

# Домашня контрольна робота

Варіант №12002

Бекешева Анастасія

Part I

ΦI-12

01.06.2022

$$1. (L_1^R L_2^R)^+ = \left( (L_1^R)^+ \cup (L_2^R)^* \right)^+$$

$$\begin{aligned} L_1^R &= \{x \in \Sigma^* \mid x^R \in L_1\} \\ L_2^R &= \{y \in \Sigma^* \mid y^R \in L_1\} \\ L_1^R L_2^R &= \{xy \in \Sigma^* \mid x^R \in L_1, y^R \in L_2^R\} \\ (L_1^R L_2^R)^+ &= \{xy \in \Sigma^* \mid \exists n \in \mathbb{N}_1, x^R \in L_1^n, y^R \in L_2^n\} \end{aligned}$$

$$\begin{aligned} L_1^R &= \{x \in \Sigma^* \mid x^R \in L_1\} \\ L_2^R &= \{y \in \Sigma^* \mid y^R \in L_1\} \\ (L_1^R)^+ &= \{x \in \Sigma^* \mid \exists n \in \mathbb{N}_1, x^R \in L_1^n\} \\ (L_2^R)^* &= \{x \in \Sigma^* \mid \exists n \in \mathbb{N}_0, x^R \in L_2^n\} \\ (L_1^R)^+ \cup (L_2^R)^* &= \{z \in \Sigma^* \mid \exists n \in \mathbb{N}_1, z^R \in L_1^n \vee z^R \in L_2^n\} \\ \left( (L_1^R)^+ \cup (L_2^R)^* \right)^+ &= \{z \in \Sigma^* \mid \exists n \in \mathbb{N}_1, z^R \in L_1^n \vee z^R \in L_2^n\} \end{aligned}$$

$$\Rightarrow (L_1^R L_2^R)^+ \neq \left( (L_1^R)^+ \cup (L_2^R)^* \right)^+$$

Let  $\Sigma = \{\alpha_1, \alpha_2, \beta_1, \beta_2\}$ ,  $L_1 = \{\alpha_1, \alpha_1\alpha_2\}$ ,  $L_2 = \{\beta_1, \beta_1\beta_2\}$ .

$$\begin{aligned} L_1^R &= \{\alpha_1, \alpha_2\alpha_1\} \\ L_2^R &= \{\beta_1, \beta_2\beta_1\} \\ L_1^R L_2^R &= \{\alpha_1\beta_1, \alpha_1\beta_2\beta_1, \alpha_2\alpha_1\beta_1, \alpha_2\alpha_2\beta_2\beta_2\} \\ (L_1^R L_2^R)^+ &= \{\alpha_1\beta_1\alpha_1\beta_1, \alpha_1\beta_2\beta_1\alpha_1\beta_2\beta_1, \alpha_2\alpha_1\beta_1\alpha_2\alpha_1\beta_1 \dots\} \end{aligned}$$

$$\begin{aligned} L_1^R &= \{\alpha_1, \alpha_2\alpha_1\} \\ L_2^R &= \{\beta_1, \beta_2\beta_1\} \\ (L_1^R)^+ &= \{\alpha_1, \alpha_2\alpha_1, \alpha_1\alpha_1, \alpha_2\alpha_1\alpha_2\alpha_1 \dots\} \\ (L_2^R)^* &= \{\varepsilon, \beta_1, \beta_2\beta_1, \beta_1\beta_1, \beta_2\beta_1\beta_2\beta_1 \dots\} \\ (L_1^R)^+ \cup (L_2^R)^* &= \{\varepsilon, \alpha_1, \alpha_2\alpha_1, \alpha_1\alpha_1, \alpha_2\alpha_1\alpha_2\alpha_1, \beta_1, \beta_2\beta_1, \beta_1\beta_1, \beta_2\beta_1\beta_2\beta_1 \dots\} \\ \left( (L_1^R)^+ \cup (L_2^R)^* \right)^+ &= \{\alpha_1, \alpha_2\alpha_1, \alpha_1\alpha_1, \alpha_1\alpha_1\alpha_1, \alpha_2\alpha_1\alpha_2\alpha_1, \beta_1, \beta_2\beta_1, \beta_1\beta_1, \beta_1\beta_1\beta_1, \beta_2\beta_1\beta_2\beta_1 \dots\} \end{aligned}$$

