ДОМАШНЯ РОБОТА №13
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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1. (a) $a_0 = \dfrac{1}{\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x) = \dfrac{1}{\pi}\left(\int\limits_{-\pi}^{0} 1\ \mathrm{d}x\ + \int\limits_{0}^{\pi} 3\ \mathrm{d}x\right) = \dfrac{1}{\pi}\left((0-(-\pi)+(3\pi-0)\right) = \dfrac{4\pi}{\pi} = 4$

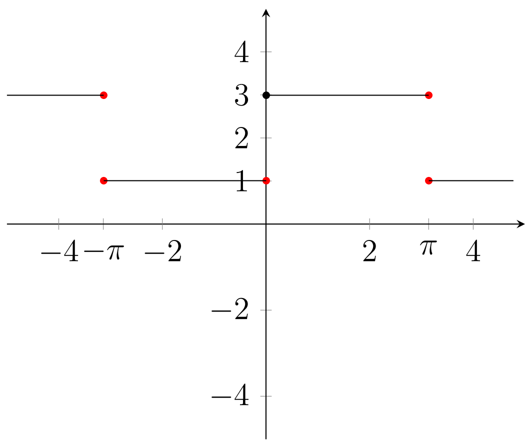
$a_n = \dfrac{1}{\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x)\cos nx = \dfrac{1}{\pi}\left(\int\limits_{-\pi}^{0} \cos nx\ \mathrm{d}x\ + \int\limits_{0}^{\pi} 3\cos nx\ \mathrm{d}x\right) = \dfrac{1}{\pi n}(\sin 0n - \sin -\pi n +$

$+ 3\sin \pi n - 3\sin 0n) = 0$

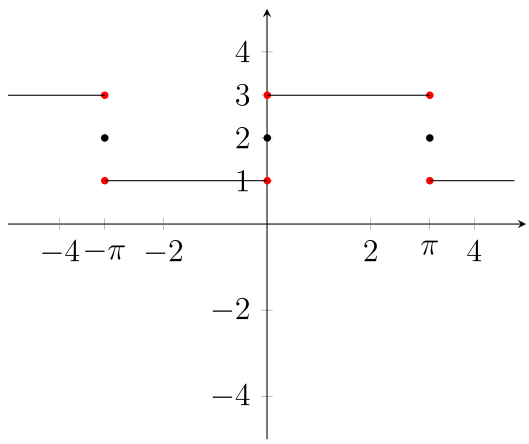
$b_n = \dfrac{1}{\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x)\sin nx = \dfrac{1}{\pi}\left(\int\limits_{-\pi}^{0} \cos nx\ \mathrm{d}x\ + \int\limits_{0}^{\pi} 3\cos nx\ \mathrm{d}x\right) = \dfrac{1}{\pi n}(-\cos 0 + \cos \pi n -$

$-\cos \pi n + \cos 0) = \dfrac{1}{\pi n}\left(-1+(-1)^n - 3(-1)^n + 3\right) = \dfrac{2}{\pi n}\left(1+(-1)^{n+1}\right)$

$f(x) = 2 + \displaystyle\sum_{n=1}^{\infty} \dfrac{2}{\pi n}(1+(-1)^{n+1})\sin nx$



$f(x)$



$S(x)$

(b) $a_0 = \dfrac{1}{\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x) = \dfrac{1}{\pi}\left(\int\limits_{0}^{\pi} \sin x\ \mathrm{d}x\right) = \dfrac{1}{\pi}(-\cos 0 - \cos \pi) = \dfrac{2}{\pi}$

$a_1 = \dfrac{1}{\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x)\cos x = \dfrac{1}{\pi}\left(\int\limits_{0}^{\pi} \sin x\cos x\ \mathrm{d}x\right) = \dfrac{1}{\pi}\left(\int\limits_{0}^{\pi} \sin x\ \mathrm{d}(\sin x)\right) =$

$= \dfrac{1}{2\pi}\left(\sin^2\pi - \sin^2 0\right) = 0$

$b_1 = \dfrac{1}{\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x)\sin x = \dfrac{1}{\pi}\left(\int\limits_{0}^{\pi} \sin^2 x\ \mathrm{d}x\right) = \dfrac{1}{2\pi}\left(\int\limits_{0}^{\pi} (1-\cos 2x)\ \mathrm{d}x\right) =$

