$ no solutions

(b)

$$379x \equiv 552 \ (\mod 914)$$

gcd(379, 914) :
$914 = 379 \cdot 2 + 156$
$379 = 156 \cdot 2 + 67$
$156 = 67 \cdot 2 + 22$
$67 = 22 \cdot 3 + 1$
$22 = 1 \cdot 22 + 0$ $\hspace{4cm}$ gcd(379, 914) = 1

q_i		2	2	2	3	
u_i	0	1	-2	5	-12	41
v_i	1	0	1	-2	5	-17

$\Rightarrow$1 solution
$a^{-1} \equiv 41 \ (\mod 914)$
$x \equiv 41 \cdot 552 \ (\mod 914) \equiv 696$
$379 \cdot 696 \equiv 552 \ (\mod 914)$

5.

$$189x \equiv 195 \ (\mod 879)$$

gcd(189, 879) :
$879 = 189 \cdot 4 + 123$
$189 = 123 \cdot 1 + 66$
$123 = 66 \cdot 1 + 57$
$66 = 57 \cdot 1 + 9$
$57 = 9 \cdot 6 + 3$
$9 = 3 \cdot 3 + 0$ $\hspace{4cm}$ gcd(189, 879) = 3

q_i		4	1	1	1	6	
u_i	0	1	-4	5	-9	14	-93
v_i	1	0	1	-1	2	-3	20

$\Rightarrow$ 3 solutions

3

$\gcd(63, 293) :$
$293 = 63 \cdot 4 + 41$
$63 = 41 \cdot 1 + 22$
$41 = 22 \cdot 1 + 19$
$22 = 19 \cdot 1 + 3$
$19 = 3 \cdot 6 + 1$
$3 = 1 \cdot 3 + 0 \Rightarrow -93 \cdot 63 + 20 \cdot 293 = 1$ $\hspace{3cm}$ $\gcd(63, 293) = 1$

q_i		4	1	1	1	6	
u_i	0	1	-4	5	-9	14	-93
v_i	1	0	1	-1	2	-3	20

$a^{-1} \equiv 200 \ (\mod 293)$
$x_0 \equiv 200 \cdot 65 \ (\mod 293) \equiv 108, \ 189 \cdot 108 \equiv 195 \ (\mod 879)$
$x \equiv x_0 + \dfrac{1 \cdot n}{\gcd(a, n)} \equiv 401, \ 189 \cdot 401 \equiv 195 \ (\mod 879)$
$x \equiv x_0 + \dfrac{2 \cdot n}{\gcd(a, n)} \equiv 694, \ 189 \cdot 694 \equiv 195 \ (\mod 879)$