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Automatic evaluation of UV and R_2 terms for beyond the Standard Model Lagrangians: A proof-of-principle☆

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ABSTRACT

The computation of renormalized one-loop amplitudes in quantum field theory requires not only the knowledge of the Lagrangian density and the corresponding Feynman rules, but also that of the ultraviolet counterterms. More in general, and depending also on the method used in the actual computation of the one-loop amplitudes, additional interactions might be needed. One example is that of the R_2 rational terms in the OPP method. In this paper, we argue that the determination of all elements necessary for loop computations in arbitrary models can be automated starting only from information on the Lagrangian at the tree-level. In particular, we show how the R_2 rational and ultraviolet counterterms for any renormalizable model can be computed with the help of a new package, which we name NLOCT and builds upon FeynRules and FeynArts. To show the potential of our approach, we calculate all additional rules that are needed to promote a Two Higgs Doublet Model Lagrangian to one-loop computations in QCD and electroweak couplings.

Program summary

Program title: FeynRules-2.1 with NLOCT

Catalogue identifier: AEDI_v2_1

Program summary URL: http://cpc.cs.qub.ac.uk/summaries/AEDI_v2_1.html

Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland

Licensing provisions: Standard CPC licence, http://cpc.cs.qub.ac.uk/licence/licence.html

No. of lines in distributed program, including test data, etc.: 58179

No. of bytes in distributed program, including test data, etc.: 506216

Distribution format: tar.gz

Programming language: Mathematica.

Computer: Platforms on which Mathematica is available.

Operating system: Operating systems on which Mathematica is available.

Classification: 11.1, 11.6.

Catalogue identifier of previous version: AEDI_v2_0

Journal reference of previous version: Comput. Phys. Comm. 185(2014)2250

External routines: FeynArts-3.7 or above

Does the new version supersede the previous version?: Yes

Nature of problem: The new version of FeynRules renormalizes and computes with the new NLOCT package the ultraviolet counterterms and the rational R_2 terms induced by one-loop amplitudes. The resulting vertices can be exported in the UFO format to event generators such as MadGraph5_aMC@NLO.

Solution method: First, the Lagrangian is renormalized in FeynRules and exported to FeynArts through the corresponding interface. Secondly, the NLOCT package computes the R_2 and *UV* vertices using the

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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amplitudes from FeynArts. Finally, the list of NLO vertices is loaded in FeynRules to be exported in the UFO format.

Reasons for new version: The new version allows the automated generation of any model in the UFO format with all the required ingredients to perform automatically one-loop computation.

Summary of revisions: This new version contains all the elements to perform the computation of the NLO ingredients: A function to perform the renormalization in FeynRules, the NLOCT package computes of the *UV* and R_2 vertices and the extended UFO interface adds them in the output.

Restrictions: The model should be renormalizable, i.e. the dimension of the operators should not exceed four, and written in the Feynman gauge.

Running time: The running time ranges from a few minutes for models like the SM to a few hours for more complicated models like the MSSM.

1. Introduction

The ability of to make accurate predictions for observables that can be compared with the corresponding measurements, has always been the key to test our understanding of the fundamental interactions. Perhaps even more now that the experimental confirmation of the Standard Model(SM) is more and more compelling and the search for new phenomena opens. Two strategies are the most commonly followed. The first is the testing of predictions of the SM by collecting precise measurements of observables possibly sensitive to new physics, an example being the campaign to determine the Higgs couplings to other SM particles. The second is the search of the effects from new particles in colliders, as now intensively done at the LHC. In this case also, the accurate knowledge of the properties, normalizations and shapes of the SM backgrounds turns out to be an essential ingredient. Furthermore, while not always needed for discovery, accurate predictions for BSM will be needed once excesses are found and the need to clarify the actual nature of the new physics will finally arise.

The need for precise predictions for hadron hadron collisions has motivated a large activity in the field of improving the techniques to compute NLO corrections, both in the QCD and electroweak sectors (see [1] for a recent review). For many years the bottleneck of reaching NLO accuracy has been that of computing loops. Recently, several techniques have been proposed to automatically deal with loop amplitude. A key ingredient for their evaluation is that any one-loop amplitude can be written as

$$A = \sum_i d_i \text{Box}_i + \sum_i c_i \text{Triangle}_i + \sum_i b_i \text{Bubble}_i + \sum_i a_i \text{Tadpole}_i + R, \tag{1.1}$$

where the Box, Triangle, Bubble and Tadpole are known scalar integrals and R is the rational term. The OPP technique [2] has provided a simple way to compute the scalar integrals coefficients further improved by the use of multiple cuts [3]. This method has been implemented in MadLoop [4] available in the MadGraph5_aMC@NLO [5] framework leading to a complete automated tool for NLO computation. So far only the SM model has been implemented despite that MadLoop is based on MadGraph5 [6] for which many BSM models are available. As a matter of fact, the evaluation of the loop corrections requires two extra ingredients that so far have been added by hand in the model. The first one is the counterterms introduced by the renormalization procedure to absorb all the UV divergences arising at the one-loop level. While the divergences can be extracted from the scalar integrals, any renormalization scheme with a non-trivial finite part in the counterterms requires a careful redefinition of the fields and of the independent parameters of the model and the resolution of the renormalization conditions. The second missing element depends on the actual method used to perform the tensor decomposition of the loop amplitudes. In the case of OPP, it is a part of the rational term. In d dimensions, any one-loop amplitude can be written as

$$\overline{A}(\overline{q}) = \frac{1}{(2\pi)^4} \int d^d\overline{q} \frac{\overline{N}(\overline{q})}{\overline{D}_0 \overline{D}_1 \cdots \overline{D}_{m-1}}, \tag{1.2}$$

with the propagator denominators given by $\overline{D}_i \equiv (\overline{q} + p_i)^2 - m_i^2$ and where m_i are the masses of the particles in the loop, q is the loop momentum and p_i are linear combinations of external momenta. All the quantities written with a bar live in d dimensions and can therefore be split in a four dimensional part x and a $d-4$ dimensional part $\tilde{x}$ as follow $\overline{x} \equiv x + \tilde{x}$. Rational terms are finite contributions generated by the part of the integrand linear in $d-4$. One then organizes the rational part in two terms, R_1 and R_2. The rational term R_1 is due to the $d-4$ component of the integrand denominators and can be computed as the four-dimensional piece but using a different set of scalar integrals [7]. The R_2 terms are defined as the finite part due to the $d-4$ component of the numerator

