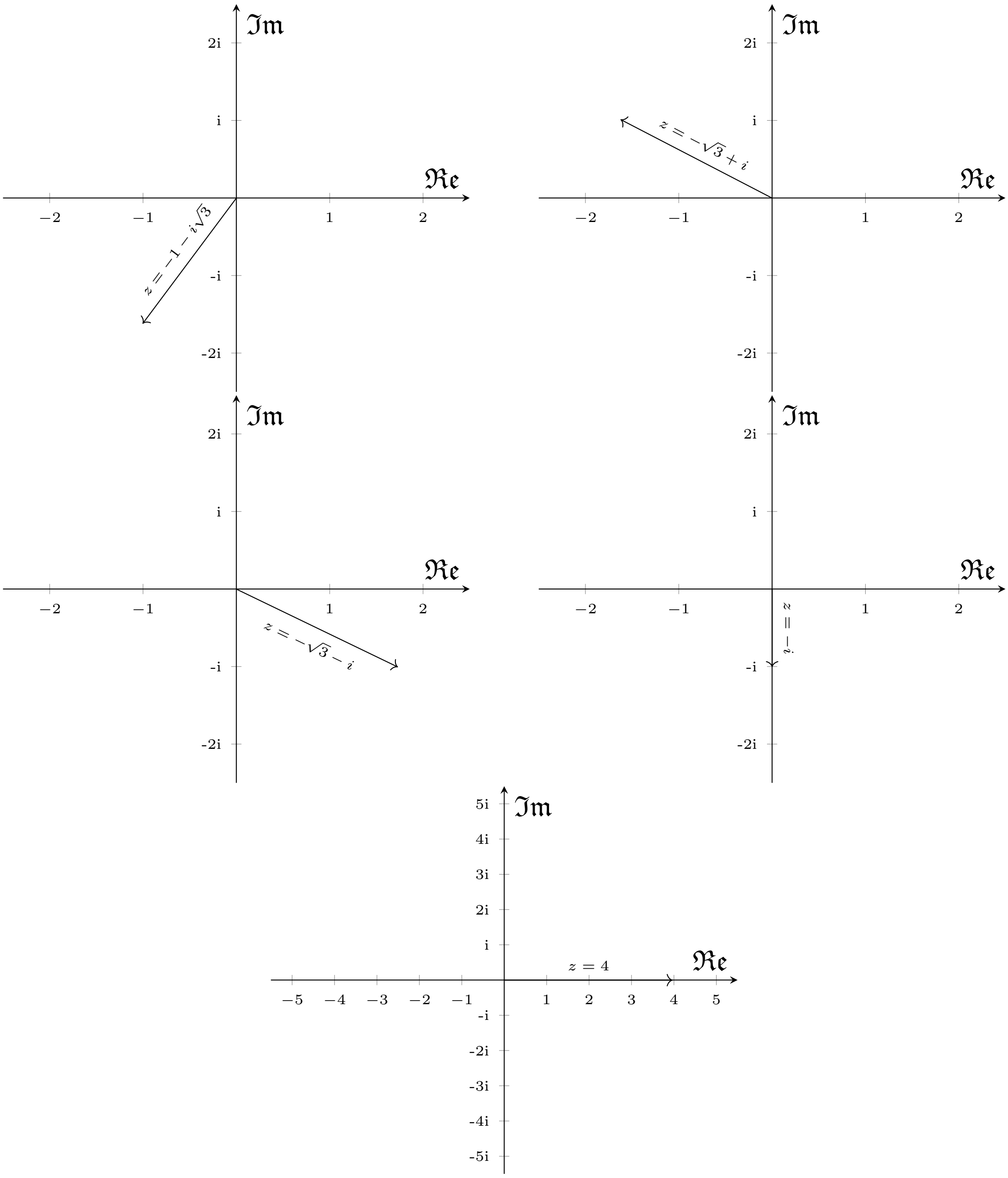
ДОМАШНЯ РОБОТА №1
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

ФІ-12 Бекешева Анастасія

1.



