ДОМАШНЯ РОБОТА №3
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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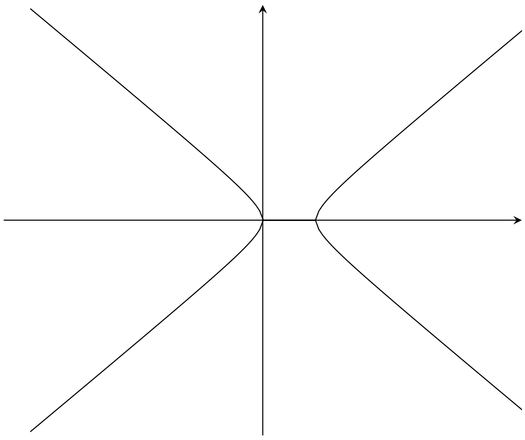
1. (a) $\mathfrak{Re}(z^2 - \bar{z}) = 0$

$z^2 - \bar{z} = (x + iy)^2 - (x - iy) = x^2 - y^2 - x + i(2xy - y), \quad \mathfrak{Re}(z^2 - \bar{z}) = x^2 - y^2 - x,$

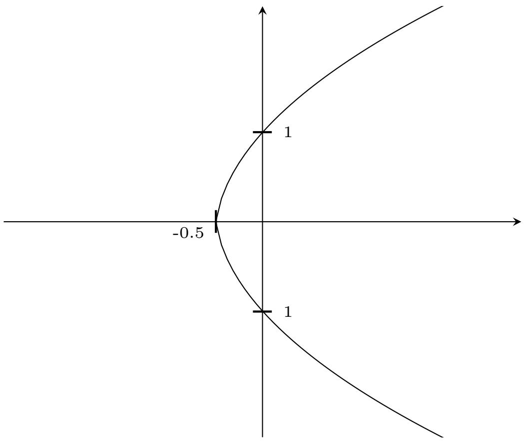
$x^2 - y^2 - x = 0, \quad y = \pm\sqrt{x^2 - x}$



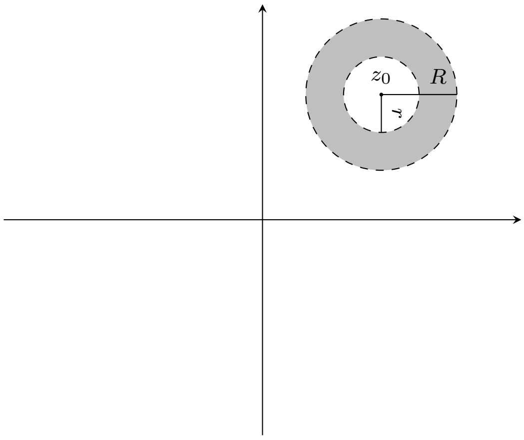
(b) $\mathfrak{Re}(1 + z) - |z| = 0$

$\mathfrak{Re}(1 + z) - |z| = \mathfrak{Re}(1 + x + iy) - \sqrt{x^2 + y^2} = 0, \quad (1 + x)^2 = x^2 - y^2,$

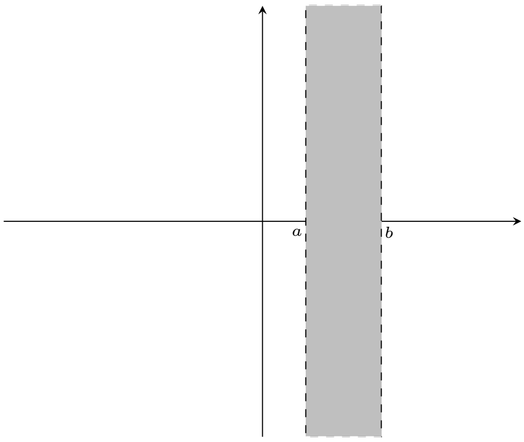
$1 + 2x + x^2 = x^2 + y^2, \quad y = \pm\sqrt{1 + 2x}$



(c) $r < |z - z_0| < R$



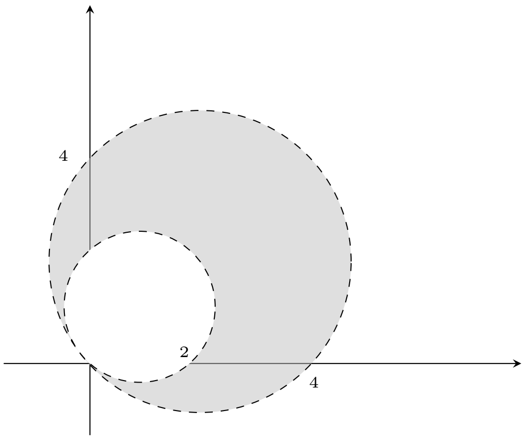
(d) $a < \mathfrak{Re} z < b$



(e) $\dfrac{1}{4} < \mathfrak{Re}\dfrac{1}{\bar{z}} + \mathfrak{Im}\dfrac{1}{\bar{z}} < \dfrac{1}{2}$