$$R_2 \equiv \lim_{\epsilon \to 0} \frac{1}{(2\pi)^4} \int d^d\overline{q} \frac{\tilde{N}(\tilde{q}, q, \epsilon)}{\overline{D}_0 \overline{D}_1 \ldots \overline{D}_{m-1}}, \tag{1.3}$$

where ϵ is defined by $d \equiv 4 - 2\epsilon$. We use here the 't Hooft–Veltman scheme [8] such that all the quantities in the loop, i.e. the loop momentum, the metric and the Dirac matrices live in d dimensions:

$$\overline{\eta}^{\overline{\mu}\,\overline{\nu}} \overline{\eta}_{\overline{\mu}\,\overline{\nu}} = d, \tag{1.4}$$

$$\overline{\gamma}^{\overline{\mu}} \overline{\gamma}_{\overline{\mu}} = d\,\mathbb{1}, \tag{1.5}$$

where $\mathbb{1}$ is the identity matrix in Dirac space. The external momenta and polarization vectors have only four dimensional components. The Dirac matrices in d dimensions $\overline{\gamma}_{\overline{\mu}}$ are chosen to anti-commute with γ_5 [9–11]. Therefore, the cyclic property of Dirac trace has to

be dropped to avoid algebraic inconsistency. The result of the evaluation of the integral in (1.3) is a set of process independent Feynman rules. As a consequence, they should only be computed once for each model. The R_2 term are the second missing ingredient as they had to be computed so far by hand for each model. The R_2 terms are known for the full SM [12,13] and for QCD corrections to the MSSM [14]. A package for the automatic computation of the R_2 terms for the SM has also been developed [15].

The purpose of this paper is to show that the procedure of determining the UV counterterms and the R_2 terms can be automated for any Lagrangian. The computation of the missing elements is done by three MATHEMATICA packages, FEYNRULES [16], NLOCT and FEYNARTS [17]. NLOCT is a completely new package, new functionalities have been added to FEYNRULES to renormalize models and to output the NLO vertices in the UFO format [18] while FEYNARTS has not been altered. The only requirement is that the model should be written in the Feynman gauge. At this stage, the package is restricted to renormalizable theories. Renormalizability is here understood strictly and not order by order like for effective field theories. Namely, the dimension of the operators in the Lagrangian should be equal to or lower than four. Although the R_2 terms are not always required, the UV counterterms are needed for any one-loop computation. Therefore, the automatically generated models can be used to provide the necessary one-loop ingredients for other NLO tools than MADGRAPH5_AMC@NLO like GOSAM [19] for example which is already using the UFO format. As an explicit example, we consider the Two Higgs Doublet Model (2DHM). The 2HDM is a simple but important extension of the SM since it provides a well defined model to search for extra scalar particles.

The paper is organized as follows. The second section focuses on the renormalization of the Lagrangian and introduces the renormalization conditions for the on-shell scheme. This scheme is easily extended to complex mass scheme to provide an appropriate treatment of the widths. The main advantage of those schemes is to avoid the evaluation of the loops on the external legs and it is used, for example, in MADLOOP to make the computation faster. The third section discusses the algorithm for the computation of the counterterms from the amplitudes. This section ends with the validation of the algorithm. The 2HDM is briefly introduced in Section 4 to fix the notation. The R_2 and UV counterterm vertices for the 2HDM are given in Sections 5 and 6 respectively. Finally, the conclusion is given in the last section.

2. Renormalization

2.1. The renormalization constants

In dimensional regularization UV-divergences appear as poles in $1/\epsilon$ where $d \equiv 4 - 2\epsilon$. In a renormalizable theory, they can be absorbed by a redefinition of the free parameters, of the fields and of the tadpoles

$$\begin{aligned} &x_0 \to x + \delta x, \\ &\phi_0 \to \left(1 + \frac{1}{2}\delta Z_{\phi\phi}\right)\phi + \sum_\chi \frac{1}{2}\delta Z_{\phi\chi}\chi, \\ &t_0^\phi \to t^\phi + \delta t_0^\phi, \end{aligned} \tag{2.6}$$

where x is an external parameter, ϕ and χ are physical fields with the same quantum numbers and t^ϕ is the tadpole for the field ϕ, i.e. the coefficient of the term linear in ϕ in the Lagrangian, the bare quantities are denoted by an additional zero subscript compared to the renormalized fields or parameters, the renormalization constants are preceded by a δ. For the fermions, each chirality is renormalized independently. The external parameters are independent parameters whose values should be fixed by experiments. On the contrary, internal parameters are functions of the external parameters. Internal parameters are also renormalized. However, their renormalization does not require the introduction of new renormalization constants and is fixed by their dependence on the external parameters. The same self renormalization constants $Z_{\phi\phi}$ are used for both the fields and their hermitian conjugates and not its conjugate as required by the complex mass scheme [20]. Their imaginary parts would otherwise disappear form the hermitian Lagrangian. For example, the kinetic term of a scalar has an imaginary part if

$$\left.\begin{aligned} \phi_0 &\to \left(1 + \frac{1}{2}\delta Z_{\phi\phi}\right)\phi \\ \phi_0^\dagger &\to \left(1 + \frac{1}{2}\delta Z_{\phi\phi}\right)\phi^\dagger \end{aligned}\right\} \Rightarrow \partial^\mu\phi_0\partial_\mu\phi_0^\dagger \to (1 + \delta Z_{\phi\phi})\partial^\mu\phi\partial_\mu\phi^\dagger \tag{2.7}$$

to absorb the imaginary part coming from the corresponding term of the two point one-loop amplitude. On the contrary, they would be no imaginary part if the conjugated field is renormalized with the conjugate of the renormalization constant, i.e.

$$\left.\begin{aligned} \phi_0 &\to \left(1 + \frac{1}{2}\delta Z_{\phi\phi}\right)\phi \\ \phi_0^\dagger &\to \left(1 + \frac{1}{2}\delta Z_{\phi\phi}^*\right)\phi^\dagger \end{aligned}\right\} \Rightarrow \partial^\mu\phi_0\partial_\mu\phi_0^\dagger \to (1 + \Re\delta Z_{\phi\phi})\partial^\mu\phi\partial_\mu\phi^\dagger. \tag{2.8}$$