$$2. A = P \wedge R \rightarrow \neg(Q \leftrightarrow \neg P \wedge R)$$

$$\begin{aligned} \text{cnf}(A) &= \neg(P \wedge R) \vee \neg(Q \leftrightarrow \neg P \wedge R) = \\ &= \neg(P \wedge R) \vee \neg((Q \vee \neg(\neg P \wedge R)) \wedge (\neg Q \vee (\neg P \wedge R))) = \\ &= \neg P \vee \neg R \vee ((\neg Q \wedge \neg P \wedge R) \vee (Q \wedge (P \vee \neg R))) = \\ &= (\neg P \vee \neg R \vee Q) \wedge (\neg P \vee \neg R \vee P \vee \neg R) \vee (\neg Q \wedge \neg P \wedge R) = \\ &= (\neg P \vee \neg R \vee Q) \vee (\neg Q \wedge \neg P \wedge R) = \\ &= ((\neg P \vee \neg R \vee Q) \vee \neg Q) \wedge ((\neg P \vee \neg R \vee Q) \vee (\neg P \vee R)) = \\ &= \neg P \vee \neg R \vee Q \end{aligned}$$

**Tseitin:**

$$\mathcal{T}(a) \setminus \mathcal{T}_0(A) = \{(\neg P), (P \wedge R), (\neg P \wedge R), (Q \leftrightarrow \neg P \wedge R), \\ (\neg(Q \leftrightarrow \neg P \wedge R)), (P \wedge R \rightarrow \neg(Q \leftrightarrow \neg P \wedge R))\}$$

t

substitute  $\neg P$  with  $S_1$

$P \wedge R$  with  $S_2$

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R)$$

$$A_1 = S_2 \rightarrow \neg(Q \leftrightarrow S_1 \wedge R)$$

substitute  $S_1 \wedge R$  with  $S_3$

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R)$$

$$A_2 = S_2 \rightarrow \neg(Q \leftrightarrow S_3)$$

substitute  $Q \leftrightarrow S_3$  with  $S_4$

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\ \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3))$$

$$A_3 = S_2 \rightarrow \neg(S_4)$$

substitute  $\neg S_4$  with  $S_5$

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\ \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3)) \wedge (S_5 \leftrightarrow \neg S_4)$$

$$A_4 = S_2 \rightarrow S_5$$

substitute  $S_2 \rightarrow S_5$  with  $S_6$

$$\tilde{\varphi}_T(A) = 1 \wedge (S_1 \leftrightarrow \neg P) \wedge (S_2 \leftrightarrow P \wedge R) \wedge (S_3 \leftrightarrow S_1 \wedge R) \wedge \\ \wedge (S_4 \leftrightarrow (Q \leftrightarrow S_3)) \wedge (S_5 \leftrightarrow \neg S_4) \wedge (S_6 \leftrightarrow (S_2 \rightarrow S_5)) \wedge S_6$$