$= \dfrac{1}{2\pi}\left(\pi - \dfrac{\sin 2\cdot\pi}{2} + 0 - \dfrac{\sin 2\cdot 0}{2}\right) = \dfrac{1}{2}$

$a_n = \dfrac{1}{\pi}\displaystyle\int\limits_{-\pi}^{\pi} f(x)\cos nx\ \mathrm{d}x\ = \dfrac{1}{\pi}\left(\int\limits_{0}^{\pi} \sin x\cos nx\ \mathrm{d}x\right) = \dfrac{1}{2\pi}\left(\int\limits_{0}^{\pi} \sin(x+nx)\ \mathrm{d}x\ +\right.$

$\left.+\displaystyle\int\limits_{0}^{\pi} \sin(x-nx)\ \mathrm{d}x\right) = \dfrac{1}{2\pi}\left(-\dfrac{\cos(x+nx)}{1+n}\Bigg|_0^{\pi} - \dfrac{\cos(x-nx)}{1-n}\Bigg|_0^{\pi}\right) =$

$= \dfrac{1}{2\pi}\left(\dfrac{(1-n)\cos(x+nx)+(1+n)\cos(x-nx)}{1-n^2}\right) =$

$$= \frac{1}{2\pi}\left(\frac{(1-n)(-1)^n + (1+n)(-1)^n + 2}{1+n^2}\right) = \frac{(-1)^n(1-n+n+1)+2}{1-n^2} = \frac{(-1)^n+1}{\pi(1-n^2)}$$

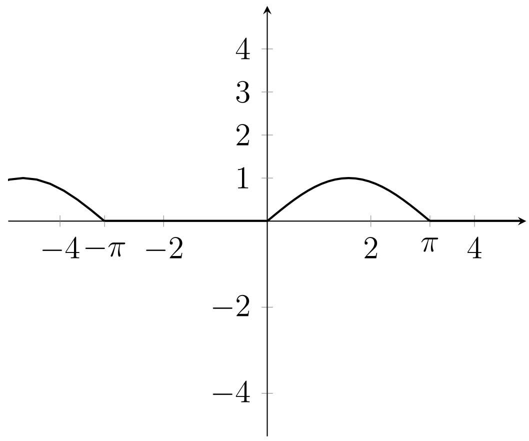
$$b_n = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} f(x)\sin nx \ \mathrm{d}x \ = \frac{1}{\pi}\left(\int\limits_{0}^{\pi} \sin x \sin nx \ \mathrm{d}x\right) = \frac{1}{2\pi}\left(\int\limits_{0}^{\pi} \cos(x-nx) \ \mathrm{d}x \ -\right.$$

$$\left.-\int\limits_{0}^{\pi} \cos(x+nx) \ \mathrm{d}x\right) = \frac{1}{2\pi}\left(-\frac{\sin(x-nx)}{1+n}\Bigg|_0^{\pi} - \frac{\sin(x+nx)}{1-n}\Bigg|_0^{\pi}\right) =$$

$$= \frac{1}{2\pi}\left(\frac{(1+n)\sin(x-nx)+(n-1)\sin(x+nx)}{1-n^2}\right) =$$

$$= \frac{1}{2\pi}\left(\frac{(1+n)\sin\pi n-(n-1)\sin\pi n}{1+n^2}\right) = 0$$

$$f(x) = \frac{1}{\pi} + \sum_{n=2}^{\infty}\frac{(-1)^n+1}{\pi(1-n^2)}\cos nx + \frac{1}{2}\sin x$$



(c) $$a_0 = \frac{1}{\pi}\int\limits_{0}^{2\pi} f(x) \ \mathrm{d}x \ = \int\limits_{0}^{2\pi} (2x-3) \ \mathrm{d}x \ = \frac{1}{\pi}(4\pi^2 - 6\pi) = 4\pi - 6$$

$$a_n = \frac{1}{\pi}\int\limits_{0}^{2\pi} f(x)\cos nx \ \mathrm{d}x \ = \frac{1}{\pi}\left(2\int\limits_{0}^{2\pi} x\cos nx \ \mathrm{d}x \ - 3\int\limits_{0}^{2\pi}\cos nx \ \mathrm{d}x\right) =$$

$$= \frac{1}{\pi}\left(\frac{2x\sin nx - 3\sin nx}{n} + \frac{2\cos nx}{n^2}\right)\Bigg|_0^{2\pi} = \frac{4\pi n\cdot 0 - 3n\cdot 0 + 2\cdot(-1) - 2}{\pi n^2} = \frac{0}{2\pi}$$