In the on-shell scheme, those renormalization constants are real and therefore also identical for both the fields and their conjugates. Therefore the field renormalization are written as in Eq. (2.7) in both scheme. However, the renormalization constants are real in the on-shell scheme and they are complex in the complex mass scheme. In the complex mass scheme, the masses and their renormalization constants also become complex but their conjugates are renormalized again with the same constants and not their conjugates. The renormalization in the two schemes is formally the same up to the fact that the renormalization constants are real in the on-shell scheme and complex in the complex mass scheme. It should be noted that only the bare Lagrangian is hermitian in the complex mass scheme since the renormalization constants are complex in this scheme. The bare Lagrangian can also be split into the renormalized one depending only on the renormalized quantities and a counterterm Lagrangian linear in the renormalization constants,

$$\mathcal{L}_0 = \mathcal{L} + \delta\mathcal{L}. \tag{2.9}$$

Table 1
Options of `OnShellRenormalization`.

`QCDOnly`	If `True`, only the fields which transform under the strong gauge group, their masses and the external couplings related to the strong interaction are renormalized. The default is `False`.
`FlavorMixing`	If `True`, all the fields with the same quantum numbers are allowed to mix for the renormalization. If `False`, no mixing is introduced for the renormalization. If the value of this option is a list of pairs of fields (`ClassMembers` should be used if present while `ClassName` should be used otherwise), only the mixing between the two particles of each pair is included. The default is `True`.
`Only2Point`	If `True`, only the fields and masses are renormalized. The default is `False`.
`Simplify2Point`	If `True`, the mass and kinetic terms are simplified using the values of the internal parameters before renormalization is performed. The default is `True`.
`Exclude4ScalarsCT`	If `True`, the four scalar terms are kept in the Lagrangian but not renormalized. The default is `False`.

The UV counterterm vertices are extracted from the counterterm Lagrangian. The renormalization is performed into FeynRules where the Lagrangian and all the parameters dependences are known.[1] The renormalization of the Lagrangian is performed by the function `OnShellRenormalization`. As indicated by the name, the renormalization is performed in the on-shell scheme or equivalently in the complex mass scheme as the domain of the renormalization constants is not specified at that stage. Therefore, the function renormalizes all the physical fields, all the masses and the remaining independent external parameters and all the tadpoles as in Eq. (2.6). The function can be called once FeynRules and the model have been loaded as follows

```
Lren = OnShellRenormalization[MyLag, options]
```

where `MyLag` is the Lagrangian, written in the FeynRules syntax, that needs to be renormalized. The options for this function are listed in Table 1. The output is the bare Lagrangian written as a function of the renormalized fields and parameters and all the renormalization constants.[2] Since each physical field and each mass have to be renormalized, the Lagrangian is expanded over the flavors in FeynRules before performing the renormalization procedure.

A first observation regards the renormalization of masses that do not appear as external parameters in the FeynRules model but as internal ones. They should be exchanged for the renormalization with an external parameter appearing in its expression, i.e. the `Value` of its parameter definition in the FeynRules model. This exchange can be done without changing the initial model (such that the param_card does not change for example) using the variable `FR$LoopSwitches`. The variable `FR$LoopSwitches` is a list of pairs of an external parameter and an internal parameter to be exchanged for the renormalization. Namely, the second parameter is renormalized and the first one is replaced by inverting the `Value` of the second to obtain its dependence on the renormalization constants of the other parameters. For example, the W-boson mass in the FeynRules SM model [21] is an internal parameter but it should be renormalized in the on-shell scheme. Consequently, one of the external parameter appearing in its expression should not have an independent renormalization constant. The command

```
FR$LoopSwitches = {{Gf, MW}};
```

replaces the Fermi constant by a function of the W-boson mass for the renormalization. Since the leading term in the expansion over a fermion mass comes from the Yukawa coupling for processes involving the Higgs boson, the FeynRules SM model defines two masses for each fermion: The Yukawa mass used to obtain the Yukawa coupling and the usual mass used anywhere else. Therefore, the mass of a fermion can be set to zero while keeping its Yukawa coupling. The Lagrangian is renormalizable and unitary only if those two external parameters are equal. More precisely, only the masses should be used since we are using the on-shell renormalization scheme. The variable `FR$RmDblExt` is a list of rules which is applied on the Lagrangian to replace the external parameters that should be removed before the renormalization. In the example of the SM, the command

```
FR$RmDblExt = { ymb -> MB, ymc -> MC, ymdo -> MD, yme -> Me,
   ymm -> MMU, yms -> MS, ymt -> MT, ymtau -> MTA, ymup -> MU};
```

ensures that the Lagrangian is renormalizable by replacing all the Yukawa masses by the usual ones before starting the renormalization procedure. The assignment of both variables should be done either in the notebook or in the FeynRules model file but before running the `OnShellRenormalization` function.

The options of the function `OnShellRenormalization` listed in Table 1 aim mainly to speed up the computation by removing unwanted terms. `QCDOnly` keeps only the fields which carry an index of a representation of the strong gauge group, their masses and the external parameters with QCD appearing in their `InteractionOrder` for the renormalization, i.e. the fields and parameters that are renormalized by the strong interaction. There is also no tadpole renormalization if `QCDOnly` is set to `True`. Similarly, the mixing between the fields with the same transformations under the symmetries of the model can be forbidden in the renormalization of the fields by setting `FlavorMixing` to `False` if the loop corrections do not mix different fields like the corrections from strong interactions in the SM for example. By default, all the fields with the same quantum numbers are allowed to mix. An intermediate situation can be obtained by setting the option `FlavorMixing` to a list of pairs of fields. The syntax for the fields in this list should be the FeynRules `ClassMembers` if present and the `ClassName` otherwise. Only the mixing between the two fields of each pair is then included. For example, only the mixing between the two light up and down quarks are included if

```
FlavorMixing -> {{u, c}, {d, s}}.
```

[1] Except for the functions and variables related to one-loop computation, we refer to the FeynRules [16] manual for a complete description of its functions and conventions.

