ДОМАШНЯ РОБОТА №11
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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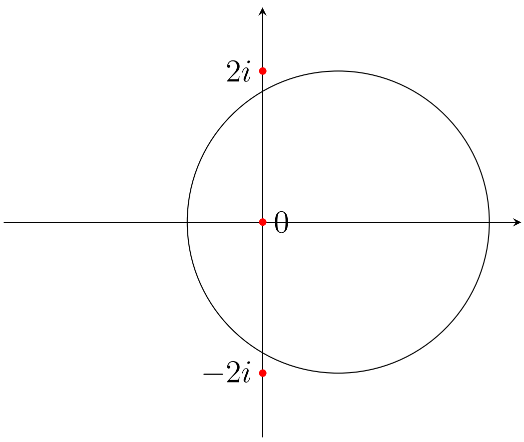
1. $\displaystyle\oint\limits_{|z-1|=2} \frac{z^2+1}{(z-2i)(z+2i)\sin\frac{z}{3}}\,\mathrm{d}z \quad =$

$z=\pm 2i, \quad \sin\dfrac{z}{3}=0, \quad z=3\pi n, \quad m_1=0, \quad m_2=1, \quad z=0$ - полюс I порядку.

$$= \quad 2\pi i\ \operatorname{Res}\frac{z^2+1}{(z-2i)(z+2i)\sin\frac{z}{3}} = 2\pi i\lim_{z\to 0}\frac{z^2+1}{(z-2i)(z+2i)\sin\frac{z}{3}}\cdot z =$$

$$= 2\pi i\lim\nolimits_{z\to 0}\frac{(z^2+1)z}{(z-2i)(z+2i)\dfrac{z}{3}} = 2\pi i\frac{3}{4} = \frac{3}{2}\pi i$$

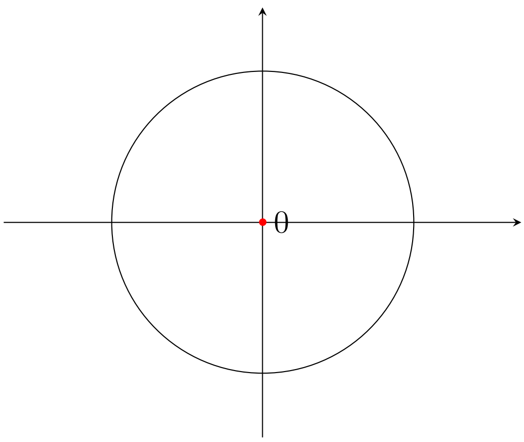


2. $\displaystyle\oint\limits_{|z|=1} \frac{z^2e^{\frac{1}{z^2}}-1}{z}\,\mathrm{d}z \quad =$

$$z=0 \text{ (знам.)}, \quad f(z)=\frac{z^2e^{\frac{1}{z^2}}-1}{z}=\frac{1}{z}\left(z^2\left(1+\frac{1}{z^2}+\frac{1}{2z^4}+\dots\right)-1\right)=\frac{1}{z}\left(z^2+\frac{1}{2z^2}+\dots\right)=$$

$$= z+\underbrace{\frac{1}{2z^3}+\dots,}_{\text{гол. час.}} \quad z=0 \text{ - істотно особлива точка.}$$

$$= \quad 2\pi i\ \operatorname{Res} f(z) = 2\pi i\cdot 0 = 0$$



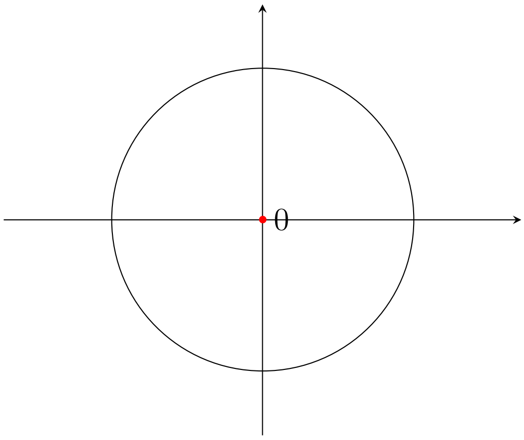
3. $\displaystyle\oint\limits_{|z|=4} \frac{\operatorname{sh} iz-\sin iz}{z^3\operatorname{sh}\frac{z}{3}}\,\mathrm{d}z \quad =$

$$z=0, \quad \operatorname{sh}\frac{z}{3}=0, \quad z=3\pi i n, \quad \operatorname{sh} iz-\sin iz = iz-\frac{iz^3}{3!}+\frac{iz^5}{5!}+\dots-iz-\frac{iz^3}{3!}-\frac{iz^5}{5!}=$$

$$-\frac{2iz^3}{3!}=z^3\left(-\frac{2i}{3!}+\dots\right),\quad z^3\operatorname{sh}\frac{z}{3}=z^3\left(\frac{z}{3}+\frac{z^3}{3^3\cdot 3!}\dots\right)=\frac{z^4}{3}\left(\frac{1}{3}+\frac{z^2}{3^3\cdot 3!}\dots\right),$$

