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PRACTICE PROBLEMS

1 The table below gives current information on the interest rates for two two-year and two eight-year maturity investments. The table also gives the maturity, liquidity, and default risk characteristics of a new investment possibility (Investment 3). All investments promise only a single payment (a payment at maturity). Assume that premiums relating to inflation, liquidity, and default risk are constant across all time horizons.

Investment	Maturity (in Years)	Liquidity	Default Risk	Interest Rate (%)
1	2	High	Low	2.0
2	2	Low	Low	2.5
3	7	Low	Low	r_3
4	8	High	Low	4.0
5	8	Low	High	6.5

Based on the information in the above table, address the following:

A Explain the difference between the interest rates on Investment 1 and Investment 2.

B Estimate the default risk premium.

C Calculate upper and lower limits for the interest rate on Investment 3, r_3.

2 A couple plans to set aside $20,000 per year in a conservative portfolio projected to earn 7 percent a year. If they make their first savings contribution one year from now, how much will they have at the end of 20 years?

3 Two years from now, a client will receive the first of three annual payments of $20,000 from a small business project. If she can earn 9 percent annually on her investments and plans to retire in six years, how much will the three business project payments be worth at the time of her retirement?

4 To cover the first year's total college tuition payments for his two children, a father will make a $75,000 payment five years from now. How much will he need to invest today to meet his first tuition goal if the investment earns 6 percent annually?

5 A client can choose between receiving 10 annual $100,000 retirement payments, starting one year from today, or receiving a lump sum today. Knowing that he can invest at a rate of 5 percent annually, he has decided to take the lump sum. What lump sum today will be equivalent to the future annual payments?

6 You are considering investing in two different instruments. The first instrument will pay nothing for three years, but then it will pay $20,000 per year for four years. The second instrument will pay $20,000 for three years and $30,000 in the fourth year. All payments are made at year-end. If your required rate of return on these investments is 8 percent annually, what should you be willing to pay for:

A The first instrument?

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B The second instrument (use the formula for a four-year annuity)?

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7 Suppose you plan to send your daughter to college in three years. You expect her to earn two-thirds of her tuition payment in scholarship money, so you estimate that your payments will be $10,000 a year for four years. To estimate whether you have set aside enough money, you ignore possible inflation in tuition payments and assume that you can earn 8 percent annually on your investments. How much should you set aside now to cover these payments?

8 A client plans to send a child to college for four years starting 18 years from now. Having set aside money for tuition, she decides to plan for room and board also. She estimates these costs at $20,000 per year, payable at the beginning of each year, by the time her child goes to college. If she starts next year and makes 17 payments into a savings account paying 5 percent annually, what annual payments must she make?

9 A couple plans to pay their child's college tuition for 4 years starting 18 years from now. The current annual cost of college is C$7,000, and they expect this cost to rise at an annual rate of 5 percent. In their planning, they assume that they can earn 6 percent annually. How much must they put aside each year, starting next year, if they plan to make 17 equal payments?

10 The nominal risk-free rate is *best* described as the sum of the real risk-free rate and a premium for:

A maturity.

B liquidity.

C expected inflation.

11 Which of the following risk premiums is most relevant in explaining the difference in yields between 30-year bonds issued by the US Treasury and 30-year bonds issued by a small private issuer?

A Inflation

B Maturity

C Liquidity

12 A bank quotes a stated annual interest rate of 4.00%. If that rate is equal to an effective annual rate of 4.08%, then the bank is compounding interest:

A daily.

B quarterly.

C semiannually.

13 The value in six years of $75,000 invested today at a stated annual interest rate of 7% compounded quarterly is *closest* to:

A $112,555.

B $113,330.

C $113,733.

14 A client requires £100,000 one year from now. If the stated annual rate is 2.50% compounded weekly, the deposit needed today is *closest* to:

A £97,500.

B £97,532.

C £97,561.

15 For a lump sum investment of ¥250,000 invested at a stated annual rate of 3% compounded daily, the number of months needed to grow the sum to ¥1,000,000 is *closest* to:

A 555.

B 563.

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C 576.

16 Given a €1,000,000 investment for four years with a stated annual rate of 3% compounded continuously, the difference in its interest earnings compared with the same investment compounded daily is *closest* to:

A €1.

B €6.

C €455.

17 An investment pays €300 annually for five years, with the first payment occurring today. The present value (PV) of the investment discounted at a 4% annual rate is *closest* to:

A €1,336.

B €1,389.

C €1,625.

18 A perpetual preferred stock makes its first quarterly dividend payment of $2.00 in five quarters. If the required annual rate of return is 6% compounded quarterly, the stock's present value is *closest* to:

A $31.

B $126.

C $133.

19 A saver deposits the following amounts in an account paying a stated annual rate of 4%, compounded semiannually:

Year	End of Year Deposits ($)
1	4,000
2	8,000
3	7,000
4	10,000

At the end of Year 4, the value of the account is *closest* to:

A $30,432

B $30,447

C $31,677

20 An investment of €500,000 today that grows to €800,000 after six years has a stated annual interest rate *closest* to:

A 7.5% compounded continuously.

B 7.7% compounded daily.

C 8.0% compounded semiannually.

21 A sweepstakes winner may select either a perpetuity of £2,000 a month beginning with the first payment in one month or an immediate lump sum payment of £350,000. If the annual discount rate is 6% compounded monthly, the present value of the perpetuity is:

A less than the lump sum.

B equal to the lump sum.

C greater than the lump sum.

22 At a 5% interest rate per year compounded annually, the present value (PV) of a 10-year ordinary annuity with annual payments of $2,000 is $15,443.47. The PV of a 10-year annuity due with the same interest rate and payments is *closest* to:

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A $14,708.

B $16,216.

C $17,443.

23 Grandparents are funding a newborn's future university tuition costs, estimated at $50,000/year for four years, with the first payment due as a lump sum in 18 years. Assuming a 6% effective annual rate, the required deposit today is *closest* to:

A $60,699.

B $64,341.

C $68,201.

24 The present value (PV) of an investment with the following year-end cash flows (CF) and a 12% required annual rate of return is *closest* to:

Year	Cash Flow (€)
1	100,000
2	150,000
5	–10,000

A €201,747.

B €203,191.

C €227,573.

25 A sports car, purchased for £200,000, is financed for five years at an annual rate of 6% compounded monthly. If the first payment is due in one month, the monthly payment is *closest* to:

A £3,847.

B £3,867.

C £3,957.

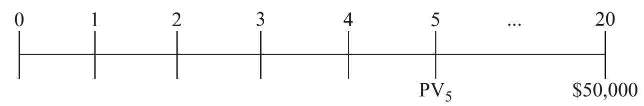
26 Given a stated annual interest rate of 6% compounded quarterly, the level amount that, deposited quarterly, will grow to £25,000 at the end of 10 years is *closest* to:

A £461.

B £474.

C £836.

27 Given the following timeline and a discount rate of 4% a year compounded annually, the present value (PV), as of the end of Year 5 (PV_5), of the cash flow received at the end of Year 20 is *closest* to:



A $22,819.

B $27,763.

C $28,873.

28 A client invests €20,000 in a four-year certificate of deposit (CD) that annually pays interest of 3.5%. The annual CD interest payments are automatically reinvested in a separate savings account at a stated annual interest rate of 2% compounded monthly. At maturity, the value of the combined asset is *closest* to:

A €21,670.

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B €22,890.

C €22,950.

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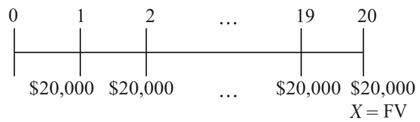
SOLUTIONS

1 **A** Investment 2 is identical to Investment 1 except that Investment 2 has low liquidity. The difference between the interest rate on Investment 2 and Investment 1 is 0.5 percentage point. This amount represents the liquidity premium, which represents compensation for the risk of loss relative to an investment's fair value if the investment needs to be converted to cash quickly.

B To estimate the default risk premium, find the two investments that have the same maturity but different levels of default risk. Both Investments 4 and 5 have a maturity of eight years. Investment 5, however, has low liquidity and thus bears a liquidity premium. The difference between the interest rates of Investments 5 and 4 is 2.5 percentage points. The liquidity premium is 0.5 percentage point (from Part A). This leaves 2.5 − 0.5 = 2.0 percentage points that must represent a default risk premium reflecting Investment 5's high default risk.

C Investment 3 has liquidity risk and default risk comparable to Investment 2, but with its longer time to maturity, Investment 3 should have a higher maturity premium. The interest rate on Investment 3, r_3, should thus be above 2.5 percent (the interest rate on Investment 2). If the liquidity of Investment 3 were high, Investment 3 would match Investment 4 except for Investment 3's shorter maturity. We would then conclude that Investment 3's interest rate should be less than the interest rate on Investment 4, which is 4 percent. In contrast to Investment 4, however, Investment 3 has low liquidity. It is possible that the interest rate on Investment 3 exceeds that of Investment 4 despite 3's shorter maturity, depending on the relative size of the liquidity and maturity premiums. However, we expect r_3 to be less than 4.5 percent, the expected interest rate on Investment 4 if it had low liquidity. Thus 2.5 percent < r_3 < 4.5 percent.

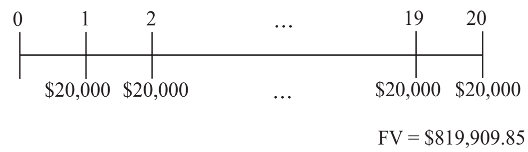
2 **i.** Draw a time line.



ii. Identify the problem as the future value of an annuity.

iii. Use the formula for the future value of an annuity.

$$
\begin{aligned}
\mathrm{FV}_N &= A\left[\frac{(1+r)^N - 1}{r}\right] \\
&= \$20{,}000\left[\frac{(1+0.07)^{20} - 1}{0.07}\right] \\
&= \$819{,}909.85
\end{aligned}
$$



iv. Alternatively, use a financial calculator.

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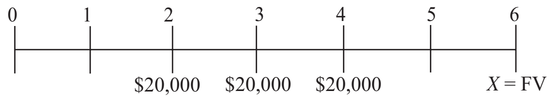
Notation Used on Most Calculators	**Numerical Value for This Problem**
N	20
%i	7
PV	n/a (= 0)
FV **compute**	X
PMT	$20,000

Enter 20 for N, the number of periods. Enter 7 for the interest rate and 20,000 for the payment size. The present value is not needed, so enter 0. Calculate the future value. Verify that you get $819,909.85 to make sure you have mastered your calculator's keystrokes.

In summary, if the couple sets aside $20,000 each year (starting next year), they will have $819,909.85 in 20 years if they earn 7 percent annually.

3 **i.** Draw a time line.



ii. Recognize the problem as the future value of a delayed annuity. Delaying the payments requires two calculations.

iii. Use the formula for the future value of an annuity (Equation 7).

$$FV_N = A\left[\frac{(1+r)^N - 1}{r}\right]$$

to bring the three $20,000 payments to an equivalent lump sum of $65,562.00 four years from today.

Notation Used on Most Calculators	**Numerical Value for This Problem**
N	3
%i	9
PV	n/a (= 0)
FV **compute**	X
PMT	$20,000

iv. Use the formula for the future value of a lump sum (Equation 2), $FV_N = PV(1 + r)^N$, to bring the single lump sum of $65,562.00 to an equivalent lump sum of $77,894.21 six years from today.

Notation Used on Most Calculators	**Numerical Value for This Problem**
N	2
%i	9
PV	$65,562.00
FV **compute**	X
PMT	n/a (= 0)

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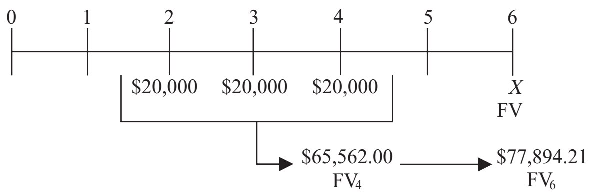
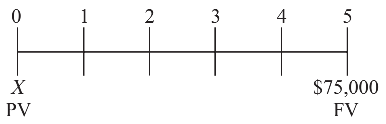


In summary, your client will have $77,894.21 in six years if she receives three yearly payments of $20,000 starting in Year 2 and can earn 9 percent annually on her investments.

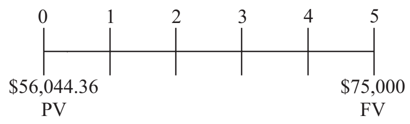
4 **i.** Draw a time line.



ii. Identify the problem as the present value of a lump sum.

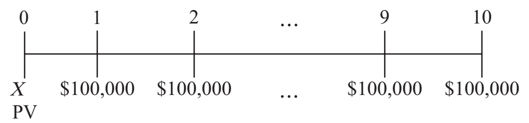
iii. Use the formula for the present value of a lump sum.

$$\begin{aligned} PV &= FV_N(1+r)^{-N} \\ &= \$75{,}000(1+0.06)^{-5} \\ &= \$56{,}044.36 \end{aligned}$$



In summary, the father will need to invest $56,044.36 today in order to have $75,000 in five years if his investments earn 6 percent annually.

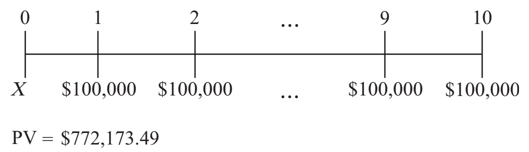
5 **i.** Draw a time line for the 10 annual payments.



ii. Identify the problem as the present value of an annuity.

iii. Use the formula for the present value of an annuity.

$$\begin{aligned} PV &= A\left[\frac{1-\dfrac{1}{(1+r)^N}}{r}\right] \\ &= \$100{,}000\left[\frac{1-\dfrac{1}{(1+0.05)^{10}}}{0.05}\right] \\ &= \$772{,}173.49 \end{aligned}$$



iv. Alternatively, use a financial calculator.

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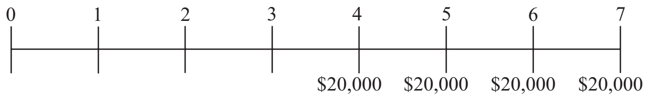
Notation Used on Most Calculators	Numerical Value for This Problem
N	10
$\%i$	5
PV **compute**	X
FV	n/a (= 0)
PMT	$100,000

In summary, the present value of 10 payments of $100,000 is $772,173.49 if the first payment is received in one year and the rate is 5 percent compounded annually. Your client should accept no less than this amount for his lump sum payment.

6 **A** To evaluate the first instrument, take the following steps:

i. Draw a time line.



ii.

$$PV_3 = A\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right]$$

$$= \$20{,}000\left[\frac{1 - \frac{1}{(1+0.08)^4}}{0.08}\right]$$

$$= \$66{,}242.54$$

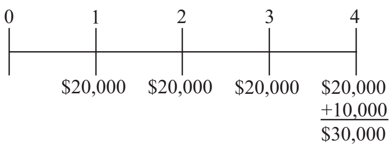
iii.

$$PV_0 = \frac{PV_3}{(1+r)^N} = \frac{\$66{,}242.54}{1.08^3} = \$52{,}585.46$$

You should be willing to pay $52,585.46 for this instrument.

B To evaluate the second instrument, take the following steps:

i. Draw a time line.



The time line shows that this instrument can be analyzed as an ordinary annuity of $20,000 with four payments (valued in Step ii below) and a $10,000 payment to be received at $t = 4$ (valued in Step iii below).

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ii.

$$\text{PV} = A\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$$

$$= \$20{,}000\left[\frac{1-\frac{1}{(1+0.08)^4}}{0.08}\right]$$

$$= \$66{,}242.54$$

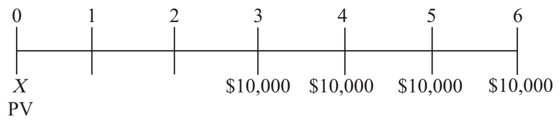
iii.

$$\text{PV} = \frac{\text{FV}_4}{(1+r)^N} = \frac{\$10{,}000}{(1+0.08)^4} = \$7{,}350.30$$

iv. Total = \$66,242.54 + \$7,350.30 = \$73,592.84

You should be willing to pay \$73,592.84 for this instrument.

7 **i.** Draw a time line.



ii. Recognize the problem as a delayed annuity. Delaying the payments requires two calculations.

iii. Use the formula for the present value of an annuity (Equation 11).

$$\text{PV} = A\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$$

to bring the four payments of \$10,000 back to a single equivalent lump sum of \$33,121.27 at t = 2. Note that we use t = 2 because the first annuity payment is then one period away, giving an ordinary annuity.

Notation Used on Most Calculators	Numerical Value for This Problem
N	4
%i	8
PV **compute**	X
PMT	\$10,000

iv. Then use the formula for the present value of a lump sum (Equation 8), PV = $\text{FV}_N(1 + r)^{-N}$, to bring back the single payment of \$33,121.27 (at t = 2) to an equivalent single payment of \$28,396.15 (at t = 0).

Notation Used on Most Calculators	Numerical Value for This Problem
N	2
%i	8
PV **compute**	X
FV	\$33,121.27
PMT	n/a (= 0)

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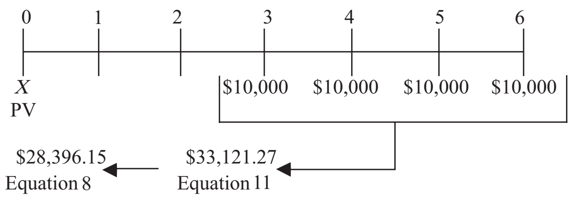
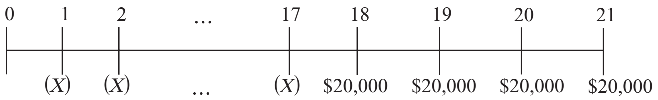


In summary, you should set aside $28,396.15 today to cover four payments of $10,000 starting in three years if your investments earn a rate of 8 percent annually.

8 **i.** Draw a time line.



ii. Recognize that you need to equate the values of two annuities.

iii. Equate the value of the four $20,000 payments to a single payment in Period 17 using the formula for the present value of an annuity (Equation 11), with $r = 0.05$. The present value of the college costs as of $t = 17$ is $70,919.

$$PV = \$20{,}000\left[\frac{1 - \dfrac{1}{(1.05)^4}}{0.05}\right] = \$70{,}919$$

Notation Used on Most Calculators	Numerical Value for This Problem
N	4
$\%i$	5
PV **compute**	X
FV	n/a (= 0)
PMT	$20,000

iv. Equate the value of the 17 investments of X to the amount calculated in Step iii, college costs as of $t = 17$, using the formula for the future value of an annuity (Equation 7). Then solve for X.

$$\$70{,}919 = \left[\frac{(1.05)^{17} - 1}{0.05}\right] = 25.840366X$$
$$X = \$2{,}744.50$$

Notation Used on Most Calculators	Numerical Value for This Problem
N	17
$\%i$	5
PV	n/a (= 0)
FV	$70,919
PMT **compute**	X

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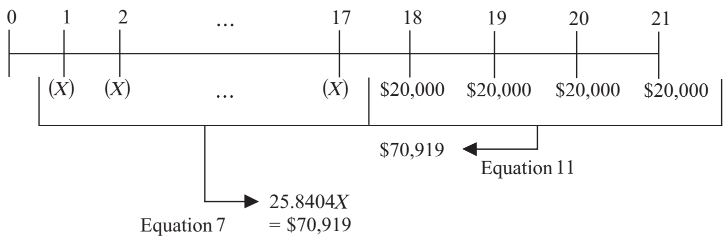
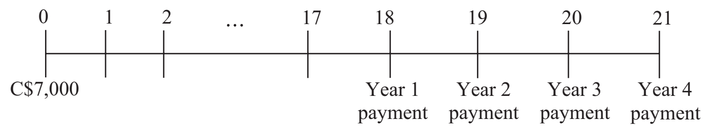


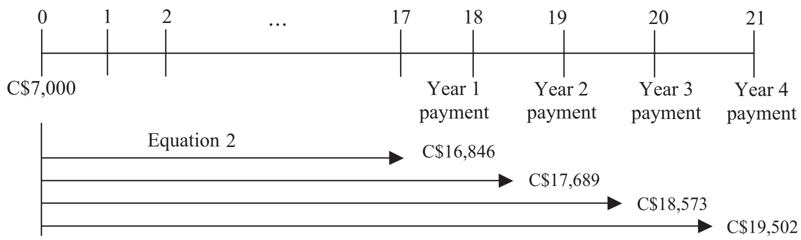
In summary, your client will have to save $2,744.50 each year if she starts next year and makes 17 payments into a savings account paying 5 percent annually.

9 **i.** Draw a time line.



ii. Recognize that the payments in Years 18, 19, 20, and 21 are the future values of a lump sum of C$7,000 in Year 0.

iii. With r = 5%, use the formula for the future value of a lump sum (Equation 2), $FV_N = PV\,(1 + r)^N$, four times to find the payments. These future values are shown on the time line below.



iv. Using the formula for the present value of a lump sum (r = 6%), equate the four college payments to single payments as of t = 17 and add them together. $C\$16{,}846(1.06)^{-1} + C\$17{,}689(1.06)^{-2} + C\$18{,}573(1.06)^{-3} + C\$19{,}502(1.06)^{-4} = C\$62{,}677$

v. Equate the sum of C$62,677 at t = 17 to the 17 payments of X, using the formula for the future value of an annuity (Equation 7). Then solve for X.

$$C\$62{,}677 = X\left[\frac{(1.06)^{17} - 1}{0.06}\right] = 28.21288X$$

$$X = C\$2{,}221.58$$

Notation Used on Most Calculators	Numerical Value for This Problem
N	17
%i	6
PV	n/a (= 0)
FV	C$62,677
PMT **compute**	X

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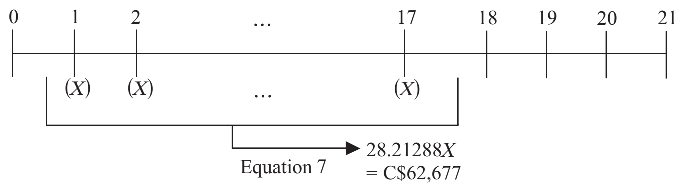

In summary, the couple will need to put aside C$2,221.58 each year if they start next year and make 17 equal payments.

