ДОМАШНЯ РОБОТА №7 З ПРЕДМЕТУ ”ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ”

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1. (a) $\displaystyle\int\limits_{-2}^{-2+i} (z+2)^2 \ \mathrm{d}z \ = \int\limits_{-2}^{-2+i} (z+2)^2 \ \mathrm{d}(z+2) \ = \left.\frac{(z+2)^3}{3}\right|_{-2}^{-2+i} = \frac{(-2+i+2)^3}{3} - \frac{(-2+2)^3}{3} =$

$\displaystyle = \frac{1}{3}i^3 - \frac{1}{3}0^3 = \frac{i^3}{3} = \frac{-i}{3}$

(b) $\displaystyle\int\limits_{0}^{\pi+2i} \cos\frac{z}{2} \ \mathrm{d}z \ = 2 \int\limits_{0}^{\pi+2i} \cos\frac{z}{2} \ \mathrm{d}\frac{z}{2} \ = 2\sin\left.\frac{z}{2}\right|_{0}^{\pi+2i} = 2\sin\left(\frac{\pi}{2}+i\right) + 2\sin 0 = 2\cos i =$

$= 2\,\mathrm{ch}\,1$

(c) $\displaystyle\int\limits_{-i}^{i} (1+4iz^3) \ \mathrm{d}z \ = (z+iz^4)\Big|_{-i}^{i} = (i + i\cdot i^4) - (-i + i \cdot i^4) = i + i - 0 = 2i$

(d) $\displaystyle\int\limits_{2e^{-\frac{\pi}{3}i}}^{2e^{\frac{\pi}{3}i}} (2-3z+z^2) \ \mathrm{d}z \ = \left.\frac{1}{3}z^3 - \frac{3}{2}z^2 + 2z\right|_{2e^{-\frac{\pi}{3}i}}^{2e^{\frac{\pi}{3}i}} = \left(\frac{1}{3}\left(2e^{\frac{\pi}{3}i}\right)^3 - \frac{3}{2}\left(2e^{\frac{\pi}{3}i}\right)^2 + 2\left(2e^{\frac{\pi}{3}i}\right)\right) -$

$\displaystyle \left(\frac{1}{3}\left(2e^{-\frac{\pi}{3}i}\right)^3 - \frac{3}{2}\left(2e^{-\frac{\pi}{3}i}\right)^2 + 2\left(2e^{-\frac{\pi}{3}i}\right)\right) = \frac{2}{3}e^{-\pi i}\left(e^{-\frac{\pi}{3}i} - 1\right)\left(3e^{\frac{\pi}{3}i} + 3e^{2\frac{\pi}{3}i} + 4e^{\pi i} + 4\right) =$

$\displaystyle = \frac{2}{3}(\cos\pi - i\sin\pi)\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3} - 1\right)\left(3\cos\frac{\pi}{3} + 3i\sin\frac{\pi}{3} + 3\cos\frac{2\pi}{3} + 3i\sin\frac{2\pi}{3} + \right.$

$\displaystyle \left. + 4\cos\pi + 4i\sin\pi\right) = \frac{2}{3}(-1 - i\cdot 0)\left(\frac{1}{2} - i\cdot\frac{\sqrt{3}}{2} - 1\right)\left(\frac{3}{2} + 3i\frac{\sqrt{3}}{2} - \frac{3}{2} + 3i\frac{\sqrt{3}}{2} - 4 + 0\right) =$

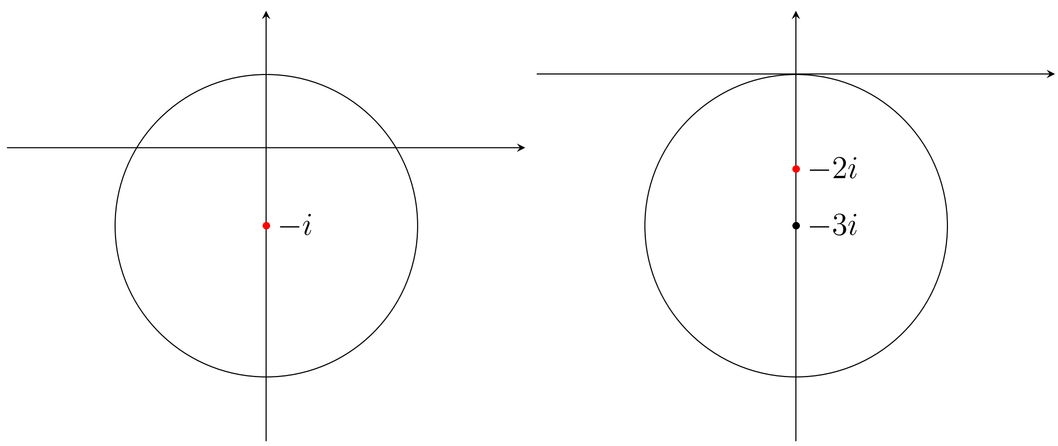
$=$

2. (a) $\displaystyle\int\limits_{|z+i|=3} \frac{\sin z}{z+i} \ \mathrm{d}z \ = \langle z \neq -i \rangle = 2\pi i \sin z\Big|_{z=-i} = 2\pi i \sin -i = 2\pi\,\mathrm{sh}\,1$

