# ДОМАШНЯ РОБОТА №6
# З ПРЕДМЕТУ
# "ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

ФІ-12 Бекешева Анастасія

1. (a) $z_1 = 0, \quad z_2 = 2 - i$

$$y = -\frac{1}{2}x \Rightarrow z = -\frac{1}{2}ix + x = x\left(1 - \frac{1}{2}i\right), \quad x : 0 \to 2$$

$$\mathrm{d}z = \left(1 - \frac{1}{2}i\right)\,\mathrm{d}x\,, \quad |z| = \left|x\left(1 - \frac{1}{2}i\right)\right| = \frac{\sqrt{5}}{2}|x|$$

$$\int\limits_C |z|\,\mathrm{d}z = \int\limits_0^2 \frac{\sqrt{5}}{2}\left(1 - \frac{1}{2}i\right)|x|\,\mathrm{d}x = \frac{\sqrt{5}}{2}\left(1 - \frac{1}{2}i\right)\frac{x^2}{2}\bigg|_0^2 = \sqrt{5}\left(1 - \frac{1}{2}i\right)$$

(b) $z_1 = -i, \quad z_2 = i$

$$x = 0, \quad y : -1 \to 1 \Rightarrow z = iy$$

$$\mathrm{d}z = i\,\mathrm{d}y\,, \quad |z| = |y|$$

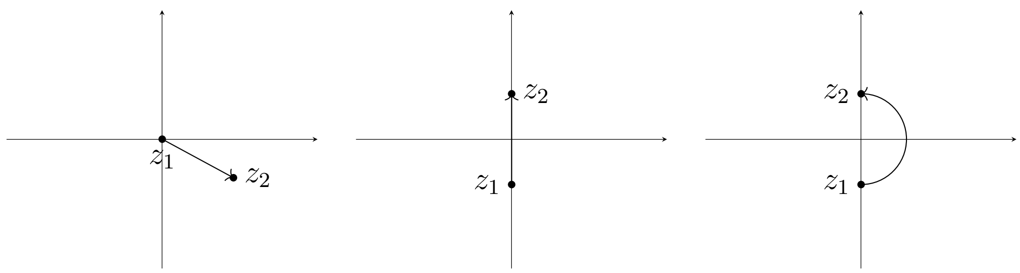
$$\int\limits_C |z|\,\mathrm{d}z = i\int\limits_{-1}^{1} |y|\,\mathrm{d}y = 2i\int\limits_0^1 y\,\mathrm{d}y = 2i \cdot \frac{y^2}{2}\bigg|_0^1 = i$$

(c) $|z| = 1, \quad -\frac{\pi}{2} \le \arg z \le \frac{\pi}{2}$ $z = z_0 + R \cdot e^{i\varphi} = 0 + 1 \cdot e^{i\varphi} = e^{i\varphi}$

$$\varphi : -\frac{\pi}{2} \to \frac{\pi}{2}, \quad \mathrm{d}z = ie^{i\varphi}\,\mathrm{d}\varphi$$

$$\int\limits_C |z|\,\mathrm{d}z = \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ie^{i\varphi}\,\mathrm{d}\varphi = e^{i\varphi}\bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = e^{i\frac{\pi}{2}} - e^{-i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} - \cos-\frac{\pi}{2} -$$

$$- i\sin-\frac{\pi}{2} = 0 + i - 0 + i = 2i$$




(a) (b) (c)

2. (a) $z_1 = 0, \quad z_2 = 1 + i$

$$y = x, \quad x : 0 \to 1, \quad z = x + ix = x(i+1), \quad \mathrm{d}z = (1+i)\,\mathrm{d}x$$

$$\int\limits_C (x - y + ix^2)\,\mathrm{d}z = \int\limits_0^1 (x - x + ix^2)\cdot(1+i)\,\mathrm{d}x = \int\limits_0^1 x^2(i-1)\,\mathrm{d}x =$$

$$= (i-1)\frac{x^3}{3}\bigg|_0^1 = \frac{1}{3}(i-1)$$

(b) $y = 0 : \quad x : 0 \to 1, \quad z = x, \quad \mathrm{d}z = \mathrm{d}x\,,$

$x = 1 : \quad y : 0 \to 1, \quad z = 1 + iy, \quad \mathrm{d}z = i\,\mathrm{d}y\,,$