$$A_5 = S_6$$

$$\text{cnf}(\tilde{\varphi}_T(A)) = (P \vee S_1) \wedge (\neg S_1 \vee \neg P) \wedge (\neg P \vee \neg R \vee S_2) \wedge \\ \wedge (\neg S_2 \vee P) \wedge (\neg S_2 \vee R) \wedge (\neg S_1 \vee \neg R \vee S_3) \wedge (\neg S_3 \vee S_1) \wedge \\ \wedge (\neg S_3 \vee R) \wedge (\neg Q \vee \neg S_3 \vee S_4) \wedge (Q \vee S_3 \vee S_4) \wedge (\neg S_3 \vee Q \vee \neg S_4) \wedge \\ \wedge (\neg Q \vee S_3 \vee \neg S_4) \wedge (S_4 \vee S_5) \wedge (\neg S_5 \vee \neg S_4) \wedge (\neg S_2 \vee S_5) \wedge S_6 = \\ = (\neg P \vee \neg R \vee S_2) \wedge (\neg P \vee \neg S_1) \wedge (P \vee \neg Q \vee \neg R \vee \neg S_5) \wedge \\ \wedge (P \vee S_1) \wedge (\neg Q \vee S_3 \vee \neg S_4) \wedge (Q \vee \neg R \vee \neg S_4) \wedge (Q \vee S_3 \vee \neg S_5) \wedge \\ \wedge (R \vee \neg S_2) \wedge (R \vee \neg S_3) \wedge (\neg S_2 \vee \neg S_3) \wedge (S_4 \vee S_5) \wedge S_6$$

$$\text{Rank} = |\{P, Q, R, S_1, S_2, S_3, S_4, S_5, S_6\}| = 9, \text{Complexity} = \sum_{\Lambda \in \text{cnf}(\tilde{\varphi}_T(A))} 1 = 11$$

$$3. W = \{P_2 \vee P_4 \vee \neg P_5, P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_2 \vee \neg P_5 \vee \neg P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

**DPLL:**

**MULT**  $(\neg P_2 \vee \neg P_5 \vee \neg P_2)$

$$W_1 = \{P_2 \vee P_4 \vee \neg P_5, P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_5 \vee \neg P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

**SUS** ( $\neg P_5 \vee P_2$ )

$$W_1 = \{P_1 \vee \neg P_2 \vee \neg P_4, \neg P_1 \vee \neg P_3 \vee P_5, P_1 \vee P_3 \vee \neg P_4, \neg P_1 \vee \neg P_4 \vee P_5, \neg P_5 \vee P_2, \neg P_1 \vee P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

**SPLIT** ( $P_1$ )

$$W_{31} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{32} = \{\neg P_3 \vee P_5, \neg P_4 \vee P_5, \neg P_5 \vee P_2, P_3, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

**UNIT** ( $P_3 \in W_{32}$ )

$$W_{41} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{42} = \{P_5, \neg P_4 \vee P_5, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

**UNIT** ( $P_5 \in W_{42}$ )

$$W_{41} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{42} = \{\neg P_4, P_2, P_2 \vee \neg P_4, \neg P_2\}$$

**UNIT** ( $P_2 \in W_{42}$ )

$$W_{51} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{52} = \{\neg P_4, \neg P_4\}$$

**SAME** ( $\neg P_4 \in W_{52}$ )

$$W_{61} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{62} = \{\neg P_4\}$$

**UNIT** ( $\neg P_4 \in W_{62}$ )

$$W_{71} = \{\neg P_2 \vee \neg P_4, P_3 \vee \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{72} = \emptyset$$

**PURE** ( $P_3 \in W_{71}$ )

$$W_{81} = \{\neg P_2 \vee \neg P_4, \neg P_4, \neg P_5 \vee P_2, P_2 \vee \neg P_4, \neg P_2 \vee P_5\}$$