[2] All the renormalization constants are multiplied by FR$CT. This variable has no physical meaning but is used to keep only terms with at most one renormalization constant and to extract the counterterms in the FeynArts interface.

If three or more fields are allowed to mix, all the possible pairs of those fields should be included in the list. The `Only2Point` option allows to remove the renormalization of the coupling since they are not needed afterwards if they are renormalized in the $\overline{MS}$ scheme (see Section 3.2). The computation of the renormalization conditions requires that the kinetic and mass terms are diagonal for the renormalized fields. Additionally, the computation of the renormalization constant is faster if they are in the canonical form. However, those terms may have more complicate expressions because they have been written in the gauge basis or before spontaneous symmetry breaking for example. The kinetic and mass terms are therefore simplified analytically using the expressions of the internal parameters. This simplification can be skip if they are already diagonal and in the canonical form by setting the option `Simplify2Point` to `False`. Finally, the counterterms for the four scalar vertices can be time consuming while they are only useful if the one of the born amplitudes contains at least one four scalar interaction. Therefore the user can remove them by setting `Exclude4ScalarsCT->False`. However, the tree-level four scalar interactions are kept as they can still appear in loops.

2.2. Renormalization conditions

While the cancellation of the divergences fixes the coefficients of the pole in the counterterm vertices, more complicated renormalization conditions may be desirable to determine their UV finite pieces. The renormalization conditions should be chosen to ease as much as possible the problem at hand or to make the physics transparent. In this respect, the renormalized mass is identified to the physical one, the real part of the pole of the propagator in the on-shell scheme such that its value is given by the mass measurement. Furthermore this scheme allows to get rid of the corrections on the external legs of the amplitudes by forcing the two-point functions to vanish on-shell. More details on the on-shell scheme can be found in Ref. [22]. In the following, we will give the renormalization conditions as they are implemented in the NLOCT package.

First, the tadpole counterterms are chosen to cancel the corresponding loop corrections such that no tadpole should be included in any computation. Secondly, the mass and the wave functions renormalization constants are fixed by the on-shell conditions on the two-point functions. Writing the renormalized fermion two-point function as

$$i\delta_{ij}\left(\not{p}-m_i\right)+i\left[f_{ij}^{L}\left(p^2\right)\not{p}\gamma_{-}+f_{ij}^{R}\left(p^2\right)\not{p}\gamma_{+}+f_{ij}^{SL}\left(p^2\right)\gamma_{-}+f_{ij}^{SR}\left(p^2\right)\gamma_{+}\right], \tag{2.10}$$

where $\gamma_{\pm}=\frac{1\pm\gamma_5}{2}$ and the f functions contain both the loop and counterterm contributions, the renormalization conditions in the on-shell scheme for the fermions are

$$\begin{aligned}
&\tilde{\Re}\left[f_{ij}^{L}\left(p^2\right)m_i+f_{ij}^{SR}\left(p^2\right)\right]\Big|_{p^2=m_i^2}=0,\\
&\tilde{\Re}\left[f_{ij}^{R}\left(p^2\right)m_i+f_{ij}^{SL}\left(p^2\right)\right]\Big|_{p^2=m_i^2}=0,\\
&\tilde{\Re}\Big[2m_i\frac{\partial}{\partial p^2}\left[\left(f_{ii}^{L}\left(p^2\right)+f_{ii}^{R}\left(p^2\right)\right)m_i+f_{ii}^{SL}\left(p^2\right)+f_{ii}^{SR}\left(p^2\right)\right]+f_{ii}^{L}\left(p^2\right)+f_{ii}^{R}\left(p^2\right)\Big]\Big|_{p^2=m_i^2}=0.
\end{aligned} \tag{2.11}$$

The function $\tilde{\Re}$ takes the real part of the loop function but not of the couplings or of the mixing parameters. The off-diagonal conditions allow to absorb the corrections that mix different flavors in the wave function renormalizations. The renormalized fields are therefore mass eigenstates. If the two fermion flavors are massless, the first two conditions are trivially satisfied and therefore are replaced by $\tilde{\Re}f_{ij}^{L}(0)=0$ and $\tilde{\Re}f_{ij}^{R}(0)=0$ to fix the renormalization constants. For a Majorana fermions Ψ, the left and right renormalization constants for the wave function should be complex conjugate of each other since the left and right handed fermion fields are related by

$$\Psi_R=e^{i\alpha}\left(\Psi_L\right)^c \tag{2.12}$$

where α is the Majorana phase. The first two conditions should therefore be equivalent for a Majorana fermion if only one renormalization constant is used. Similarly, if the renormalized two-point function for a scalar is

$$i\delta_{ij}\left(p^2-m_i^2\right)+if_{ij}^{S}\left(p^2\right), \tag{2.13}$$

the renormalization conditions read

$$\begin{aligned}
&\tilde{\Re}\left[f_{ij}^{S}\left(p^2\right)\right]\Big|_{p^2=m_i^2}=0\\
&\tilde{\Re}\left[f_{ij}^{S}\left(p^2\right)\right]\Big|_{p^2=m_j^2}=0\\
&\tilde{\Re}\left[\frac{\partial}{\partial p^2}f_{ii}^{S}\left(p^2\right)\right]\Big|_{p^2=m_i^2}=0.
\end{aligned} \tag{2.14}$$

Finally, if the renormalized two-point function of a vector is written as

$$-i\delta_{ij}\eta_{\mu\nu}\left(p^2-m_i^2\right)-if_{ij}^{T}\left(p^2\right)\left(\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)-if_{ij}^{VL}\left(p^2\right)\frac{p_\mu p_\nu}{p^2}, \tag{2.15}$$

the corresponding renormalization conditions are

$$\tilde{\Re}\left[f_{ij}^{T}\left(p^2\right)\right]\Big|_{p^2=m_i^2}=0$$

$$\tilde{\Re}\left[f_{ij}^{T}\left(p^{2}\right)\right]\Big|_{p^{2}=m_{j}^{2}}=0$$

$$\tilde{\Re}\left[\frac{\partial}{\partial p^{2}}f_{ii}^{T}\left(p^{2}\right)\right]\Big|_{p^{2}=m_{i}^{2}}=0. \tag{2.16}$$

The complex mass scheme allows the renormalized masses and the wave functions to be complex and is obtained by removing the $\tilde{\Re}$ [20].