(b) $C: x^2 + y^2 + 6y = 0, \quad y = \pm\sqrt{9 - x^2} - 3, \quad x = \pm 3 : y = -3, \quad x = 0 : y = 0, -6,$

$\displaystyle\int\limits_{C} \frac{\sin z \ \mathrm{d}z}{z^2+4} = \int\limits_{C} \frac{(z-2i)^{-1}\sin z \ \mathrm{d}z}{(z+2i)} = \langle z \neq \pm 2i \rangle = \left.\frac{2\pi i}{(z-2i)}\sin z\right|_{z=-2i} = -\frac{\pi}{2}\cdot$

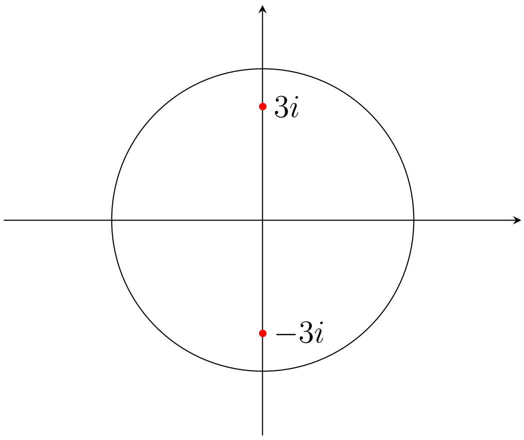
$\displaystyle\cdot \sin(-2i) = \frac{\pi}{2} i\,\mathrm{sh}\,2$



(a) (b)

3. $\displaystyle\int\limits_{|z|=4} \frac{\mathrm{d}z}{(z^2+9)(z+9)} = \langle z \neq \pm 3i, -9 \rangle = \int\limits_{z\neq 3i} \frac{(z+9)^{-1} \ \mathrm{d}z}{(z^2+9)} = \int\limits_{|z|=4} \frac{\mathrm{d}z}{(z^2+9)(z+9)} +$

$$+\int\limits_{z\neq -3i}\frac{\mathrm{d}z}{(z^2+9)(z+9)}=(z+9)^{-1}(z+3i)^{-1}\Big|_{z=3i}+(z+9)^{-1}(z-3i)^{-1}\Big|_{z=-3i}=$$
$$=\frac{2\pi i}{(3i+9)6i}-\frac{2\pi i}{(-3i+9)6i}=\frac{2\pi i}{6i}\cdot\frac{-i+3-i-3}{3(i+3)(-i+3)}=\frac{2\pi i}{6i}\frac{-2i}{(9-i^2)}=\frac{2\pi i}{9(9-(-1))}=$$
$$-\frac{2\pi i}{90}$$

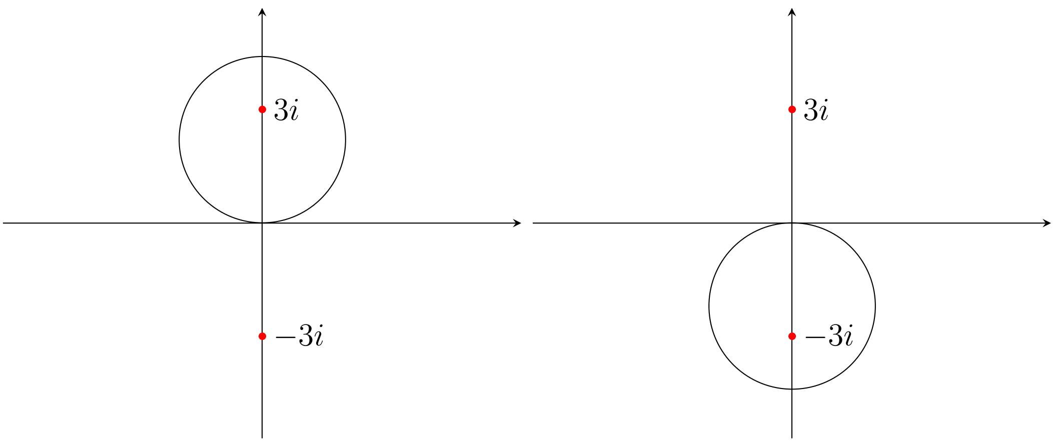


4. (a) $C:|z-2i|=2,\quad \int\limits_C\frac{\mathrm{d}z}{z^2+9}=\langle z\neq \pm 3i\rangle=\int\limits_C\frac{(z+3i)^{-1}\ \mathrm{d}z}{z-3i}=\frac{2\pi i}{z+3i}\Big|_{z=3i}=$

$$=\frac{2\pi i}{3i+3i}=\frac{\pi}{3}$$

(b) $C:|z+2i|=2,\quad \int\limits_C\frac{\mathrm{d}z}{z^2+9}=\langle z\neq \pm 3i\rangle=\int\limits_C\frac{(z-3i)^{-1}\ \mathrm{d}z}{z+3i}=\frac{2\pi i}{z-3i}\Big|_{z=-3i}=$

$$=\frac{2\pi i}{-3i-3i}=-\frac{\pi}{3}$$



(a) (b)