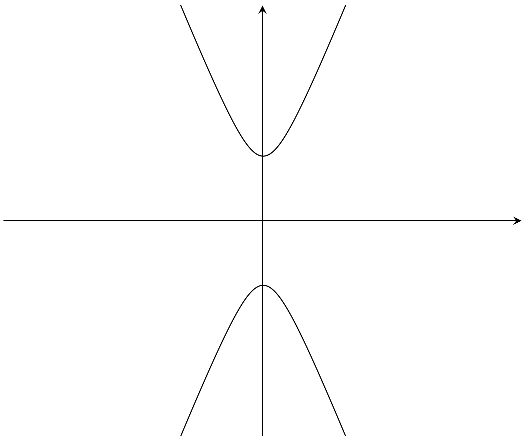
$\dfrac{1}{\bar{z}} = \dfrac{1}{x-iy} = \dfrac{x}{x^2+y^2} + \dfrac{iy}{x^2+y^2}, \quad \dfrac{1}{4} < \dfrac{x}{x^2+y^2} + \dfrac{y}{x^2+y^2} < \dfrac{1}{2},$

$$\begin{cases} \dfrac{x+y}{x^2+y^2} > \dfrac{1}{4} \\ \dfrac{x+y}{x^2+y^2} < \dfrac{1}{2} \end{cases} \qquad \begin{cases} 0 > (x^2-4x+4)+(y^2-4y+4)-8 \\ 0 < (x^2-2x+1)+(y^2-2y+1)-2 \end{cases} \qquad \begin{cases} 8 > (x-2)^2+(y-2)^2 \\ 2 < (x-1)^2+(y-1)^2 \end{cases}$$



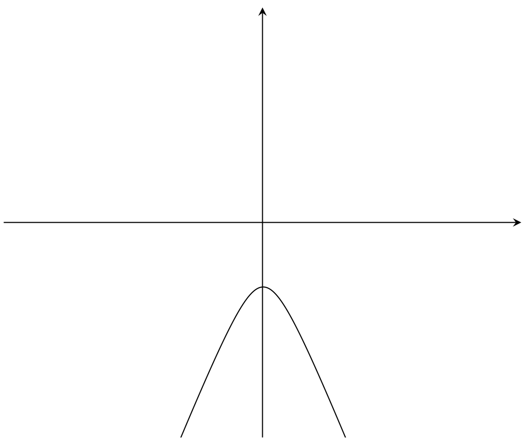
2. (a) $z = -\operatorname{ctg} t + i3 \operatorname{cosec} t$

$$\begin{cases} x = -\dfrac{\cos t}{\sin t} \\ \dfrac{y}{3} = \dfrac{1}{\sin t} \end{cases} , \quad \dfrac{y^2}{9} - x^2 = \dfrac{1-\cos^2 t}{\sin^2 t} = 1$$



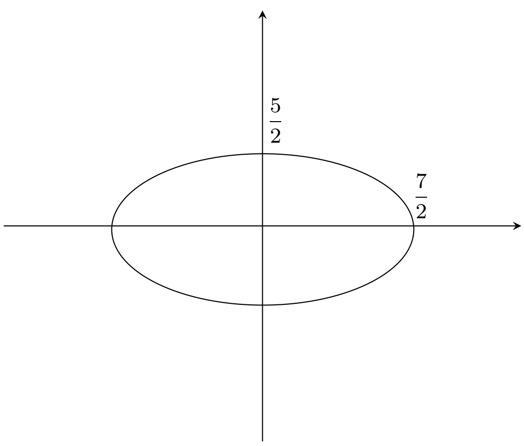
(b) $z = -2\operatorname{sh} 5t - i5\operatorname{ch} 5t$

$$\begin{cases} x = -2\operatorname{sh} 5t \\ y = -5\operatorname{ch} 5t \end{cases}, \quad \begin{cases} \dfrac{y^2}{25} - \dfrac{x^2}{4} = \operatorname{ch}^2 t - \operatorname{sh}^2 t = 1 \\ y < 0 \end{cases}$$



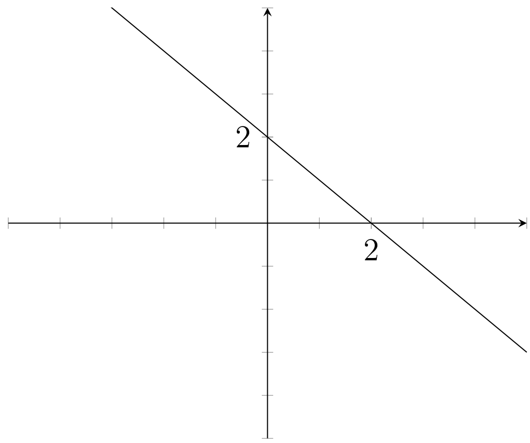
(c) $z = 3e^{it} - \dfrac{1}{2^{it}} = 3(\cos t + i\sin t) - \dfrac{1}{2}(\cos t - i\sin t) = \dfrac{5}{2}\cos t + \dfrac{7}{2} i\sin t$

$$\begin{cases} x = \dfrac{5}{2}\cos t \\ y = \dfrac{7}{2}\sin t \end{cases}$$



(d) $z = \frac{1+i}{1-t} + \frac{t}{1-t}(2-4i) = \frac{1+i+2t-4ti}{1-t} = \frac{1+2t}{1-t} + \frac{i(1-4t)}{1-t}$

$$\begin{cases} x = \frac{1+2t}{1-t} \\ y = \frac{1-4t}{1-t} \end{cases}, \quad x + y = \frac{1+2t+1-4t}{1-t} = 2$$



(e) $z = t^2 + 4t + 20 - i(t^2 + 4t + 4)$

$$\begin{cases} x = t^2 + 4t + 20 \\ y = -(t^2 + 4t + 4) \end{cases}, \quad \begin{cases} x + y = (t+2)^2 - (t+2)^2 + 16 = 16 \\ y < 0 \end{cases}$$