Finally, all the external parameters but the masses are renormalized in the $\overline{MS}$ scheme by default. Namely, only the pole in

$$\frac{1}{\bar{\epsilon}}\equiv\frac{1}{\epsilon}-\gamma+\log\left(4\pi\right) \tag{2.17}$$

where γ is the Euler–Mascheroni constant, is included in the counterterms. This scheme will be used for example for the Yukawa couplings to the scalar doublet without vev in the generic 2DHM. It is no longer true in a type I or II 2HDM where all the Yukawa depend on the masses in a similar way as in the SM. This scheme will be used to fix all the renormalization constants of the external parameters but the masses. Alternatively, the zero-momentum scheme is commonly used for the renormalization of the gauge coupling constant and should be used in MadGraph5_aMC@NLO for the strong coupling constant. The renormalized coupling is then fixed by requiring that one vertex between two fermions and the gauge boson associated to the coupling is equal to the tree-level one when the momentum of the boson vanishes. Writing the renormalized vectorial gauge interactions of a fermion as

$$\begin{aligned}\Gamma_{FFV}^{\mu}\left(p_{1},p_{2}\right)=igT^{a}\delta_{f_{1},f_{2}}\Bigg[&\gamma^{\mu}\left(\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{1}{2}\delta Z_{FF}^{R}+\frac{1}{2}\delta Z_{FF}^{L}+\frac{g_{V}^{\prime}}{2g}\delta Z_{V^{\prime}V}\right)+\gamma^{\mu}\gamma_{5}\left(\frac{1}{2}\delta Z_{FF}^{R}-\frac{1}{2}\delta Z_{FF}^{L}+\frac{g_{A}^{\prime}}{2g}\delta Z_{V^{\prime}V}\right)\\&+\left(\gamma^{\mu}h^{V}\left(k^{2}\right)+\gamma^{\mu}\gamma_{5}h^{A}\left(k^{2}\right)+\frac{\left(p_{1}-p_{2}\right)^{\mu}}{2m}h^{S}\left(k^{2}\right)+\frac{k_{\mu}}{2m}h^{P}\left(k^{2}\right)\right)\Bigg],\end{aligned} \tag{2.18}$$

where p_1, p_2 and k are the incoming momenta of the two fermions and the vector boson, the h functions contain the one-loop contributions from the triangle diagrams, g is the gauge coupling constant and T^a are the generators of the gauge group and should be replaced by the charge for an abelian group. The first two terms are due to the renormalization of the tree-level vertex. The last pieces of the first two lines are due to the mixing with another vector boson V' (g_V' and g_A' are its vector and axial couplings to the fermions). The renormalization conditions at zero momentum, i.e. $k=-p_1-p_2=0$, then read

$$\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{1}{2}\delta Z_{FF}^{R}+\frac{1}{2}\delta Z_{FF}^{L}+\frac{g_{V}^{\prime}}{2g}\delta Z_{V^{\prime}V}+h^{V}\left(0\right)+h^{S}\left(0\right)=0 \tag{2.19}$$

$$\frac{1}{2}\delta Z_{FF}^{R}-\frac{1}{2}\delta Z_{FF}^{L}+\frac{g_{A}^{\prime}}{2g}\delta Z_{V^{\prime}V}+h^{A}\left(0\right)=0. \tag{2.20}$$

Gauge invariance implies [22,23] that the second is always satisfied. In addition,

$$\frac{1}{2}\delta Z_{FF}^{R}+\frac{1}{2}\delta Z_{FF}^{L}+h^{V}\left(0\right)+h^{S}\left(0\right)+\frac{g_{A}^{\prime}}{2g}\delta Z_{V^{\prime}V}=0 \tag{2.21}$$

if either the gauge group is unbroken or if only the left-handed fermions are charged under the non-abelian component of the broken gauge group.[3] In the first case, $\delta Z_{V'V}$ vanishes as the gauge boson do not mix. Those two cases cover respectively the strong and electromagnetic interactions in the SM. This second relation requires that the vector boson V do not have an axial coupling to the fermion. Consequently, the renormalization of the gauge coupling is fixed by

$$\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{g_{V}^{\prime}}{2g}\delta Z_{V^{\prime}V}-\frac{g_{A}^{\prime}}{2g}\delta Z_{V^{\prime}V}=0. \tag{2.22}$$

In practice, the contribution from the triangle loop diagrams should not and are not computed since the coupling renormalization can be obtained from the wave-function renormalization constants only thanks to gauge invariance. Therefore, the zero-momentum scheme can only be used for the gauge bosons of an unbroken gauge symmetry[4] with vectorial interactions to the fermions and satisfying one of the two cases mentioned above.

In principle, other renormalization schemes can be used. In that case, however, the corresponding renormalization conditions should be implemented. In particular, renormalization schemes which only require the computation of the two-point functions available in NLOCT like the M_Z scheme [24–26] can easily be included. However, a user friendly way to introduce new renormalization schemes is left for a future version. As explained in Section 3.2, the $\overline{MS}$ scheme can alternatively also be used for the two-point functions.

3. Computation of the R_2 and UV counterterms

The computation of the one-loop amplitudes is performed in NLOCT using FeynArts [17] version 3.7 or above to write the amplitudes. NLOCT evaluates the terms of the amplitudes required for the computations of the R_2 and UV counterterms and solves the renormalization conditions. Before loading those packages, the renormalized Lagrangian should be passed to FeynArts through the corresponding interface, i.e.

```
WriteFeynArtsOutput[Lren,options].
```

[3] The sign of the last term change if only right-handed fermions are charged under the non-abelian subgroup.

[4] Which can be the subgroup of a larger broken symmetry.

Table 2
Options of WriteCT.

