ДОМАШНЯ РОБОТА №11
З ПРЕДМЕТУ
”ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ”

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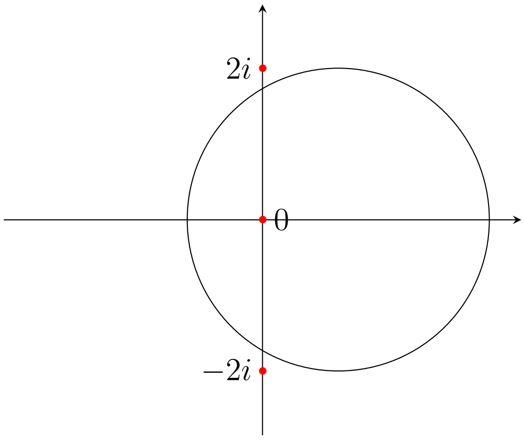
1. $\displaystyle\oint\limits_{|z-1|=2} \frac{z^2+1}{(z-2i)(z+2i)\sin\frac{z}{3}}\,\mathrm{d}z \quad =$

$z=\pm 2i, \quad \sin\dfrac{z}{3}=0, \quad z=3\pi n, \quad m_1=0, \quad m_2=1, \quad z=0$ - полюс I порядку.

$$= \quad 2\pi i\ \operatorname{Res}\frac{z^2+1}{(z-2i)(z+2i)\sin\frac{z}{3}} = 2\pi i\lim_{z\to 0}\frac{z^2+1}{(z-2i)(z+2i)\sin\frac{z}{3}}\cdot z =$$

$$= 2\pi i\lim_{z\to 0}\frac{(z^2+1)z}{(z-2i)(z+2i)\frac{z}{3}} = 2\pi i\frac{3}{4} = \frac{3}{2}\pi i$$

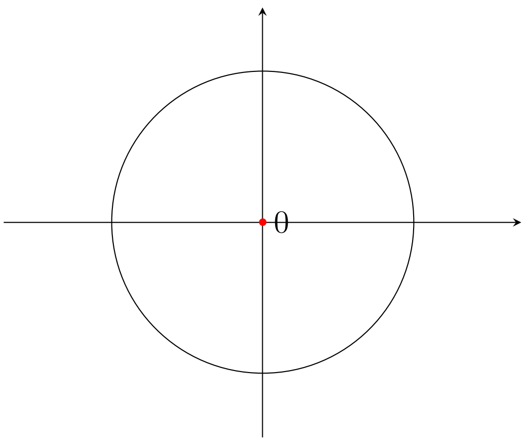


2. $\displaystyle\oint\limits_{|z|=1} \frac{z^2e^{\frac{1}{z^2}}-1}{z}\,\mathrm{d}z \quad =$

$$z=0\ (\text{знам.}), \quad f(z)=\frac{z^2e^{\frac{1}{z^2}}-1}{z} = \frac{1}{z}\left(z^2\left(1+\frac{1}{z^2}+\frac{1}{2z^4}+\dots\right)-1\right) = \frac{1}{z}\left(z^2+\frac{1}{2z^2}+\dots\right) =$$

$= z + \underbrace{\dfrac{1}{2z^3}+\dots,}_{\text{гол. час.}} \quad z=0$ - істотно особлива точка.

$$= \quad 2\pi i\ \operatorname{Res} f(z) = 2\pi i\cdot 0 = 0$$



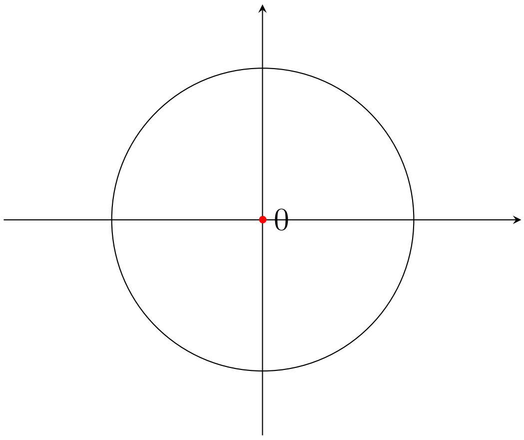
3. $\displaystyle\oint\limits_{|z|=4} \frac{\operatorname{sh} iz-\sin iz}{z^3\operatorname{sh}\frac{z}{3}}\,\mathrm{d}z \quad =$

$$z=0, \quad \operatorname{sh}\frac{z}{3}=0, \quad z=3\pi i n, \quad \operatorname{sh} iz-\sin iz = iz-\frac{iz^3}{3!}+\frac{iz^5}{5!}+\dots-iz-\frac{iz^3}{3!}-\frac{iz^5}{5!} =$$

$$-\frac{2iz^3}{3!} = z^3\left(-\frac{2i}{3!} + \dots\right), \quad z^3 \operatorname{sh}\frac{z}{3} = z^3\left(\frac{z}{3} + \frac{z^3}{3^3\cdot 3!}\dots\right) = \frac{z^4}{3}\left(\frac{1}{3} + \frac{z^2}{3^3\cdot 3!}\dots\right),$$

$$f(z) = \frac{z^3 g_1(z)}{\frac{z^4}{3} g_2(z)}, \quad m_1 = 3, \quad m_2 = 4, \quad z = 0 \text{ - полюс I порядку.}$$

$$= \quad 2\pi i \lim_{z\to 0} \frac{z^3 g_1(z)}{\frac{z^4}{3} g_2(z)} = 2\pi i \frac{\frac{2i}{3!}}{\frac{1}{3}} = 2\pi$$