10 C is correct. The sum of the real risk-free interest rate and the inflation premium is the nominal risk-free rate.

11 C is correct. US Treasury bonds are highly liquid, whereas the bonds of small issuers trade infrequently and the interest rate includes a liquidity premium. This liquidity premium reflects the relatively high costs (including the impact on price) of selling a position.

12 A is correct. The effective annual rate (EAR) when compounded daily is 4.08%.

EAR = (1 + Periodic interest rate)^m – 1

EAR = (1 + 0.04/365)^{365} – 1

EAR = (1.0408) – 1 = 0.04081 ≈ 4.08%.

13 C is correct, as shown in the following (where FV is future value and PV is present value):

$$\text{FV} = \text{PV}\left(1 + \frac{r_s}{m}\right)^{mN}$$

$$\text{FV}_6 = \$75{,}000\left(1 + \frac{0.07}{4}\right)^{(4\times6)}$$

FV_6 = $113,733.21.

14 B is correct because £97,531 represents the present value (PV) of £100,000 received one year from today when today's deposit earns a stated annual rate of 2.50% and interest compounds weekly, as shown in the following equation (where FV is future value):

$$\text{PV} = \text{FV}_N\left(1 + \frac{r_s}{m}\right)^{-mN}$$

$$\text{PV} = £100{,}000\left(1 + \frac{0.025}{52}\right)^{-52}$$

PV = £97,531.58.

15 A is correct. The effective annual rate (EAR) is calculated as follows:

EAR = (1 + Periodic interest rate)^m – 1

EAR = (1 + 0.03/365)^{365} – 1

EAR= (1.03045) – 1 = 0.030453 ≈ 3.0453%.

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Solving for N on a financial calculator results in (where FV is future value and PV is present value):

$(1 + 0.030453)^N = FV_N/PV = ¥1,000,000/¥250,000$

= 46.21 years, which multiplied by 12 to convert to months results in 554.5, or ≈ 555 months.

16 B is correct. The difference between continuous compounding and daily compounding is

€127,496.85 – €127,491.29 = €5.56, or ≈ €6, as shown in the following calculations.

With continuous compounding, the investment earns (where PV is present value)

$$\begin{aligned} PVe^{r_sN} - PV &= €1,000,000e^{0.03(4)} - €1,000,000 \\ &= €1,127,496.85 - €1,000,000 \\ &= €127,496.85 \end{aligned}$$

With daily compounding, the investment earns:

$€1,000,000(1 + 0.03/365)^{365(4)} - €1,000,000 = €1,127,491.29 - €1,000,000 = €127,491.29$.

17 B is correct, as shown in the following calculation for an annuity (A) due:

$$PV = A\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right](1+r)$$

where A = €300, r = 0.04, and N = 5.

$$PV = €300\left[\frac{1 - \frac{1}{(1+.04)^5}}{.04}\right](1.04)$$

PV = €1,388.97, or ≈ €1,389.

18 B is correct. The value of the perpetuity one year from now is calculated as:

PV = A/r, where PV is present value, A is annuity, and r is expressed as a quarterly required rate of return because the payments are quarterly.

PV = \$2.00/(0.06/4)

PV = \$133.33.

The value today is (where FV is future value)

$PV = FV_N(1 + r)^{-N}$

$PV = \$133.33(1 + 0.015)^{-4}$

PV = \$125.62 ≈ \$126.

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19 B is correct. To solve for the future value of unequal cash flows, compute the future value of each payment as of Year 4 at the semiannual rate of 2%, and then sum the individual future values, as follows:

Year	End of Year Deposits ($)	Factor	Future Value ($)
1	4,000	$(1.02)^6$	4,504.65
2	8,000	$(1.02)^4$	8,659.46
3	7,000	$(1.02)^2$	7,282.80
4	10,000	$(1.02)^0$	10,000.00
		Sum =	30,446.91

20 C is correct, as shown in the following (where FV is future value and PV is present value):

If:

$$\mathrm{FV}_N = \mathrm{PV}\left(1 + \frac{r_s}{m}\right)^{mN}$$

Then:

$$\left(\frac{\mathrm{FV}_N}{\mathrm{PV}}\right)^{\frac{1}{mN}} - 1 = \frac{r_s}{m}$$

$$\left(\frac{800{,}000}{500{,}000}\right)^{\frac{1}{2\times 6}} - 1 = \frac{r_s}{2}$$

r_s = 0.07988 (rounded to 8.0%).

21 C is correct. As shown below, the present value (PV) of a £2,000 per month perpetuity is worth approximately £400,000 at a 6% annual rate compounded monthly. Thus, the present value of the annuity (A) is worth more than the lump sum offers.

A = £2,000

r = (6%/12) = 0.005

PV = (A/r)

PV = (£2,000/0.005)

PV = £400,000

22 B is correct.

The present value of a 10-year annuity (A) due with payments of \$2,000 at a 5% discount rate is calculated as follows:

$$\mathrm{PV} = A\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right] + \$2{,}000$$

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$$\text{PV} = \$2{,}000\left[\frac{1-\frac{1}{(1+0.05)^{9}}}{0.05}\right] + \$2{,}000$$

PV = $16,215.64.

Alternatively, the PV of a 10-year annuity due is simply the PV of the ordinary annuity multiplied by 1.05:

PV = $15,443.47 × 1.05

PV = $16,215.64.

23 B is correct. First, find the present value (PV) of an ordinary annuity in Year 17 that represents the tuition costs:

$$\$50{,}000\left[\frac{1-\frac{1}{(1+0.06)^{4}}}{0.06}\right]$$

= $50,000 × 3.4651

= $173,255.28.

Then, find the PV of the annuity in today's dollars (where FV is future value):

$$\text{PV}_0 = \frac{\text{FV}}{(1+0.06)^{17}}$$

$$\text{PV}_0 = \frac{\$173{,}255.28}{(1+0.06)^{17}}$$

PV_0 = $64,340.85 ≈ $64,341.

24 B is correct, as shown in the following table.

Year	Cash Flow (€)	Formula CF × (1 + *r*)^t	PV at Year 0
1	100,000	$100{,}000(1.12)^{-1}$ =	89,285.71
2	150,000	$150{,}000(1.12)^{-2}$ =	119,579.08
5	–10,000	$-10{,}000(1.12)^{-5}$ =	–5,674.27
			203,190.52

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25 B is correct, calculated as follows (where A is annuity and PV is present value):

$$\text{A} = (\text{PV of annuity})\Bigg/\left[\frac{1-\dfrac{1}{(1+r_s/m)^{mN}}}{r_s/m}\right]$$

$$= (£200{,}000)\Bigg/\left[\frac{1-\dfrac{1}{(1+r_s/m)^{mN}}}{r_s/m}\right]$$

$$(£200{,}000)\Bigg/\left[\frac{1-\dfrac{1}{(1+0.06/12)^{12(5)}}}{0.06/12}\right]$$

$$= (£200{,}000)/51.72556$$

$$= £3{,}866.56$$

26 A is correct. To solve for an annuity (A) payment, when the future value (FV), interest rate, and number of periods is known, use the following equation:

$$\text{FV} = \text{A}\left[\frac{\left(1+\dfrac{r_s}{m}\right)^{mN}-1}{\dfrac{r}{m}}\right]$$

$$£25{,}000 = \text{A}\left[\frac{\left(1+\dfrac{0.06}{4}\right)^{4\times 10}-1}{\dfrac{0.06}{4}}\right]$$

$$\text{A} = £460.68$$

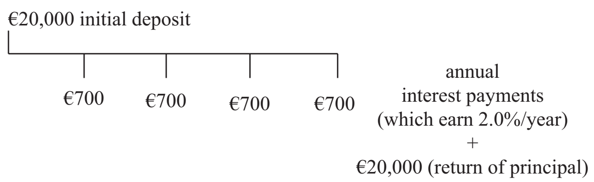
27 B is correct. The PV in Year 5 of a $50,000 lump sum paid in Year 20 is $27,763.23 (where FV is future value):

$$\text{PV} = \text{FV}_N(1+r)^{-N}$$

$$\text{PV} = \$50{,}000(1+0.04)^{-15}$$

$$\text{PV} = \$27{,}763.23$$

28 B is correct, as the following cash flows show:



The four annual interest payments are based on the CD's 3.5% annual rate.

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The first payment grows at 2.0% compounded monthly for three years (where FV is future value):

$$\mathrm{FV}_N = €700\left(1 + \frac{0.02}{12}\right)^{3\times12}$$

$$\mathrm{FV}_N = 743.25$$

The second payment grows at 2.0% compounded monthly for two years:

$$\mathrm{FV}_N = €700\left(1 + \frac{0.02}{12}\right)^{2\times12}$$

$$\mathrm{FV}_N = 728.54$$

The third payment grows at 2.0% compounded monthly for one year:

$$\mathrm{FV}_N = €700\left(1 + \frac{0.02}{12}\right)^{1\times12}$$

$$\mathrm{FV}_N = 714.13$$

The fourth payment is paid at the end of Year 4. Its future value is €700.

The sum of all future value payments is as follows:

€20,000.00	CD
€743.25	First payment's *FV*
€728.54	Second payment's *FV*
€714.13	Third payment's *FV*
€700.00	Fourth payment's *FV*
€22,885.92	Total *FV*

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PRACTICE PROBLEMS

1 Which of the following groups *best* illustrates a sample?

A The set of all estimates for Exxon Mobil's EPS for next financial year

B The FTSE Eurotop 100 as a representation of the European stock market

C UK shares traded on Wednesday of last week that also closed above £120/share on the London Stock Exchange

2 Published ratings on stocks ranging from 1 (strong sell) to 5 (strong buy) are examples of which measurement scale?

A Ordinal

B Interval

C Nominal

3 Which of the following groups *best* illustrates a population?

A The 500 companies in the S&P 500 Index

B The NYSE-listed stocks in the Dow Jones Industrial Average

C The Lehman Aggregate Bond Index as a representation of the US bond market

4 In descriptive statistics, an example of a parameter is the:

A median of a population.

B mean of a sample of observations.

C standard deviation of a sample of observations.

5 A mutual fund has the return frequency distribution shown in the following table.

Return Interval (%)	Absolute Frequency
−10.0 to −7.0	3
−7.0 to −4.0	7
−4.0 to −1.0	10
−1.0 to +2.0	12
+2.0 to +5.0	23
+5.0 to +8.0	5

Which of the following statements is correct?

A The relative frequency of the interval "−1.0 to +2.0" is 20%.

B The relative frequency of the interval "+2.0 to +5.0" is 23%.

C The cumulative relative frequency of the interval "+5.0 to +8.0" is 91.7%.

6 An analyst is using the data in the following table to prepare a statistical report.

Portfolio's Deviations from Benchmark Return, 2003–2014 (%)

Year 1	2.48	**Year 7**	−9.19
Year 2	−2.59	**Year 8**	−5.11
Year 3	9.47	**Year 9**	1.33

(continued)

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(Continued)			
Year 4	–0.55	**Year 10**	6.84
Year 5	–1.69	**Year 11**	3.04
Year 6	–0.89	**Year 12**	4.72

The cumulative relative frequency for the interval $-1.71\% \le x < 2.03\%$ is *closest* to:

A 0.250.

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B 0.333.

C 0.583.

7 Frequency distributions summarize data in:

A a tabular display.

B overlapping intervals.

C a relatively large number of intervals.

8 Based on the table below, which of the following statements is correct?

Frequency Distributions of Sample Returns

Interval	Range	Absolute Frequency
A	–10% ≤ Observation < –5%	2
B	–5% ≤ Observation < 0%	7
C	0% ≤ Observation < 5%	15
D	5% ≤ Observation < 10%	2

A The relative frequency of Interval C is 15.

B The cumulative frequency of Interval D is 100%.

C The cumulative relative frequency of Interval C is 92.3%.

The following information relates to Questions 9–10

The following histogram shows a distribution of the S&P 500 Index annual returns for a 50-year period:

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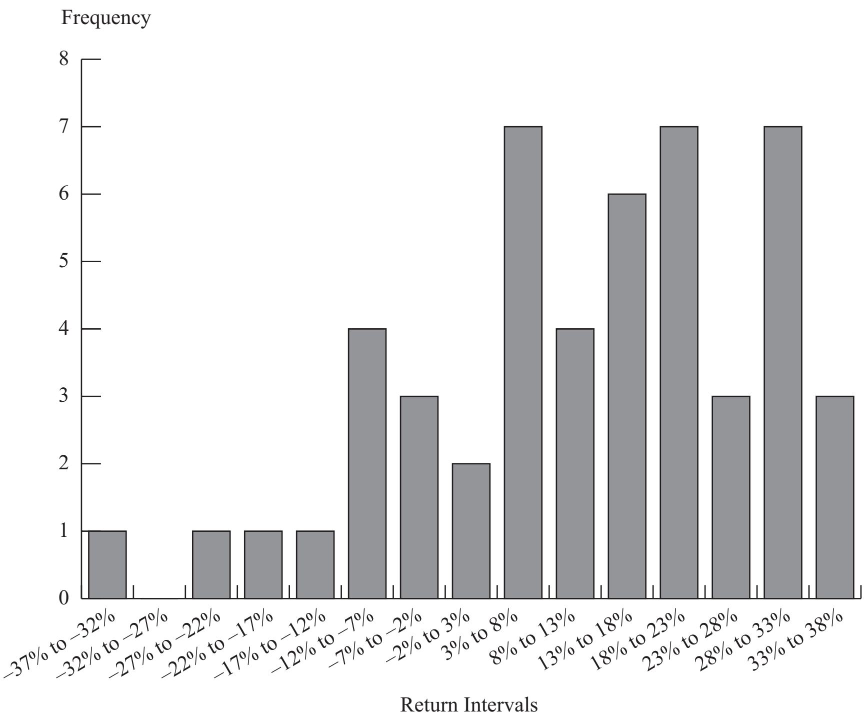

9 The interval containing the median return is:

A 3% to 8%.

B 8% to 13%.

C 13% to 18%.

10 Based on the previous histogram, the distribution is *best* described as having:

A one mode.

B two modes.

C three modes.

---

11 The following is a frequency polygon of monthly exchange rate changes in the US dollar/Japanese yen spot exchange rate for a four-year period. A positive change represents yen appreciation (the yen buys more dollars), and a negative change represents yen depreciation (the yen buys fewer dollars).

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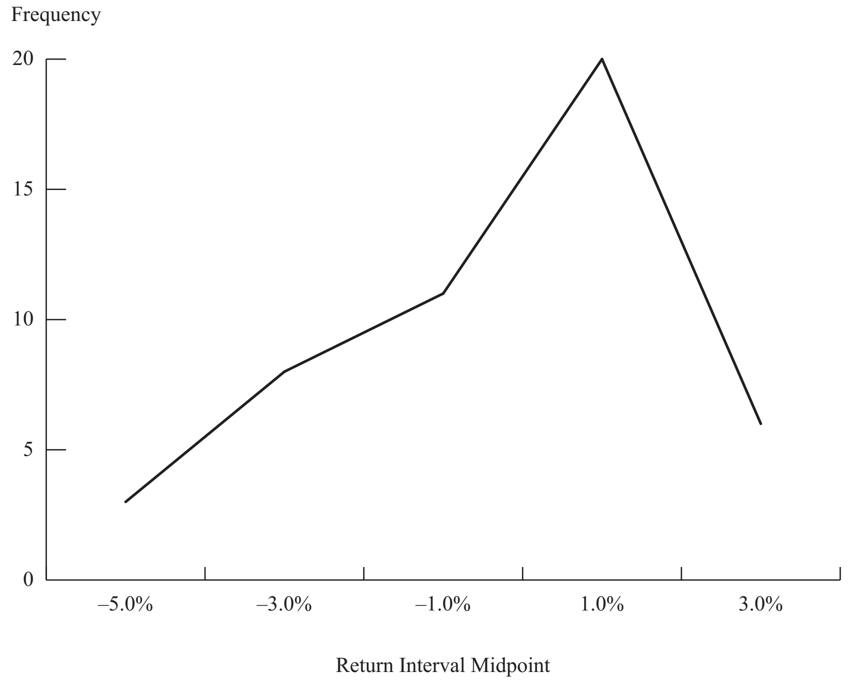
Monthly Changes in the US Dollar/Japanese Yen Spot Exchange Rate



Based on the chart, yen appreciation:

A occurred more than 50% of the time.

B was less frequent than yen depreciation.

C in the 0.0 to 2.0 interval occurred 20% of the time.

12 The height of a bar in a histogram represents the matching data interval's:

A relative frequency.

B absolute frequency.

C cumulative frequency.

The following table relates to Questions 13 and 14

Equity Returns for Six Companies

Company	Total Equity Return (%)
A	–4.53
B	–1.40
C	–1.20
D	–1.20

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(Continued)

Company	Total Equity Return (%)
E	0.70
F	8.90

13 Based on the table, the arithmetic mean of the equity returns is *closest* to the return of:

A Company B.

B Company C.

C Company E.

14 Using the data from the table, the difference between the median and the mode is *closest* to:

A –1.41.

B 0.00.

C 1.41.

---

15 The annual returns for three portfolios are shown in the following table. Portfolios P and R were created in Year 1, Portfolio Q in Year 2.

	Annual Portfolio Returns (%)				
	Year 1	Year 2	Year 3	Year 4	Year 5
Portfolio P	–3.0	4.0	5.0	3.0	7.0
Portfolio Q		–3.0	6.0	4.0	8.0
Portfolio R	1.0	–1.0	4.0	4.0	3.0

The median annual return from portfolio creation to 2013 for:

A Portfolio P is 4.5%.

B Portfolio Q is 4.0%.

C Portfolio R is higher than its arithmetic mean annual return.

16 Last year, an investor allocated his retirement savings in the asset classes shown in the following table.

Asset Class	Asset Allocation (%)	Asset Class Return (%)
Large-cap US equities	20.0	8.0
Small-cap US equities	40.0	12.0
Emerging market equities	25.0	–3.0
High-yield bonds	15.0	4.0

The portfolio return in 2015 is *closest to*:

A 5.1%.

B 5.3%.

C 6.3%.

17 The following table shows the annual returns for Fund Y.

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	Fund Y (%)
Year 1	19.5
Year 2	–1.9
Year 3	19.7
Year 4	35.0
Year 5	5.7

The geometric mean for Fund Y is *closest* to:

A 14.9%.

B 15.6%.

C 19.5%.

18 A manager invests €5,000 annually in a security for four years at the prices shown in the following table.

	Purchase Price of Security (€)
Year 1	62.00
Year 2	76.00
Year 3	84.00
Year 4	90.00

The average price paid for the security is *closest* to:

A €76.48.

B €77.26.

C €78.00.

The following information relates to Questions 19–20

The following exhibit shows the annual MSCI World Index total returns for a 10-year period.

Year 1	15.25%	**Year 6**	30.79%
Year 2	10.02%	**Year 7**	12.34%
Year 3	20.65%	**Year 8**	–5.02%
Year 4	9.57%	**Year 9**	16.54%
Year 5	–40.33%	**Year 10**	27.37%

19 The fourth quintile return for the MSCI World Index is *closest* to:

A 20.65%.

B 26.03%.

C 27.37%.

20 For Year 6 to Year 10, the mean absolute deviation of the MSCI World Index total returns is *closest* to:

A 10.20%.

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B 12.74%.

C 16.40%.

The following table relates to questions 21 and 22

10 Years of S&P 500 Total Returns (in Ascending Order)
Returns
–38.49%
–0.73%
0.00%
9.54%
11.39%
12.78%
13.41%
19.42%
23.45%
29.60%

21 The third quartile percentage of total returns is *closest* to:

A 19.42%.

B 20.43%.

C 23.45%.

22 Complete the missing entries in the table below to answer this question.

Overall Risk Measures, S&P 500 vs. Sample Portfolio

	S&P 500	Sample Portfolio
Mean	8.04%	8.54%
Range	–	67.09%
MAD	–	11.78%

An analyst does a performance measurement to compare the risk of a contemporaneous sample portfolio with that of the S&P 500 by determining the ranges and mean absolute deviations (MAD) of the two investments. The comparison shows that the S&P 500 appears riskier in terms of the:

A range only.

B MAD only.

C MAD and range.

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23 Annual returns and summary statistics for three funds are listed in the following table:

	Annual Returns (%)		
Year	**Fund ABC**	**Fund XYZ**	**Fund PQR**
Year 1	–20.0	–33.0	–14.0
Year 2	23.0	–12.0	–18.0
Year 3	–14.0	–12.0	6.0
Year 4	5.0	–8.0	–2.0
Year 5	–14.0	11.0	3.0
Mean	–4.0	–10.8	–5.0
Standard deviation	17.8	15.6	10.5

The fund that shows the highest dispersion is:

A Fund PQR if the measure of dispersion is the range.

B Fund XYZ if the measure of dispersion is the variance.

C Fund ABC if the measure of dispersion is the mean absolute deviation.

24 Using the information in the following table, the sample standard deviation for VWIGX is *closest* to:

2015–2017 Total Return for VWIGX

Year	Vanguard International Growth Fund (VWIGX)
2015	–0.67%
2016	1.71%
2017	42.96%

A 6.02%.