Output	The name of the output file written as a string to which the.nlo extension is added. The default is Automatic and the FeynRules model name is used.
QCDOnly	If True, only QCD corrections are kept. The default is False.
Assumptions	A list of assumptions that will be used to simplify the expression of the UV parts. The default is an empty list.
ComplexMass	If True, the complex mass scheme is used for the renormalization of the two-point functions. The default is False.
MSbar	If True, the $\overline{MS}$ scheme is used for the renormalization of the two-point functions. The default is False.
ZeroMom	A list of pairs of an external parameter and the list of particles of the vertex which finite part should vanish to renormalize the external parameter in the zero-momentum scheme.
LabelInternal	If True, the contribution of each diagram is multiplied by IPL[part] where part is the list of the particles appearing in the loop. Each particle appears only once in the list even its propagator has more than one occurrence in the loop. Similarly, the list of particles in the loop only contains particles and not antiparticles. The default is True.
KeptIndices	The list of indices that should be kept in the argument of the IPL functions like a flavor index and unlike a color index. The default is an empty list.
Exclude4ScalarsCT	If True, the four scalar counterterms are not computed. The default is False.
CTparameters	If True, the vertex are expressed in term of internal parameters if the size of the expressions of the renormalization constants are large. Those internal parameters and their expressions are stored in the variable FR$CTparam in the output file. The default is False.

The default generic file of FeynArts Lorentz.gen contains optimized Lorentz structure vectors and should be used whenever possible to fasten the computation. This can only be done if the option GenericFile has been set to False when calling the WriteFeynArtsOutput function. If this option is not set to False, FeynRules will create a generic file that have to be used with the model file created simultaneously. Although the computation time may differ, both ways lead to the same result if the model Lorentz structures match those of Lorentz.gen. Since Lorentz.gen does not contain tadpole vertices, it cannot be used if there is at least one tadpole renormalization. However, the optimized Lorentz structures can still be used by setting GenericFile to False and by using LorentzTadpole.gen as the generic file afterwards. This LorentzTadpole.gen only has the tadpole vertex in addition to Lorentz.gen and can be found in the directory Interfaces/FeynArts of FeynRules. FeynArts should be loaded prior to the NLOCT package since the latter uses FeynArts to write the amplitudes. Both packages should be loaded in a new kernel as FeynArts and FeynRules cannot run on the same Mathematica kernel. They can however be used on the same Mathematica session as long as they run on two different parallel kernels. The computation of the R_2 and UV counterterms vertices is done by the WriteCT function of the NLOCT package called as follows,

```
WriteCT[<model>,<genericfile>,options]
```

where <model> and <genericfile> are the name of the FeynArts model files without extension (.mod and.gen) written as a string, the options are listed in Table 2 and will be further detailed in the following. The genericfile should be set to "Lorentz" or to "LorentzTadpole" if the GenericFile option has been set to False in FeynRules or to the same value as model otherwise. If the generic file is not given, the default value is to "Lorentz". The FeynArts model files should be in the FeynArts model directory by default. The output file is written in the current directory and is named by the FeynRules model name with a .nlo extension by default.

3.1. Computation of the one-loop amplitude contributions

The algorithm starts with the evaluation of the R_2 parts of the one-loop amplitudes as well as the terms required for the resolution of the renormalization conditions, i.e. the UV divergences and the UV finite contributions for amplitudes with at most two external particles. The latter will be referred to as the UV parts in the following. The evaluation of those terms relies on the assumptions that the model is renormalizable and written in Feynman Gauge. This gauge is also used by MadLoop. Although the computation can in principle be done in any gauge, the algorithm uses the Feynman gauge assumption to speed up and simplify the code. The user should be aware that the result returned by the current version of NLOCT will not be correct for another gauge as diagrams would be wrongly discarded as explain in the following. Since the maximal dimension of the operators in the Lagrangian is then four, only the vertices with one vector and two scalar or ghost fields or with three vector fields depend on momenta. Furthermore, only the fermion propagators bring additional momenta in the numerator of the integrand in the Feynman gauge. As a consequence, the power of the loop momentum in the numerator is bound by the number of propagators. Only diagrams with at most four propagators and with the sum of the dimension of the external fields lower or equal to four can diverge and therefore generate a R_2 and/or UV part. Furthermore, the amplitudes with four external particles do not have a UV divergence if a three scalars vertex or a scalar–vector–vector vertex is present. As a matter of fact, those vertices do not contain any momenta such that the highest power of the loop momentum in the numerator is lower than the number of propagator denominators. Similarly, the amplitudes with a ghost in the loop and four external scalar fields do not diverge. Therefore, all those diagrams are not generated.

Since the sum of the dimensions of the external fields of an amplitude with a R_2 and/or a UV part is lower or equal to four, only the amplitudes with two fermions and zero or one boson and with one to four bosons have to be generated. All the irreducible amplitudes are produced simultaneously according to their numbers of external fermion, scalar and vector fields. The R_2 and UV parts are computed this way once for each set of amplitudes at the generic level, i.e. when only the spin of the particles in the diagrams is specified. The vertices in FeynArts are written as

$$\vec{c} \cdot \vec{L} = \sum_i c_i L_i, \tag{3.23}$$

where $\vec{c}$ is a vector containing the couplings and $\vec{L}$ is the vector of the Lorentz structures.[5] The Lorentz structures contain all the kinematic information: Dirac matrices, metric tensors, momenta, Levi-Civita tensors. The vector of Lorentz structures is common for all the vertices

[5] They are stored respectively in the model and generic files.

involving the same number of fermion, ghost, scalar and vector fields. On the contrary, the couplings are free from those elements but are functions of the parameters and the gauge group representations of the model specific to each vertex. As a result, all the elements of the amplitudes are fixed at the generic level except for the masses and those couplings. They will be replaced by their actual values later.