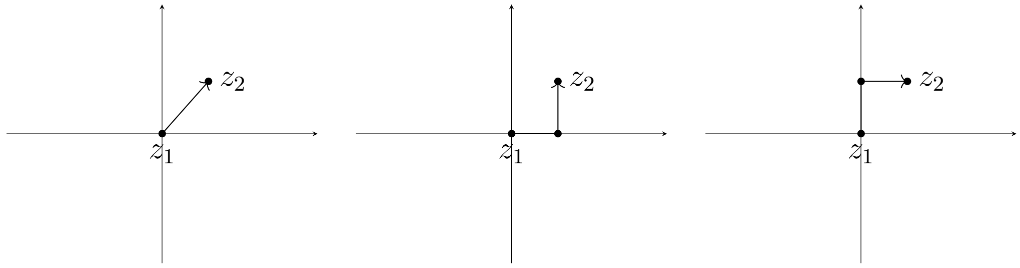
$$\int\limits_C (x - y + ix^2)\,\mathrm{d}z = \int\limits_0^1 (x + ix^2)\,\mathrm{d}x + \int\limits_0^1 (1 - y + i)i\,\mathrm{d}y = \frac{x^2}{2}\bigg|_0^1 + i\frac{x^3}{3}\bigg|_0^1 - \frac{iy^2}{2}\bigg|_0^1 +$$

$$+ (1+i)iy\bigg|_0^1 = \frac{1}{2} + i\frac{1}{3} + i - \frac{i}{2} - 1 = 1 + i\left(\frac{1}{3} + 1 - \frac{1}{2}\right) = -\frac{1}{2} + i\frac{5}{6}$$

(c) $x = 0 : \quad y : 0 \to 1, \quad z = iy, \quad \mathrm{d}z = i\,\mathrm{d}y\,,$

$y = 1 : \quad x : 0 \to 1, \quad z = x + i, \quad \mathrm{d}z = \mathrm{d}x\,,$

$$\int\limits_C (x-y+ix^2)\ \mathrm{d}z\ = \int\limits_0^1 (-iy)\ \mathrm{d}y\ + \int\limits_0^1 (x-1+ix^2)\ \mathrm{d}x\ = -i\frac{y^2}{2}\bigg|_0^1 + \frac{x^2}{2}\bigg|_0^1 - x\bigg|_0^1 +$$
$$-\frac{1}{2} - \frac{i}{6}$$




(a) (b) (c)

3. (a) $z_1, = -2, \quad z_2 = 2,$

$z = 2e^{i\varphi}, \quad \varphi : \pi \to 2\pi, \qquad \mathrm{d}z\ = 2ie^{i\varphi}\ \mathrm{d}\varphi\ , \quad |z| = 2,$

$$\int\limits_L z|z|\ \mathrm{d}z\ = \int\limits_\pi^{2\pi} 2\cdot 2e^{i\varphi}\cdot 2ie^{i\varphi}\ \mathrm{d}\varphi\ = 8i\int\limits_\pi^{2\pi} e^{2i\varphi}\ \mathrm{d}\varphi\ = -\frac{i}{2}\cdot 8i\int\limits_\pi^{2\pi} e^{2i\varphi}\ \mathrm{d}(2i\varphi)\ =$$
$$= 4e^{2i\varphi}\bigg|_\pi^{2\pi} = 4(e^{4\pi i} - e^{2\pi i}) = 4e^{2\pi i}(e^{2\pi i} - 1) = 4\cdot(\cos 2\pi + i\sin 2\pi)\cdot(\cos 2\pi + i\sin 2\pi - 1)$$
$$= 4\cdot(1 + i\cdot 0)\cdot(1 + i\cdot 0 - 1) = 4\cdot 1\cdot 0 = 0$$