B 12.04%.

C 24.54%.

25 Over the past 240 months, an investor's portfolio had a mean monthly return of 0.79%, with a standard deviation of monthly returns of 1.16%. According to Chebyshev's inequality, the minimum number of the 240 monthly returns that fall into the range of –0.95% to 2.53% is *closest* to:

A 80.

B 107.

C 133.

26 For a distribution of 2,000 observations with finite variance, sample mean of 10.0%, and standard deviation of 4.0%, what is the minimum number of observations that will lie within 8.0% around the mean according to Chebyshev's Inequality?

A 720

B 1,500

C 1,680

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27 The mean monthly return and the standard deviation for three industry sectors are shown in the following exhibit.

Sector	Mean Monthly Return (%)	Standard Deviation of Return (%)
Utilities (UTIL)	2.10	1.23
Materials (MATR)	1.25	1.35
Industrials (INDU)	3.01	1.52

Based on the coefficient of variation, the riskiest sector is:

A utilities.

B materials.

C industrials.

The following information relates to Questions 28–29

The following table shows various statistics for Portfolios 1, 2, and 3.

	Mean Return (%)	Standard Deviation of Returns (%)	Skewness	Excess Kurtosis
Portfolio 1	7.8	15.1	0.0	0.7
Portfolio 2	10.2	20.5	0.9	–1.8
Portfolio 3	12.9	29.3	–1.5	6.2

28 The skewness of Portfolio 1 indicates its mean return is *most likely*:

A less than its median.

B equal to its median.

C greater than its median.

29 Compared with a normal distribution, the distribution of returns for Portfolio 3 *most likely*:

A has less weight in the tails.

B has a greater number of extreme returns.

C has fewer small deviations from its mean.

---

30 Two portfolios have unimodal return distributions. Portfolio 1 has a skewness of 0.77, and Portfolio 2 has a skewness of –1.11.

Which of the following is correct?

A For Portfolio 1, the median is less than the mean.

B For Portfolio 1, the mode is greater than the mean.

C For Portfolio 2, the mean is greater than the median.

31 A return distribution with frequent small gains and a few extreme losses is *most likely* to be called:

A leptokurtic.

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B positively skewed.

C negatively skewed.

32 Which of the following sequences *best* represents the relative sizes of the mean, median, and mode for a positively skewed unimodal distribution?

A mode ≤ median ≤ mean

B mode < median < mean

C mean < median < mode

33 A distribution with excess kurtosis less than zero is termed:

A mesokurtic.

B platykurtic.

C leptokurtic.

34 When analyzing investment returns, which of the following statements is correct?

A The geometric mean will exceed the arithmetic mean for a series with non-zero variance.

B The geometric mean measures an investment's compound rate of growth over multiple periods.

C The arithmetic mean accurately estimates an investment's terminal value over multiple periods.

35 Which of the following statistical means *best* measures a mutual fund's past performance?

A Harmonic

B Geometric

C Arithmetic

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SOLUTIONS

1 B is correct. The FTSE Eurotop 100 represents a sample of all European stocks. It is a subset of the population of all European stocks.

2 A is correct. Ordinal scales sort data into categories that are ordered with respect to some characteristic and may involve numbers to identify categories but do not assure that the differences between scale values are equal. The buy rating scale indicates that a stock ranked 5 is expected to perform better than a stock ranked 4, but it tells us nothing about the performance difference between stocks ranked 4 and 5 compared with the performance difference between stocks ranked 1 and 2, and so on.

3 A is correct. A population is defined as all members of a specified group. The S&P 500 Index consists of 500 companies, so this group is the population of companies in the index.

B is incorrect because there are several Dow Jones component stocks that are not traded on the NYSE, making the NYSE group a subset of the total population of stocks included in the Dow Jones average.

C is incorrect because although the Lehman Aggregate Bond Index is representative of the US bond market, it is a sampling of bonds in that market and not the entire population of bonds in that market.

4 A is correct. Any descriptive measure of a population characteristic is referred to as a parameter.

5 A is correct. The relative frequency is the absolute frequency of each interval divided by the total number of observations. Here, the relative frequency is calculated as: (12/60) × 100 = 20%. B is incorrect because the relative frequency of this interval is (23/60) × 100 = 38.33%. C is incorrect because the cumulative relative frequency of the last interval must equal 100%.

6 C is correct. The cumulative relative frequency of an interval identifies the fraction of observations that are less than the upper limit of the given interval. It is determined by summing the relative frequencies from the lowest interval up to and including the given interval. The following exhibit shows the relative frequencies for all the intervals of the data from the previous exhibit:

Lower Limit (%)	Upper Limit (%)	Absolute Frequency	Relative Frequency	Cumulative Relative Frequency
−9.19 ≤	< −5.45	1	0.083	0.083
−5.45 ≤	< −1.71	2	0.167	0.250
−1.71 ≤	< 2.03	4	0.333	0.583
2.03 ≤	< 5.77	3	0.250	0.833
5.77 ≤	≥ 9.51	2	0.167	1.000

The interval $-1.71\% \le x < 2.03\%$ has a cumulative relative frequency of 0.583.

7 A is correct. A frequency distribution is a tabular display of data summarized into a relatively small number of intervals.

B is incorrect because intervals cannot overlap. Each observation is placed uniquely into one interval.

C is incorrect because a frequency distribution is summarized into a relatively small number of intervals.

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8 C is correct because the cumulative relative frequency of an interval tells us the fraction of all observations that are less than the upper limit of an interval. For Interval C, that would be (2 + 7 + 15)/26 = 92.3%.

A is incorrect because the relative frequency of an interval is the absolute frequency of that interval divided by the total number of observations, here 15/26 = 57.7%. The number 15 represents Interval C's absolute frequency (also known as frequency), which is simply the actual number of observations in a given interval.

B is incorrect because the cumulative frequency tells us the number of observations that are less than the upper limit of a return interval, not the percentage of observations meeting that criteria. Because Interval D is the uppermost return interval, its cumulative frequency is the total number of observations for all intervals, yielding 2 + 7 + 15 + 2 = 26 and not 100%, which is the cumulative relative frequency for Interval D.

9 C is correct. Because there are 50 data points in the histogram, the median return would be the mean of the 50/2 = 25th and (50 + 2)/2 = 26th positions. The sum of the return interval frequencies to the left of the 13% to 18% interval is 24. As a result, the 25th and 26th returns will fall in the 13% to 18% interval.

10 C is correct. The mode of a distribution with data grouped in intervals is the interval with the highest frequency. The three intervals of 3% to 8%, 18% to 23%, and 28% to 33% all have a high frequency of 7.

11 A is correct. Twenty observations lie in the interval "0.0 to 2.0," and six observations lie in the 2.0 to 4.0 interval. Together, they represent 26/48, or 54.17% of all observations, which is more than 50%.

12 B is correct. In a histogram, the height of each bar represents the absolute frequency of its associated data interval.

A is incorrect because the height of each bar in a histogram represents the absolute (not relative) frequency.

C is incorrect because the height of each bar in a histogram represents the absolute (not cumulative) frequency. 需要最新cfa/frm网课+微信286982279，全网最低价

13 C is correct. The arithmetic mean equals the sum of the observations divided by the number of observations. In this case, (–4.53 –1.40 –1.20 –1.20 + 0.70 + 8.90)/6 = 1.27/6 = 0.21.

The arithmetic mean is closest to the total equity return of Company E at 0.70 for a difference of (0.70 – 0.21) = 0.49.

A is incorrect because compared with the arithmetic mean, Company B's total equity return has a difference of (–1.40 –0.21) = –1.61, which is a wider distance from the mean than Company E's total equity return.

B is incorrect because compared with the arithmetic mean, Company C's total equity return has a difference of (–1.20 – 0.21) = –1.41, which is a wider distance from the mean than Company E's total equity return.

14 B is correct. The median is the value of the middle item of a set of items sorted into ascending or descending order. In an even-numbered sample, we define the median as the mean of the values of items occupying the $n/2$ and $(n + 2)/2$ positions (the two middle items). Given Table 2 has six observations, the median is the mean of the third and fourth observations. Because both are–1.20, the median is –1.20.

The mode is the most frequently occurring value in a distribution. The only value occurring more than once is –1.20.

Because the median and the mode both equal –1.20, their difference is zero.

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A is incorrect because –1.41 is the difference between both the identical mode and median with the arithmetic mean. Both differences are: [–1.20 – (0.21)] = –1.41.

C is incorrect because 1.41 is the difference between the arithmetic mean with both the identical mode and median. Both differences are: [0.21 – (–1.20)] = 1.41.

15 C is correct. The median of Portfolio R is 0.8% higher than the mean for Portfolio R.

16 C is correct. The portfolio return must be calculated as the weighted mean return, where the weights are the allocations in each asset class:

$$(0.20 \times 8\%) + (0.40 \times 12\%) + (0.25 \times -3\%) + (0.15 \times 4\%) = 6.25\%, \text{ or } \approx 6.3\%.$$

17 A is correct. The geometric mean return for Fund Y is found as follows:

$$\begin{aligned}\text{Fund Y} &= [(1 + 0.195) \times (1 - 0.019) \times (1 + 0.197) \times (1 + 0.350) \times (1 + 0.057)]^{(1/5)} - 1 \\ &= 14.9\%.\end{aligned}$$

18 A is correct. The harmonic mean is appropriate for determining the average price per unit. It is calculated by summing the reciprocals of the prices; then averaging that sum by dividing by the number of prices; and finally, taking the reciprocal of the average:

$$4/[(1/62.00) + (1/76.00) + (1/84.00) + (1/90.00)] = €76.48.$$

19 B is correct. Quintiles divide a distribution into fifths, with the fourth quintile occurring at the point at which 80% of the observations lie below it. The fourth quintile is equivalent to the 80th percentile. To find the yth percentile (P_y), we first must determine its location. The formula for the location (L_y) of a yth percentile in an array with n entries sorted in ascending order is $L_y = (n + 1) \times (y/100)$. In this case, $n = 10$ and $y = 80\%$, so

$$L_{80} = (10 + 1) \times (80/100) = 11 \times 0.8 = 8.8.$$

With the data arranged in ascending order (–40.33%, –5.02%, 9.57%, 10.02%, 12.34%, 15.25%, 16.54%, 20.65%, 27.37%, and 30.79%), the 8.8th position would be between the 8th and 9th entries, 20.65% and 27.37%, respectively. Using linear interpolation, $P_{80} = X_8 + (L_y - 8) \times (X_9 - X_8)$,

$$\begin{aligned}P_{80} &= 20.65 + (8.8 - 8) \times (27.37 - 20.65) \\ &= 20.65 + (0.8 \times 6.72) = 20.65 + 5.38 \\ &= 26.03\%.\end{aligned}$$

20 A is correct. The formula for mean absolute deviation (MAD) is

$$\text{MAD} = \frac{\sum_{i=1}^{n}\left|X_i - \bar{X}\right|}{n}$$

Column 1: Sum annual returns and divide by n to find the arithmetic mean $\left(\bar{X}\right)$ of 16.40%.

Column 2: Calculate the absolute value of the difference between each year's return and the mean from Column 1. Sum the results and divide by n to find the MAD.

These calculations are shown in the following exhibit:

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	Column 1		Column 2
Year	**Return**		$\lvert X_i - \bar{X} \rvert$
Year 6	30.79%		14.39%
Year 7	12.34%		4.06%
Year 8	–5.02%		21.42%
Year 9	16.54%		0.14%
Year 10	27.37%		10.97%
Sum:	82.02%	Sum:	50.98%
n:	5	*n*:	5
$\bar{X}$:	16.40%	MAD:	10.20%

21 B is correct. Quartiles divide a distribution into quarters, with the third quartile occurring at the point at which 75% of the observations lie below it. The third quartile is equivalent to the 75th percentile. The formula for the location (L_y) of the *y*th percentile in an array with *n* entries sorted in ascending order is $L_y = (n + 1) \times (y/100)$. In this case, $n = 10$ and $y = 75$, so $L_{75} = (11) \times (75/100) = 11 \times 0.75 = 8.25$.

Rearranging the data in ascending order (i.e., with the lowest value at the top), the 8.25th position would be between the eighth and ninth rank order entries, 19.42% and 23.45%, respectively. Using linear interpolation, $P_{75} = X_8 + (L_{75} - 8) \times (X_9 - X_8)$, so P_{75} = 19.42% + (8.25 – 8) × (23.45% – 19.42%) = 20.428%, or 20.43%.

A is incorrect because it is the non-interpolated value of the eighth observation without the adjustment for placement at the location of the third quartile.

C is incorrect because it is the non-interpolated value of the ninth observation without the adjustment for placement at the location of the third quartile.

22 C is correct. Both the range and MAD of the S&P 500 are greater than the range and MAD of the sample portfolio. Thus both measures indicate the S&P 500 is riskier.

The range for the S&P 500 equals the distance between the lowest and highest values in the dataset. That distance for the S&P 500 is [29.60% – (–38.49%)] = 68.09%. Given that this range is larger than the range of the sample portfolio at 67.09%, the S&P 500 appears riskier than the sample portfolio.

The MAD for the S&P 500 returns equals the sum of the absolute deviations from the mean return divided by the number of observations.

$$\text{MAD} = \frac{\sum_{i=1}^{n} \lvert X_i - \bar{X} \rvert}{n}$$, where $\bar{X}$ is the sample mean and n is the number of observations in the sample.

Use the 10 observed S&P 500 returns from the table (sample mean = 8.04%) to calculate the MAD for the S&P 500 as follows:

$$\text{MAD}_{\text{S\&P500}} = \frac{\lvert 19.42 - \bar{x} \rvert + \lvert 9.54 - \bar{x} \rvert + \lvert -0.73 - \bar{x} \rvert + \lvert 11.39 - \bar{x} \rvert + \lvert 29.60 - \bar{x} \rvert + \lvert 13.41 - \bar{x} \rvert + \lvert 0.00 - \bar{x} \rvert + \lvert 12.78 - \bar{x} \rvert + \lvert 23.45 - \bar{x} \rvert + \lvert -38.49 - \bar{x} \rvert}{10}$$

$$\text{MAD}_{\text{S\&P500}} = 12.67\%$$

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Given that the MAD for the S&P 500 is greater than the MAD for the sample portfolio (12.67% versus 11.78%), the S&P 500 appears riskier than the sample portfolio.

A is incorrect because although the S&P 500 is correctly identified as having the larger range, the sample portfolio has a smaller MAD.

B is incorrect because although the S&P 500 is correctly identified as having the larger MAD, the Sample Portfolio has a smaller range.

23 C is correct. The mean absolute deviation (MAD) of Fund ABC's returns is greater than the MAD of both of the other funds.

$$\text{MAD} = \frac{\sum_{i=1}^{n}\left|X_i - \bar{X}\right|}{n}, \text{ where } \bar{X} \text{ is the arithmetic mean of the series.}$$

MAD for Fund ABC =

$$\frac{\left|-20-(-4)\right| + \left|23-(-4)\right| + \left|-14-(-4)\right| + \left|5-(-4)\right| + \left|-14-(-4)\right|}{5} = 14.4\%$$

MAD for Fund XYZ =

$$\frac{\left|-33-(-10.8)\right| + \left|-12-(-10.8)\right| + \left|-12-(-10.8)\right| + \left|-8-(-10.8)\right| + \left|11-(-10.8)\right|}{5} = 9.8\%$$

MAD for Fund PQR =

$$\frac{\left|-14-(-5)\right| + \left|-18-(-5)\right| + \left|6-(-5)\right| + \left|-2-(-5)\right| + \left|3-(-5)\right|}{5} = 8.8\%$$

A and B are incorrect because the range and variance of the three funds are as follows:

	Fund ABC	**Fund XYZ**	**Fund PQR**
Range	43%	44%	24%
Variance	317	243	110

The numbers shown for variance are understood to be in "percent squared" terms so that when taking the square root, the result is standard deviation in percentage terms. Alternatively, by expressing standard deviation and variance in decimal form, one can avoid the issue of units; in decimal form, the variances for Fund ABC, Fund XYZ, and Fund PQR are 0.0317, 0.0243, and 0.0110, respectively.

24 C is correct. The sample variance is defined as sum of the squared deviations from the sample mean divided by the sample size minus one, and the sample standard deviation equals the square root of the sample variance.

The following figure summarizes the inputs for the calculation of VWGIX sample variance.

Year	**VWIGX**	$\left(X_i - \bar{X}\right)^2$
2015	–0.67%	2.35
2016	1.71%	1.68
2017	42.96%	8.00

(continued)

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Year	VWIGX	$\left(X_i - \bar{X}\right)^2$
Sample mean $\left(\bar{X}\right)$	14.67%	
$\sum\left(X - \bar{X}\right)^2$		12.04
$\left[\sum\left(X - \bar{X}\right)^2\right] / (n-1) = \sigma^2$	[12.04/2]	6.02
$\sqrt{\left[\sum\left(X - \bar{X}\right)^2\right] / (n-1)} = \sigma$	$\sqrt{(12.04/2)}$	24.54%

The sample variance is thus calculated as $\dfrac{\left[\sum\left(X - \bar{X}\right)^2\right]}{(n-1)} = \dfrac{12.04}{2} = 6.02.$

The square root of the sample variance is the sample standard deviation.

That number is $\left(\sqrt{6.02}\right) = 24.54\%$.

A is incorrect because it is the sample variance for VWGIX, not its sample standard deviation.

B is incorrect because it represents the sum of the squared deviations from the mean, not the sample standard deviation.

25 C is correct. According to Chebyshev's inequality, the proportion of the observations within k standard deviations of the arithmetic mean is at least $1 - 1/k^2$ for all $k > 1$.

The upper limit of the range is 2.53%, which is 2.53 – 0.79 = 1.74% above the mean. The lower limit is –0.95, which is 0.79 – (–0.95) = 1.74% below the mean. As a result, k = 1.74/1.16 = 1.50 standard deviations.

Because k = 1.50, the proportion of observations within the interval is at least $1 - 1/1.5^2 = 1 - 0.444 = 0.556$, or 55.6%. Thus, the number of observations in the given range is at least 240 × 55.6%, which is ≈ 133.

26 B is correct. Observations within 8% of the sample mean will cover an interval of 8/4 or two standard deviations. Chebyshev's Inequality says the proportion of the observations P within k standard deviations of the arithmetic mean is at least $1 - 1/k^2$ for all $k > 1$. So, solving for $k = 2$: P = 1 – ¼ = 75%. Given 2,000 observations, this implies at least 1,500 will lie within 8.0% of the mean.

A is incorrect because 720 shows P = 720/2,000 = 36.0% of the observations. Using P to solve for k implies $36.0\% = 1 - 1/k^2$, where k = 1.25. This result would cover an interval only 4% × 1.25 or 5% around the mean (i.e. less than two standard deviations).

C is incorrect because 1,680 shows P = 1,680/2,000 = 84.0% of the observations. Using P to solve for k implies $84.0\% = 1 - 1/k^2$, where k = 2.50. This result would cover an interval of 4% × 2.5, or 10% around the mean (i.e., more than two standard deviations).

27 B is correct. The coefficient of variation (CV) is the ratio of the standard deviation to the mean, where a higher CV implies greater risk per unit of return.

$$CV_{UTIL} = \frac{s}{\bar{X}} = \frac{1.23\%}{2.10\%} = 0.59$$

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$$CV_{MATR} = \frac{s}{\overline{X}} = \frac{1.35\%}{1.25\%} = 1.08$$

$$CV_{INDU} = \frac{s}{\overline{X}} = \frac{1.52\%}{3.01\%} = 0.51$$

28 B is correct. Portfolio 1 has a skewness of 0.0, which indicates that the portfolio's return distribution is symmetrical and thus its mean and median are equal.

29 B is correct. Portfolio 3 has positive excess kurtosis (i.e., kurtosis greater than 3), which indicates that its return distribution is leptokurtic and has fatter tails than the normal. The fatter tails mean Portfolio 3 has a greater number of extreme returns.

30 A is correct. Portfolio 1 is positively skewed, so the mean is greater than the median, which is greater than the mode.

31 C is correct. A return distribution with negative skew has frequent small gains and a few extreme losses.

A is incorrect because a leptokurtic distribution is more peaked with fatter tails, which exhibit both extreme gains and losses.

B is incorrect because a return distribution with positive skew has frequent small losses and a few extreme gains.

32 B is correct. For the positively skewed unimodal distribution, the mode is less than the median, which is less than the mean.

A is incorrect because, for the positively skewed unimodal distribution, the mode is less than the median (not less than or equal to), which is less than (not less than or equal to) the mean.

C is incorrect because, for the negatively (not positively) skewed unimodal distribution, the mean is less than the median, which is less than the mode.

33 B is correct. A platykurtic distribution has excess kurtosis less than zero.

A is incorrect because a normal or other mesokurtic distribution has excess kurtosis equal to zero.

C is incorrect because a leptokurtic distribution has excess kurtosis greater than zero.

34 B is correct. The geometric mean compounds the periodic returns of every period, giving the investor a more accurate measure of the terminal value of an investment.

35 B is correct. The geometric mean is an excellent measure of past performance. For reporting historical returns, the geometric mean has considerable appeal because it is the rate of growth or return a fund would have had to earn each year to match the actual, cumulative investment performance. To estimate the average returns over more than one period, the geometric mean captures how the total returns are linked over time.

A is incorrect because the harmonic mean is more appropriate for determining the average price per unit, not evaluating a mutual fund's return history. The average price paid is in fact the harmonic mean of the asset's prices at the purchase dates. The harmonic mean is applicable when ratios are repeatedly applied to a fixed quantity to yield a variable number of units, such as in cost averaging, which involves the periodic investment of a fixed amount of money.