(a) $\mathfrak{Re} z = -1, \quad \mathfrak{Im} z = -\sqrt{3}, \quad |z| = 2, \quad \arg z = \arctan \dfrac{-\sqrt{3}}{-1} - \pi = -\dfrac{2\pi}{3},$

$z = 2\left(\cos -\dfrac{2\pi}{3} + i \sin -\dfrac{2\pi}{3}\right), \quad z = 2e^{-\dfrac{2\pi}{3}i}$

(b) $\mathfrak{Re} z = -\sqrt{3}, \quad \mathfrak{Im} z = 1, \quad |z| = 2, \quad \arg z = \arctan \dfrac{1}{-\sqrt{3}} + \pi = \dfrac{5\pi}{6},$

$z = 2\left(\cos \dfrac{5\pi}{6} + i \sin \dfrac{5\pi}{6}\right), \quad z = 2e^{\dfrac{5\pi}{6}i}$

(c) $\mathfrak{Re} z = \sqrt{3}, \quad \mathfrak{Im} z = -1, \quad |z| = 2, \quad \arg z = \arctan \dfrac{-1}{\sqrt{3}} = -\dfrac{\pi}{6},$

$z = 2\left(\cos -\dfrac{\pi}{6} + i \sin -\dfrac{\pi}{6}\right), \quad z = 2e^{-\dfrac{\pi}{6}i}$

(d) $\mathfrak{Re} z = 0, \quad \mathfrak{Im} z = -1, \quad |z| = 1, \quad \arg z = \arctan -\frac{1}{0} = -\frac{\pi}{2},$

$z = \left(\cos -\frac{\pi}{2} + i \sin -\frac{\pi}{2}\right), \quad z = e^{-\frac{\pi}{2} i}$

(e) $\mathfrak{Re} z = 4, \quad \mathfrak{Im} z = 0, \quad |z| = 4, \quad \arg z = \arctan \frac{0}{4} = 0,$

$z = 4\left(\cos 0 + i \sin 0\right), \quad z = 4e^{0 \cdot i}$

2. (a) $z = 2 \pm 5i, \quad |z| = \sqrt{29}, \quad \arg z = \arctan \pm \frac{5}{2}$

(b) $z = -2 \pm 5i, \quad |z| = \sqrt{29}, \quad \arg z = \arctan \mp \frac{5}{2} \pm \pi$

(c) $z = ib, b \in \mathbb{R}, b \neq 0, \quad |z| = |b|, \quad \arg z = \arctan \frac{b}{0} = \pm \frac{\pi}{2}$

(d) $z = e^{-5i}, \quad |z| = 1, \quad \arg z = 5$

3. $z = \frac{5+i}{1+2i} = \frac{7-9i}{5} = \frac{7}{5} - \frac{9}{5} i, \quad \mathfrak{Re} z = \frac{7}{5}, \quad \mathfrak{Im} z = -\frac{9}{5}$

4. $x(1-2i) + y(2i-3) = 4-8i, \quad x - 2ix + 2iy - 3y = 4-8i, \quad (x-3y) + i(2x-2y) = 4-8i,$

$\begin{cases} -4 = x - y \\ 4 = x - 3y \end{cases}, \quad \begin{cases} y = -4 \\ x = -8 \end{cases}$

5. $z_1 = 1 - 2i, z_2 = -2 + i$

$$\begin{aligned} z_1 + z_2 &= (1-2) + i(-2+1) = -1 - i \\ z_1 - z_2 &= (1+2) + i(-2-1) = 3 - 3i \\ z_1 \cdot z_2 &= -2 + 2 + i(1+4) = 5i \\ \frac{z_1}{z_2} &= \frac{1-2i}{-2+i} \cdot \frac{-2-i}{-2-i} = \frac{-4+3i}{5} = -\frac{4}{5} + \frac{3}{5} i \end{aligned}$$

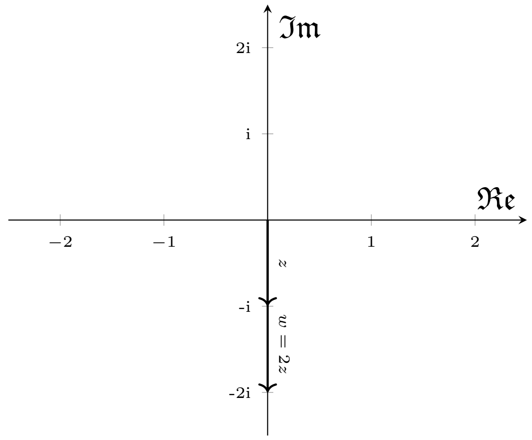
6. $z_i = 3 + 3i, \quad z_2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}, \quad z_3 = 2\sqrt{3} - 2i,$