$$W_{72} = \emptyset$$

**UNIT** ( $\neg P_4 \in W_{81}$ )

$$W_{91} = \{\neg P_2, \neg P_5 \vee P_2, P_2, \neg P_2 \vee P_5\}$$

$$W_{72} = \emptyset$$

**UNIT** ( $P_2 \in W_{91}$ )

$$W_{101} = \{\neg P_5, P_5\}$$

$$W_{72} = \emptyset$$

**UNIT** ( $P_5 \in W_{101}$ )

$$W_{111} = \emptyset$$

$$W_{72} = \emptyset$$

W - unsatisfiable

**Resolution:**

- |                                       |  |                         |
|---------------------------------------|--|-------------------------|
| (1) $P_2 \vee P_4 \vee \neg P_5$      | (4) $P_1 \vee P_3 \vee \neg P_4$           | (7) $\neg P_1 \vee P_3$ |
| (2) $P_1 \vee \neg P_2 \vee \neg P_4$ | (5) $\neg P_1 \vee \neg P_4 \vee P_5$      | (8) $P_2 \vee \neg P_4$ |
| (3) $\neg P_1 \vee \neg P_3 \vee P_5$ | (6) $\neg P_2 \vee \neg P_5 \vee \neg P_2$ | (9) $\neg P_2 \vee P_5$ |
| (a)                                   | (b)  |                         |

$$P_2 \vee \neg P_5$$

$$(1, 8, P_4)$$

$$P_3 \vee P_4$$

$$(4, 7, P_1)$$

(c)	$P_2 \vee P_3$	(8, b, P <sub>4</sub> )	(g)	$\neg P_4$	(8, d, P <sub>2</sub> )
(d)	$\neg P_2$	(6, 9, P <sub>5</sub> )	(h)		
(e)	$\neg P_5$	(a, d, P <sub>2</sub> )		$\neg P_1 \vee P_5$	(3, f, P <sub>3</sub> )
(f)	$P_3$	(c, d, P <sub>3</sub> )	(i)	$\neg P_1$	(e, h, P <sub>5</sub> )

**Resolution:**

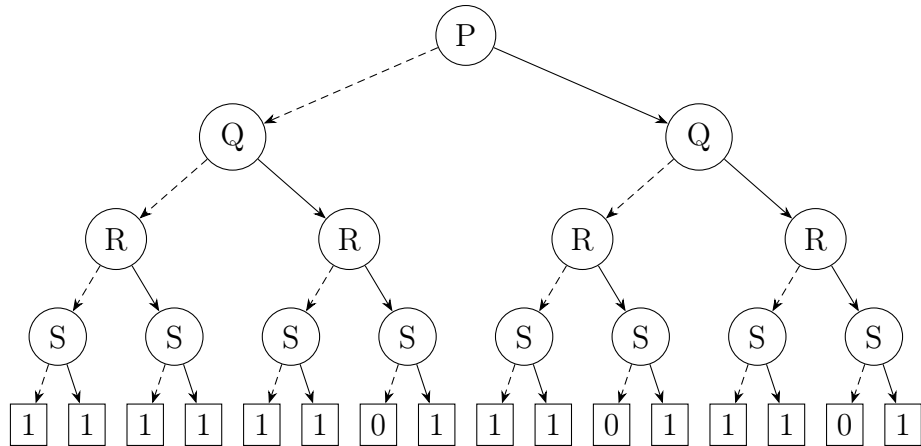
(1)	$P_2 \vee P_4 \vee \neg P_5$	(4)	$P_1 \vee P_3 \vee \neg P_4$	(7)	$\neg P_1 \vee P_3$
(2)	$P_1 \vee \neg P_2 \vee \neg P_4$	(5)	$\neg P_1 \vee \neg P_4 \vee P_5$	(8)	$P_2 \vee \neg P_4$
(3)	$\neg P_1 \vee \neg P_3 \vee P_5$	(6)	$\neg P_2 \vee \neg P_5 \vee \neg P_2$	(9)	$\neg P_2 \vee P_5$
(a)	$\neg P_4 \vee \neg P_2$	(8, 9, P <sub>2</sub> )	(k)	$\neg P_1 \vee \neg P_4$	(8, j, P <sub>2</sub> )
(b)	$P_1 \vee \neg P_2 \vee P_3$	(4, a, P <sub>4</sub> )	(l)	$\neg P_1 \vee P_2 \vee \neg P_5$	(1, k, P <sub>4</sub> )
(c)	$P_1 \vee P_4 \vee P_3 \vee \neg P_5$	(1, b, P <sub>2</sub> )	(m)	$\neg P_1 \vee P_2 \vee \neg P_4$	(5, j, P <sub>5</sub> )
(d)	$P_4 \vee P_3 \vee \neg P_5$	(7, c, P <sub>1</sub> )	(n)	$P_2 \vee P_3 \vee \neg P_4$	(4, m, P <sub>1</sub> )
(e)	$\neg P_2 \vee P_3 \vee P_4$	(9, d, P <sub>5</sub> )	(o)	$\neg P_1 \vee \neg P_4 \vee P_5$	(3, o, P <sub>3</sub> )
(f)	$\neg P_1 \vee \neg P_2 \vee P_3 \vee P_5$	(5, e, P <sub>4</sub> )	(p)	$\neg P_1 \vee \neg P_2 \vee \neg P_4$	(6, p, P <sub>5</sub> )
(g)	$\neg P_2 \vee P_3 \vee \neg P_4 \vee P_5$	(2, e, P <sub>1</sub> )	(q)	$\neg P_2 \vee P_3 \vee \neg P_4$	(4, p, P <sub>1</sub> )
(h)	$\neg P_1 \vee \neg P_2 \vee \neg P_4 \vee P_5$	(3, h, P <sub>3</sub> )	(r)	$P_3 \vee \neg P_4$	(8, q, P <sub>2</sub> )
(i)	$\neg P_1 \vee \neg P_4 \vee P_5$	(9, h, P <sub>2</sub> )	(s)	$\neg P_1 \vee \neg P_4 \vee P_5$	(3, s, P <sub>3</sub> )
(j)	$\neg P_1 \vee \neg P_2 \vee \neg P_4$	(6, i, P <sub>5</sub> )	(t)	$\neg P_1 \vee \neg P_2 \vee \neg P_4$	(6, t, P <sub>5</sub> )