After shifting the loop momentum using Feynman parameters, the denominator becomes an even function of the loop momentum and therefore the terms of the numerator with an odd power of the loop momentum vanish after integration. Terms with less than two (four) occurrences of the loop momentum for amplitudes with three (four) propagators are also dropped as they do not induce a UV divergence. Loop momenta are then gathered in scalar products using one of the following replacements

$$q^\mu q^\nu q^\rho q^\sigma \to q^4 \frac{1}{d(d+2)} \left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right), \tag{3.24}$$

$$q^\mu q^\nu \to q^2 \frac{1}{d}\eta^{\mu\nu}. \tag{3.25}$$

The Dirac algebra and the contractions of the metric tensors are then performed using Eqs. (1.4) and (1.5) to obtain the two lower terms of the ϵ expansion of the numerator. The integration over the loop momentum generates the following R_2

$$\int d^d q \frac{\epsilon}{q^2 - m^2}\bigg|_{R_2} = i\pi^2 m^2, \tag{3.26}$$

$$\int d^d q \frac{\epsilon}{\left(q^2 - \Delta\right)^2}\bigg|_{R_2} = i\pi^2, \tag{3.27}$$

$$\int d^d q \frac{q^2\,(a\epsilon + b)}{\left(q^2 - \Delta\right)^2}\bigg|_{R_2} = i\pi^2 (2a - b)\Delta, \tag{3.28}$$

$$\int d^d q \frac{q^2\,(a\epsilon + b)}{\left(q^2 - \Delta\right)^3}\bigg|_{R_2} = i\pi^2 \left(a - \frac{1}{2}b\right), \tag{3.29}$$

$$\int d^d q \frac{q^4\,(a\epsilon + b)}{\left(q^2 - \Delta\right)^4}\bigg|_{R_2} = i\pi^2 \left(a - \frac{5}{6}b\right), \tag{3.30}$$

where a and b do not depend on ϵ or the loop momentum but are polynomials of the Feynman parameters. The UV parts are given by

$$\mu^{2\epsilon} \int d^d q \frac{a\epsilon + b}{q^2 - m^2}\bigg|_{UV} = i\pi^2 m^2 \left(\frac{b}{\bar{\epsilon}} + a + b - b\log\left(\frac{m^2}{\mu^2}\right)\right), \tag{3.31}$$

$$\mu^{2\epsilon} \int d^d q \frac{a\epsilon + b}{\left(q^2 - \Delta\right)^2}\bigg|_{UV} = i\pi^2\,(a\epsilon + b)\left(\frac{1}{\bar{\epsilon}} - \log\left(\frac{\Delta}{\mu^2}\right)\right), \tag{3.32}$$

$$\mu^{2\epsilon} \int d^d q \frac{q^2\,(a\epsilon + b)}{\left(q^2 - \Delta\right)^2}\bigg|_{UV} = i\pi^2\,(2a\epsilon + b\epsilon + 2b)\left(\frac{1}{\bar{\epsilon}} - \log\left(\frac{\Delta}{\mu^2}\right)\right)\Delta, \tag{3.33}$$

$$\mu^{2\epsilon} \int d^d q \frac{q^2\,(a\epsilon + b)}{\left(q^2 - \Delta\right)^3}\bigg|_{UV} = i\pi^2 \frac{b}{\bar{\epsilon}}, \tag{3.34}$$

$$\mu^{2\epsilon} \int d^d q \frac{q^4\,(a\epsilon + b)}{\left(q^2 - \Delta\right)^4}\bigg|_{UV} = i\pi^2 \frac{b}{\bar{\epsilon}}, \tag{3.35}$$

where the finite parts of the integrals are kept for cases with one and two propagators since only those are relevant for the one- and two-point functions. Those terms are removed if the number of external particles is bigger than two. The users can remove those terms by setting the options `MSbar` to `True`. Only the UV divergence are then kept and therefore all the quantities are renormalized in the $\overline{MS}$ scheme. For a single propagator, the logarithm is set to zero if the mass in its argument vanishes. Since the logarithms can induce an infrared pole for massless particles after the integration over the Feynman parameters, the terms linear in ϵ in the coefficient of the logarithm are kept when there are two propagators. The integration over the Feynman parameters is then performed for the cases of multiple propagators except for the terms with a logarithm. Those integrations are kept unevaluated until the external momentum is fixed by the renormalization conditions to handle properly massless particles.

After the computation of the required parts of the amplitudes, the masses and couplings are finally replaced by their values for each field insertion at the class level, i.e. after fixing the type for each fermion, ghost, scalar and vector field in the diagrams. The `QCDOnly` option allows to keep only the QCD contributions in the same spirit as what was done for the renormalization. Since the aim is to speed up the code, the earlier non-QCD contributions are removed the more efficient it is. Therefore some diagrams are already discarded before actually writing the corresponding amplitudes, i.e. at the field insertion level. Since QCD interactions involve particles transforming under $SU(3)_c$, all the diagrams without any vertex with at least three fields with a non-trivial representation under the strong gauge group are removed if this option is set to `True`. After the replacement of the generic couplings by their expressions, only the terms with a power of

g_s higher than two and higher the number of external colored particles if at least one external particle is in the adjoint representation of $SU(3)_c$ are kept. Finally, the color algebra is performed for the triplet and octet ending the computation for the R_2 vertices. For any other representation like a sextet, the products of the color generators are left unevaluated.

3.2. Resolution of the renormalization conditions

The renormalization conditions are solved at each order in $\overline{\epsilon}_{UV}$ separately. The subscript UV has been added to $\overline{\epsilon}$ to emphasize that it is only the pole due to the UV divergence. First, the UV divergent part of the UV counterterm vertices is simply given by the divergence of the corresponding one-loop amplitudes up to the overall sign. Writing the sum of the one-loop amplitudes as

$$A^{loop} = A^{UV}\frac{1}{\overline{\epsilon}_{UV}} + A^{UV\,fin}, \tag{3.36}$$

where $A^{UV\,fin}$ contains the UV finite part of the amplitudes,[6] the UV divergent part of the corresponding counterterm vertex is $-A^{UV}\frac{1}{\overline{\epsilon}_{UV}}$. In so doing, the computation UV divergence of the renormalization constant is bypassed. The renormalization constants are therefore not used for quantities renormalized in the $\overline{MS}$ scheme as mentioned earlier. Secondly, the renormalization conditions of Section 2.2 are solved for the UV finite parts of the renormalized two-point functions to obtain the UV finite parts of the renormalization constants $\delta X^{UV\,fin}$. Namely, only $A^{UV\,fin}$ is kept instead of the full sum of loop amplitudes in the renormalization conditions. The counterterm vertices with the renormalization constants replaced by their UV finite parts are finally added the UV divergent parts of the UV counterterm vertices to obtain the full UV counterterm vertices. For example, the gluon–gluon counterterm vertex is then written as

$$-i\delta^{a_1a_2}\delta Z_{gg}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1\cdot p_2\eta^{\mu_1\mu_2}\right) = -A^{UV}\frac{1}{\overline{\epsilon}_{UV}} - i\delta^{a_1a_2}\delta Z_{gg}^{UV\,fin}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1\cdot p_2\eta^{\mu_1\mu_2}\right). \tag{3.37}$$

The contribution of each diagram to the UV divergence of a vertex can this way be associated with the particle in its loop. This information is kept for both the UV and R_2 vertices if the `LabelInternal` option is set to `True` and will be later included in the UFO output by the associated FeynRules interface. The loop particles for the UV finite parts of the UV counterterms are however not well defined. For the UV finite parts of the UV counterterms, the particles in the loop are chosen to be those of the one- or two-point amplitudes from which the renormalization constants have been computed.