$f(z)=\dfrac{z^3g_1(z)}{\frac{z^4}{3}g_2(z)},\quad m_1=3,\quad m_2=4,\quad z=0$ - полюс I порядку.

$$= \quad 2\pi i\lim_{z\to 0}\frac{z^3g_1(z)}{\frac{z^4}{3}g_2(z)}=2\pi i\frac{\frac{-2i}{3!}}{\frac{1}{3}}=2\pi$$



4. $$\oint\limits_{|z+2i|=3}\left(\underbrace{\frac{\pi}{e^{\frac{\pi z}{2}}+1}}_{\mathfrak{I}_1}+\underbrace{\frac{6\operatorname{ch}\frac{\pi iz}{2-2i}}{(z-2+2i)^2(z-4+2i))}}_{\mathfrak{I}_2}\right)\,\mathrm{d}z=\mathfrak{I}_1+6\mathfrak{I}_2$$

$(\mathfrak{I}_1)$ $e^{\frac{\pi z}{2}}+1=0,\quad e^{\frac{\pi z}{2}}=-1,\quad \frac{\pi z}{2}=\ln(-1),\quad z_n=2i(2n+1),$

$z=-2i(n=-1),\quad m_1=0,\quad m_2=1,\quad z=-2i$ - полюс I порядку.

$$f(z)=\frac{\pi}{e^{\frac{\pi z}{2}}+1},\quad \left(e^{\frac{\pi z}{2}}+1\right)'=\frac{\pi}{2}e^{\frac{\pi z}{2}}\neq 0$$

$$\operatorname{Res} f(z)=\left.\frac{\pi}{\frac{\pi}{2}e^{\frac{\pi z}{2}}}\right|_{z=-2i}=\frac{2}{e^{-\pi i}}=-2,\quad \mathfrak{I}_1=-4\pi i$$

$(\mathfrak{I}_2)$ $(z-2+2i)^2(z-4+2i)=0$

$\left[\begin{array}{l} z-2+2i=0\\ z-4+2i=0\end{array}\right.\ \left[\begin{array}{l} z=2-2i \text{ (нуль знам., не нуль чис.)}\\ z=4-2i\end{array}\right.$

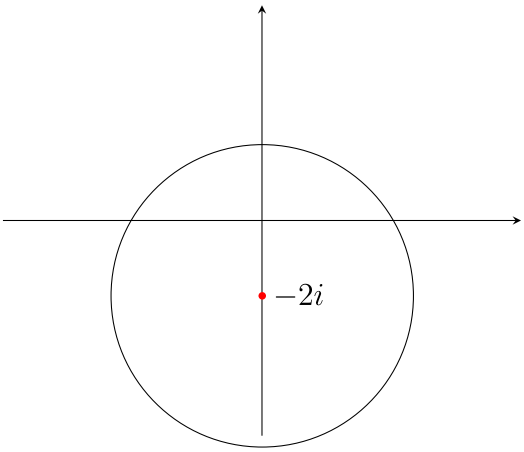
$$f(z)=\frac{\operatorname{ch}\frac{\pi iz}{2-2i}}{(z-2+2i)^2(z-4+2i))},\quad z=2-2i,\quad m_1=0,\quad m_2=2,$$

$z=2-2i$ - - полюс II порядку. $\operatorname{Res} f(z)=\lim\limits_{z\to 2-2i}\dfrac{\partial}{\partial z}\left(f(z)(z-2+2i)^2\right)=$

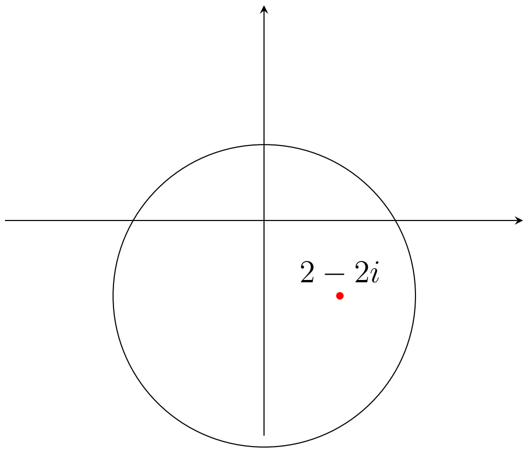
$$=\lim_{z\to 2-2i}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\operatorname{ch}\frac{\pi iz}{2-2i}}{(z-4+2i))}\right)=\lim_{z\to 2-2i}\frac{\operatorname{sh}\frac{\pi iz}{2-2i}\cdot\frac{\pi i}{2-2i}(z-4+2i)-\operatorname{ch}\frac{\pi iz}{2-2i}}{(z-4+2i)^2}=$$

$$=\langle z=2-2i\rangle=\lim_{z\to 2-2i}\frac{\operatorname{sh}\pi i\cdot\frac{\pi i}{2-2i}-\operatorname{ch}\pi i}{4}=\frac{1}{4}$$

$\mathfrak{I} = \mathfrak{I}_1 + 6\mathfrak{I}_2 = -4\pi i + 3\pi i = -\pi i$



$\mathfrak{I}_1$



$\mathfrak{I}_2$