$$4. A = ((P \rightarrow \neg R) \leftrightarrow (Q \wedge \neg P \wedge R)) \rightarrow S. \quad P < Q < R < S$$

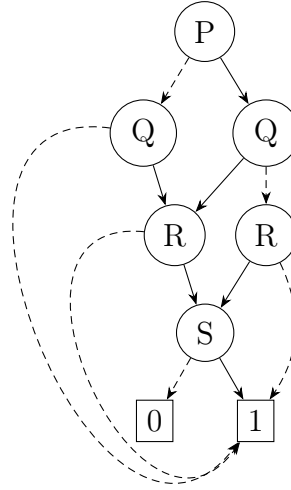
**Shannon:**

$$\begin{aligned}
A[1/P] &= ((1 \rightarrow \neg R) \leftrightarrow (Q \wedge \neg 1 \wedge R)) \rightarrow S = ((1 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S \\
A[0/P] &= ((0 \rightarrow \neg R) \leftrightarrow (Q \wedge \neg 0 \wedge R)) \rightarrow S = (1 \leftrightarrow (Q \wedge R)) \rightarrow S \\
A &= (P \Rightarrow A[1/P], A[0/P]) \\
A_1[1/Q] &= ((1 \rightarrow \neg R) \leftrightarrow (1 \wedge \neg 1 \wedge R)) \rightarrow S = ((1 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S \\
A_1[0/Q] &= ((1 \rightarrow \neg R) \leftrightarrow (0 \wedge \neg 1 \wedge R)) \rightarrow S = ((1 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S \\
A_1 &= (Q \Rightarrow A_1[1/Q], A_1[0/Q]) \\
A_0[1/Q] &= ((0 \rightarrow \neg R) \leftrightarrow (1 \wedge \neg 0 \wedge R)) \rightarrow S = (1 \leftrightarrow R) \rightarrow S \\
A_0[0/Q] &= ((0 \rightarrow \neg R) \leftrightarrow (0 \wedge \neg 0 \wedge R)) \rightarrow S = ((0 \rightarrow \neg R) \leftrightarrow 0) \rightarrow S = 1 \\
A_1 &= (Q \Rightarrow A_1[1/Q], A_1[0/Q]) \\
A_{11}[1/R] &= ((1 \rightarrow \neg 1) \leftrightarrow (1 \wedge \neg 1 \wedge 1)) \rightarrow S = S \\
A_{11}[0/R] &= ((1 \rightarrow \neg 0) \leftrightarrow (1 \wedge \neg 1 \wedge 0)) \rightarrow S = 1 \\
A_{10}[1/R] &= ((1 \rightarrow \neg 1) \leftrightarrow (0 \wedge \neg 1 \wedge 1)) \rightarrow S = S \\
A_{10}[0/R] &= ((1 \rightarrow \neg 0) \leftrightarrow (0 \wedge \neg 1 \wedge 0)) \rightarrow S = 1 \\
A_{11} &= (R \Rightarrow A_{11}[1/R], A_{11}[0/R]) \\
A_{10} &= (R \Rightarrow A_{10}[1/R], A_{10}[0/R]) \\
A_{01}[1/R] &= ((0 \rightarrow \neg 1) \leftrightarrow (1 \wedge \neg 0 \wedge 1)) \rightarrow S = S \\
A_{01}[0/R] &= ((0 \rightarrow \neg 0) \leftrightarrow (1 \wedge \neg 0 \wedge 0)) \rightarrow S = 1 \\