C is incorrect because the arithmetic mean is more appropriate for making investment statements in a forward-looking context, not for historical returns. It can distort the assessment of historical performance, so it is better applied to estimate the average return over a one-period horizon.

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PRACTICE PROBLEMS

1 Suppose that 5 percent of the stocks meeting your stock-selection criteria are in the telecommunications (telecom) industry. Also, dividend-paying telecom stocks are 1 percent of the total number of stocks meeting your selection criteria. What is the probability that a stock is dividend paying, given that it is a telecom stock that has met your stock selection criteria?

2 You are using the following three criteria to screen potential acquisition targets from a list of 500 companies:

Criterion	Fraction of the 500 Companies Meeting the Criterion
Product lines compatible	0.20
Company will increase combined sales growth rate	0.45
Balance sheet impact manageable	0.78

If the criteria are independent, how many companies will pass the screen?

3 You apply both valuation criteria and financial strength criteria in choosing stocks. The probability that a randomly selected stock (from your investment universe) meets your valuation criteria is 0.25. Given that a stock meets your valuation criteria, the probability that the stock meets your financial strength criteria is 0.40. What is the probability that a stock meets both your valuation and financial strength criteria?

4 Suppose the prospects for recovering principal for a defaulted bond issue depend on which of two economic scenarios prevails. Scenario 1 has probability 0.75 and will result in recovery of \$0.90 per \$1 principal value with probability 0.45, or in recovery of \$0.80 per \$1 principal value with probability 0.55. Scenario 2 has probability 0.25 and will result in recovery of \$0.50 per \$1 principal value with probability 0.85, or in recovery of \$0.40 per \$1 principal value with probability 0.15.

A Compute the probability of each of the four possible recovery amounts: \$0.90, \$0.80, \$0.50, and \$0.40.

B Compute the expected recovery, given the first scenario.

C Compute the expected recovery, given the second scenario.

D Compute the expected recovery.

E Graph the information in a tree diagram.

5 You have developed a set of criteria for evaluating distressed credits. Companies that do not receive a passing score are classed as likely to go bankrupt within 12 months. You gathered the following information when validating the criteria:

- Forty percent of the companies to which the test is administered will go bankrupt within 12 months: $P(nonsurvivor) = 0.40$.
- Fifty-five percent of the companies to which the test is administered pass it: $P(pass\ test) = 0.55$.
- The probability that a company will pass the test given that it will subsequently survive 12 months, is 0.85: $P(pass\ test \mid survivor) = 0.85$.

A What is $P(pass\ test \mid nonsurvivor)$?

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B Using Bayes' formula, calculate the probability that a company is a survivor, given that it passes the test; that is, calculate P(survivor | pass test).

C What is the probability that a company is a *nonsurvivor*, given that it fails the test?

D Is the test effective?

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6 In probability theory, exhaustive events are *best* described as events:

A with a probability of zero.

B that are mutually exclusive.

C that include all potential outcomes.

7 Which probability estimate *most likely* varies greatly between people?

A An *a priori* probability

B An empirical probability

C A subjective probability

8 If the probability that Zolaf Company sales exceed last year's sales is 0.167, the odds for exceeding sales are *closest* to:

A 1 to 5.

B 1 to 6.

C 5 to 1.

9 The probability of an event given that another event has occurred is a:

A joint probability.

B marginal probability.

C conditional probability.

10 After estimating the probability that an investment manager will exceed his benchmark return in each of the next two quarters, an analyst wants to forecast the probability that the investment manager will exceed his benchmark return over the two-quarter period in total. Assuming that each quarter's performance is independent of the other, which probability rule should the analyst select?

A Addition rule

B Multiplication rule

C Total probability rule

11 Which of the following is a property of two dependent events?

A The two events must occur simultaneously.

B The probability of one event influences the probability of the other event.

C The probability of the two events occurring is the product of each event's probability.

12 Which of the following *best* describes how an analyst would estimate the expected value of a firm under the scenarios of bankruptcy and survivorship? The analyst would use:

A the addition rule.

B conditional expected values.

C the total probability rule for expected value.

13 An analyst developed two scenarios with respect to the recovery of $100,000 principal from defaulted loans:

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Scenario	Probability of Scenario (%)	Amount Recovered ($)	Probability of Amount (%)
1	40	50,000	60
		30,000	40
2	60	80,000	90
		60,000	10

The amount of the expected recovery is *closest* to:

A $36,400.

B $63,600.

C $81,600.

14 US and Spanish bonds have return standard deviations of 0.64 and 0.56, respectively. If the correlation between the two bonds is 0.24, the covariance of returns is *closest* to:

A 0.086.

B 0.670.

C 0.781.

15 The covariance of returns is positive when the returns on two assets tend to:

A have the same expected values.

B be above their expected value at different times.

C be on the same side of their expected value at the same time.

16 Which of the following correlation coefficients indicates the weakest linear relationship between two variables?

A –0.67

B –0.24

C 0.33

17 An analyst develops the following covariance matrix of returns:

	Hedge Fund	Market Index
Hedge fund	256	110
Market index	110	81

The correlation of returns between the hedge fund and the market index is *closest* to:

A 0.005.

B 0.073.

C 0.764.

18 All else being equal, as the correlation between two assets approaches +1.0, the diversification benefits:

A decrease.

B stay the same.

C increase.

19 Given a portfolio of five stocks, how many unique covariance terms, excluding variances, are required to calculate the portfolio return variance?

A 10

B 20

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C 25

20 The probability distribution for a company's sales is:

Probability	Sales ($ millions)
0.05	70
0.70	40
0.25	25

The standard deviation of sales is *closest* to:

A $9.81 million.

B $12.20 million.

C $32.40 million.

21 Which of the following statements is *most* accurate? If the covariance of returns between two assets is 0.0023, then:

A the assets' risk is near zero.

B the asset returns are unrelated.

C the asset returns have a positive relationship.

22 An analyst produces the following joint probability function for a foreign index (FI) and a domestic index (DI).

	R_{DI} = 30%	R_{DI} = 25%	R_{DI} = 15%
R_{FI} = 25%	0.25		
R_{FI} = 15%		0.50	
R_{FI} = 10%			0.25

The covariance of returns on the foreign index and the returns on the domestic index is *closest* to:

A 26.39.

B 26.56.

C 28.12.

23 A manager will select 20 bonds out of his universe of 100 bonds to construct a portfolio. Which formula provides the number of possible portfolios?

A Permutation formula

B Multinomial formula

C Combination formula

24 A firm will select two of four vice presidents to be added to the investment committee. How many different groups of two are possible?

A 6

B 12

C 24

25 From an approved list of 25 funds, a portfolio manager wants to rank 4 mutual funds from most recommended to least recommended. Which formula is *most* appropriate to calculate the number of possible ways the funds could be ranked?

A Permutation formula

B Multinomial formula

C Combination formula

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SOLUTIONS

1 Use Equation 1 to find this conditional probability: P(*stock is dividend paying* | *telecom stock that meets criteria*) = P(*stock is dividend paying and telecom stock that meets criteria*)/P(*telecom stock that meets criteria*) = 0.01/0.05 = 0.20.

2 According to the multiplication rule for independent events, the probability of a company meeting all three criteria is the product of the three probabilities. Labeling the event that a company passes the first, second, and third criteria, A, B, and C, respectively $P(ABC) = P(A)P(B)P(C) = (0.20)(0.45)(0.78) = 0.0702$. As a consequence, (0.0702)(500) = 35.10, so 35 companies pass the screen.

3 Use Equation 2, the multiplication rule for probabilities $P(AB) = P(A \mid B)P(B)$, defining A as the event that *a stock meets the financial strength criteria* and defining B as the event that *a stock meets the valuation criteria*. Then $P(AB) = P(A \mid B)P(B) = 0.40 \times 0.25 = 0.10$. The probability that a stock meets both the financial and valuation criteria is 0.10.

4 A *Outcomes associated with Scenario 1:* With a 0.45 probability of a \$0.90 recovery per \$1 principal value, given Scenario 1, and with the probability of Scenario 1 equal to 0.75, the probability of recovering \$0.90 is 0.45 (0.75) = 0.3375. By a similar calculation, the probability of recovering \$0.80 is 0.55(0.75) = 0.4125.

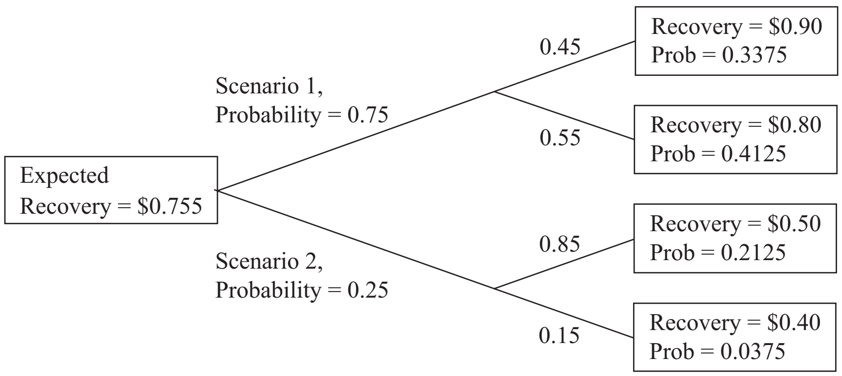
Outcomes associated with Scenario 2: With a 0.85 probability of a \$0.50 recovery per \$1 principal value, given Scenario 2, and with the probability of Scenario 2 equal to 0.25, the probability of recovering \$0.50 is 0.85(0.25) = 0.2125. By a similar calculation, the probability of recovering \$0.40 is 0.15(0.25) = 0.0375.

B E(*recovery* | *Scenario 1*) = 0.45(\$0.90) + 0.55(\$0.80) = \$0.845

C E(*recovery* | *Scenario 2*) = 0.85(\$0.50) + 0.15(\$0.40) = \$0.485

D E(*recovery*) = 0.75(\$0.845) + 0.25(\$0.485) = \$0.755

E



5 A We can set up the equation using the total probability rule:

$$P(\textit{pass test}) = P(\textit{pass test} \mid \textit{survivor})P(\textit{survivor}) + P(\textit{pass test} \mid \textit{nonsurvivor})P(\textit{nonsurvivor})$$

We know that P(*survivor*) = 1 – P(*nonsurvivor*) = 1 – 0.40 = 0.60. Therefore, P(*pass test*) = 0.55 = 0.85(0.60) + P(*pass test* | *nonsurvivor*)(0.40). Thus P(*pass test* | *nonsurvivor*) = [0.55 – 0.85(0.60)]/0.40 = 0.10.

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B $P(survivor \mid pass\ test) = \left[P(pass\ test \mid survivor)/P(pass\ test)\right]P(survivor)$

$$= (0.85/0.55)0.60 = 0.927273$$

The information that a company passes the test causes you to update your probability that it is a survivor from 0.60 to approximately 0.927.

C According to Bayes' formula, $P(nonsurvivor \mid fail\ test) = [P(fail\ test \mid nonsurvivor)/ P(fail\ test)]P(nonsurvivor) = [P(fail\ test \mid nonsurvivor)/0.45]0.40$.

We can set up the following equation to obtain $P(fail\ test \mid nonsurvivor)$:

$$P(fail\ test) = P(fail\ test \mid nonsurvivor)P(nonsurvivor) + P(fail\ test \mid survivor)P(survivor)$$

$$0.45 = P(fail\ test \mid nonsurvivor)0.40 + 0.15(0.60)$$

where $P(fail\ test \mid survivor) = 1 - P(pass\ test \mid survivor) = 1 - 0.85 = 0.15$. So $P(fail\ test \mid nonsurvivor) = [0.45 - 0.15(0.60)]/0.40 = 0.90$. Using this result with the formula above, we find $P(nonsurvivor \mid fail\ test) = (0.90/0.45)0.40 = 0.80$. Seeing that a company fails the test causes us to update the probability that it is a nonsurvivor from 0.40 to 0.80.

D A company passing the test greatly increases our confidence that it is a survivor. A company failing the test doubles the probability that it is a nonsurvivor. Therefore, the test appears to be useful.

6 C is correct. The term "exhaustive" means that the events cover all possible outcomes.

7 C is correct. A subjective probability draws on personal or subjective judgment that may be without reference to any particular data.

8 A is correct. Given odds for E of a to b, the implied probability of $E = a/(a + b)$. Stated in terms of odds a to b with $a = 1$, $b = 5$, the probability of $E = 1/(1 + 5) = 1/6 = 0.167$. This result confirms that a probability of 0.167 for beating sales is odds of 1 to 5.

9 C is correct. A conditional probability is the probability of an event given that another event has occurred.

10 B is correct. Because the events are independent, the multiplication rule is most appropriate for forecasting their joint probability. The multiplication rule for independent events states that the joint probability of both A and B occurring is P(AB) = P(A)P(B).

11 B is correct. The probability of the occurrence of one is related to the occurrence of the other. If we are trying to forecast one event, information about a dependent event may be useful.

12 C is correct. The total probability rule for expected value is used to estimate an expected value based on mutually exclusive and exhaustive scenarios.

13 B is correct. If Scenario 1 occurs, the expected recovery is 60% ($50,000) + 40% ($30,000) = $42,000, and if Scenario 2 occurs, the expected recovery is 90% ($80,000) + 10%($60,000) = $78,000. Weighting by the probability of each scenario, the expected recovery is 40%($42,000) + 60%($78,000) = $63,600. Alternatively, first calculating the probability of each amount occurring, the expected recovery is (40%)(60%)($50,000) + (40%)(40%)($30,000) + (60%)(90%)($80,000) + (60%)(10%)($60,000) = $63,600.

14 A is correct. The covariance is the product of the standard deviations and correlation using the formula Cov(US bond returns, Spanish bond returns) = σ(US bonds) × σ (Spanish bonds) × ρ(US bond returns, Spanish bond returns) = 0.64 × 0.56 × 0.24 = 0.086.

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15 C is correct. The covariance of returns is positive when the returns on both assets tend to be on the same side (above or below) their expected values at the same time, indicating an average positive relationship between returns.

16 B is correct. Correlations near +1 exhibit strong positive linearity, whereas correlations near −1 exhibit strong negative linearity. A correlation of 0 indicates an absence of any linear relationship between the variables. The closer the correlation is to 0, the weaker the linear relationship.

17 C is correct. The correlation between two random variables R_i and R_j is defined as $\rho(R_i,R_j) = \text{Cov}(R_i,R_j)/[\sigma(R_i)\sigma(R_j)]$. Using the subscript i to represent hedge funds and the subscript j to represent the market index, the standard deviations are $\sigma(R_i) = 256^{1/2} = 16$ and $\sigma(R_j) = 81^{1/2} = 9$. Thus, $\rho(R_i,R_j) = \text{Cov}(R_i,R_j)/[\sigma(R_i)\sigma(R_j)] = 110/(16 \times 9) = 0.764$.

18 A is correct. As the correlation between two assets approaches +1, diversification benefits decrease. In other words, an increasingly positive correlation indicates an increasingly strong positive linear relationship and fewer diversification benefits.

19 A is correct. A covariance matrix for five stocks has 5 × 5 = 25 entries. Subtracting the 5 diagonal variance terms results in 20 off-diagonal entries. Because a covariance matrix is symmetrical, only 10 entries are unique (20/2 = 10).

20 A is correct. The analyst must first calculate expected sales as 0.05 × \$70 + 0.70 × \$40 + 0.25 × \$25 = \$3.50 million + \$28.00 million + \$6.25 million = \$37.75 million.

After calculating expected sales, we can calculate the variance of sales:

$$
\begin{aligned}
&= \sigma^2 \text{ (Sales)} \\
&= P(\$70)[\$70 - E(\text{Sales})]^2 + P(\$40)[\$40 - E(\text{Sales})]^2 + P(\$25) \\
&\quad [\$25 - E(\text{Sales})]^2 \\
&= 0.05(\$70 - 37.75)^2 + 0.70(\$40 - 37.75)^2 + 0.25(\$25 - 37.75)^2 \\
&= \$52.00 \text{ million} + \$3.54 \text{ million} + \$40.64 \text{ million} = \$96.18 \text{ million.}
\end{aligned}
$$

The standard deviation of sales is thus $\sigma = (\$96.18)^{1/2} = \$9.81$ million.

21 C is correct. The covariance of returns is positive when the returns on both assets tend to be on the same side (above or below) their expected values at the same time.

22 B is correct. The covariance is 26.56, calculated as follows. First, expected returns are

$$
\begin{aligned}
E(R_{FI}) &= (0.25 \times 25) + (0.50 \times 15) + (0.25 \times 10) \\
&= 6.25 + 7.50 + 2.50 = 16.25 \text{ and} \\
E(R_{DI}) &= (0.25 \times 30) + (0.50 \times 25) + (0.25 \times 15) \\
&= 7.50 + 12.50 + 3.75 = 23.75.
\end{aligned}
$$

Covariance is

$$
\begin{aligned}
\text{Cov}(R_{FI},R_{DI}) &= \sum_i \sum_j P\left(R_{FI,i}, R_{DI,j}\right)\left(R_{FI,i} - ER_{FI}\right)\left(R_{DI,j} - ER_{DI}\right) \\
&= 0.25[(25 - 16.25)(30 - 23.75)] + 0.50[(15 - 16.25) \\
&\quad (25 - 23.75)] + 0.25[(10 - 16.25)(15 - 23.75)] \\
&= 13.67 + (-0.78) + 13.67 = 26.56.
\end{aligned}
$$

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23 C is correct. The combination formula provides the number of ways that r objects can be chosen from a total of n objects, when the order in which the r objects are listed does not matter. The order of the bonds within the portfolio does not matter.

24 A is correct. The answer is found using the combination formula

$${}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Here, $n = 4$ and $r = 2$, so the answer is $4!/[(4 - 2)!2!] = 24/[(2) \times (2)] = 6$. This result can be verified by assuming there are four vice presidents, VP1–VP4. The six possible additions to the investment committee are VP1 and VP2, VP1 and VP3, VP1 and VP4, VP2 and VP3, VP2 and VP4, and VP3 and VP4.

25 A is correct. The permutation formula is used to choose r objects from a total of n objects when order matters. Because the portfolio manager is trying to rank the four funds from most recommended to least recommended, the order of the funds matters; therefore, the permutation formula is most appropriate.

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PRACTICE PROBLEMS

1 A European put option on stock conveys the right to sell the stock at a prespecified price, called the exercise price, at the maturity date of the option. The value of this put at maturity is (exercise price – stock price) or \$0, whichever is greater. Suppose the exercise price is \$100 and the underlying stock trades in ticks of \$0.01. At any time before maturity, the terminal value of the put is a random variable.

   A Describe the distinct possible outcomes for terminal put value. (Think of the put's maximum and minimum values and its minimum price increments.)

   B Is terminal put value, at a time before maturity, a discrete or continuous random variable?

   C Letting Y stand for terminal put value, express in standard notation the probability that terminal put value is less than or equal to \$24. No calculations or formulas are necessary.

2 Define the term "binomial random variable." Describe the types of problems for which the binomial distribution is used.

3 The value of the cumulative distribution function $F(x)$, where x is a particular outcome, for a discrete uniform distribution:

   A sums to 1.

   B lies between 0 and 1.

   C decreases as x increases.

4 For a binomial random variable with five trials, and a probability of success on each trial of 0.50, the distribution will be:

   A skewed.

   B uniform.

   C symmetric.

5 In a discrete uniform distribution with 20 potential outcomes of integers 1 to 20, the probability that X is greater than or equal to 3 but less than 6, $P(3 \le X < 6)$, is:

   A 0.10.

   B 0.15.

   C 0.20.

6 Over the last 10 years, a company's annual earnings increased year over year seven times and decreased year over year three times. You decide to model the number of earnings increases for the next decade as a binomial random variable.

   A What is your estimate of the probability of success, defined as an increase in annual earnings?

   For Parts B, C, and D of this problem, assume the estimated probability is the actual probability for the next decade.

   B What is the probability that earnings will increase in exactly 5 of the next 10 years?

   C Calculate the expected number of yearly earnings increases during the next 10 years.

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D Calculate the variance and standard deviation of the number of yearly earnings increases during the next 10 years.

E The expression for the probability function of a binomial random variable depends on two major assumptions. In the context of this problem, what must you assume about annual earnings increases to apply the binomial distribution in Part B? What reservations might you have about the validity of these assumptions?

7 A portfolio manager annually outperforms her benchmark 60% of the time. Assuming independent annual trials, what is the probability that she will outperform her benchmark four or more times over the next five years?

A 0.26

B 0.34

C 0.48

8 You are examining the record of an investment newsletter writer who claims a 70 percent success rate in making investment recommendations that are profitable over a one-year time horizon. You have the one-year record of the newsletter's seven most recent recommendations. Four of those recommendations were profitable. If all the recommendations are independent and the newsletter writer's skill is as claimed, what is the probability of observing four or fewer profitable recommendations out of seven in total?

9 You are forecasting sales for a company in the fourth quarter of its fiscal year. Your low-end estimate of sales is €14 million, and your high-end estimate is €15 million. You decide to treat all outcomes for sales between these two values as equally likely, using a continuous uniform distribution.

A What is the expected value of sales for the fourth quarter?

B What is the probability that fourth-quarter sales will be less than or equal to €14,125,000?

10 State the approximate probability that a normal random variable will fall within the following intervals:

A Mean plus or minus one standard deviation.

B Mean plus or minus two standard deviations.

C Mean plus or minus three standard deviations.

