ДОМАШНЯ РОБОТА №4
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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1. (a) $\sin\left(\frac{\pi}{6}-3i\right)=\frac{1}{2i}\left(e^{i\cdot\left(\frac{\pi}{6}-3i\right)}-e^{-i\cdot\left(\frac{\pi}{6}-3i\right)}\right)=\frac{1}{2i}\left(e^{\left(i\frac{\pi}{6}+3\right)}-e^{\left(-i\frac{\pi}{6}-3\right)}\right)$

   (b) $\cos\left(\frac{\pi}{3}+3i\right)\frac{1}{2}\left(e^{i\cdot\left(\frac{\pi}{3}+3i\right)}-e^{-i\cdot\left(\frac{\pi}{3}+3i\right)}\right)=\frac{1}{2}\left(e^{\left(i\frac{\pi}{3}-3\right)}-e^{\left(-i\frac{\pi}{3}+3\right)}\right)$

   (c) $\operatorname{Arcsin} i=-i\ \operatorname{Ln}\left(i\cdot i\pm\sqrt{1-i^2}\right)=-i\ \operatorname{Ln}\left(-1\pm\sqrt{2}\right)=-i\ \operatorname{Ln}\left(-1+\sqrt{2}\right)=$
   $=-i\left(\ln(-1+\sqrt{2})+2i\pi k\right)$

   (d) $\operatorname{Arccos} 2=-i\ \operatorname{Ln}\left(2\pm\sqrt{4-1}\right)=-i\ \operatorname{Ln}\left(2\pm\sqrt{3}\right)=-i\left(\ln(2\pm\sqrt{3})+2i\pi k\right)$

   (e) $\operatorname{Arctan}\left(1+2i\right)=-\frac{i}{2}\ \operatorname{Ln}\frac{1+i(1+2i)}{1-i(1+2i)}=-\frac{i}{2}\ \operatorname{Ln}\frac{i-1}{3-i}=-\frac{i}{2}\left(\ln\frac{1}{\sqrt{5}}+\right.$
   $\left.+i\left(\arctan-\frac{1}{2}+2\pi k\right)\right)$

   (f) $\operatorname{Arth}\left(1-i\right)=\frac{1}{2}\ \operatorname{Ln}\frac{1+(1-i)}{1-(1-i)}=\frac{1}{2}\ \operatorname{Ln}\frac{2-i}{i}=\frac{1}{2}\left(\ln\sqrt{5}+i\left(\arctan 2-\pi+2\pi k\right)\right)$

   (g) $\operatorname{Arch} 2i=\ \operatorname{Ln}\left(2i\pm\sqrt{-4-1}\right)=\ \operatorname{Ln}\left(2i\pm(\pm\sqrt{5}i)\right)=\ \operatorname{Ln}\left(2i+\sqrt{5}i\right)=\ln(9+4\sqrt{5})+$
   $+i\left(\frac{\pi}{2}+2\pi k\right)$

   (h) $\operatorname{Ln}\left(-i\right)=\ln 1+i\left(-\frac{\pi}{2}+2\pi k\right)=i\left(-\frac{\pi}{2}+2\pi k\right)$

2. (a) $i^{1+i}=\exp((1+i)\ \operatorname{Ln} i)=\exp\left((1+i)\cdot\left(\ln 1+i\frac{\pi}{2}+2\pi k\right)\right)=\exp\left(2\pi k-\frac{\pi}{2}+\right.$
   $\left.+i\left(\frac{\pi}{2}+2\pi k\right)\right)=e^{2\pi k-\frac{\pi}{2}}\left(\cos\left(\frac{\pi}{2}+2\pi k\right)+i\sin\left(\frac{\pi}{2}+2\pi k\right)\right)$

   (b) $(1+i)^i=\exp(i\ \operatorname{Ln}\left(1+i\right))=\exp\left(i\ln\sqrt{2}+i\cdot i\left(\arctan 1+2\pi k\right)\right)=\exp(-\arctan 1-$
   $-2\pi k+i\ln\sqrt{2})=e^{-\arctan 1-2\pi k}(\cos\ln\sqrt{2}+i\sin\ln\sqrt{2})$

   (c) $3^i=\exp(i\ \operatorname{Ln} 3)=\exp(i(\ln 3+2\pi ki))=\exp(-2\pi k+i\ln 3)=e^{-2\pi k}(\cos\ln 3+$
   $+i\sin\ln 3)$

   (d) $2^{1+i}=\exp((1+i)\ \operatorname{Ln} 2)=\exp((1+i)\cdot(\ln 2+2\pi ki))=\exp(\ln 2-2\pi k+i(2\pi k+\ln 2))=$
   $=e^{\ln 2-2\pi k}(\cos(2\pi k+\ln 2)+i\sin(2\pi k+\ln 2))$

   (e) $(-1)^{\sqrt{3}}=\exp(\sqrt{3}\ \operatorname{Ln}\left(-1\right))=\exp(\sqrt{3}(\ln 1+i(2\pi k+2\pi)))=\exp(\sqrt{3}\cdot i(2\pi k+2\pi))=$
   $=e^0(\cos\sqrt{3}\cdot(2\pi k+2\pi)+i\sin\sqrt{3}\cdot(2\pi k+2\pi))$