5. (a) $L:|z-1|=\frac{1}{3},\quad \int\limits_L\frac{\operatorname{sh} z\pi\ \mathrm{d}z}{(z-1)(z^2+4)^2}=\langle z\neq 1,\pm 2i\rangle=\int\limits_L\frac{(z^2+4)^{-2}\operatorname{sh} z\pi\ \mathrm{d}z}{z-1}=$

$$=2\pi i(z^2+4)^{-2}\operatorname{sh} z\pi\Big|_{z=1}=2\pi\frac{\operatorname{sh} z\pi}{25}$$

(b) $L: |z+2i| = \frac{1}{3}, \quad \int\limits_L \frac{\operatorname{sh} z\pi \; \mathrm{d}z}{(z-1)(z^2+4)^2} = \langle z \neq 1, \pm 2i \rangle = \int\limits_L \frac{\operatorname{sh} z\pi (z+2i)^{-2}}{(z-1)(z-2i)^2} =$

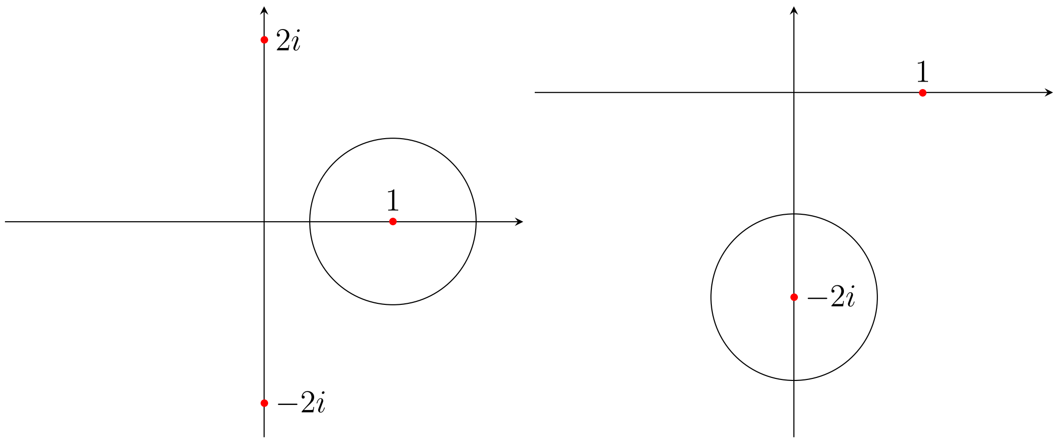
$$= 2\pi i \frac{\mathrm{d}}{\mathrm{d}z} \left(\operatorname{sh} z\pi (z-1)^{-1}(z-2i)^{-2} \right) \Big|_{z=-2i} =$$

$$= 2\pi i \left(\frac{\pi \operatorname{ch}(\pi z)(z-1)(z-2i)^2 - \operatorname{sh}(\pi z)(2(z-1)(z-2i) + (z-2i)^2)}{(z-1)^2(z-2i)^4} \right) =$$

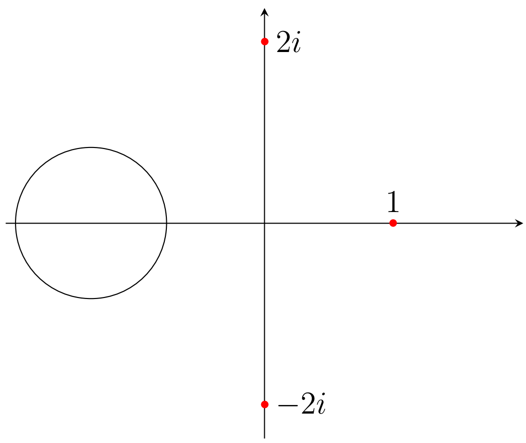
$$= 2\pi i \frac{\pi \operatorname{ch}(-2\pi i)(-16)(-2i-1) + \operatorname{sh}(2\pi i)(-8(-2i-1)-16)}{(-16)^2(-3+4i)} = \frac{2\pi^2 i(16+32i) + 0(16i-8)}{(16+32i)^2} =$$

$$= \frac{\pi^2 i}{8+16i} = \frac{\pi^2 i \cdot 8 - \pi^2 i \cdot 16i}{320} = \frac{\pi^2(i+2)}{40}$$

(d) $L: |z+2| = 1, \quad \int\limits_L \frac{\operatorname{sh} z \; \mathrm{d}z}{(z-1)(z^2+4)^2} = \langle z \neq 1, \pm 2i \rangle = 0 (z \notin D)$



(a) (b)



(a)