4. $$\oint\limits_{|z+2i|=3} \left(\underbrace{\frac{\pi}{e^{\frac{\pi z}{2}} + 1}}_{\mathfrak{I}_1} + \underbrace{\frac{6\operatorname{ch}\frac{\pi i z}{2-2i}}{(z-2+2i)^2(z-4+2i))}}_{\mathfrak{I}_2} \right) \mathrm{d}z = \mathfrak{I}_1 + 6\mathfrak{I}_2$$

$(\mathfrak{I}_1)$ $e^{\frac{\pi z}{2}} + 1 = 0, \quad e^{\frac{\pi z}{2}} = -1, \quad \frac{\pi z}{2} = \ln(-1), \quad z_n = 2i(2n+1),$

$z = -2i(n=-1), \quad m_1 = 0, \quad m_2 = 1, \quad z = -2i$ - полюс I порядку.

$$f(z) = \frac{\pi}{e^{\frac{\pi z}{2}} + 1}, \quad \left(e^{\frac{\pi z}{2}} + 1\right)' = \frac{\pi}{2} e^{\frac{\pi z}{2}} \neq 0$$

$$\operatorname{Res} f(z) = \left.\frac{\pi}{\frac{\pi}{2} e^{\frac{\pi z}{2}}}\right|_{z=-2i} = \frac{2}{e^{-\pi i}} = -2, \quad \mathfrak{I}_1 = -4\pi i$$

$(\mathfrak{I}_2)$ $(z-2+2i)^2(z-4+2i) = 0$

$$\left[\begin{array}{l} z-2+2i = 0 \\ z-4+2i = 0 \end{array}\right. \left[\begin{array}{l} z = 2-2i \text{ (нуль знам., не нуль чис.)} \\ z = 4-2i \end{array}\right.$$

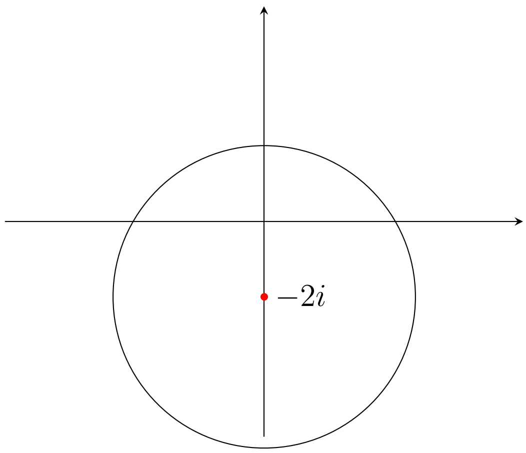
$$f(z) = \frac{\operatorname{ch}\frac{\pi i z}{2-2i}}{(z-2+2i)^2(z-4+2i))}, \quad z = 2-2i, \quad m_1 = 0, \quad m_2 = 2,$$

$z = 2-2i$ - - полюс II порядку. $\operatorname{Res} f(z) = \lim\limits_{z\to 2-2i} \frac{\partial}{\partial z}\left(f(z)(z-2+2i)^2\right) =$

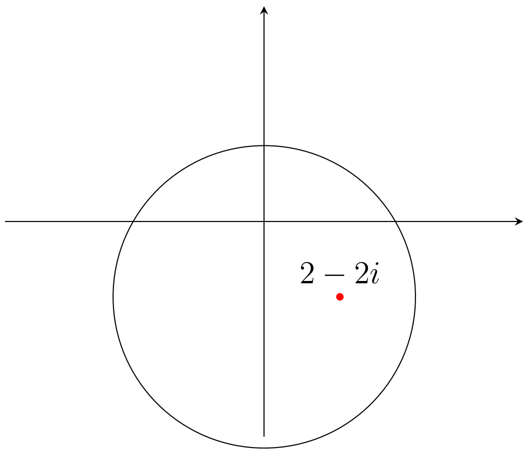
$$= \lim_{z\to 2-2i} \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\operatorname{ch}\frac{\pi i z}{2-2i}}{(z-4+2i))}\right) = \lim_{z\to 2-2i} \frac{\operatorname{sh}\frac{\pi i z}{2-2i}\cdot\frac{\pi i}{2-2i}(z-4+2i) - \operatorname{ch}\frac{\pi i z}{2-2i}}{(z-4+2i)^2} =$$

$$= \langle z = 2-2i \rangle = \lim_{z\to 2-2i} \frac{\operatorname{sh}\pi i \cdot \frac{\pi i}{2-2i} - \operatorname{ch}\pi i}{4} = \frac{1}{4}$$

$\mathfrak{I} = \mathfrak{I}_1 + 6\mathfrak{I}_2 = -4\pi i + 3\pi i = -\pi i$



$\mathfrak{I}_1$



$\mathfrak{I}_2$