11 Find the area under the normal curve up to $z = 0.36$; that is, find $P(Z \le 0.36)$. Interpret this value.

12 If the probability that a portfolio outperforms its benchmark in any quarter is 0.75, the probability that the portfolio outperforms its benchmark in three or fewer quarters over the course of a year is *closest* to:

A 0.26

B 0.42

C 0.68

13 In futures markets, profits or losses on contracts are settled at the end of each trading day. This procedure is called marking to market or daily resettlement. By preventing a trader's losses from accumulating over many days, marking to market reduces the risk that traders will default on their obligations. A futures markets trader needs a liquidity pool to meet the daily mark to market. If liquidity is exhausted, the trader may be forced to unwind his position at an unfavorable time.

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Suppose you are using financial futures contracts to hedge a risk in your portfolio. You have a liquidity pool (cash and cash equivalents) of λ dollars per contract and a time horizon of T trading days. For a given size liquidity pool, λ, Kolb, Gay, and Hunter (1985) developed an expression for the probability stating that you will exhaust your liquidity pool within a T-day horizon as a result of the daily mark to market. Kolb et al. assumed that the expected change in futures price is 0 and that futures price changes are normally distributed. With σ representing the standard deviation of daily futures price changes, the standard deviation of price changes over a time horizon to day T is $\sigma\sqrt{T}$, given continuous compounding. With that background, the Kolb et al. expression is

Probability of exhausting liquidity pool = $2[1 - N(x)]$

where $x = \lambda/\left(\sigma\sqrt{T}\right)$. Here x is a standardized value of λ. $N(x)$ is the standard normal cumulative distribution function. For some intuition about $1 - N(x)$ in the expression, note that the liquidity pool is exhausted if losses exceed the size of the liquidity pool at any time up to and including T; the probability of that event happening can be shown to be proportional to an area in the right tail of a standard normal distribution, $1 - N(x)$.

Using the Kolb et al. expression, answer the following questions:

A Your hedging horizon is five days, and your liquidity pool is \$2,000 per contract. You estimate that the standard deviation of daily price changes for the contract is \$450. What is the probability that you will exhaust your liquidity pool in the five-day period?

B Suppose your hedging horizon is 20 days, but all the other facts given in Part A remain the same. What is the probability that you will exhaust your liquidity pool in the 20-day period?

14 Which of the following is characteristic of the normal distribution?

A Asymmetry

B Kurtosis of 3

C Definitive limits or boundaries

15 Which of the following assets *most likely* requires the use of a multivariate distribution for modeling returns?

A A call option on a bond

B A portfolio of technology stocks

C A stock in a market index

16 The total number of parameters that fully characterizes a multivariate normal distribution for the returns on two stocks is:

A 3.

B 4.

C 5.

17 A client has a portfolio of common stocks and fixed-income instruments with a current value of £1,350,000. She intends to liquidate £50,000 from the portfolio at the end of the year to purchase a partnership share in a business. Furthermore, the client would like to be able to withdraw the £50,000 without reducing the initial capital of £1,350,000. The following table shows four alternative asset allocations.

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Mean and Standard Deviation for Four Allocations (in Percent)

	A	B	C	D
Expected annual return	16	12	10	9
Standard deviation of return	24	17	12	11

Address the following questions (assume normality for Parts B and C):

A Given the client's desire not to invade the £1,350,000 principal, what is the shortfall level, R_L? Use this shortfall level to answer Part B.

B According to the safety-first criterion, which of the allocations is the best?

C What is the probability that the return on the safety-first optimal portfolio will be less than the shortfall level, R_L?

Please refer to Exhibit 1 for Questions 18 and 19

Exhibit 1 ***Z*-Table Values, $P(Z \le z) = N(z)$ for $z \ge 0$**

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

18 A portfolio has an expected mean return of 8 percent and standard deviation of 14 percent. The probability that its return falls between 8 and 11 percent is *closest* to:

A 8.3%

B 14.8%.

C 58.3%.

19 A portfolio has an expected return of 7% with a standard deviation of 13%. For an investor with a minimum annual return target of 4%, the probability that the portfolio return will fail to meet the target is *closest* to:

A 33%.

B 41%.

C 59%.

20 **A** Define Monte Carlo simulation and explain its use in finance.

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B Compared with analytical methods, what are the strengths and weaknesses of Monte Carlo simulation for use in valuing securities?

21 A standard lookback call option on stock has a value at maturity equal to (Value of the stock at maturity – Minimum value of stock during the life of the option prior to maturity) or $0, whichever is greater. If the minimum value reached prior to maturity was $20.11 and the value of the stock at maturity is $23, for example, the call is worth $23 – $20.11 = $2.89. Briefly discuss how you might use Monte Carlo simulation in valuing a lookback call option.

22 Which of the following is a continuous random variable?

A The value of a futures contract quoted in increments of $0.05

B The total number of heads recorded in 1 million tosses of a coin

C The rate of return on a diversified portfolio of stocks over a three-month period

23 X is a discrete random variable with possible outcomes $X = \{1,2,3,4\}$. Three functions $f(x)$, $g(x)$, and $h(x)$ are proposed to describe the probabilities of the outcomes in X.

	Probability Function		
X = x	**f(x) = P(X = x)**	**g(x) = P(X = x)**	**h(x) = P(X = x)**
1	–0.25	0.20	0.20
2	0.25	0.25	0.25
3	0.50	0.50	0.30
4	0.25	0.05	0.35

The conditions for a probability function are satisfied by:

A $f(x)$.

B $g(x)$.

C $h(x)$.

24 The cumulative distribution function for a discrete random variable is shown in the following table.

X = x	**Cumulative Distribution Function F(x) = P(X ≤ x)**
1	0.15
2	0.25
3	0.50
4	0.60
5	0.95
6	1.00

The probability that X will take on a value of either 2 or 4 is *closest* to:

A 0.20.

B 0.35.

C 0.85.

25 Which of the following events can be represented as a Bernoulli trial?

A The flip of a coin

B The closing price of a stock

C The picking of a random integer between 1 and 10

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26 The weekly closing prices of Mordice Corporation shares are as follows:

Date	Closing Price (€)
1 August	112
8 August	160
15 August	120

The continuously compounded return of Mordice Corporation shares for the period August 1 to August 15 is *closest to*:

A 6.90%

B 7.14%

C 8.95%

27 A stock is priced at $100.00 and follows a one-period binomial process with an up move that equals 1.05 and a down move that equals 0.97. If 1 million Bernoulli trials are conducted, and the average terminal stock price is $102.00, the probability of an up move (p) is *closest* to:

A 0.375.

B 0.500.

C 0.625.

28 A call option on a stock index is valued using a three-step binomial tree with an up move that equals 1.05 and a down move that equals 0.95. The current level of the index is $190, and the option exercise price is $200. If the option value is positive when the stock price exceeds the exercise price at expiration and $0 otherwise, the number of terminal nodes with a positive payoff is:

A one.

B two.

C three.

29 A random number between zero and one is generated according to a continuous uniform distribution. What is the probability that the first number generated will have a value of exactly 0.30?

A 0%

B 30%

C 70%

30 A Monte Carlo simulation can be used to:

A directly provide precise valuations of call options.

B simulate a process from historical records of returns.

C test the sensitivity of a model to changes in assumptions.

31 A limitation of Monte Carlo simulation is:

A its failure to do "what if" analysis.

B that it requires historical records of returns

C its inability to independently specify cause-and-effect relationships.

32 Which parameter equals zero in a normal distribution?

A Kurtosis

B Skewness

C Standard deviation

33 An analyst develops the following capital market projections.

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	Stocks	Bonds
Mean Return	10%	2%
Standard Deviation	15%	5%

Assuming the returns of the asset classes are described by normal distributions, which of the following statements is correct?

A Bonds have a higher probability of a negative return than stocks.

B On average, 99% of stock returns will fall within two standard deviations of the mean.

C The probability of a bond return less than or equal to 3% is determined using a Z-score of 0.25.

34 A client holding a £2,000,000 portfolio wants to withdraw £90,000 in one year without invading the principal. According to Roy's safety-first criterion, which of the following portfolio allocations is optimal?

	Allocation A	Allocation B	Allocation C
Expected annual return	6.5%	7.5%	8.5%
Standard deviation of returns	8.35%	10.21%	14.34%

A Allocation A

B Allocation B

C Allocation C

35 In contrast to normal distributions, lognormal distributions:

A are skewed to the left.

B have outcomes that cannot be negative.

C are more suitable for describing asset returns than asset prices.

36 The lognormal distribution is a more accurate model for the distribution of stock prices than the normal distribution because stock prices are:

A symmetrical.

B unbounded.

C non-negative.

37 The price of a stock at t = 0 is $208.25 and at t = 1 is $186.75. The continuously compounded rate of return for the stock from t = 0 to t = 1 is *closest* to:

A –10.90%.

B –10.32%.

C 11.51%.

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SOLUTIONS

1 **A** The put's minimum value is \$0. The put's value is \$0 when the stock price is at or above \$100 at the maturity date of the option. The put's maximum value is \$100 = \$100 (the exercise price) – \$0 (the lowest possible stock price). The put's value is \$100 when the stock is worthless at the option's maturity date. The put's minimum price increments are \$0.01. The possible outcomes of terminal put value are thus \$0.00, \$0.01, \$0.02, …, \$100.

B The price of the underlying has minimum price fluctuations of \$0.01: These are the minimum price fluctuations for terminal put value. For example, if the stock finishes at \$98.20, the payoff on the put is \$100 – \$98.20 = \$1.80. We can specify that the nearest values to \$1.80 are \$1.79 and \$1.81. With a continuous random variable, we cannot specify the nearest values. So, we must characterize terminal put value as a discrete random variable.

C The probability that terminal put value is less than or equal to \$24 is $P(Y \leq 24)$ or $F(24)$, in standard notation, where F is the cumulative distribution function for terminal put value.

2 A binomial random variable is defined as the number of successes in n Bernoulli trials (a trial that produces one of two outcomes). The binomial distribution is used to make probability statements about a record of successes and failures or about anything with binary (twofold) outcomes.

3 B is correct. The value of the cumulative distribution function lies between 0 and 1 for any x: $0 \leq F(x) \leq 1$.

4 C is correct. The binomial distribution is symmetric when the probability of success on a trial is 0.50, but it is asymmetric or skewed otherwise. Here it is given that $p = 0.50$

5 B is correct. The probability of any outcome is 0.05, $P(1) = 1/20 = 0.05$. The probability that X is greater than or equal to 3 but less than 6, which is expressed as $P(3 \leq X < 6) = P(3) + P(4) + P(5) = 0.05 + 0.05 + 0.05 = 0.15$

6 **A** The probability of an earnings increase (success) in a year is estimated as 7/10 = 0.70 or 70 percent, based on the record of the past 10 years.

B The probability that earnings will increase in 5 out of the next 10 years is about 10.3 percent. Define a binomial random variable X, counting the number of earnings increases over the next 10 years. From Part A, the probability of an earnings increase in a given year is $p = 0.70$ and the number of trials (years) is $n = 10$. Equation 1 gives the probability that a binomial random variable has x successes in n trials, with the probability of success on a trial equal to p.

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x}$$

For this example,

$$\binom{10}{5} 0.7^5 0.3^{10-5} = \frac{10!}{(10-5)!5!} 0.7^5 0.3^{10-5}$$
$$= 252 \times 0.16807 \times 0.00243 = 0.102919$$

We conclude that the probability that earnings will increase in exactly 5 of the next 10 years is 0.1029, or approximately 10.3 percent.

C The expected number of yearly increases is $E(X) = np = 10 \times 0.70 = 7$.

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D The variance of the number of yearly increases over the next 10 years is $\sigma^2 = np(1 - p) = 10 \times 0.70 \times 0.30 = 2.1$. The standard deviation is 1.449 (the positive square root of 2.1).

E You must assume that 1) the probability of an earnings increase (success) is constant from year to year and 2) earnings increases are independent trials. If current and past earnings help forecast next year's earnings, Assumption 2 is violated. If the company's business is subject to economic or industry cycles, neither assumption is likely to hold.

7 B is correct. To calculate the probability of 4 years of outperformance, use the formula:

$$p(x) = P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} = \frac{n!}{(n - x)!x!} p^x (1 - p)^{n-x}$$

Using this formula to calculate the probability in 4 of 5 years, $n = 5$, $x = 4$ and $p = 0.60$.

Therefore,

$$p(4) = \frac{5!}{(5 - 4)!4!} 0.6^4 (1 - 0.6)^{5-4} = [120/24](0.1296)(0.40) = 0.2592$$

$$p(5) = \frac{5!}{(5 - 5)!5!} 0.6^5 (1 - 0.6)^{5-5} = [120/120](0.0778)(1) = 0.0778$$

The probability of outperforming 4 or more times is $p(4) + p(5) = 0.2592 + 0.0778 = 0.3370$

8 The observed success rate is 4/7 = 0.571, or 57.1 percent. The probability of four or fewer successes is $F(4) = p(4) + p(3) + p(2) + p(1) + p(0)$, where $p(4)$, $p(3)$, $p(2)$, $p(1)$, and $p(0)$ are respectively the probabilities of 4, 3, 2, 1, and 0 successes, according to the binomial distribution with $n = 7$ and $p = 0.70$. We have

$p(4) = (7!/4!3!)(0.70^4)(0.30^3) = 35(0.006483) = 0.226895$

$p(3) = (7!/3!4!)(0.70^3)(0.30^4) = 35(0.002778) = 0.097241$

$p(2) = (7!/2!5!)(0.70^2)(0.30^5) = 21(0.001191) = 0.025005$

$p(1) = (7!/1!6!)(0.70^1)(0.30^6) = 7(0.000510) = 0.003572$

$p(0) = (7!/0!7!)(0.70^0)(0.30^7) = 1(0.000219) = 0.000219$

Summing all these probabilities, you conclude that $F(4) = 0.226895 + 0.097241 + 0.025005 + 0.003572 + 0.000219 = 0.352931$, or 35.3 percent.

9 **A** The expected value of fourth-quarter sales is €14,500,000, calculated as (€14,000,000 + €15,000,000)/2. With a continuous uniform random variable, the mean or expected value is the midpoint between the smallest and largest values. (See Example 7.)

B The probability that fourth-quarter sales will be less than €14,125,000 is 0.125 or 12.5 percent, calculated as (€14,125,000 − €14,000,000)/(€15,000,000 − €14,000,000).

10 **A** Approximately 68 percent of all outcomes of a normal random variable fall within plus or minus one standard deviation of the mean.

B Approximately 95 percent of all outcomes of a normal random variable fall within plus or minus two standard deviations of the mean.

C Approximately 99 percent of all outcomes of a normal random variable fall within plus or minus three standard deviations of the mean.

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11 The area under the normal curve for $z = 0.36$ is 0.6406 or 64.06 percent. The following table presents an excerpt from the tables of the standard normal cumulative distribution function in the back of this volume. To locate $z = 0.36$, find 0.30 in the fourth row of numbers, then look at the column for 0.06 (the second decimal place of 0.36). The entry is 0.6406.

$P(Z \leq x) = N(x)$ for $x \geq 0$ or $P(Z \leq z) = N(z)$ for $z \geq 0$

x or *z*	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	**0.6406**	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

The interpretation of 64.06 percent for $z = 0.36$ is that 64.06 percent of observations on a standard normal random variable are smaller than or equal to the value 0.36. (So 100% − 64.06% = 35.94% of the values are greater than 0.36.)

12 C is correct. The probability that the performance is at or below the expectation is calculated by finding $F(3) = p(3) + p(2) + p(1)$ using the formula:

$$p(x) = P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} = \frac{n!}{(n - x)!x!} p^x (1 - p)^{n-x}$$

Using this formula,

$$p(3) = \frac{4!}{(4 - 3)!3!} 0.75^3 (1 - 0.75)^{4-3} = [24/6](0.42)(0.25) = 0.42$$

$$p(2) = \frac{4!}{(4 - 2)!2!} 0.75^2 (1 - 0.75)^{4-2} = [24/4](0.56)(0.06) = 0.20$$

$$p(1) = \frac{4!}{(4 - 1)!1!} 0.75^1 (1 - 0.75)^{4-1} = [24/6](0.75)(0.02) = 0.06$$

$$p(0) = \frac{4!}{(4 - 0)!0!} 0.75^0 (1 - 0.75)^{4-0} = [24/24](1)(0.004) = 0.004$$

Therefore,

$F(3) = p(3) + p(2) + p(1) + p(0) = 0.42 + 0.20 + 0.06 + 0.004 = 0.684$ or approximately 68 percent

13 **A** The probability of exhausting the liquidity pool is 4.7 percent. First calculate $x = \lambda / (\sigma\sqrt{T}) = \$2{,}000 / (\$450\sqrt{5}) = 1.987616$. We can round this value to 1.99 to use the standard normal tables in the back of this book. Using those tables, we find that $N(1.99) = 0.9767$. Thus, the probability of exhausting the liquidity pool is $2[1 - N(1.99)] = 2(1 - 0.9767) = 0.0466$ or about 4.7 percent.

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B The probability of exhausting the liquidity pool is now 32.2 percent. The calculation follows the same steps as those in Part A. We calculate $x = \lambda/\left(\sigma\sqrt{T}\right) = \$2{,}000/\left(\$450\sqrt{20}\right) = 0.993808$. We can round this value to 0.99 to use the standard normal tables in the back of this book. Using those tables, we find that $N(0.99) = 0.8389$. Thus, the probability of exhausting the liquidity pool is $2[1 - N(0.99)] = 2(1 - 0.8389) = 0.3222$ or about 32.2 percent. This is a substantial probability that you will run out of funds to meet mark to market.

In their paper, Kolb et al. call the probability of exhausting the liquidity pool the probability of ruin, a traditional name for this type of calculation.

14 B is correct. The normal distribution has a skewness of 0, a kurtosis of 3, and a mean, median and mode that are all equal.

15 B is correct. Multivariate distributions specify the probabilities for a group of related random variables. A portfolio of technology stocks represents a group of related assets. Accordingly, statistical interrelationships must be considered, resulting in the need to use a multivariate normal distribution.

16 C is correct. A bivariate normal distribution (two stocks) will have two means, two variances and one correlation. A multivariate normal distribution for the returns on n stocks will have n means, n variances and $n(n - 1)/2$ distinct correlations.

17 **A** Because £50,000/£1,350,000 is 3.7 percent, for any return less than 3.7 percent the client will need to invade principal if she takes out £50,000. So R_L = 3.7 percent.

B To decide which of the allocations is safety-first optimal, select the alternative with the highest ratio $[E(R_P) - R_L]/\sigma_P$:

Allocation A: 0.5125 = (16 – 3.7)/24

Allocation B: 0.488235 = (12 – 3.7)/17

Allocation C: 0.525 = (10 – 3.7)/12

Allocation D: 0.481818 = (9 – 3.7)/11

Allocation C, with the largest ratio (0.525), is the best alternative according to the safety-first criterion.

C To answer this question, note that $P(R_C < 3.7) = N(-0.525)$. We can round 0.525 to 0.53 for use with tables of the standard normal cdf. First, we calculate $N(-0.53) = 1 - N(0.53) = 1 - 0.7019 = 0.2981$ or about 30 percent. The safety-first optimal portfolio has a roughly 30 percent chance of not meeting a 3.7 percent return threshold.

18 A is correct. P(8% ≤ Portfolio return ≤ 11%) = N(Z corresponding to 11%) – N(Z corresponding to 8%). For the first term, Z = (11% – 8%)/14% = 0.21 approximately, and using the table of cumulative normal distribution given in the problem, $N(0.21) = 0.5832$. To get the second term immediately, note that 8 percent is the mean, and for the normal distribution 50 percent of the probability lies on either side of the mean. Therefore, N(Z corresponding to 8%) must equal 50 percent. So P(8% ≤ Portfolio return ≤ 11%) = 0.5832 – 0.50 = 0.0832 or approximately 8.3 percent.

19 B is correct. There are three steps, which involve standardizing the portfolio return: First, subtract the portfolio mean return from each side of the inequality: P(Portfolio return – 7%) ≤ 4% – 7%). Second, divide each side of the inequality by the standard deviation of portfolio return: P[(Portfolio return

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− 7%)/13% ≤ (4% − 7%)/13%] = $P(Z \le -0.2308) = N(-0.2308)$. Third, recognize that on the left-hand side we have a standard normal variable, denoted by Z and $N(-x) = 1 - N(x)$. Rounding −0.2308 to −0.23 for use with the cumulative distribution function (cdf) table, we have $N(-0.23) = 1 - N(0.23) = 1 - 0.5910 = 0.409$, approximately 41 percent. The probability that the portfolio will underperform the target is about 41 percent.

20 **A** Elements that should appear in a definition of Monte Carlo simulation are that it makes use of a computer; that it is used to represent the operation of a complex system, or in some applications, to find an approximate solution to a problem; and that it involves the generation of a large number of random samples from a specified probability distribution. The exact wording can vary, but one definition follows:

Monte Carlo simulation in finance involves the use of a computer to represent the operation of a complex financial system. In some important applications, Monte Carlo simulation is used to find an approximate solution to a complex financial problem. An integral part of Monte Carlo simulation is the generation of a large number of random samples from a probability distribution.

B *Strengths.* Monte Carlo simulation can be used to price complex securities for which no analytic expression is available, particularly European-style options.

Weaknesses. Monte Carlo simulation provides only statistical estimates, not exact results. Analytic methods, when available, provide more insight into cause-and-effect relationships than does Monte Carlo simulation.

21 In the text, we described how we could use Monte Carlo simulation to value an Asian option, a complex European-style option. Just as we can calculate the average value of the stock over a simulation trial to value an Asian option, we can also calculate the minimum value of the stock over a simulation trial. Then, for a given simulation trial, we can calculate the terminal value of the call, given the minimum value of the stock for the simulation trial. We can then discount back this terminal value to the present to get the value of the call today ($t = 0$). The average of these $t = 0$ values over all simulation trials is the Monte Carlo simulated value of the lookback call option.

