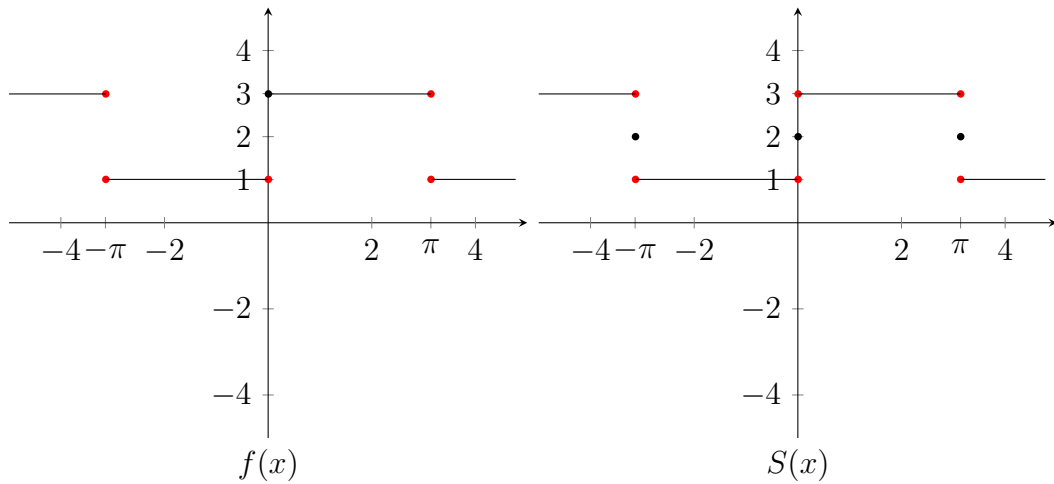
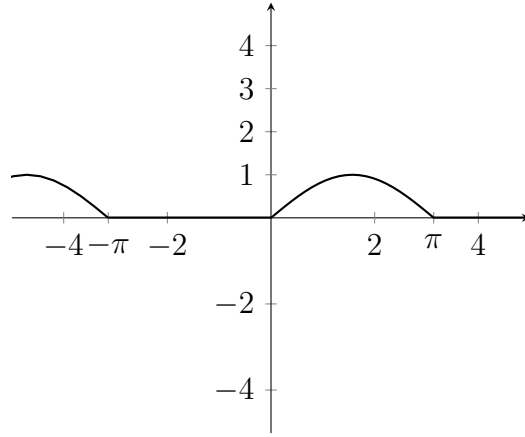

ДОМАШНЯ РОБОТА №13
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"
ФІ-12 Бекешева Анастасія

$$\begin{aligned}
1. \quad (a) \quad a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \, dx = \frac{1}{\pi} \left(\int_{-\pi}^0 1 \, dx + \int_0^{\pi} 3 \, dx \right) = \frac{1}{\pi} ((0 - (-\pi)) + (3\pi - 0)) = \frac{4\pi}{\pi} = 4 \\
a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = \frac{1}{\pi} \left(\int_{-\pi}^0 \cos nx \, dx + \int_0^{\pi} 3 \cos nx \, dx \right) = \frac{1}{\pi n} (\sin 0n - \sin -\pi n + \\
&\quad + 3 \sin \pi n - 3 \sin 0n) = 0 \\
b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = \frac{1}{\pi} \left(\int_{-\pi}^0 \cos nx \, dx + \int_0^{\pi} 3 \cos nx \, dx \right) = \frac{1}{\pi n} (-\cos 0 + \cos \pi n - \\
&\quad - \cos \pi n + \cos 0) = \frac{1}{\pi n} (-1 + (-1)^n - 3(-1)^n + 3) = \frac{2}{\pi n} (1 + (-1)^{n+1}) \\
f(x) &= 2 + \sum_{n=1}^{\infty} \frac{2}{\pi n} (1 + (-1)^{n+1}) \sin nx
\end{aligned}$$



