Домашня робота 7

5.15 $$\int \frac{\mathbf{d}x}{x^3\sqrt{x^2+1}} = \int x^{-3}(1+x^2)^{-\frac{1}{2}}\mathbf{d}x = \left| \begin{array}{l} m=-3,\ n=2,\ p=-\frac{1}{2} \\ t^2=1+x^2,\ x=\sqrt{t^2-1} \\ \mathbf{d}x = \dfrac{t}{\sqrt{t^2-1}}\,\mathbf{d}t \end{array} \right| = \int \frac{\mathbf{d}t}{\sqrt{t^2-1}^4} =$$

$$= \int (t^2-1)^{-2}\mathbf{d}t = \int \frac{1}{(t^2+1)(t^2-1)}\mathbf{d}t = \frac{1}{4}\int \left(\frac{1}{t+1} + \frac{1}{(t+1)^2} + \frac{1}{t-1} + \frac{1}{(t-1)^2} \right) =$$

$$= \frac{1}{4}\left(\ln|t+1| - \frac{1}{t+1} + \ln|t-1| - \frac{1}{t-1} \right) + c$$

5.16 $$\int x\sqrt{x^2-2+2} = \left| \begin{array}{l} \sqrt{x^2-2x+2} = t+x \\ x = -\dfrac{t^2-2}{2+2t} \\ \mathbf{d}x = \dfrac{2t^2+4t+4}{(2+2t)^2}\mathbf{d}t \end{array} \right| = \int -\frac{2t^6+6t^5+8t^4-16t^2-24t-16}{(2+2t)^4}\mathbf{d}t$$

5.17 $$\frac{x^5\mathbf{d}x}{\sqrt{1-x^2}} = \left| \begin{array}{l} m=5,\ n=2,\ p=-\frac{1}{2} \\ t^2=1-x^2 \\ \mathbf{d}x = -\dfrac{t}{\sqrt{1-t^2}}\,\mathbf{d}t \end{array} \right| = \int \sqrt{1-t^2}^4\mathbf{d}t = \int -(1-t^2)^2\mathbf{d}t = -t + \frac{2t^3}{3} - \frac{t^5}{5} =$$

$$= -\sqrt{1-x^2} + \frac{2\sqrt{1-x^2}^3}{3} - \frac{\sqrt{1-x^2}^5}{5} + c$$