22 C is correct. The rate of return is a random variable because the future outcomes are uncertain, and it is continuous because it can take on an unlimited number of outcomes.

23 B is correct. The function $g(x)$ satisfies the conditions of a probability function. All of the values of $g(x)$ are between 0 and 1, and the values of $g(x)$ all sum to 1.

24 A is correct. The probability that X will take on a value of 4 or less is: $F(4) = P(X \le 4) = p(1) + p(2) + p(3) + p(4) = 0.60$. The probability that X will take on a value of 3 or less is: $F(3) = P(X \le 3) = p(1) + p(2) + \text{p}(3) = 0.50$. So, the probability that X will take on a value of 4 is: $F(4) - F(3) = p(4) = 0.10$. The probability of $X = 2$ can be found using the same logic: $F(2) - F(1) = p(2) = 0.25 - 0.15 = 0.10$. The probability of X taking on a value of 2 or 4 is: $p(2) + p(4) = 0.10 + 0.10 = 0.20$.

25 A is correct. A trial, such as a coin flip, will produce one of two outcomes. Such a trial is a Bernoulli trial.

26 A is correct. The continuously compounded return of an asset over a period is equal to the natural log of period's change. In this case:

$$\ln(120/112) = 6.90\%$$

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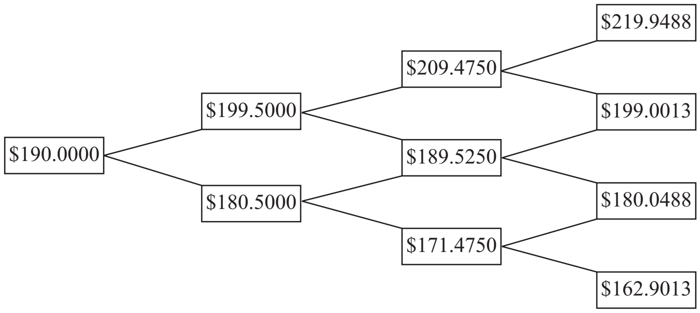
27 C is correct. The probability of an up move (p) can be found by solving the equation: $(p)uS + (1 - p)dS = (p)105 + (1 - p)97 = 102$. Solving for p gives $8p = 5$, so that $p = 0.625$.

28 A is correct. Only the top node value of \$219.9488 exceeds \$200.



29 A is correct. The probability of generating a random number equal to any fixed point under a continuous uniform distribution is zero.

30 C is correct. A characteristic feature of Monte Carlo simulation is the generation of a large number of random samples from a specified probability distribution or distributions to represent the role of risk in the system.

31 C is correct. Monte Carlo simulation is a complement to analytical methods. Monte Carlo simulation provides statistical estimates and not exact results. Analytical methods, when available, provide more insight into cause-and-effect relationships

32 B is correct. A normal distribution has a skewness of zero (it is symmetrical around the mean). A non-zero skewness implies asymmetry in a distribution.

33 A is correct. The chance of a negative return falls in the area to the left of 0% under a standard normal curve. By standardizing the returns and standard deviations of the two assets, the likelihood of either asset experiencing a negative return may be determined: Z-score (standardized value) = $(X - \mu)/\sigma$

Z-score for a bond return of 0% = $(0 - 2)/5 = -0.40$.

Z-score for a stock return of 0% = $(0 - 10)/15 = -0.67$.

For bonds, a 0% return falls 0.40 standard deviations below the mean return of 2%. In contrast, for stocks, a 0% return falls 0.67 standard deviations below the mean return of 10%. A standard deviation of 0.40 is less than a standard deviation of 0.67. Negative returns thus occupy more of the left tail of the bond distribution than the stock distribution. Thus, bonds are more likely than stocks to experience a negative return.

34 B is correct. Allocation B has the highest safety-first ratio. The threshold return level R_L for the portfolio is £90,000/£2,000,000 = 4.5%, thus any return less than R_L = 4.5% will invade the portfolio principal. To compute the allocation that is safety-first optimal, select the alternative with the highest ratio:

$$\frac{\left[E\left(R_P - R_L\right)\right]}{\sigma_P}$$

$$\text{Allocation A} = \frac{6.5 - 4.5}{8.35} = 0.240$$

$$\text{Allocation B} = \frac{7.5 - 4.5}{10.21} = 0.294$$

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$$\text{Allocation C} = \frac{8.5 - 4.5}{14.34} = 0.279$$

35 B is correct. By definition, lognormal random variables cannot have negative values.

36 C is correct. A lognormal distributed variable has a lower bound of zero. The lognormal distribution is also right skewed, which is a useful property in describing asset prices.

37 A is correct. The continuously compounded return from $t = 0$ to $t = 1$ is $r_{0,1} = \ln(S_1/S_0) = \ln(186.75/208.25) = -0.10897 = -10.90\%$.

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PRACTICE PROBLEMS

1 Peter Biggs wants to know how growth managers performed last year. Biggs assumes that the population cross-sectional standard deviation of growth manager returns is 6 percent and that the returns are independent across managers.

A How large a random sample does Biggs need if he wants the standard deviation of the sample means to be 1 percent?

B How large a random sample does Biggs need if he wants the standard deviation of the sample means to be 0.25 percent?

2 Petra Munzi wants to know how value managers performed last year. Munzi estimates that the population cross-sectional standard deviation of value manager returns is 4 percent and assumes that the returns are independent across managers.

A Munzi wants to build a 95 percent confidence interval for the mean return. How large a random sample does Munzi need if she wants the 95 percent confidence interval to have a total width of 1 percent?

B Munzi expects a cost of about $10 to collect each observation. If she has a $1,000 budget, will she be able to construct the confidence interval she wants?

3 Assume that the equity risk premium is normally distributed with a population mean of 6 percent and a population standard deviation of 18 percent. Over the last four years, equity returns (relative to the risk-free rate) have averaged −2.0 percent. You have a large client who is very upset and claims that results this poor should *never* occur. Evaluate your client's concerns.

A Construct a 95 percent confidence interval around the population mean for a sample of four-year returns.

B What is the probability of a −2.0 percent or lower average return over a four-year period?

4 Compare the standard normal distribution and Student's t-distribution.

5 Find the reliability factors based on the t-distribution for the following confidence intervals for the population mean (df = degrees of freedom, n = sample size):

A A 99 percent confidence interval, df = 20.

B A 90 percent confidence interval, df = 20.

C A 95 percent confidence interval, n = 25.

D A 95 percent confidence interval, n = 16.

6 Assume that monthly returns are normally distributed with a mean of 1 percent and a sample standard deviation of 4 percent. The population standard deviation is unknown. Construct a 95 percent confidence interval for the sample mean of monthly returns if the sample size is 24.

7 Ten analysts have given the following fiscal year earnings forecasts for a stock:

Forecast (X_i)	Number of Analysts (n_i)
1.40	1
1.43	1
1.44	3

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Forecast (X_i)	Number of Analysts (n_i)
1.45	2
1.47	1
1.48	1
1.50	1

Because the sample is a small fraction of the number of analysts who follow this stock, assume that we can ignore the finite population correction factor. Assume that the analyst forecasts are normally distributed.

A What are the mean forecast and standard deviation of forecasts?

B Provide a 95 percent confidence interval for the population mean of the forecasts.

8 Thirteen analysts have given the following fiscal-year earnings forecasts for a stock:

Forecast (X_i)	Number of Analysts (n_i)
0.70	2
0.72	4
0.74	1
0.75	3
0.76	1
0.77	1
0.82	1

Because the sample is a small fraction of the number of analysts who follow this stock, assume that we can ignore the finite population correction factor.

A What are the mean forecast and standard deviation of forecasts?

B What aspect of the data makes us uncomfortable about using *t*-tables to construct confidence intervals for the population mean forecast?

9 Explain the differences between constructing a confidence interval when sampling from a normal population with a known population variance and sampling from a normal population with an unknown variance.

10 An exchange rate has a given expected future value and standard deviation.

A Assuming that the exchange rate is normally distributed, what are the probabilities that the exchange rate will be at least 2 or 3 standard deviations away from its mean?

B Assume that you do not know the distribution of exchange rates. Use Chebyshev's inequality (that at least $1 - 1/k^2$ proportion of the observations will be within k standard deviations of the mean for any positive integer k greater than 1) to calculate the maximum probabilities that the exchange rate will be at least 2 or 3 standard deviations away from its mean.

11 Although he knows security returns are not independent, a colleague makes the claim that because of the central limit theorem, if we diversify across a large number of investments, the portfolio standard deviation will eventually approach zero as n becomes large. Is he correct?

12 Why is the central limit theorem important?

13 What is wrong with the following statement of the central limit theorem?

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Central Limit Theorem. "If the random variables $X_1, X_2, X_3, \ldots, X_n$ are a random sample of size *n* from any distribution with finite mean μ and variance σ^2, then the distribution of $\bar{X}$ will be approximately normal, with a standard deviation of $\sigma/\sqrt{n}$."

14 Suppose we take a random sample of 30 companies in an industry with 200 companies. We calculate the sample mean of the ratio of cash flow to total debt for the prior year. We find that this ratio is 23 percent. Subsequently, we learn that the population cash flow to total debt ratio (taking account of all 200 companies) is 26 percent. What is the explanation for the discrepancy between the sample mean of 23 percent and the population mean of 26 percent?

A Sampling error.

B Bias.

C A lack of consistency.

15 Alcorn Mutual Funds is placing large advertisements in several financial publications. The advertisements prominently display the returns of 5 of Alcorn's 30 funds for the past 1-, 3-, 5-, and 10-year periods. The results are indeed impressive, with all of the funds beating the major market indexes and a few beating them by a large margin. Is the Alcorn family of funds superior to its competitors?

16 Julius Spence has tested several predictive models in order to identify undervalued stocks. Spence used about 30 company-specific variables and 10 market-related variables to predict returns for about 5,000 North American and European stocks. He found that a final model using eight variables applied to telecommunications and computer stocks yields spectacular results. Spence wants you to use the model to select investments. Should you? What steps would you take to evaluate the model?

17 The *best* approach for creating a stratified random sample of a population involves:

A drawing an equal number of simple random samples from each subpopulation.

B selecting every *k*th member of the population until the desired sample size is reached.

C drawing simple random samples from each subpopulation in sizes proportional to the relative size of each subpopulation.

18 A population has a non-normal distribution with mean μ and variance σ^2. The sampling distribution of the sample mean computed from samples of large size from that population will have:

A the same distribution as the population distribution.

B its mean approximately equal to the population mean.

C its variance approximately equal to the population variance.

19 A sample mean is computed from a population with a variance of 2.45. The sample size is 40. The standard error of the sample mean is *closest* to:

A 0.039.

B 0.247.

C 0.387.

20 An estimator with an expected value equal to the parameter that it is intended to estimate is described as:

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A efficient.

B unbiased.

C consistent.

21 If an estimator is consistent, an increase in sample size will increase the:

A accuracy of estimates.

B efficiency of the estimator.

C unbiasedness of the estimator.

22 For a two-sided confidence interval, an increase in the degree of confidence will result in:

A a wider confidence interval.

B a narrower confidence interval.

C no change in the width of the confidence interval.

23 As the *t*-distribution's degrees of freedom decrease, the *t*-distribution *most likely*:

A exhibits tails that become fatter.

B approaches a standard normal distribution.

C becomes asymmetrically distributed around its mean value.

24 For a sample size of 17, with a mean of 116.23 and a variance of 245.55, the width of a 90% confidence interval using the appropriate *t*-distribution is *closest to*:

A 13.23.

B 13.27.

C 13.68.

25 For a sample size of 65 with a mean of 31 taken from a normally distributed population with a variance of 529, a 99% confidence interval for the population mean will have a lower limit *closest* to:

A 23.64.

B 25.41.

C 30.09.

26 An increase in sample size is *most likely* to result in a:

A wider confidence interval.

B decrease in the standard error of the sample mean.

C lower likelihood of sampling from more than one population.

27 A report on long-term stock returns focused exclusively on all currently publicly traded firms in an industry is *most likely* susceptible to:

A look-ahead bias.

B survivorship bias.

C intergenerational data mining.

28 Which sampling bias is *most likely* investigated with an out-of-sample test?

A Look-ahead bias

B Data-mining bias

C Sample selection bias

29 Which of the following characteristics of an investment study *most likely* indicates time-period bias?

A The study is based on a short time-series.

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B Information not available on the test date is used.

C A structural change occurred prior to the start of the study's time series.

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SOLUTIONS

1 A The standard deviation or standard error of the sample mean is $\sigma_{\bar{X}} = \sigma/\sqrt{n}$. Substituting in the values for $\sigma_{\bar{X}}$ and σ, we have 1% = $6\%/\sqrt{n}$, or $\sqrt{n}$ = 6. Squaring this value, we get a random sample of n = 36.

B As in Part A, the standard deviation of sample mean is $\sigma_{\bar{X}} = \sigma/\sqrt{n}$. Substituting in the values for $\sigma_{\bar{X}}$ and σ, we have 0.25% = $6\%/\sqrt{n}$, or $\sqrt{n}$ = 24. Squaring this value, we get a random sample of n = 576, which is substantially larger than for Part A of this question.

2 A Assume the sample size will be large and thus the 95 percent confidence interval for the mean of a sample of manager returns is $\bar{X} \pm 1.96 s_{\bar{X}}$, where $s_{\bar{X}} = s/\sqrt{n}$. Munzi wants the distance between the upper limit and lower limit in the confidence interval to be 1 percent, which is

$$\left(\bar{X} + 1.96 s_{\bar{X}}\right) - \left(\bar{X} - 1.96 s_{\bar{X}}\right) = 1\%$$

Simplifying this equation, we get $2\left(1.96 s_{\bar{X}}\right)$ = 1%. Finally, we have $3.92 s_{\bar{X}}$ = 1%, which gives us the standard deviation of the sample mean, $s_{\bar{X}}$ = 0.255%. The distribution of sample means is $s_{\bar{X}} = s/\sqrt{n}$. Substituting in the values for $s_{\bar{X}}$ and s, we have 0.255% = $4\%/\sqrt{n}$, or $\sqrt{n}$ = 15.69. Squaring this value, we get a random sample of n = 246.

B With her budget, Munzi can pay for a sample of up to 100 observations, which is far short of the 246 observations needed. Munzi can either proceed with her current budget and settle for a wider confidence interval or she can raise her budget (to around $2,460) to get the sample size for a 1 percent width in her confidence interval.

3 A This is a small-sample problem in which the sample comes from a normal population with a known standard deviation; thus we use the z-distribution in the solution. For a 95 percent confidence interval (and 2.5 percent in each tail), the critical z-value is 1.96. For returns that are normally distributed, a 95 percent confidence interval is of the form

$$\mu + 1.96\frac{\sigma}{\sqrt{n}}$$

The lower limit is $X_1 = \mu - 1.96\frac{\sigma}{\sqrt{n}} = 6\% - 1.96\frac{18\%}{\sqrt{4}}$ = 6% − 1.96(9%) = −11.64%.

The upper limit is $X_u = \mu + 1.96\frac{\sigma}{\sqrt{n}} = 6\% + 1.96\frac{18\%}{\sqrt{4}}$ = 6% + 1.96(9%) = 23.64%.

There is a 95 percent probability that four-year average returns will be between −11.64 percent and +23.64 percent.

B The critical z-value associated with the −2.0 percent return is

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{-2\% - 6\%}{18\%/\sqrt{4}} = \frac{-8\%}{9\%} = -0.89$$

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Using a normal table, the probability of a z-value less than −0.89 is $P(Z < -0.89) = 0.1867$. Unfortunately, although your client is unhappy with the investment result, a four-year average return of −2.0 percent or lower should occur 18.67 percent of the time.

4 (Refer to Figure 1 to help visualize the answer to this question.) Basically, only one standard normal distribution exists, but many t-distributions exist—one for every different number of degrees of freedom. The normal distribution and the t-distribution for a large number of degrees of freedom are practically the same. The lower the degrees of freedom, the flatter the t-distribution becomes. The t-distribution has less mass (lower probabilities) in the center of the distribution and more mass (higher probabilities) out in both tails. Therefore, the confidence intervals based on t-values will be wider than those based on the normal distribution. Stated differently, the probability of being within a given number of standard deviations (such as within ±1 standard deviation or ±2 standard deviations) is lower for the t-distribution than for the normal distribution.

5 **A** For a 99 percent confidence interval, the reliability factor we use is $t_{0.005}$; for df = 20, this factor is 2.845.

B For a 90 percent confidence interval, the reliability factor we use is $t_{0.05}$; for df = 20, this factor is 1.725.

C Degrees of freedom equals $n - 1$, or in this case $25 - 1 = 24$. For a 95 percent confidence interval, the reliability factor we use is $t_{0.025}$; for df = 24, this factor is 2.064.

D Degrees of freedom equals $16 - 1 = 15$. For a 95 percent confidence interval, the reliability factor we use is $t_{0.025}$; for df = 15, this factor is 2.131.

6 Because this is a small sample from a normal population and we have only the sample standard deviation, we use the following model to solve for the confidence interval of the population mean:

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where we find $t_{0.025}$ (for a 95 percent confidence interval) for df = $n - 1 = 24 - 1 = 23$; this value is 2.069. Our solution is $1\% \pm 2.069(4\%)/\sqrt{24} = 1\% \pm 2.069(0.8165) = 1\% \pm 1.69$. The 95 percent confidence interval spans the range from −0.69 percent to +2.69 percent.

7 The following table summarizes the calculations used in the answers.

Forecast (X_i)	Number of Analysts (n_i)	$X_i n_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$(X_i - \bar{X})^2 n_i$
1.40	1	1.40	−0.05	0.0025	0.0025
1.43	1	1.43	−0.02	0.0004	0.0004
1.44	3	4.32	−0.01	0.0001	0.0003
1.45	2	2.90	0.00	0.0000	0.0000
1.47	1	1.47	0.02	0.0004	0.0004
1.48	1	1.48	0.03	0.0009	0.0009
1.50	1	1.50	0.05	0.0025	0.0025
Sums	10	14.50			0.0070

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A With $n = 10, \overline{X} = \sum_{i=1}^{10} X_i/n = 14.50/10 = 1.45$. The variance is $s^2 = \left[\sum_{i=1}^{10}\left(X_i - \overline{X}\right)^2\right] \Big/ (n-1) = 0.0070/9 = 0.0007778$. The sample standard deviation is $s = \sqrt{0.0007778} = 0.02789$.

B The confidence interval for the mean can be estimated by using $\overline{X} \pm t_{\alpha/2}\left(s/\sqrt{n}\right)$. For 9 degrees of freedom, the reliability factor, $t_{0.025}$, equals 2.262 and the confidence interval is

$$\begin{aligned} 1.45 \pm 2.262 \times 0.02789/\sqrt{10} &= 1.45 \pm 2.262(0.00882) \\ &= 1.45 \pm 0.02 \end{aligned}$$

The confidence interval for the population mean ranges from 1.43 to 1.47.

8 The following table summarizes the calculations used in the answers.

Forecast (X_i)	Number of Analysts (n_i)	$X_i n_i$	$\left(X_i - \overline{X}\right)$	$\left(X_i - \overline{X}\right)^2$	$\left(X_i - \overline{X}\right)^2 n_i$
0.70	2	1.40	−0.04	0.0016	0.0032
0.72	4	2.88	−0.02	0.0004	0.0016
0.74	1	0.74	0.00	0.0000	0.0000
0.75	3	2.25	0.01	0.0001	0.0003
0.76	1	0.76	0.02	0.0004	0.0004
0.77	1	0.77	0.03	0.0009	0.0009
0.82	1	0.82	0.08	0.0064	0.0064
Sums	13	9.62			0.0128

A With $n = 13, \overline{X} = \sum_{i=1}^{13} X_i/n = 9.62/13 = 0.74$. The variance is $s^2 = \left[\sum_{i=1}^{13}\left(X_i - \overline{X}\right)^2\right] \Big/ (n-1) = 0.0128/12 = 0.001067$. The sample standard deviation is $s = \sqrt{0.001067} = 0.03266$.

B The sample is small, and the distribution appears to be bimodal. We cannot compute a confidence interval for the population mean because we have probably sampled from a distribution that is not normal.

9 If the population variance is known, the confidence interval is

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

The confidence interval for the population mean is centered at the sample mean, $\overline{X}$. The population standard deviation is σ, and the sample size is n. The population standard deviation divided by the square root of n is the standard error of the estimate of the mean. The value of z depends on the desired degree of confidence. For a 95 percent confidence interval, $z_{0.025} = 1.96$ and the confidence interval estimate is

$$\overline{X} \pm 1.96\frac{\sigma}{\sqrt{n}}$$

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If the population variance is not known, we make two changes to the technique used when the population variance is known. First, we must use the sample standard deviation instead of the population standard deviation. Second, we use the t-distribution instead of the normal distribution. The critical t-value will depend on degrees of freedom $n - 1$. If the sample size is large, we have the alternative of using the z-distribution with the sample standard deviation.

10 **A** The probabilities can be taken from a normal table, in which the critical z-values are 2.00 or 3.00 and we are including the probabilities in both tails. The probabilities that the exchange rate will be at least 2 or 3 standard deviations away from the mean are

$$P\left(\left|X - \mu\right| \geq 2\sigma\right) = 0.0456$$
$$P\left(\left|X - \mu\right| \geq 3\sigma\right) = 0.0026$$

B With Chebyshev's inequality, the maximum probability of the exchange rate being at least k standard deviations from the mean is $P(| X - \mu | \geq k\sigma) \leq (1/k)^2$. The maximum probabilities of the rate being at least 2 or 3 standard deviations away from the mean are

$$P\left(\left|X - \mu\right| \geq 2\sigma\right) \leq \left(1/2\right)^2 = 0.2500$$
$$P\left(\left|X - \mu\right| \geq 3\sigma\right) \leq \left(1/3\right)^2 = 0.1111$$

The probability of the rate being outside 2 or 3 standard deviations of the mean is much smaller with a known normal distribution than when the distribution is unknown and we are relying on Chebyshev's inequality.

