ДОМАШНЯ РОБОТА №14
З ПРЕДМЕТУ
"ТЕОРІЯ ФУНКЦІЇ КОМПЛЕКСНОЇ ЗМІННОЇ"

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1. $c_n = \frac{1}{2\pi}\int\limits_{-\pi}^{\pi} f(x)e^{-inx}\ \mathrm{d}x\ = \frac{1}{2\pi}\left(\int\limits_{-\pi}^{0} e^{-inx}\ \mathrm{d}x\ + 3\int\limits_{0}^{\pi} e^{-inx}\ \mathrm{d}x\right) = \frac{1}{2\pi}\left(\frac{i}{n}e^{-inx}\Big|_{-\pi}^{0} + \right.$

$\left. + \frac{i}{n}e^{-3i\frac{nx}{2}}\Big|_{0}^{\pi}\right) = \frac{1}{2\pi}\left(\frac{i}{n}e^{0} - \frac{i}{n}e^{i\pi n} + \frac{3i}{n}e^{-i\pi n} - \frac{3i}{n}e^{0}\right) = \frac{i}{2\pi n}\left(-2 - e^{i\pi n} + 3e^{-i\pi n}\right) =$

$= \frac{i}{\pi n}\left(-2 - \cos(\pi n) - i\sin(\pi n) + 3\cos(\pi n) + 3i\sin(\pi n)\right) = \frac{i}{\pi n}\left(-2 - (-1)^n + 3\cdot(-1)^n\right)$

$f(x) = \sum\limits_{n=-\infty}^{\infty} \frac{2i}{\pi n}(-1 + (-1)^n)e^{i\pi n x}$

2. $a_0 = \frac{2}{l}\int\limits_{0}^{l} f(x)\ \mathrm{d}x\ = \frac{2}{l}\int\limits_{0}^{l} |x|\ \mathrm{d}x\ = \frac{2}{l}\int\limits_{0}^{l} x\ \mathrm{d}x\ = \frac{1}{l}\left(l^2 - 0\right) = l$

$a_n = \frac{2}{l}\int\limits_{0}^{l} f(x)\cos\left(\frac{\pi n x}{l}\right) = \frac{2}{l}\int\limits_{0}^{l} x\cos\left(\frac{\pi n x}{l}\right) = \left\langle \begin{matrix} u = x, & \mathrm{d}v\ = \cos\left(\frac{\pi n x}{l}\right) \\ \mathrm{d}u\ =\ \mathrm{d}x\ , & v = \frac{l}{\pi n}\sin\left(\frac{\pi n x}{l}\right) \end{matrix} \right\rangle =$

$= \frac{2}{l}\left(\frac{xl}{\pi n}\sin\left(\frac{\pi n x}{l}\right)\Big|_{0}^{l} - \frac{l}{\pi n}\int\limits_{0}^{l}\sin\left(\frac{\pi n x}{l}\right)\ \mathrm{d}x\right) = \frac{2}{l}\left(\frac{xl}{\pi n}\sin\left(\frac{\pi n x}{l}\right) + \frac{l^2}{\pi^2 n^2}\cos\left(\frac{\pi n x}{l}\right)\right)\Big|_{0}^{l} =$

$= \frac{2}{l}\left(\frac{l^2}{\pi n}\sin(\pi n) + \frac{l^2}{\pi^2 n^2}\cos(\pi n) - 1\right) = \frac{2l}{\pi^2 n^2}(-1)^n - \frac{2}{l}$

$f(x) = \frac{l}{2} + \sum\limits_{n=1}^{\infty}\left(\frac{2l}{\pi^2 n^2}(-1)^n - \frac{2}{l}\right)\cos\left(\frac{\pi n x}{l}\right)$

3. $a_0 = \frac{2}{2}\int\limits_{0}^{2} f(x)\ \mathrm{d}x\ = \int\limits_{0}^{1}\ \mathrm{d}x\ + \int\limits_{1}^{2} 0\ \mathrm{d}x\ = x\big|_0^1 = 1$

$a_n = \frac{2}{2}\int\limits_{0}^{2} f(x)\cos\left(\frac{\pi n x}{2}\right)\ \mathrm{d}x\ = \int\limits_{0}^{1}\cos\left(\frac{\pi n x}{2}\right)\ \mathrm{d}x + \int\limits_{1}^{2} 0\cdot\cos\left(\frac{\pi n x}{2}\right)\ \mathrm{d}x\ = \frac{2}{\pi n}\sin\left(\frac{\pi n x}{2}\right)\Big|_{0}^{1} =$

$= \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right) - \frac{2}{\pi n}\sin\left(\frac{\pi n\cdot 0}{2}\right) = \frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)$

$f(x) = \frac{1}{2} + \sum\limits_{n=1}^{\infty}\frac{2}{\pi n}\sin\left(\frac{\pi n}{2}\right)\cos\left(\frac{\pi n x}{2}\right)$

4. $b_n = \frac{2}{\pi}\int\limits_{0}^{\pi} f(x)\sin(nx)\ \ \mathrm{d}x\ = \frac{2}{\pi}\int\limits_{0}^{\pi}\operatorname{ch}x\sin(nx)\ \mathrm{d}x\ = \left\langle \begin{matrix} u = \sin(nx) & \mathrm{d}v\ = \operatorname{ch}x\ \mathrm{d}x \\ \mathrm{d}u\ = n\cos x\ \mathrm{d}x & v = \operatorname{sh}x \end{matrix} \right\rangle =$

$= \frac{2}{\pi}\left(\operatorname{sh}x\sin(nx) - n\int\limits_{0}^{\pi}\operatorname{sh}x\cos x\ \mathrm{d}x\right) = \left\langle \begin{matrix} u = \cos(nx), & \mathrm{d}v\ = \operatorname{sh}x\ \mathrm{d}x \\ \mathrm{d}u\ = -n\sin x\ \mathrm{d}x\ , & v = \operatorname{ch}x \end{matrix} \right\rangle =$

$= \frac{2}{\pi}\left(\operatorname{sh}x\sin(nx) - n\operatorname{ch}x\cos(nx) - n^2\int\limits_{0}^{\pi}\operatorname{ch}x\sin(nx)\right) =$

$\int\limits_{0}^{\pi}\operatorname{ch}x\sin(nx)\ \mathrm{d}x\ + n^2\int\limits_{0}^{\pi}\operatorname{ch}x\sin(nx) = \operatorname{sh}x\sin(nx) - n\operatorname{ch}x\cos(nx)$

$$= \quad \frac{2}{\pi}\,\frac{\operatorname{sh} x \sin(nx) - n \operatorname{ch} x \cos(nx)}{n^2+1}\Bigg|_0^{\pi} = \frac{2}{\pi(n^2+1)}(\operatorname{sh}\pi \sin(\pi n) - n \operatorname{ch}\pi \cos(\pi n) - \operatorname{sh} 0 \sin 0 +$$

$$+ 0 \cdot \operatorname{ch} 0 \cos 0) = \frac{2}{\pi(n^2+1)}(-n \operatorname{ch}\pi (-1)^n)$$

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{2}{\pi(n^2+1)}(-n \operatorname{ch}\pi(-1)^n)\sin(nx)\right)$$