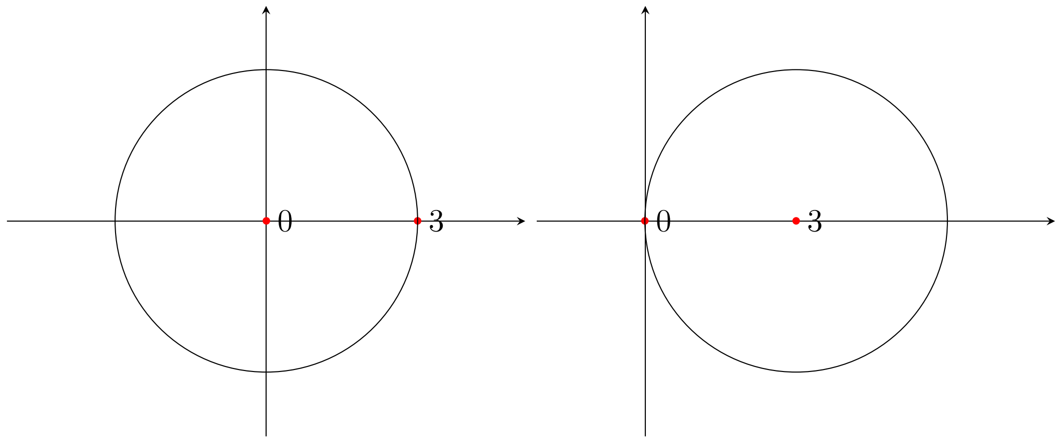
ДОМАШНЯ РОБОТА №9
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

ФІ-12 Бекешева Анастасія

2. $f(z) = \dfrac{1}{z(z-3)},\ z=0,\ z=3$



$$z_0 = 0, \quad |z| < 3, \quad \frac{1}{z-3} = -\frac{1}{3}\cdot\frac{1}{1-\frac{z}{3}} = \left\langle \left|\frac{z}{3}\right| < 1 \right\rangle = -\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^n z^n}{3^{n+1}}$$

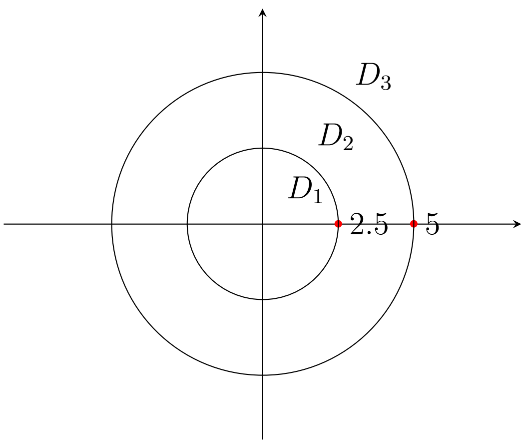
$$f(z) = \frac{1}{z}\sum_{n=0}^{\infty}\frac{(-1)^n z^n}{3^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n z^{n-1}}{3^{n+1}}$$

$$z_0 = 3, \quad |z-3| < 3, \quad \frac{1}{z} = -\frac{1}{-3-(z-3)} = \frac{1}{3}\cdot\frac{1}{1-\left(-\frac{z-3}{3}\right)} = \frac{1}{3}\sum_{n=0}^{\infty}\left(-\frac{z-3}{3}\right)^n =$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n (z-3)^n}{3^{n+1}}$$

$$f(z) = \frac{1}{z-3}\sum_{n=0}^{\infty}\frac{(-1)^n (z-3)^n}{3^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n (z-3)^{n-1}}{3^{n+1}}$$

3. $f(z) = \dfrac{5z-50}{2z^3+5z^2-25z}, \quad z_0 = 0$



$$\left[\begin{array}{lll} z_1 = 0, & D_1: & 0 < |z| < 2.5 \\ z_2 = 2.5 & D_2: & 2.5 < |z| < 5 \\ z_3 = -5 & D_3: & |z| > 5 \end{array}\right.,$$

$$f(z) = \frac{5}{z}\cdot\frac{z-10}{(z+5)(2z-5)} = \frac{5}{z}\cdot\left(\frac{A}{z+5}+\frac{B}{2z-5}\right) = \frac{5}{z}\cdot\left(\frac{1}{z+5}+\frac{1}{2z-5}\right)$$

$$\frac{1}{z+5} = \frac{1}{5}\cdot\frac{1}{1-\left(-\frac{z}{5}\right)} = \frac{1}{5}\sum_{n=0}^{\infty}\left(-\frac{z}{5}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n z^n}{5^{n+1}} \in D_1, D_2$$

$$\frac{1}{z+5} = \frac{1}{z}\cdot\frac{1}{1-\left(-\frac{5}{z}\right)} = \frac{1}{z}\sum_{n=0}^{\infty}\left(-\frac{5}{z}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n 5^n}{z^{n+1}} \in D_3$$

$$\frac{1}{2z-5} = -\frac{1}{5}\cdot\frac{1}{1-\frac{2z}{5}} = -\frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{2z}{5}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n z^n}{5^{n+1}} \in D_1$$

$$\frac{1}{2z-5} = \frac{1}{2z}\cdot\frac{1}{1-\frac{5}{2z}} = \frac{1}{2z}\sum_{n=0}^{\infty}\left(\frac{5}{2z}\right) = \sum_{n=0}^{\infty}\frac{5^n}{2^{n+1}z^{n+1}} \in D_1, D_3$$

$$D_1: \qquad 0 < |z| < 2.5: \qquad f(z) = \frac{5}{z}\left(\sum_{n=0}^{\infty}\frac{(-1)^n z^n}{5^{n+1}} + \sum_{n=0}^{\infty}\frac{2^n z^n}{5^{n+1}}\right) = \sum_{n=0}^{\infty}\frac{((-1)^n + 2^n)z^{n-1}}{5^n}$$

$$D_2: \qquad 2.5 < |z| < 5: \qquad f(z) = \sum_{n=0}^{\infty}\frac{(-1)^n z^{n-1}}{5^n} - \sum_{n=0}^{\infty}\frac{5^{n+1}}{2^{n+1}z^{n+2}}$$

$$D_3: \qquad |z| > 5: \qquad f(z) = \sum_{n=0}^{\infty}\frac{(-1)^n - 2^{-n-1}5^{n+2}}{z^{n+2}}$$