11 No. First the conclusion on the limit of zero is wrong; second, the support cited for drawing the conclusion (i.e., the central limit theorem) is not relevant in this context.

12 In many instances, the distribution that describes the underlying population is not normal or the distribution is not known. The central limit theorem states that if the sample size is large, regardless of the shape of the underlying population, the distribution of the sample mean is approximately normal. Therefore, even in these instances, we can still construct confidence intervals (and conduct tests of inference) as long as the sample size is large (generally $n \geq 30$).

13 The statement makes the following mistakes:

- Given the conditions in the statement, the distribution of $\bar{X}$ will be approximately normal only for large sample sizes.
- The statement omits the important element of the central limit theorem that the distribution of $\bar{X}$ will have mean μ.

14 A is correct. The discrepancy arises from sampling error. Sampling error exists whenever one fails to observe every element of the population, because a sample statistic can vary from sample to sample. As stated in the reading, the sample mean is an unbiased estimator, a consistent estimator, and an efficient estimator of the population mean. Although the sample mean is an unbiased estimator of the population mean—the expected value of the sample mean equals the population mean—because of sampling error, we do not expect the sample mean to exactly equal the population mean in any one sample we may take.

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15 No, we cannot say that Alcorn Mutual Funds as a group is superior to competitors. Alcorn Mutual Funds' advertisement may easily mislead readers because the advertisement does not show the performance of all its funds. In particular, Alcorn Mutual Funds is engaging in sample selection bias by presenting the investment results from its best-performing funds only.

16 Spence may be guilty of data mining. He has used so many possible combinations of variables on so many stocks, it is not surprising that he found some instances in which a model worked. In fact, it would have been more surprising if he had not found any. To decide whether to use his model, you should do two things: First, ask that the model be tested on out-of-sample data—that is, data that were not used in building the model. The model may not be successful with out-of-sample data. Second, examine his model to make sure that the relationships in the model make economic sense, have a story, and have a future.

17 C is correct. Stratified random sampling involves dividing a population into subpopulations based on one or more classification criteria. Then, simple random samples are drawn from each subpopulation in sizes proportional to the relative size of each subpopulation. These samples are then pooled to form a stratified random sample.

18 B is correct. Given a population described by any probability distribution (normal or non-normal) with finite variance, the central limit theorem states that the sampling distribution of the sample mean will be approximately normal, with the mean approximately equal to the population mean, when the sample size is large.

19 B is correct. Taking the square root of the known population variance to determine the population standard deviation (σ) results in:

$$\sigma = \sqrt{2.45} = 1.565$$

The formula for the standard error of the sample mean (σ_X), based on a known sample size (n), is:

$$\sigma_X = \frac{\sigma}{\sqrt{n}}$$

Therefore,

$$\sigma_X = \frac{1.565}{\sqrt{40}} = 0.247$$

20 B is correct. An unbiased estimator is one for which the expected value equals the parameter it is intended to estimate.

21 A is correct. A consistent estimator is one for which the probability of estimates close to the value of the population parameter increases as sample size increases. More specifically, a consistent estimator's sampling distribution becomes concentrated on the value of the parameter it is intended to estimate as the sample size approaches infinity.

22 A is correct. As the degree of confidence increases (e.g., from 95% to 99%), a given confidence interval will become wider. A confidence interval is a range for which one can assert with a given probability $1 - \alpha$, called the degree of confidence, that it will contain the parameter it is intended to estimate.

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23 A is correct. A standard normal distribution has tails that approach zero faster than the t-distribution. As degrees of freedom increase, the tails of the t-distribution become less fat and the t-distribution begins to look more like a standard normal distribution. But as degrees of freedom decrease, the tails of the t-distribution become fatter.

24 B is correct. The confidence interval is calculated using the following equation:

$$\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

Sample standard deviation (s) = $\sqrt{245.55}$ = 15.670.

For a sample size of 17, degrees of freedom equal 16, so $t_{0.05}$ = 1.746.

The confidence interval is calculated as

$$116.23 \pm 1.746\frac{15.67}{\sqrt{17}} = 116.23 \pm 6.6357$$

Therefore, the interval spans 109.5943 to 122.8656, meaning its width is equal to approximately 13.271. (This interval can be alternatively calculated as 6.6357 × 2).

25 A is correct. To solve, use the structure of Confidence interval = Point estimate ± Reliability factor × Standard error, which, for a normally distributed population with known variance, is represented by the following formula:

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

For a 99% confidence interval, use $z_{0.005}$ = 2.58.

Also, $\sigma = \sqrt{529}$ = 23.

Therefore, the lower limit = $31 - 2.58\frac{23}{\sqrt{65}} = 23.6398$.

26 B is correct. All else being equal, as the sample size increases, the standard error of the sample mean decreases and the width of the confidence interval also decreases.

27 B is correct. A report that uses a current list of stocks does not account for firms that failed, merged, or otherwise disappeared from the public equity market in previous years. As a consequence, the report is biased. This type of bias is known as survivorship bias.

28 B is correct. An out-of-sample test is used to investigate the presence of data-mining bias. Such a test uses a sample that does not overlap the time period of the sample on which a variable, strategy, or model was developed.

29 A is correct. A short time series is likely to give period-specific results that may not reflect a longer time period.

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PRACTICE PROBLEMS

1 Which of the following statements about hypothesis testing is correct?

A The null hypothesis is the condition a researcher hopes to support.

B The alternative hypothesis is the proposition considered true without conclusive evidence to the contrary.

C The alternative hypothesis exhausts all potential parameter values not accounted for by the null hypothesis.

2 Identify the appropriate test statistic or statistics for conducting the following hypothesis tests. (Clearly identify the test statistic and, if applicable, the number of degrees of freedom. For example, "We conduct the test using an *x*-statistic with *y* degrees of freedom.")

A H_0: μ = 0 versus H_a: μ ≠ 0, where μ is the mean of a normally distributed population with unknown variance. The test is based on a sample of 15 observations.

B H_0: μ = 0 versus H_a: μ ≠ 0, where μ is the mean of a normally distributed population with unknown variance. The test is based on a sample of 40 observations.

C H_0: μ ≤ 0 versus H_a: μ > 0, where μ is the mean of a normally distributed population with known variance σ^2. The sample size is 45.

D H_0: σ^2 = 200 versus H_a: σ^2 ≠ 200, where σ^2 is the variance of a normally distributed population. The sample size is 50.

E $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 \neq \sigma_2^2$, where σ_1^2 is the variance of one normally distributed population and σ_2^2 is the variance of a second normally distributed population. The test is based on two independent random samples.

F H_0: (Population mean 1) – (Population mean 2) = 0 versus H_a: (Population mean 1) – (Population mean 2) ≠ 0, where the samples are drawn from normally distributed populations with unknown variances. The observations in the two samples are correlated.

G H_0: (Population mean 1) – (Population mean 2) = 0 versus H_a: (Population mean 1) – (Population mean 2) ≠ 0, where the samples are drawn from normally distributed populations with unknown but assumed equal variances. The observations in the two samples (of size 25 and 30, respectively) are independent.

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3 For each of the following hypothesis tests concerning the population mean, μ, state the rejection point condition or conditions for the test statistic (e.g., t > 1.25); n denotes sample size.

A H_0: μ = 10 versus H_a: μ ≠ 10, using a t-test with n = 26 and α = 0.05

B H_0: μ = 10 versus H_a: μ ≠ 10, using a t-test with n = 40 and α = 0.01

C H_0: μ ≤ 10 versus H_a: μ > 10, using a t-test with n = 40 and α = 0.01

D H_0: μ ≤ 10 versus H_a: μ > 10, using a t-test with n = 21 and α = 0.05

E H_0: μ ≥ 10 versus H_a: μ < 10, using a t-test with n = 19 and α = 0.10

F H_0: μ ≥ 10 versus H_a: μ < 10, using a t-test with n = 50 and α = 0.05

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4 For each of the following hypothesis tests concerning the population mean, μ, state the rejection point condition or conditions for the test statistic (e.g., $z > 1.25$); n denotes sample size.

A H_0: μ = 10 versus H_a: μ ≠ 10, using a z-test with n = 50 and α = 0.01

B H_0: μ = 10 versus H_a: μ ≠ 10, using a z-test with n = 50 and α = 0.05

C H_0: μ = 10 versus H_a: μ ≠ 10, using a z-test with n = 50 and α = 0.10

D H_0: μ ≤ 10 versus H_a: μ > 10, using a z-test with n = 50 and α = 0.05

5 Willco is a manufacturer in a mature cyclical industry. During the most recent industry cycle, its net income averaged $30 million per year with a standard deviation of $10 million ($n$ = 6 observations). Management claims that Willco's performance during the most recent cycle results from new approaches and that we can dismiss profitability expectations based on its average or normalized earnings of $24 million per year in prior cycles.

A With μ as the population value of mean annual net income, formulate null and alternative hypotheses consistent with testing Willco management's claim.

B Assuming that Willco's net income is at least approximately normally distributed, identify the appropriate test statistic.

C Identify the rejection point or points at the 0.05 level of significance for the hypothesis tested in Part A.

D Determine whether or not to reject the null hypothesis at the 0.05 significance level.

The following information relates to Questions 6–7

Performance in Forecasting Quarterly Earnings per Share

	Number of Forecasts	Mean Forecast Error (Predicted – Actual)	Standard Deviations of Forecast Errors
Analyst A	101	0.05	0.10
Analyst B	121	0.02	0.09

6 Investment analysts often use earnings per share (EPS) forecasts. One test of forecasting quality is the zero-mean test, which states that optimal forecasts should have a mean forecasting error of 0. (Forecasting error = Predicted value of variable − Actual value of variable.)

You have collected data (shown in the table above) for two analysts who cover two different industries: Analyst A covers the telecom industry; Analyst B covers automotive parts and suppliers.

A With μ as the population mean forecasting error, formulate null and alternative hypotheses for a zero-mean test of forecasting quality.

B For Analyst A, using both a t-test and a z-test, determine whether to reject the null at the 0.05 and 0.01 levels of significance.

C For Analyst B, using both a t-test and a z-test, determine whether to reject the null at the 0.05 and 0.01 levels of significance.

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7 Reviewing the EPS forecasting performance data for Analysts A and B, you want to investigate whether the larger average forecast errors of Analyst A are due to chance or to a higher underlying mean value for Analyst A. Assume that the forecast errors of both analysts are normally distributed and that the samples are independent.

A Formulate null and alternative hypotheses consistent with determining whether the population mean value of Analyst A's forecast errors (μ_1) are larger than Analyst B's (μ_2).

B Identify the test statistic for conducting a test of the null hypothesis formulated in Part A.

C Identify the rejection point or points for the hypothesis tested in Part A, at the 0.05 level of significance.

D Determine whether or not to reject the null hypothesis at the 0.05 level of significance.

---

8 The table below gives data on the monthly returns on the S&P 500 and small-cap stocks for a forty-year period and provides statistics relating to their mean differences. Furthermore, the entire sample period is split into two subperiods of 20 years each and the returns data for these subperiods is also given in the table.

Measure	S&P 500 Return (%)	Small-Cap Stock Return (%)	Differences (S&P 500–Small-Cap Stock)
Entire sample period, 480 months			
Mean	1.0542	1.3117	–0.258
Standard deviation	4.2185	5.9570	3.752
First subperiod, 240 months			
Mean	0.6345	1.2741	–0.640
Standard deviation	4.0807	6.5829	4.096
Second subperiod, 240 months			
Mean	1.4739	1.3492	0.125
Standard deviation	4.3197	5.2709	3.339

Let μ_d stand for the population mean value of difference between S&P 500 returns and small-cap stock returns. Use a significance level of 0.05 and suppose that mean differences are approximately normally distributed.

A Formulate null and alternative hypotheses consistent with testing whether any difference exists between the mean returns on the S&P 500 and small-cap stocks.

B Determine whether or not to reject the null hypothesis at the 0.05 significance level for the entire sample period.

C Determine whether or not to reject the null hypothesis at the 0.05 significance level for the first subperiod.

D Determine whether or not to reject the null hypothesis at the 0.05 significance level for the second subperiod.

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9 During a 10-year period, the standard deviation of annual returns on a portfolio you are analyzing was 15 percent a year. You want to see whether this record is sufficient evidence to support the conclusion that the portfolio's underlying variance of return was less than 400, the return variance of the portfolio's benchmark.

A Formulate null and alternative hypotheses consistent with the verbal description of your objective.

B Identify the test statistic for conducting a test of the hypotheses in Part A.

C Identify the rejection point or points at the 0.05 significance level for the hypothesis tested in Part A.

D Determine whether the null hypothesis is rejected or not rejected at the 0.05 level of significance.

10 You are investigating whether the population variance of returns on the S&P 500/BARRA Growth Index changed subsequent to the October 1987 market crash. You gather the following data for 120 months of returns before October 1987 and for 120 months of returns after October 1987. You have specified a 0.05 level of significance.

Time Period	*n*	Mean Monthly Return (%)	Variance of Returns
Before October 1987	120	1.416	22.367
After October 1987	120	1.436	15.795

A Formulate null and alternative hypotheses consistent with the verbal description of the research goal.

B Identify the test statistic for conducting a test of the hypotheses in Part A.

C Determine whether or not to reject the null hypothesis at the 0.05 level of significance. (Use the *F*-tables in the back of this volume.)

11 The following table shows the sample correlations between the monthly returns for four different mutual funds and the S&P 500. The correlations are based on 36 monthly observations. The funds are as follows:

Fund 1	Large-cap fund
Fund 2	Mid-cap fund
Fund 3	Large-cap value fund
Fund 4	Emerging markets fund
S&P 500	US domestic stock index

	Fund 1	Fund 2	Fund 3	Fund 4	S&P 500
Fund 1	1				
Fund 2	0.9231	1			
Fund 3	0.4771	0.4156	1		
Fund 4	0.7111	0.7238	0.3102	1	
S&P 500	0.8277	0.8223	0.5791	0.7515	1

Test the null hypothesis that each of these correlations, individually, is equal to zero against the alternative hypothesis that it is not equal to zero. Use a 5 percent significance level.

12 In the step "stating a decision rule" in testing a hypothesis, which of the following elements must be specified?

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A Critical value

B Power of a test

C Value of a test statistic

13 Which of the following statements is correct with respect to the null hypothesis?

A It is considered to be true unless the sample provides evidence showing it is false.

B It can be stated as "not equal to" provided the alternative hypothesis is stated as "equal to."

C In a two-tailed test, it is rejected when evidence supports equality between the hypothesized value and population parameter.

14 An analyst is examining a large sample with an unknown population variance. To test the hypothesis that the historical average return on an index is less than or equal to 6%, which of the following is the *most* appropriate test?

A One-tailed z-test

B Two-tailed z-test

C One-tailed F-test

15 A hypothesis test for a normally-distributed population at a 0.05 significance level implies a:

A 95% probability of rejecting a true null hypothesis.

B 95% probability of a Type I error for a two-tailed test.

C 5% critical value rejection region in a tail of the distribution for a one-tailed test.

16 Which of the following statements regarding a one-tailed hypothesis test is correct?

A The rejection region increases in size as the level of significance becomes smaller.

B A one-tailed test more strongly reflects the beliefs of the researcher than a two-tailed test.

C The absolute value of the rejection point is larger than that of a two-tailed test at the same level of significance.

17 The value of a test statistic is *best* described as the basis for deciding whether to:

A reject the null hypothesis.

B accept the null hypothesis.

C reject the alternative hypothesis.

18 Which of the following is a Type I error?

A Rejecting a true null hypothesis

B Rejecting a false null hypothesis

C Failing to reject a false null hypothesis

19 A Type II error is *best* described as:

A rejecting a true null hypothesis.

B failing to reject a false null hypothesis.

C failing to reject a false alternative hypothesis.

20 The level of significance of a hypothesis test is *best* used to:

A calculate the test statistic.

B define the test's rejection points.

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C specify the probability of a Type II error.

21 You are interested in whether excess risk-adjusted return (alpha) is correlated with mutual fund expense ratios for US large-cap growth funds. The following table presents the sample.

Mutual Fund	1	2	3	4	5	6	7	8	9
Alpha (*X*)	−0.52	−0.13	−0.60	−1.01	−0.26	−0.89	−0.42	−0.23	−0.60
Expense Ratio (*Y*)	1.34	0.92	1.02	1.45	1.35	0.50	1.00	1.50	1.45

A Formulate null and alternative hypotheses consistent with the verbal description of the research goal.

B Identify the test statistic for conducting a test of the hypotheses in Part A.

C Justify your selection in Part B.

D Determine whether or not to reject the null hypothesis at the 0.05 level of significance.

22 All else equal, is specifying a smaller significance level in a hypothesis test likely to increase the probability of a:

	Type I error?	Type II error?
A	No	No
B	No	Yes
C	Yes	No

23 The probability of correctly rejecting the null hypothesis is the:

A *p*-value.

B power of a test.

C level of significance.

24 The power of a hypothesis test is:

A equivalent to the level of significance.

B the probability of not making a Type II error.

C unchanged by increasing a small sample size.

25 When making a decision in investments involving a statistically significant result, the:

A economic result should be presumed meaningful.

B statistical result should take priority over economic considerations.

C economic logic for the future relevance of the result should be further explored.

26 An analyst tests the profitability of a trading strategy with the null hypothesis being that the average abnormal return before trading costs equals zero. The calculated *t*-statistic is 2.802, with critical values of ± 2.756 at significance level $\alpha = 0.01$. After considering trading costs, the strategy's return is near zero. The results are *most likely*:

A statistically but not economically significant.

B economically but not statistically significant.

C neither statistically nor economically significant.

27 Which of the following statements is correct with respect to the *p*-value?

A It is a less precise measure of test evidence than rejection points.

B It is the largest level of significance at which the null hypothesis is rejected.

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C It can be compared directly with the level of significance in reaching test conclusions.

28 Which of the following represents a correct statement about the *p*-value?

A The *p*-value offers less precise information than does the rejection points approach.

B A larger *p*-value provides stronger evidence in support of the alternative hypothesis.

C A *p*-value less than the specified level of significance leads to rejection of the null hypothesis.

29 Which of the following statements on *p*-value is correct?

A The *p*-value is the smallest level of significance at which H_0 can be rejected.

B The *p*-value indicates the probability of making a Type II error.

C The lower the p-value, the weaker the evidence for rejecting the H_0.

30 The following table shows the significance level (α) and the *p*-value for three hypothesis tests.

	α	*p*-value
Test 1	0.05	0.10
Test 2	0.10	0.08
Test 3	0.10	0.05

The evidence for rejecting H_0 is strongest for:

A Test 1.

B Test 2.

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C Test 3.

31 Which of the following tests of a hypothesis concerning the population mean is *most* appropriate?

A A *z*-test if the population variance is unknown and the sample is small

B A *z*-test if the population is normally distributed with a known variance

C A *t*-test if the population is non-normally distributed with unknown variance and a small sample

32 For a small sample with unknown variance, which of the following tests of a hypothesis concerning the population mean is most appropriate?

A A *t*-test if the population is normally distributed

B A *t*-test if the population is non-normally distributed

C A *z*-test regardless of the normality of the population distribution

33 For a small sample from a normally distributed population with unknown variance, the *most* appropriate test statistic for the mean is the:

A *z*-statistic.

B *t*-statistic.

C χ^2 statistic.

34 An investment consultant conducts two independent random samples of 5-year performance data for US and European absolute return hedge funds. Noting a 50 basis point return advantage for US managers, the consultant decides to test whether the two means are statistically different from one another at a 0.05 level of significance. The two populations are assumed to be normally distributed with unknown but equal variances. Results of the hypothesis test are contained in the tables below.

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	Sample Size	Mean Return %	Standard Deviation
US Managers	50	4.7	5.4
European Managers	50	4.2	4.8

Null and Alternative Hypotheses	H_0: $\mu_{US} - \mu_E = 0$; H_a: $\mu_{US} - \mu_E \neq 0$
Test Statistic	0.4893
Critical Value Rejection Points	±1.984

μ_{US} is the mean return for US funds and μ_E is the mean return for European funds.

The results of the hypothesis test indicate that the:

A null hypothesis is not rejected.

B alternative hypothesis is statistically confirmed.

C difference in mean returns is statistically different from zero.

35 A pooled estimator is used when testing a hypothesis concerning the:

A equality of the variances of two normally distributed populations.

B difference between the means of two at least approximately normally distributed populations with unknown but assumed equal variances.

C difference between the means of two at least approximately normally distributed populations with unknown and assumed unequal variances.

36 When evaluating mean differences between two dependent samples, the *most* appropriate test is a:

A chi-square test.

B paired comparisons test.

C z-test.

37 A fund manager reported a 2% mean quarterly return over the past ten years for its entire base of 250 client accounts that all follow the same investment strategy. A consultant employing the manager for 45 client accounts notes that their mean quarterly returns were 0.25% less over the same period. The consultant tests the hypothesis that the return disparity between the returns of his clients and the reported returns of the fund manager's 250 client accounts are significantly different from zero.

Assuming normally distributed populations with unknown population variances, the *most* appropriate test statistic is:

A a paired comparisons t-test.

B a t-test of the difference between the two population means.

C an approximate t-test of mean differences between the two populations.

38 A chi-square test is *most* appropriate for tests concerning:

A a single variance.