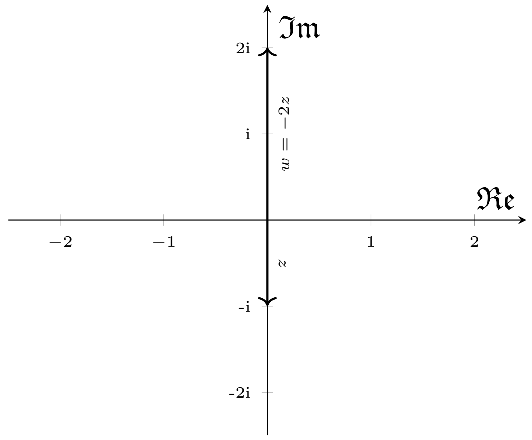
$z_1 = 3\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right), \quad z_2 = \cos -\frac{2\pi}{3} + i \sin -\frac{2\pi}{3}, \quad z_3 = 4\left(\cos -\frac{\pi}{6} + i \sin -\frac{\pi}{6}\right)$

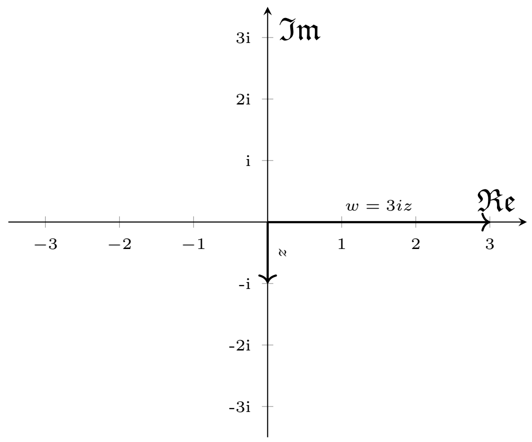
$z_1 \cdot z_2 = 3\sqrt{2}\left(\cos -\frac{5\pi}{12} + i \sin -\frac{5\pi}{12}\right) = \frac{3\sqrt{3}-3}{2} - i\frac{3\sqrt{3}+3}{2},$

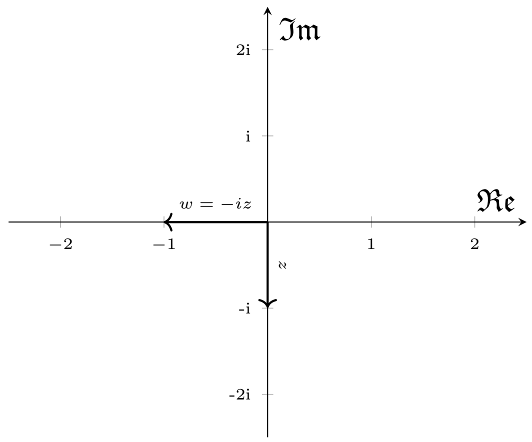
$\frac{z_1 \cdot z_2}{z_3} = \frac{3\sqrt{2}}{4}\left(\cos -\frac{7\pi}{12} + i \sin -\frac{7\pi}{12}\right) = \frac{3\sqrt{3}-3}{8} - i\frac{3\sqrt{3}+3}{8}$

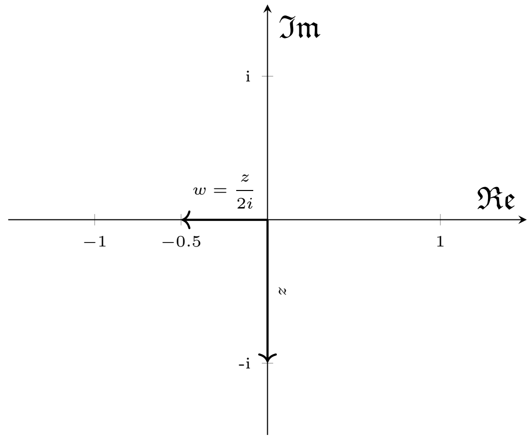
7. $z = iy, y < 0$


ℑ𝔪
ℜ𝔢
2i
i
-i
-2i
−2
−1
1
2
z
w = 2z


ℑ𝔪
ℜ𝔢
2i
i
-i
-2i
−2
−1
1
2
z
w = −2z


ℑ𝔪
ℜ𝔢
3i
2i
i
-i
-2i
-3i
−3
−2
−1
1
2
3
z
w = 3iz


ℑ𝔪
ℜ𝔢
2i
i
-i
-2i
−2
−1
1
2
z
w = −iz


ℑ𝔪
ℜ𝔢
i
-i
−1
−0.5
1
z
w = z/2i