ДОМАШНЯ РОБОТА №4
З ПРЕДМЕТУ
”ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ”

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1. (a) $\sin\left(\frac{\pi}{6}-3i\right)=\frac{1}{2i}\left(e^{i\cdot\left(\frac{\pi}{6}-3i\right)}-e^{-i\cdot\left(\frac{\pi}{6}-3i\right)}\right)=\frac{1}{2i}\left(e^{\left(i\frac{\pi}{6}+3\right)}-e^{\left(-i\frac{\pi}{6}-3\right)}\right)=\sin\frac{\pi}{6}\cos 3i-$
$-\cos\frac{\pi}{6}\sin 3i=\frac{1}{2}\operatorname{ch}3-\frac{\sqrt{3}}{2}i\operatorname{sh}3$

(b) $\cos\left(\frac{\pi}{3}+3i\right)\frac{1}{2}\left(e^{i\cdot\left(\frac{\pi}{3}+3i\right)}-e^{-i\cdot\left(\frac{\pi}{3}+3i\right)}\right)=\frac{1}{2}\left(e^{\left(i\frac{\pi}{3}-3\right)}-e^{\left(-i\frac{\pi}{3}+3\right)}\right)=\cos\frac{\pi}{3}\cos 3i-$
$-\sin\frac{\pi}{3}\sin 3i=\frac{1}{2}\operatorname{ch}3-\frac{\sqrt{3}}{2}i\operatorname{sh}3$

(c) $\operatorname{Arcsin} i=-i\ \operatorname{Ln}(i\cdot i\pm\sqrt{1-i^2})=-i\ \operatorname{Ln}(-1\pm\sqrt{2})=$
$=\begin{cases}-i(\ln|-1-\sqrt{2}|+i(\pi+2\pi k))\\-i(\ln|-1-\sqrt{2}|+i2\pi k)\end{cases}=\begin{cases}-i\ln(\ln+\sqrt{2})+\pi+2\pi k\\-i\ln(-1+\sqrt{2})+2\pi k\end{cases},k\in\mathbb{Z}$

(d) $\operatorname{Arccos}2=-i\ \operatorname{Ln}(2\pm\sqrt{4-1})=-i\ \operatorname{Ln}(2\pm\sqrt{3})=-i\left(\ln(2\pm\sqrt{3})+2i\pi k\right)=$
$=-i\ln(2\pm\sqrt{3})+2\pi k,k\in\mathbb{Z}$

(e) $\operatorname{Arctan}(1+2i)=-\frac{i}{2}\ \operatorname{Ln}\frac{1+i(1+2i)}{1-i(1+2i)}=-\frac{i}{2}\ \operatorname{Ln}\frac{i-1}{3-i}=-\frac{i}{2}\ \operatorname{Ln}\frac{-3-i+3i-1}{10}=$
$=-\frac{i}{2}\ \operatorname{Ln}\frac{i-2}{5}=-\frac{i}{2}\left(\ln\frac{1}{\sqrt{5}}+i\left(\arctan-\frac{1}{2}+\pi+2\pi k\right)\right)=\frac{1}{2}\left(\arctan-\frac{1}{2}+\pi+2\pi k\right)-$
$-\frac{i}{2}\ln\frac{1}{\sqrt{5}},k\in\mathbb{Z}$

(f) $\operatorname{Arth}(1-i)=\frac{1}{2}\ \operatorname{Ln}\frac{1+(1-i)}{1-(1-i)}=\frac{1}{2}\ \operatorname{Ln}\frac{2-i}{i}=\frac{1}{2}\ \operatorname{Ln}(-1-2i)=\frac{1}{2}\left(\ln\sqrt{5}+\right.$
$\left.+i\left(\arctan 2-\pi+2\pi k\right)\right)=\frac{1}{2}\ln\sqrt{5}+\frac{i}{2}(\arctan 2-\pi+2\pi k),k\in\mathbb{Z}$

(g) $\operatorname{Arch}2i=\ \operatorname{Ln}(2i\pm\sqrt{-4-1})=\ \operatorname{Ln}(2i\pm\sqrt{-5}=\ \operatorname{Ln}(i|2\pm\sqrt{5}|)=$
$=\begin{cases}\ln(2+\sqrt{5})+i\left(\frac{\pi}{2}+2\pi k\right)\\\ln(-2+\sqrt{5})+i\left(\frac{\pi}{2}+2\pi k\right)\end{cases}$

(h) $\operatorname{Ln}(-i)=\ln 1+i\left(-\frac{\pi}{2}+2\pi k\right)=i\left(-\frac{\pi}{2}+2\pi k\right)$

2. (a) $i^{1+i}=\exp((1+i)\ \operatorname{Ln} i)=\exp\left((1+i)\cdot\left(\ln 1+i\frac{\pi}{2}+2\pi k\right)\right)=\exp\left(2\pi k-\frac{\pi}{2}+\right.$
$\left.+i\left(\frac{\pi}{2}+2\pi k\right)\right)=e^{2\pi k-\frac{\pi}{2}}\left(\cos\left(\frac{\pi}{2}+2\pi k\right)+i\sin\left(\frac{\pi}{2}+2\pi k\right)\right)=e^{2\pi k-\frac{\pi}{2}}(\sin(2\pi k)+$
$+i\cos(2\pi k))=e^{2\pi k-\frac{\pi}{2}}\cdot i,k\in\mathbb{Z}$

(b) $(1+i)^i=\exp(i\ \operatorname{Ln}(1+i))=\exp\left(i\ln\sqrt{2}+i\cdot i\left(\arctan 1+2\pi k\right)\right)=\exp(-\arctan 1-$
$-2\pi k+i\ln\sqrt{2})=e^{-\arctan 1-2\pi k}(\cos\ln\sqrt{2}+i\sin\ln\sqrt{2})=e^{-\frac{\pi}{4}-2\pi k}(\cos\ln\sqrt{2}+i\sin\ln\sqrt{2})$

(c) $3^i=\exp(i\ \operatorname{Ln}3)=\exp(i(\ln 3+2\pi ki))=\exp(-2\pi k+i\ln 3)=e^{-2\pi k}(\cos\ln 3+$
$+i\sin\ln 3)$

(d) $2^{1+i}=\exp((1+i)\ \operatorname{Ln}2)=\exp((1+i)\cdot(\ln 2+2\pi ki))=\exp(\ln 2-2\pi k+i(2\pi k+\ln 2))=$
$=e^{\ln 2-2\pi k}(\cos(2\pi k+\ln 2)+i\sin(2\pi k+\ln 2))=e^{\ln 2-2\pi k}(\cos(\ln 2)+i\sin(\ln 2))$

(e) $(-1)^{\sqrt{3}}=\exp(\sqrt{3}\ \operatorname{Ln}(-1))=\exp(\sqrt{3}(\ln 1+i(2\pi k+2\pi)))=\exp(\sqrt{3}\cdot i(2\pi k+2\pi))=$
$=e^0(\cos\sqrt{3}\cdot(2\pi k+2\pi)+i\sin\sqrt{3}\cdot(2\pi k+2\pi))$