B differences between two population means with variances assumed to be equal.

C differences between two population means with variances assumed to not be equal.

39 Which of the following should be used to test the difference between the variances of two normally distributed populations?

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- **A** *t*-test
- **B** *F*-test
- **C** Paired comparisons test

40 Jill Batten is analyzing how the returns on the stock of Stellar Energy Corp. are related with the previous month's percent change in the US Consumer Price Index for Energy (CPIENG). Based on 248 observations, she has computed the sample correlation between the Stellar and CPIENG variables to be −0.1452. She also wants to determine whether the sample correlation is statistically significant. The critical value for the test statistic at the 0.05 level of significance is approximately 1.96. Batten should conclude that the statistical relationship between Stellar and CPIENG is:

- **A** significant, because the calculated test statistic has a lower absolute value than the critical value for the test statistic.
- **B** significant, because the calculated test statistic has a higher absolute value than the critical value for the test statistic.
- **C** not significant, because the calculated test statistic has a higher absolute value than the critical value for the test statistic.

41 In which of the following situations would a non-parametric test of a hypothesis *most likely* be used?

- **A** The sample data are ranked according to magnitude.
- **B** The sample data come from a normally distributed population.
- **C** The test validity depends on many assumptions about the nature of the population.

42 An analyst is examining the monthly returns for two funds over one year. Both funds' returns are non-normally distributed. To test whether the mean return of one fund is greater than the mean return of the other fund, the analyst can use:

- **A** a parametric test only.
- **B** a nonparametric test only.
- **C** both parametric and nonparametric tests.

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SOLUTIONS

1 C is correct. Together, the null and alternative hypotheses account for all possible values of the parameter. Any possible values of the parameter not covered by the null must be covered by the alternative hypothesis (e.g., H_0: θ ≤ 5 versus H_a: θ > 5).

2 **A** The appropriate test statistic is a *t*-statistic with $n - 1 = 15 - 1 = 14$ degrees of freedom. A *t*-statistic is theoretically correct when the sample comes from a normally distributed population with unknown variance. When the sample size is also small, there is no practical alternative.

B The appropriate test statistic is a *t*-statistic with 40 − 1 = 39 degrees of freedom. A *t*-statistic is theoretically correct when the sample comes from a normally distributed population with unknown variance. When the sample size is large (generally, 30 or more is a "large" sample), it is also possible to use a *z*-statistic, whether the population is normally distributed or not. A test based on a *t*-statistic is more conservative than a *z*-statistic test.

C The appropriate test statistic is a *z*-statistic because the sample comes from a normally distributed population with known variance. (The known population standard deviation is used to compute the standard error of the mean using Equation 2 in the text.)

D The appropriate test statistic is chi-square (χ^2) with 50 − 1 = 49 degrees of freedom.

E The appropriate test statistic is the *F*-statistic (the ratio of the sample variances).

F The appropriate test statistic is a *t*-statistic for a paired observations test (a paired comparisons test), because the samples are correlated.

G The appropriate test statistic is a *t* –statistic using a pooled estimate of the population variance. The *t*-statistic has 25 + 30 − 2 = 53 degrees of freedom. This statistic is appropriate because the populations are normally distributed with unknown variances; because the variances are assumed equal, the observations can be pooled to estimate the common variance. The requirement of independent samples for using this statistic has been met.

3 **A** With degrees of freedom (df) $n - 1 = 26 - 1 = 25$, the rejection point conditions for this two-sided test are $t > 2.060$ and $t < -2.060$. Because the significance level is 0.05, 0.05/2 = 0.025 of the probability is in each tail. The tables give one-sided (one-tailed) probabilities, so we used the 0.025 column. Read across df = 25 to the α = 0.025 column to find 2.060, the rejection point for the right tail. By symmetry, −2.060 is the rejection point for the left tail.

B With df = 39, the rejection point conditions for this two-sided test are $t > 2.708$ and $t < -2.708$. This is a two-sided test, so we use the 0.01/2 = 0.005 column. Read across df = 39 to the α = 0.005 column to find 2.708, the rejection point for the right tail. By symmetry, −2.708 is the rejection point for the left tail.

C With df = 39, the rejection point condition for this one-sided test is $t > 2.426$. Read across df = 39 to the α = 0.01 column to find 2.426, the rejection point for the right tail. Because we have a "greater than" alternative, we are concerned with only the right tail.

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D With df = 20, the rejection point condition for this one-sided test is $t > 1.725$. Read across df = 20 to the $\alpha = 0.05$ column to find 1.725, the rejection point for the right tail. Because we have a "greater than" alternative, we are concerned with only the right tail.

E With df = 18, the rejection point condition for this one-sided test is $t < -1.330$. Read across df = 18 to the $\alpha = 0.10$ column to find 1.330, the rejection point for the right tail. By symmetry, the rejection point for the left tail is −1.330.

F With df = 49, the rejection point condition for this one-sided test is $t < -1.677$. Read across df = 49 to the $\alpha = 0.05$ column to find 1.677, the rejection point for the right tail. By symmetry, the rejection point for the left tail is −1.677.

4 Recall that with a z-test (in contrast to the t-test), we do not employ degrees of freedom. The standard normal distribution is a single distribution applicable to all z-tests. You should refer to "Rejection Points for a z-Test" in Section 3.1 to answer these questions.

A This is a two-sided test at a 0.01 significance level. In Part C of "Rejection Points for a z-Test," we find that the rejection point conditions are $z > 2.575$ and $z < -2.575$.

B This is a two-sided test at a 0.05 significance level. In Part B of "Rejection Points for a z-Test," we find that the rejection point conditions are $z > 1.96$ and $z < -1.96$.

C This is a two-sided test at a 0.10 significance level. In Part A of "Rejection Points for a z-Test," we find that the rejection point conditions are $z > 1.645$ and $z < -1.645$.

D This is a one-sided test at a 0.05 significance level. In Part B of "Rejection Points for a z-Test," we find that the rejection point condition for a test with a "greater than" alternative hypothesis is $z > 1.645$.

5 **A** As stated in the text, we often set up the "hoped for" or "suspected" condition as the alternative hypothesis. Here, that condition is that the population value of Willco's mean annual net income exceeds \$24 million. Thus we have H_0: $\mu \leq 24$ versus H_a: $\mu > 24$.

B Given that net income is normally distributed with unknown variance, the appropriate test statistic is t with $n - 1 = 6 - 1 = 5$ degrees of freedom.

C In the t-distribution table in the back of the book, in the row for df = 5 under $\alpha = 0.05$, we read the rejection point (critical value) of 2.015. We will reject the null if $t > 2.015$.

D The t-test is given by Equation 4:

$$t_5 = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{30 - 24}{10/\sqrt{6}} = \frac{6}{4.082483} = 1.469694$$

or 1.47. Because 1.47 does not exceed 2.015, we do not reject the null hypothesis. The difference between the sample mean of \$30 million and the hypothesized value of \$24 million under the null is not statistically significant.

6 **A** H_0: $\mu = 0$ versus H_a: $\mu \neq 0$.

B The t-test is based on $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}}$ with $n - 1 = 101 - 1 = 100$ degrees of freedom. At the 0.05 significance level, we reject the null if $t > 1.984$ or if $t < -1.984$. At the 0.01 significance level, we reject the null if $t > 2.626$ or if $t <$

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−2.626. For Analyst A, we have $t = (0.05 - 0)\Big/\left(0.10/\sqrt{101}\right)$ = 0.05/0.00995 = 5.024938 or 5.025. We clearly reject the null hypothesis at both the 0.05 and 0.01 levels.

The calculation of the z-statistic with unknown variance, as in this case, is the same as the calculation of the t-statistic. The rejection point conditions for a two-tailed test are as follows: $z > 1.96$ and $z < -1.96$ at the 0.05 level; and $z > 2.575$ and $z < -2.575$ at the 0.01 level. Note that the z-test is a less conservative test than the t-test, so when the z-test is used, the null is easier to reject. Because $z = 5.025$ is greater than 2.575, we reject the null at the 0.01 level; we also reject the null at the 0.05 level.

In summary, Analyst A's EPS forecasts appear to be biased upward—they tend to be too high.

C For Analyst B, the t-test is based on t with 121 − 1 = 120 degrees of freedom. At the 0.05 significance level, we reject the null if $t > 1.980$ or if $t < -1.980$. At the 0.01 significance level, we reject the null if $t > 2.617$ or if $t < -2.617$. We calculate $t = (0.02 - 0)\Big/\left(0.09/\sqrt{121}\right)$ = 0.02/0.008182 = 2.444444 or 2.44. Because 2.44 > 1.98, we reject the null at the 0.05 level. However, 2.44 is not larger than 2.617, so we do not reject the null at the 0.01 level.

For a z-test, the rejection point conditions are the same as given in Part B, and we come to the same conclusions as with the t-test. Because 2.44 > 1.96, we reject the null at the 0.05 significance level; however, because 2.44 is not greater than 2.575, we do not reject the null at the 0.01 level.

The mean forecast error of Analyst B is only \$0.02; but because the test is based on a large number of observations, it is sufficient evidence to reject the null of mean zero forecast errors at the 0.05 level.

7 **A** Stating the suspected condition as the alternative hypothesis, we have

$$H_0: \mu_1 - \mu_2 \le 0 \text{ versus } H_a: \mu_1 - \mu_2 > 0$$

where

μ_1 = the population mean value of Analyst A's forecast errors
μ_2 = the population mean value of Analyst B's forecast errors

B We have two normally distributed populations with unknown variances. Based on the samples, it is reasonable to assume that the population variances are equal. The samples are assumed to be independent; this assumption is reasonable because the analysts cover quite different industries. The appropriate test statistic is t using a pooled estimate of the common variance. The number of degrees of freedom is

$$n_1 + n_2 - 2 = 101 + 121 - 2 = 222 - 2 = 220.$$

C For df = 200 (the closest value to 220), the rejection point for a one-sided test at the 0.05 significance level is 1.653.

D We first calculate the pooled estimate of variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(101 - 1)(0.10)^2 + (121 - 1)(0.09)^2}{101 + 121 - 2}$$

$$= \frac{1.972}{220} = 0.008964$$

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Then

$$t = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\left(\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}\right)^{1/2}} = \frac{(0.05 - 0.02) - 0}{\left(\dfrac{0.008964}{101} + \dfrac{0.008964}{121}\right)^{1/2}}$$

$$= \frac{0.03}{0.01276} = 2.351018$$

or 2.35. Because 2.35 > 1.653, we reject the null hypothesis in favor of the alternative hypothesis that the population mean forecast error of Analyst A is greater than that of Analyst B.

8 **A** We test H_0: $\mu_d = 0$ versus H_a: $\mu_d \neq 0$.

B This is a paired comparisons t-test with $n - 1 = 480 - 1 = 479$ degrees of freedom. At the 0.05 significance level, we reject the null hypothesis if either $t > 1.96$ or $t < -1.96$. We use df = ∞ in the t-distribution table under α = 0.025 because we have a very large sample and a two-sided test.

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}} = \frac{-0.258 - 0}{3.752/\sqrt{480}} = \frac{-0.258}{0.171255} = -1.506529 \text{ or } -1.51$$

At the 0.05 significance level, because neither rejection point condition is met, we do not reject the null hypothesis that the mean difference between the returns on the S&P 500 and small-cap stocks during the entire sample period was 0.

C This t-test now has $n - 1 = 240 - 1 = 239$ degrees of freedom. At the 0.05 significance level, we reject the null hypothesis if either $t > 1.972$ or $t < -1.972$, using df = 200 in the t-distribution tables.

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}} = \frac{-0.640 - 0}{4.096/\sqrt{240}} = \frac{-0.640}{0.264396} = -2.420615 \text{ or } -2.42$$

Because −2.42 < −1.972, we reject the null hypothesis at the 0.05 significance level. During this subperiod, small-cap stocks significantly outperformed the S&P 500.

D This t-test has $n - 1 = 240 - 1 = 239$ degrees of freedom. At the 0.05 significance level, we reject the null hypothesis if either $t > 1.972$ or $t < -1.972$, using df = 200 in the t-distribution tables.

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}} = \frac{0.125 - 0}{3.339/\sqrt{240}} = \frac{0.125}{0.215532} = 0.579962 \text{ or } 0.58$$

At the 0.05 significance level, because neither rejection point condition is met, we do not reject the null hypothesis that for the second subperiod, the mean difference between the returns on the S&P 500 and small-cap stocks was zero.

9 **A** We have a "less than" alternative hypothesis, where σ^2 is the variance of return on the portfolio. The hypotheses are H_0: $\sigma^2 \geq 400$ versus H_a: $\sigma^2 < 400$, where 400 is the hypothesized value of variance, σ_0^2.

B The test statistic is chi-square with 10 − 1 = 9 degrees of freedom.

C The rejection point is found across degrees of freedom of 9, under the 0.95 column (95 percent of probability above the value). It is 3.325. We will reject the null hypothesis if we find that $\chi^2 < 3.325$.

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D The test statistic is calculated as

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{9 \times 15^2}{400} = \frac{2{,}025}{400} = 5.0625 \text{ or } 5.06$$

Because 5.06 is not less than 3.325, we do not reject the null hypothesis.

10 **A** We have a "not equal to" alternative hypothesis:

$$H_0: \sigma^2_{\text{Before}} = \sigma^2_{\text{After}} \text{ versus } H_a: \sigma^2_{\text{Before}} \neq \sigma^2_{\text{After}}$$

B To test a null hypothesis of the equality of two variances, we use an *F*-test:

$$F = \frac{s_1^2}{s_2^2}$$

C The "before" sample variance is larger, so following a convention for calculating *F*-statistics, the "before" sample variance goes in the numerator. $F = 22.367/15.795 = 1.416$, with $120 - 1 = 119$ numerator and denominator degrees of freedom. Because this is a two-tailed test, we use *F*-tables for the 0.025 level (df = 0.05/2). Using the tables in the back of the volume, the closest value to 119 is 120 degrees of freedom. At the 0.05 level, the rejection point is 1.43. (Using the Insert/Function/Statistical feature on a Microsoft Excel spreadsheet, we would find FINV(0.025, 119, 119) = 1.434859 as the critical *F*-value.) Because 1.416 is not greater than 1.43, we do not reject the null hypothesis that the "before" and "after" variances are equal.

11 The critical *t*-value for $n - 2 = 34$ df, using a 5 percent significance level and a two-tailed test, is 2.032. First, take the smallest correlation in the table, the correlation between Fund 3 and Fund 4, and see if it is significantly different from zero. Its calculated *t*-value is

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.3102\sqrt{36-2}}{\sqrt{1-0.3102^2}} = 1.903$$

This correlation is not significantly different from zero. If we take the next lowest correlation, between Fund 2 and Fund 3, this correlation of 0.4156 has a calculated *t*-value of 2.664. So this correlation is significantly different from zero at the 5 percent level of significance. All of the other correlations in the table (besides the 0.3102) are greater than 0.4156, so they too are significantly different from zero.

12 A is correct. The critical value in a decision rule is the rejection point for the test. It is the point with which the test statistic is compared to determine whether to reject the null hypothesis, which is part of the fourth step in hypothesis testing.

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13 A is correct. The null hypothesis is the hypothesis to be tested. The null hypothesis is considered to be true unless the evidence indicates that it is false, in which case the alternative hypothesis is accepted.

14 A is correct. If the population sampled has unknown variance and the sample is large, a *z*-test may be used. Hypotheses involving "greater than" or "less than" postulations are one-sided (one-tailed). In this situation, the null and alternative hypotheses are stated as H_0: $\mu \leq 6\%$ and H_a: $\mu > 6\%$, respectively. A one-tailed *t*-test is also acceptable in this case.

15 C is correct. For a one-tailed hypothesis test, there is a 5% critical value rejection region in one tail of the distribution.

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16 B is correct. One-tailed tests in which the alternative is "greater than" or "less than" represent the beliefs of the researcher more firmly than a "not equal to" alternative hypothesis.

17 A is correct. Calculated using a sample, a test statistic is a quantity whose value is the basis for deciding whether to reject the null hypothesis.

18 A is correct. The definition of a Type I error is when a true null hypothesis is rejected.

19 B is correct. A Type II error occurs when a false null hypothesis is not rejected.

20 B is correct. The level of significance is used to establish the rejection points of the hypothesis test.

21 **A** We have a "not equal to" alternative hypothesis:

$$H_0: \rho = 0 \text{ versus } H_a: \rho \neq 0$$

B We would use the nonparametric Spearman rank correlation coefficient to conduct the test.

C Mutual fund expense ratios are bounded from above and below, and in practice there is at least a lower bound on alpha (as any return cannot be less than −100 percent). These variables are markedly non-normally distributed, and the assumptions of a parametric test are not likely to be fulfilled. Thus a nonparametric test appears to be appropriate.

D The calculation of the Spearman rank correlation coefficient is given in the following table.

Mutual Fund	1	2	3	4	5	6	7	8	9
Alpha (X)	−0.52	−0.13	−0.60	−1.01	−0.26	−0.89	−0.42	−0.23	−0.60
Expense Ratio (Y)	1.34	0.92	1.02	1.45	1.35	0.50	1.00	1.50	1.45
X Rank	5	1	6.5	9	3	8	4	2	6.5
Y Rank	5	8	6	2.5	4	9	7	1	2.5
d_i	0	−7	0.5	6.5	−1	−1	−3	1	4
d_i^2	0	49	0.25	42.25	1	1	9	1	16

$$r_s = 1 - \frac{6\sum d_i^2}{n\left(n^2 - 1\right)} = 1 - \frac{6(119.50)}{9(81 - 1)} = 0.0042$$

We use Table 11 to tabulate the rejection points for a test on the Spearman rank correlation. Given a sample size of 9 in a two-tailed test at a 0.05 significance level, the upper-tail rejection point is 0.6833 (we use the 0.025 column). Thus we reject the null hypothesis if the Spearman rank correlation coefficient is less than −0.6833 or greater than 0.6833. Because r_S is equal to 0.0042, we do not reject the null hypothesis.

22 B is correct. Specifying a smaller significance level decreases the probability of a Type I error (rejecting a true null hypothesis), but increases the probability of a Type II error (not rejecting a false null hypothesis). As the level of significance decreases, the null hypothesis is less frequently rejected.

23 B is correct. The power of a test is the probability of rejecting the null hypothesis when it is false.

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24 B is correct. The power of a hypothesis test is the probability of correctly rejecting the null when it is false. Failing to reject the null when it is false is a Type II error. Thus, the power of a hypothesis test is the probability of not committing a Type II error.

25 C is correct. When a statistically significant result is also economically meaningful, one should further explore the logic of why the result might work in the future.

26 A is correct. The hypothesis is a two-tailed formulation. The t-statistic of 2.802 falls outside the critical rejection points of less than −2.756 and greater than 2.756, therefore the null hypothesis is rejected; the result is statistically significant. However, despite the statistical results, trying to profit on the strategy is not likely to be economically meaningful because the return is near zero after transaction costs

27 C is correct. When directly comparing the p-value with the level of significance, it can be used as an alternative to using rejection points to reach conclusions on hypothesis tests. If the p-value is smaller than the specified level of significance, the null hypothesis is rejected. Otherwise, the null hypothesis is not rejected.

28 C is correct. The p-value is the smallest level of significance at which the null hypothesis can be rejected for a given value of the test statistic. The null hypothesis is rejected when the p-value is less than the specified significance level.

29 A is correct. The p-value is the smallest level of significance (α) at which the null hypothesis can be rejected.

30 C is correct. The p-value is the smallest level of significance (α) at which the null hypothesis can be rejected. If the p-value is less than α, the null can be rejected. The smaller the p-value, the stronger the evidence is against the null hypothesis and in favor of the alternative hypothesis. Thus, the evidence for rejecting the null is strongest for Test 3.

31 B is correct. The z-test is theoretically the correct test to use in those limited cases when testing the population mean of a normally distributed population with known variance.

32 A is correct. A t-test is used if the sample is small and drawn from a normally or approximately normally distributed population.

33 B is correct. A t-statistic is the most appropriate for hypothesis tests of the population mean when the variance is unknown and the sample is small but the population is normally distributed.

34 A is correct. The t-statistic value of 0.4893 does not fall into the critical value rejection regions (≤ -1.984 or > 1.984). Instead it falls well within the acceptance region. Thus, H_0 cannot be rejected; the result is not statistically significant at the 0.05 level.

35 B is correct. The assumption that the variances are equal allows for the combining of both samples to obtain a pooled estimate of the common variance.

36 B is correct. A paired comparisons test is appropriate to test the mean differences of two samples believed to be dependent.

37 A is correct. The sample sizes for both the fund manager and the consultant's accounts consists of forty quarterly periods of returns. However, the consultant's client accounts are a subset of the fund manager's entire account base. As such, they are not independent samples. When samples are dependent, a paired comparisons test is appropriate to conduct tests of the differences in dependent items.

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38 A is correct. A chi-square test is used for tests concerning the variance of a single normally distributed population.

39 B is correct. An *F*-test is used to conduct tests concerning the difference between the variances of two normally distributed populations with random independent samples.

40 B is correct. The calculated test statistic is

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$= \frac{-0.1452\sqrt{248-2}}{\sqrt{1-(-0.1452)^2}} = -2.30177$$

Because the absolute value of $t = -2.30177$ is greater than 1.96, the correlation coefficient is statistically significant.

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41 A is correct. A non-parametric test is used when the data are given in ranks.

42 B is correct. There are only 12 (monthly) observations over the one year of the sample and thus the samples are small. Additionally, the funds' returns are non-normally distributed. Therefore, the samples do not meet the distributional assumptions for a parametric test. The Mann–Whitney U test (a nonparametric test) could be used to test the differences between population means.