A_{00}[1/R] &= ((0 \rightarrow \neg 1) \leftrightarrow (0 \wedge \neg 0 \wedge 1)) \rightarrow S = 1 \\
A_{00}[0/R] &= ((0 \rightarrow \neg 0) \leftrightarrow (0 \wedge \neg 0 \wedge 0)) \rightarrow S = 1 \\
A_{01} &= (R \Rightarrow A_{01}[1/R], A_{01}[0/R]) \\
A_{00} &= (R \Rightarrow A_{00}[1/R], A_{00}[0/R]) \\
A_{111}[1/S] &= 1, A_{111}[0/S] = 0, A_{110}[1/S] = 1, A_{110}[0/S] = 1 \\
A_{101}[1/S] &= 1, A_{101}[0/S] = 0, A_{100}[1/S] = 1, A_{100}[0/S] = 1 \\
A_{011}[1/S] &= 1, A_{011}[0/S] = 0, A_{010}[1/S] = 1, A_{010}[0/S] = 1 \\
A_{001}[1/S] &= 1, A_{001}[0/S] = 1, A_{000}[1/S] = 1, A_{000}[0/S] = 1
\end{aligned}$$

**BDT:**



**ROBDD:**



$$\begin{aligned}
\mathcal{T}(A) &= \{P, Q, R, S\}, |\mathcal{T}(A)| = 4, \text{weight}(P) = 2^4 = 16 \\
A : & \{P, Q, R, S\}, \{P, Q, \neg R, S\}, \{P, Q, \neg R, \neg S\}, \{P, \neg Q, R, S\}, \{P, \neg Q, \neg R, S\}, \\
& \{P, \neg Q, \neg R, \neg S\}, \{P, \neg Q, \neg R, S\}, \{\neg P, Q, R, S\}, \{\neg P, Q, \neg R, S\}, \{\neg P, Q, \neg R, \neg S\}, \\
& \{\neg P, \neg Q, R, S\}, \{\neg P, \neg Q, R, \neg S\}, \{\neg P, \neg Q, \neg R, S\}, \{\neg P, \neg Q, \neg R, \neg S\} \\
& (P \Rightarrow (Q \Rightarrow 1, (R \Rightarrow 1, (S \Rightarrow 1, 0))), (Q \Rightarrow (R \Rightarrow 1, (S \Rightarrow 1, 0)), (R \Rightarrow 1, (S \Rightarrow 1, 0)))) \\
& = (P \Rightarrow (Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0)))) \quad \text{- ordered} \\
& = (P \Rightarrow (Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0), (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0))), \\
& \quad (Q \Rightarrow (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0), (R \Rightarrow (S \Rightarrow 1, 0), (S \Rightarrow 1, 0)))) \quad \text{- full unordered} \\
& = (P \Rightarrow (Q \Rightarrow 1, (R \Rightarrow 1, (S \Rightarrow 1, 0)))) \quad \text{- full ordered}
\end{aligned}$$

