ДОМАШНЯ РОБОТА №14
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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1. $c_n = \dfrac{1}{2\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x)e^{-inx}\ \mathrm{d}x\ = \dfrac{1}{2\pi}\left(\displaystyle\int\limits_{-\pi}^{0} e^{-inx}\ \mathrm{d}x\ + 3\displaystyle\int\limits_{0}^{\pi} e^{-inx}\ \mathrm{d}x\right) = \dfrac{1}{2\pi}\left(\dfrac{i}{n}e^{-inx}\Big|_{-\pi}^{0} +\right.$

$$\left. + \ \frac{i}{n}e^{-3i\frac{nx}{2}}\Big|_{0}^{\pi}\right) = \frac{1}{2\pi}\left(\frac{i}{n}e^0 - \frac{i}{n}e^{i\pi n} + \frac{3i}{n}e^{-i\pi n} - \frac{3i}{n}e^0\right) = \frac{i}{2\pi n}\left(-2 - e^{i\pi n} + 3e^{-i\pi n}\right) =$$

$$= \frac{i}{2\pi n}\left(-2 - \cos(\pi n) - i\sin(\pi n) + 3\cos(\pi n) + 3i\sin(\pi n)\right) = \frac{i}{2\pi n}\left(-2 - (-1)^n + 3\cdot(-1)^n\right) =$$

$$= \frac{((-1)^n - 1)i}{\pi n}, \quad c_0 = \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} f(x)\ \mathrm{d}x\ = \frac{1}{2\pi}\int\limits_{-\pi}^{0}\ \mathrm{d}x\ + \frac{1}{2\pi}\int\limits_{0}^{\pi} 3\ \mathrm{d}x\ = \frac{1}{2\pi}(\pi + 3\pi) = 2$$

$$f(x) = 2 + \sum_{n\in\mathbb{Z}\setminus\{0\}} \frac{((-1)^n - 1)i}{\pi n}e^{inx}$$

2. $a_0 = \dfrac{2}{l}\displaystyle\int\limits_0^l f(x)\ \mathrm{d}x\ = \dfrac{2}{l}\displaystyle\int\limits_0^l |x|\ \mathrm{d}x\ = \dfrac{2}{l}\displaystyle\int\limits_0^l x\ \mathrm{d}x\ = \dfrac{1}{l}\left(l^2 - 0\right) = l$

$$a_n = \frac{2}{l}\int\limits_0^l f(x)\cos\left(\frac{\pi n x}{l}\right) = \frac{2}{l}\int\limits_0^l x\cos\left(\frac{\pi n x}{l}\right) = \left\langle \begin{matrix} u = x, & \mathrm{d}v\ = \cos\left(\dfrac{\pi n x}{l}\right) \\ \mathrm{d}u\ =\ \mathrm{d}x\ , & v = \dfrac{l}{\pi n}\sin\left(\dfrac{\pi n x}{l}\right) \end{matrix} \right\rangle =$$

$$= \frac{2}{l}\left(\frac{xl}{\pi n}\sin\left(\frac{\pi n x}{l}\right)\Big|_0^l - \frac{l}{\pi n}\int\limits_0^l \sin\left(\frac{\pi n x}{l}\right)\ \mathrm{d}x\right) = \frac{2}{l}\left(\frac{xl}{\pi n}\sin\left(\frac{\pi n x}{l}\right) + \frac{l^2}{\pi^2 n^2}\cos\left(\frac{\pi n x}{l}\right)\right)\Big|_0^l =$$

$$= \frac{2}{l}\left(\frac{l^2}{\pi n}\sin(\pi n) + \frac{l^2}{\pi^2 n^2}\cos(\pi n) - 0 - \frac{l^2}{\pi^2 n^2}\cos 0\right) = \frac{2l}{\pi^2 n^2}((-1)^n - 1)$$

$$f(x) = \frac{l}{2} + \sum_{n=1}^{\infty}\frac{2l}{\pi^2 n^2}((-1)^n - 1)\cos\left(\frac{\pi n x}{l}\right)$$

3. $a_0 = \dfrac{2}{2}\displaystyle\int\limits_0^2 f(x)\ \mathrm{d}x\ = \displaystyle\int\limits_0^1\ \mathrm{d}x\ + \displaystyle\int\limits_1^2 0\ \mathrm{d}x\ = x\big|_0^1 = 1$

$$a_n = \frac{2}{2}\int\limits_0^2 f(x)\cos\left(\frac{\pi n x}{2}\right)\ \mathrm{d}x\ = \int\limits_0^1 \cos\left(\frac{\pi n x}{2}\right)\ \mathrm{d}x + \int\limits_1^2 0\cdot\cos\left(\frac{\pi n x}{2}\right)\ \mathrm{d}x\ = \frac{2}{\pi n}\sin\left(\frac{\pi n x}{2}\right)\Big|_0^1 =$$

$$= \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right) - \frac{2}{\pi n}\sin\left(\frac{\pi n\cdot 0}{2}\right) = \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)\cos\left(\frac{\pi n x}{2}\right)$$

4. $b_n = \dfrac{2}{\pi}\displaystyle\int\limits_0^{\pi} f(x)\sin(nx)\ \mathrm{d}x\ = \dfrac{2}{\pi}\displaystyle\int\limits_0^{\pi} \operatorname{ch}x\sin(nx)\ \mathrm{d}x\ = \left\langle \begin{matrix} u = \sin(nx) & \mathrm{d}v\ = \operatorname{ch}x\ \mathrm{d}x \\ \mathrm{d}u\ = n\cos x\ \mathrm{d}x & v = \operatorname{sh}x \end{matrix} \right\rangle =$

$$= \frac{2}{\pi}\left(\operatorname{sh}x\sin(nx) - n\int\limits_0^{\pi}\operatorname{sh}x\cos x\ \mathrm{d}x\right) = \left\langle \begin{matrix} u = \cos(nx), & \mathrm{d}v\ = \operatorname{sh}x\ \mathrm{d}x \\ \mathrm{d}u\ = -n\sin x\ \mathrm{d}x\ , & v = \operatorname{ch}x \end{matrix} \right\rangle =$$

$$= \frac{2}{\pi}\left(\operatorname{sh}x\sin(nx) - n\operatorname{ch}x\cos(nx) - n^2\int\limits_0^{\pi}\operatorname{ch}x\sin(nx)\right) =$$

$$\int\limits_0^{\pi} \operatorname{ch} x \sin(nx) \ \mathrm{d}x \ + n^2 \int\limits_0^{\pi} \operatorname{ch} x \sin(nx) = \operatorname{sh} x \sin(nx) - n \operatorname{ch} x \cos(nx)$$

$$= \quad \frac{2}{\pi} \left. \frac{\operatorname{sh} x \sin(nx) - n \operatorname{ch} x \cos(nx)}{n^2+1} \right|_0^{\pi} = \frac{2}{\pi(n^2+1)} (\operatorname{sh} \pi \sin(\pi n) - n \operatorname{ch} \pi \cos(\pi n) - \operatorname{sh} 0 \sin 0 +$$

$$+ n \cdot \operatorname{ch} 0 \cos 0) = \frac{2}{\pi(n^2+1)} (n - n \operatorname{ch} \pi (-1)^n)$$

$$f(x) = \sum_{n=1}^{\infty} \left(\frac{2n}{\pi(n^2+1)} (1 - \operatorname{ch} \pi (-1)^n) \sin(nx) \right)$$