(c) $z_1 = 0, \quad z_2 = \dfrac{1}{2} + i\dfrac{\sqrt{3}}{2},$

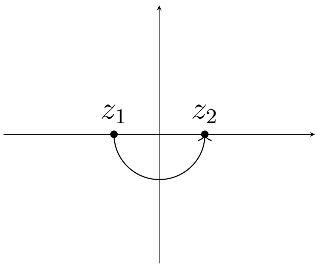
$y = \sqrt{3}x, \quad x : 0 \to \dfrac{1}{2}, \quad z = x + i\sqrt{3}x, \qquad \mathrm{d}z\ = (1 + i\sqrt{3})\ \mathrm{d}x\ ,$

$$\int\limits_L e^{|z|^2}\mathfrak{Re}z\ \mathrm{d}z\ = \int\limits_0^{\frac12} x(1+i\sqrt{3})e^{\sqrt{x^2+3x^2}^2}\ \mathrm{d}x\ = (1+i\sqrt{3})\int\limits_0^{\frac12} e^{4x^2}\ \mathrm{d}x\ = \frac{(1+i\sqrt{3})}{2}\cdot$$
$$\cdot\int\limits_0^{\frac12}\frac{1}{x}e^{x^2}\ \mathrm{d}(4x^2)\ = \frac{(1+i\sqrt{3})}{2}\cdot\frac{e^{4x^2}}{4}\bigg|_0^{\frac12} = \frac{(1+i\sqrt{3})}{8}(e-1)$$

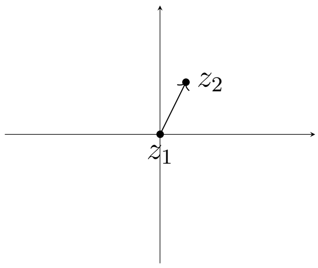
(d) $|z-1| = 1, \quad z = e^{i\varphi}, \quad \varphi : 0 \to 2\pi, \qquad \mathrm{d}z\ = ie^{i\varphi}\ \mathrm{d}\varphi\ ,$

$\mathfrak{Re}(z) = \mathfrak{Re}(e^{i\varphi}) = \mathfrak{Re}(\cos\varphi + i\sin\varphi) = \cos\varphi$

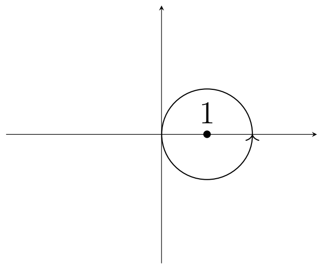
$$\oint\limits_{|z-1|=1}\mathfrak{Re}(z)\ \mathrm{d}z\ = \int\limits_0^{2\pi}\cos\varphi\cdot ie^{i\varphi}\ \mathrm{d}\varphi\ = -i\cdot i\int\limits_0^{2\pi} e^{i\varphi}\operatorname{ch} i\varphi\ \mathrm{d}(i\varphi)\ = \int\limits_0^{2\pi}\frac{1}{4}\cdot 2e^{2i\varphi}\cdot$$
$$\cdot e^{-2i\varphi}(e^{2i\varphi}+1)\ \mathrm{d}(i\varphi)\ = \left\langle t = e^{2i\varphi}, \qquad \mathrm{d}(i\varphi)\ = \frac{1}{2}e^{-2i\varphi}\ \mathrm{d}t\ \right\rangle = \frac{1}{4}\int\limits_0^{2\pi}\frac{t+1}{t}\ \mathrm{d}t\ =$$
$$= \frac{1}{4}\int\limits_0^{2\pi}\left(\frac{1}{t}+1\right)\ \mathrm{d}t\ = \frac{1}{4}\ln t + \frac{1}{4}t\bigg|_0^{2\pi} = \frac{1}{2}i\varphi + \frac{1}{4}e^{2i\varphi}\bigg|_0^{2\pi} = i\pi + \frac{1}{4}e^{4i\pi} - 0 - \frac{1}{4}e^0 =$$
$$= i\pi - \frac{1}{4} + \frac{1}{4}(\cos 4\pi + i\sin 4\pi) = i\pi - \frac{1}{4} + \frac{1}{4} = i\pi$$




(a)




(c)




(a)