5.  $P$  - Serhiy and Boris are about the same age,  $Q$  - Serhiy is older than Boris,  $R$  - Nadiya and Boris are of different age,  $S$  - Boris is older than Fedor

$$\begin{aligned}
A &= (P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S) \wedge (R \rightarrow S) = \\
&= (P \vee Q) \wedge (\neg P \vee R) \wedge (\neg Q \vee S) \wedge (\neg R \vee S) = \\
&= (\neg P \vee R) \wedge (P \vee Q) \wedge S
\end{aligned}$$

$$W = \{P \vee Q, \neg P \vee R, \neg Q \vee S, \neg R \vee S\}$$

**DPLL:**

**SPLIT** ( $P$ )

$$W_{11} = \{Q, \neg Q \vee S, \neg R \vee S\}$$

$$W_{12} = \{R, \neg Q \vee S, \neg R \vee S\}$$

**UNIT** ( $Q \in W_{11}, R \in W_{12}$ )

$$W_{21} = \{S, \neg R \vee S\}$$

$$W_{22} = \{\neg Q \vee S, S\}$$

**UNIT**  $(S \in W_{21}, S \in W_{22})$   
 $W_{31} = \{\neg R\}$   
 $W_{32} = \{\neg Q\}$

**UNIT**  $(R \in W_{21}, Q \in W_{22})$   
 $W_{41} = \emptyset$   
 $W_{42} = \emptyset$

W - unsatisfiable

**Resolution:**

(1) $P \vee Q$	(2) $\neg P \vee R$	(3) $\neg Q \vee S$	(4) $\neg R \vee S$
(a)		(c)	
	$Q \vee R \quad (1, 2, P)$		$P \vee S \quad (1, 3, Q)$
(b)		(d)	
	$\neg P \vee S \quad (2, 4, R)$		$S \quad (b, c, P)$

**Shannon:**

$A = (P \vee Q) \wedge (\neg P \vee R) \wedge (\neg Q \vee S) \wedge (\neg R \vee S), P < Q < R < S$

$A[1/P] = (1 \vee Q) \wedge (\neg 1 \vee R) \wedge (\neg Q \vee S) \wedge (\neg R \vee S)$   
 $A[0/P] = (0 \vee Q) \wedge (\neg 0 \vee R) \wedge (\neg Q \vee S) \wedge (\neg R \vee S)$   
 $A = (P \Rightarrow A[1/Q], A[0/Q])$

$A_1[1/Q] = (1 \vee 1) \wedge (\neg 1 \vee R) \wedge (\neg 1 \vee S) \wedge (\neg R \vee S)$   
 $A_1[0/Q] = (1 \vee 0) \wedge (\neg 1 \vee R) \wedge (\neg 0 \vee S) \wedge (\neg R \vee S)$   
 $A_1 = (Q \Rightarrow A_1[1/Q], A_1[0/Q])$

$A_0[1/Q] = (0 \vee 1) \wedge (\neg 0 \vee R) \wedge (\neg 1 \vee S) \wedge (\neg R \vee S)$   
 $A_0[0/Q] = (0 \vee 0) \wedge (\neg 0 \vee R) \wedge (\neg 0 \vee S) \wedge (\neg R \vee S) = 0$   
 $A_0 = (Q \Rightarrow A_0[1/Q], A_0[0/Q])$