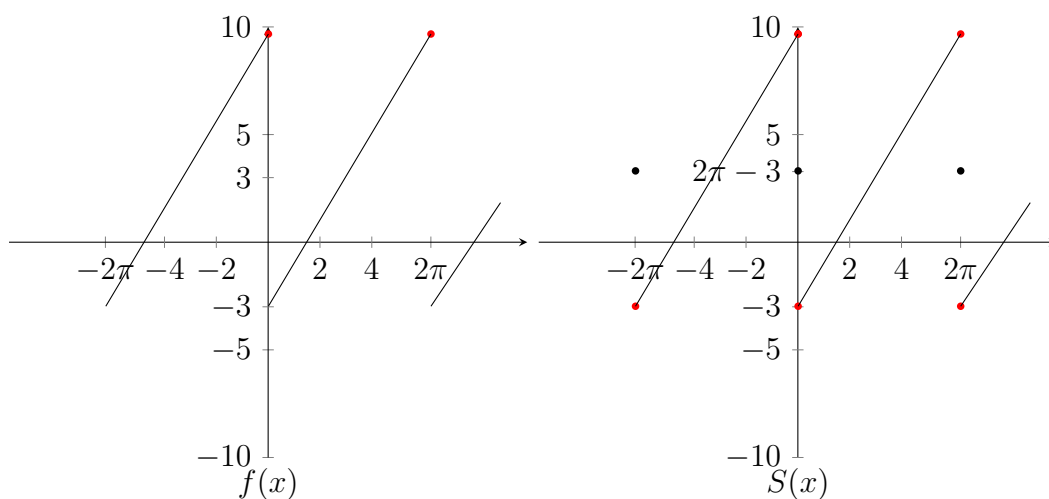
$$\begin{aligned}
(b) \quad a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \, dx = \frac{1}{\pi} \left(\int_0^{\pi} \sin x \, dx \right) = \frac{1}{\pi} (-\cos 0 - \cos \pi) = \frac{2}{\pi} \\
a_1 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos x \, dx = \frac{1}{\pi} \left(\int_0^{\pi} \sin x \cos x \, dx \right) = \frac{1}{\pi} \left(\int_0^{\pi} \sin x \, d(\sin x) \right) = \\
&= \frac{1}{2\pi} (\sin^2 \pi - \sin^2 0) = 0 \\
b_1 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin x \, dx = \frac{1}{\pi} \left(\int_0^{\pi} \sin^2 x \, dx \right) = \frac{1}{2\pi} \left(\int_0^{\pi} (1 - \cos 2x) \, dx \right) = \\
&= \frac{1}{2\pi} \left(\pi - \frac{\sin 2 \cdot \pi}{2} + 0 - \frac{\sin 2 \cdot 0}{2} \right) = \frac{1}{2} \\
a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = \frac{1}{\pi} \left(\int_0^{\pi} \sin x \cos nx \, dx \right) = \frac{1}{2\pi} \left(\int_0^{\pi} \sin(x + nx) \, dx + \right. \\
&\quad \left. + \int_0^{\pi} \sin(x - nx) \, dx \right) = \frac{1}{2\pi} \left(-\frac{\cos(x + nx)}{1 + n} \Big|_0^{\pi} - \frac{\cos(x - nx)}{1 - n} \Big|_0^{\pi} \right) = \\
&= \frac{1}{2\pi} \left(\frac{(1 - n) \cos(x + nx) + (1 + n) \cos(x - nx)}{1 - n^2} \right) =
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\pi} \left(\frac{(1-n)(-1)^n + (1+n)(-1)^n + 2}{1+n^2} \right) = \frac{(-1)^n(1-n+n+1) + 2}{1-n^2} = \frac{(-1)^n + 1}{\pi(1-n^2)} \\
b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = \frac{1}{\pi} \left(\int_0^{\pi} \sin x \sin nx \, dx - \int_0^{\pi} \cos(x-nx) \, dx - \int_0^{\pi} \cos(x+nx) \, dx \right) \\
&= \frac{1}{2\pi} \left(-\frac{\sin(x-nx)}{1+n} \Big|_0^{\pi} - \frac{\sin(x+nx)}{1-n} \Big|_0^{\pi} \right) = \\
&= \frac{1}{2\pi} \left(\frac{(1+n)\sin(x-nx) + (n-1)\sin(x+nx)}{1-n^2} \right) = \\
&= \frac{1}{2\pi} \left(\frac{(1+n)\sin \pi n - (n-1)\sin \pi n}{1-n^2} \right) = 0 \\
f(x) &= \frac{1}{\pi} + \sum_{n=2}^{\infty} \frac{(-1)^n + 1}{\pi(1-n^2)} \cos nx + \frac{1}{2} \sin x
\end{aligned}$$



$$\begin{aligned}
\text{(c) } a_0 &= \frac{1}{\pi} \int_0^{2\pi} f(x) \, dx = \int_0^{2\pi} (2x-3) \, dx = \frac{1}{\pi} (4\pi^2 - 6\pi) = 4\pi - 6 \\
a_n &= \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx \, dx = \frac{1}{\pi} \left(2 \int_0^{2\pi} x \cos nx \, dx - 3 \int_0^{2\pi} \cos nx \, dx \right) = \\
&= \frac{1}{\pi} \left(\frac{2x \sin nx - 3 \sin nx}{n} + \frac{2 \cos nx}{n^2} \right) \Big|_0^{2\pi} = \frac{4\pi n \cdot 0 - 3n \cdot 0 + 2 \cdot (-1) - 2}{\pi n^2} = \frac{0}{2\pi} \\
b_n &= \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx \, dx = \frac{1}{\pi} \left(2 \int_0^{2\pi} x \sin nx \, dx - 3 \int_0^{2\pi} \sin nx \, dx \right) = \\
&= \frac{1}{\pi} \left(\frac{2x \sin nx}{n} + \frac{2 \cos nx}{n^2} - \frac{3 \sin nx}{n} \right) \Big|_0^{2\pi} = \frac{4\pi n \sin 2\pi n - 3n \sin 2\pi n - 4\pi n - \cos 2\pi n}{\pi n^2} = \\
&= -\frac{4n}{\pi n} = -\frac{4}{n} \\
f(x) &= 2\pi - 3 + \sum_{n=1}^{\infty} -\frac{4}{n} \sin nx
\end{aligned}$$

$$\text{(d) } a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \, dx = \frac{1}{\pi} \cdot \frac{x^3}{3} \Big|_{-\pi}^{\pi} = \frac{2}{3} \pi^2$$



$$\begin{aligned}
 a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = \langle t = nx \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{t^2 \cos t}{n^3} \, dt = \frac{1}{\pi n^3} \left(t^2 \sin t - \int_{-\pi}^{\pi} 2t \sin t \, dt \right) = \\
 &= \frac{1}{\pi n^3} \left(t^2 \sin t + 2t \cos t + \int_{-\pi}^{\pi} \cos t \, dt \right) = \frac{1}{\pi n^3} ((nx^2) \sin nx - 2(nx(-\cos nx) + \sin nx)) \Big|_{-\pi}^{\pi} = \\
 &= \frac{1}{\pi n^3} (4\pi^2 n^2 \sin \pi n + 4\pi n \cos \pi n - 4 \sin \pi n) = \frac{4(-1)^n}{n} \\
 f(x) &= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos n(x)
 \end{aligned}$$

