Домашня робота 3

3.13 $\int \frac{x^2\mathbf{d}x}{\sqrt{x^2-2}} = \left| \begin{array}{ll} x=\sqrt{2}\operatorname{ch}t & t=\operatorname{arcch}\frac{x}{\sqrt{2}} \\ \mathbf{d}x = \sqrt{2}\operatorname{sh}t\mathbf{d}t & \mathbf{d}t = \frac{\mathbf{d}x}{\sqrt{2}\operatorname{sh}t}\end{array}\right| = \int \frac{2\operatorname{ch}^2 t\cdot\sqrt{2}\operatorname{sh}t\mathbf{d}t}{\sqrt{2\operatorname{ch}^2t-2}} = 2\int\frac{\operatorname{ch}^2t\operatorname{sh}t\mathbf{d}t}{\sqrt{\operatorname{ch}^2t-1}} = 2\int \operatorname{ch}^2 t =$

$= \frac{\operatorname{ch}t\operatorname{sh}t}{2} + \frac{1}{2}\int 1\mathbf{d}t = \frac{1}{2}(\operatorname{ch}t\operatorname{sh}t + t) + c = \frac{1}{2}(\frac{x}{\sqrt{2}}\operatorname{sh}(\operatorname{arcch}\frac{x}{\sqrt{2}}) + \operatorname{arcch}\frac{x}{\sqrt{2}}) + c$

3.14 $\int \frac{x^2}{\sqrt{x^2+a^2}}\mathbf{d}x = \left| \begin{array}{ll} x=a\operatorname{sh}t & t=\operatorname{arcsh}\frac{x}{a} \\ \mathbf{d}x = a\operatorname{ch}t\mathbf{d}t & \mathbf{d}t = \frac{\mathbf{d}x}{a\operatorname{ch}t}\end{array}\right| = \int \frac{a^3\operatorname{sh}^2 t\operatorname{ch}t\mathbf{d}t}{\sqrt{a^2\operatorname{sh}^t+a^2}} = a^2\int \operatorname{sh}^t\mathbf{d}t = \frac{a^2}{2}(\operatorname{ch}t\operatorname{sh}t -$

$- \int 1\mathbf{d}t) = \frac{a^2}{2}(\operatorname{ch}t\operatorname{sh}t + t) + c = \frac{a^2}{2}(\operatorname{ch}(\operatorname{arcsh}\frac{x}{a}))\cdot\frac{x}{a} + \operatorname{arcsh}\frac{x}{a}$

3.15 $\int \frac{2x+3}{(x-2)(x+5)}\mathbf{d}x = \left|\frac{A}{x-2} + \frac{B}{x+5}\right| =$

$x = A(x+5) + B(x-2) = Ax + 5A + Bx - 2B$

$1\cdot x + 0 = x(A+B) + (5A-2B)$

$\begin{cases} A+B=1 \\ 5A-2B=0 \end{cases} \Rightarrow \begin{cases} A=\frac{2}{7} \\ B=\frac{5}{7} \end{cases}$

3.16 $\int \frac{x^3+1}{x^3-5x^2+6x} =$

$$\begin{array}{rrrrrrrl} & x^3 & + & x^2\cdot 0 & + & x\cdot 0 & + & 1 \;\big|\; x^3-5x+6x \\ - & x^3 & + & 5x^2 & - & 6x & & \quad 1+ \\ \hline & & & 5x^2 & - & 6x & + & 1 \end{array}$$

$= 1 + \int \frac{5x^2-6x+1}{x(x-2)(x-3)} = \left|\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-3}\right| =$

$A = \left.\frac{5x^2-6x+1}{(x-2)(x-3)}\right|_{x=0} = \frac{1}{6}$

$B = \left.\frac{5x^2-6x+1}{x(x-3)}\right|_{x=2} = -\frac{9}{2}$

$C = \left.\frac{5x^2-6x+1}{x(x-2)}\right|_{x=3} = \frac{28}{3}$

3.17 $\int \frac{\mathbf{d}x}{(x+1)(x^2+1)} = \left|\frac{A}{x+1} + \frac{Bx+c}{x^2+1}\right| =$

$1 = (x^2+1)A + (x+1)(Bx+C)$

$0\cdot x^2 + 0\cdot x + 1 = (A+B)x^2 + (C+B)x + (A+C)$

$\begin{cases} 0=A+B \\ 0=C+B \\ 1=A+C \end{cases} \Rightarrow \begin{cases} A=\frac{1}{2} \\ B=-\frac{1}{2} \\ C=\frac{1}{2} \end{cases}$

3.18 $\int \frac{\mathbf{d}x}{x(x+1)(x^2+x+1)} = \left|\frac{A}{x} + \frac{B}{x+1} + \frac{Cx+D}{x^2+x+1}\right| =$

$1 = (x+1)(x^2+x+1)A + x(x^2+x+1)B + x(x+1)C$

$0\cdot x^3 + 0\cdot x^2 + 0\cdot x + 1 = (A+B+C)x^3 + (2A+B+D+C)x^2 + (2A+B+D) + A$

$\begin{cases} 0=A+B+C \\ 0=2A+B+C+D \\ 0=2A+B+D \\ 1=A \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=0 \\ D=-1 \end{cases}$

3.19 $\int \frac{\mathbf{d}x}{x^5+x^4-2x^3-2x^2+x+1} = \int \frac{\mathbf{d}x}{(x+1)(x+1)(x+1)(x-1)(x-1)} = \left|\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{x-1} + \frac{E}{(x-1)^2}\right| =$

$1 = (x+1)^5(x-1)^3A + (x+1)^4(x-1)^3B + (x+1)^3(x-1)^3C + (x+1)^6(x-1)^2D + (x+1)^6(x-1)E$

3.00 $\int x\sqrt{x-1}\mathbf{d}x \left| \begin{array}{l} \sqrt{x}=\operatorname{ch} t \\ \mathbf{d}x = \operatorname{sh} 2t\mathbf{d}t \end{array} \right| = \int \operatorname{ch}^2 t \operatorname{sh} 2t\sqrt{\operatorname{ch}^2 t - 1}\mathbf{d}t = 2\int \operatorname{ch} t \operatorname{sh}^2 t(\operatorname{sh}^2 t + 1)\mathbf{d}t =$
$= 4\int \operatorname{sh} 2^t(\operatorname{sh}^2 t + 1)\mathbf{d}(\operatorname{sh} t) = 4(\int \operatorname{sh}^4 \mathbf{d}(\operatorname{sh} t) + \int \operatorname{sh}^2 \mathbf{d}(\operatorname{sh} t)) = 4(\frac{\operatorname{sh}^5 t}{5} + \frac{\operatorname{sh}^3 t}{3}) + c =$
$= \frac{4}{5}\operatorname{sh}^5(\operatorname{arcch}\sqrt{x}) + \frac{4}{3}\operatorname{sh}^3(\operatorname{arcch}\sqrt{x}) + c$