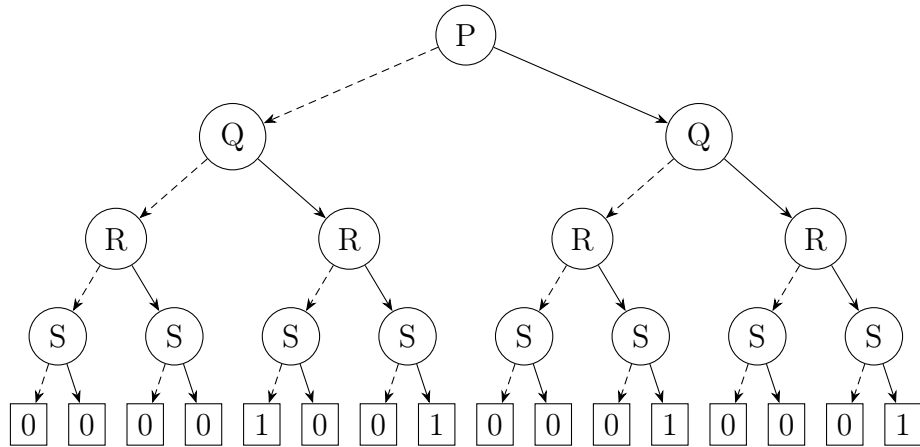
$A_{11}[1/R] = (1 \vee 1) \wedge (\neg 1 \vee 1) \wedge (\neg 1 \vee S) \wedge (\neg 1 \vee S) = S$   
 $A_{11}[0/R] = (1 \vee 1) \wedge (\neg 1 \vee 0) \wedge (\neg 1 \vee S) \wedge (\neg 0 \vee S) = 0$   
 $A_{10}[1/R] = (1 \vee 0) \wedge (\neg 1 \vee 1) \wedge (\neg 0 \vee S) \wedge (\neg 1 \vee S) = S$   
 $A_{10}[0/R] = (1 \vee 0) \wedge (\neg 1 \vee 0) \wedge (\neg 0 \vee S) \wedge (\neg 0 \vee S) = 0$   
 $A_{11} = (R \Rightarrow A_{11}[1/R], A_{11}[0/R])$   
 $A_{10} = (R \Rightarrow A_{10}[1/R], A_{10}[0/R])$

$A_{01}[1/R] = (0 \vee 1) \wedge (\neg 0 \vee 1) \wedge (\neg 1 \vee S) \wedge (\neg 1 \vee S) = S$   
 $A_{01}[0/R] = (0 \vee 1) \wedge (\neg 0 \vee 0) \wedge (\neg 1 \vee S) \wedge (\neg 0 \vee S) = S$   
 $A_{00}[1/R] = (0 \vee 0) \wedge (\neg 0 \vee 1) \wedge (\neg 0 \vee S) \wedge (\neg 1 \vee S) = 0$   
 $A_{00}[0/R] = (0 \vee 0) \wedge (\neg 0 \vee 0) \wedge (\neg 0 \vee S) \wedge (\neg 0 \vee S) = 0$   
 $A_{01} = (R \Rightarrow A_{01}[1/R], A_{01}[0/R])$   
 $A_{00} = (R \Rightarrow A_{00}[1/R], A_{00}[0/R])$



$$\begin{aligned}
&A_{111}[1/S] = 1, A_{111}[0/S] = 0, A_{101}[1/S] = 0, A_{101}[0/S] = 0 \\
&A_{101}[1/S] = 1, A_{101}[0/S] = 0, A_{100}[1/S] = 0, A_{100}[0/S] = 0 \\
&A_{011}[1/S] = 1, A_{011}[0/S] = 0, A_{010}[1/S] = 1, A_{010}[0/S] = 0 \\
&A_{001}[1/S] = 0, A_{001}[0/S] = 0, A_{000}[1/S] = 0, A_{000}[0/S] = 0
\end{aligned}$$

***BDT:***



***ROBDD:***