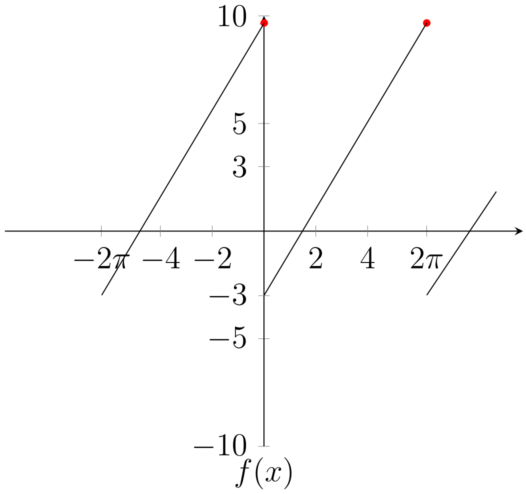
$$b_n = \frac{1}{\pi}\int\limits_{0}^{2\pi} f(x)\sin nx \ \mathrm{d}x \ = \frac{1}{\pi}\left(2\int\limits_{0}^{2\pi} x\sin nx \ \mathrm{d}x \ - 3\int\limits_{0}^{2\pi}\sin nx \ \mathrm{d}x\right) =$$

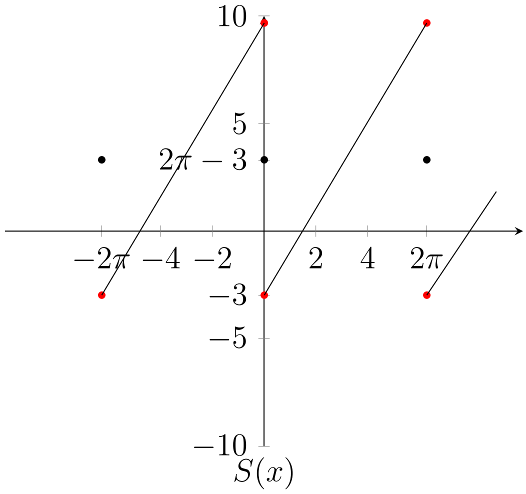
$$= \frac{1}{\pi}\left(\frac{2x\sin nx}{n} + \frac{2\cos nx}{n^2} - \frac{3\sin nx}{n}\right)\Bigg|_0^{2\pi} = \frac{4\pi n\sin 2\pi n - 3n\sin 2\pi n - 4\pi n - \cos 2\pi n}{\pi n^2} =$$

$$= -\frac{4n}{\pi n} = -\frac{4}{n}$$

$$f(x) = 2\pi - 3 + \sum_{n=1}^{\infty} -\frac{4}{n}\sin nx$$

(d) $$a_0 = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} f(x) \ \mathrm{d}x \ = \frac{1}{\pi}\cdot\frac{x^3}{3}\Bigg|_{-\pi}^{\pi} = \frac{2}{3}\pi^2$$





$$a_n = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} f(x)\cos nx \ \mathrm{d}x \ = \langle t = nx\rangle = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} \frac{t^2\cos t}{n^3} \ \mathrm{d}t \ = \frac{1}{\pi n^3}\left(t^2\sin t - \int\limits_{-\pi}^{\pi} 2t\sin t\right) =$$

$$= \frac{1}{\pi n^3}\left(t^2\sin t + 2t\cos t + \int\limits_{-\pi}^{\pi}\cos t dt\right) = \frac{1}{\pi n^3}((nx^2)\sin nx - 2(nx(-\cos nx) + \sin nx))\Big|_{-\pi}^{\pi} =$$

$$= \frac{1}{\pi n^3}(4\pi^2 n^2 \sin \pi n + 4\pi n\cos \pi n - 4\sin \pi n) = \frac{4(-1)^n}{n}$$

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos n(x)$$