The integration over the Feynman parameter for the bubbles logarithms is performed after writing the renormalization conditions for the UV finite part of the renormalized amplitudes, namely after the derivative over the external momentum has been performed if needed and after replacing the external momentum squared by the square of one of the external masses. The UV finite part of the renormalized one- and two-point functions are obtained by adding the UV finite part of the loop amplitudes and the corresponding counterterm amplitudes, i.e. the tree-level amplitudes with the vertices from $\delta\mathcal{L}$. Those logarithms can be written as

$$b_0\left(p^2, m_1, m_2\right) \equiv \int_0^1 dx \log\left(\frac{p^2(x-1)x + x\left(m_1^2 - m_2^2\right) + m_2^2 - i\epsilon_p}{\mu^2}\right), \tag{3.38}$$

where m_1 and m_2 are the masses of the particles in the loop, p is the external momentum and ϵ_p is coming from the prescription for the propagators and is used to choose the appropriate side of the branch cut when $p^2 \geq \left(m_1^2 + m_2^2\right)$, i.e. when the external particle is kinematically allowed to decay into the loop particles. This integral is nothing more than the finite part of the scalar two-point functions $B_0 = 1/\bar{\epsilon} - b_0$. Consequently, its expression is

$$b_0\left(p^2, m_1, m_2\right) = \log\frac{m_1m_2}{\mu^2} + \frac{m_2^2 - m_1^2}{p^2}\log\frac{m_2}{m_1} + \frac{m_1m_2}{p^2}\left(\frac{1}{r} - r\right)\log r - 2 \tag{3.39}$$

with r and $\frac{1}{r}$ being the roots of

$$x^2 - \frac{m_1^2 + m_2^2 - p^2 - i\epsilon_p}{m_1m_2}x + 1 = 0 \tag{3.40}$$

and its derivative expression is given by

$$\frac{\partial b_0\left(p^2, m_1, m_2\right)}{\partial p^2} = \frac{m_1^2 - m_2^2}{p^4}\log\frac{m_2}{m_1} + \frac{m_1m_2}{p^2}\left(\frac{1}{r} - r\right)\log r - \frac{1}{p^2}\left(1 + \frac{r^2+1}{r^2-1}\log r\right). \tag{3.41}$$

r never crosses the branch cut in the complex mass scheme. However, the logarithm can have a negative argument for real masses depending on the spectrum and are then replaced by

$$\log(-x) \to \log(x) \pm i\pi, \tag{3.42}$$

where the sign of the imaginary part is fixed by ϵ_p. However, the $i\pi$ term is dropped in the on-shell scheme due to the $\tilde{\Re}$. For $m_1 = 0$ and/or $p^2 = 0$, Eqs. (3.39) and (3.41) seem divergent. However, the b_0 and its derivative expressions reduce to

$$b_0\left(p^2, 0, m_2\right) = -2 + 2\frac{m_2^2}{p^2}\log\frac{m_2}{\mu} + \frac{p^2 - m_2^2}{p^2}\log\frac{m_2^2 - p^2}{\mu^2} \tag{3.43}$$

$$\frac{\partial b_0\left(p^2, 0, m_2\right)}{\partial p^2} = \frac{m_2^2\log\frac{m_2^2-p^2}{\mu^2} - 2m_2^2\log\frac{m_2}{\mu} + p^2}{p^4} \tag{3.44}$$

[6] Which may contain an IR divergence.

when $m_1 = 0$, to

$$b_0\left(0, m_1, m_2\right) = -1 + \frac{2m_1^2 \log\frac{m_1}{\mu} - 2m_2^2 \log\frac{m_2}{\mu}}{m_1^2 - m_2^2} \tag{3.45}$$

$$\frac{\partial b_0\left(0, m_1, m_2\right)}{\partial p^2} = \frac{-m_1^4 + m_2^4 + 4m_2^2 m_1^2 \log\left(\frac{m_1}{m_2}\right)}{2\left(m_1^2 - m_2^2\right)^3} \tag{3.46}$$

when $p^2 = 0$ and to

$$b_0\left(0, 0, m_2\right) = -1 + 2\log\frac{m_2}{\mu} \tag{3.47}$$

$$\frac{\partial b_0\left(0, 0, m_2\right)}{\partial p^2} = \frac{-1}{2m_2^2} \tag{3.48}$$

when both vanish. The b_0 function only has an infrared divergence when all the arguments vanish,

$$b_0\left(0, 0, 0\right) = \frac{1}{\bar{\epsilon}}. \tag{3.49}$$

When the two particles in the loop are massless, its derivative only appears multiplied by the external momenta squared for dimensional reason. That product vanishes when the external momenta is light like. The b_0 derivative is also IR divergent when one of the loop particle is massless and the other has the same mass as the external particle,

$$\frac{\partial b_0\left(m^2, 0, m\right)}{\partial p^2} = \frac{1}{2m^2}\left(\frac{1}{\bar{\epsilon}} + 2 - \log\frac{m^2}{\mu^2}\right). \tag{3.50}$$

Consequently, the expansion in ϵ is only done after the evaluation of those functions and just before solving the renormalization conditions to get the finite part and IR divergence of the renormalization constants. This UV finite part is computed by default to cover all possible cases by inserting if statement in the expressions, i.e. for all mass hierarchies and all non-zero masses are still allowed to vanish. As a result, the expressions can become quite long. The list of assumptions passed by the `Assumptions` option is used to remove the cases that do not satisfy them. The expressions are therefore shorter and the computation is faster. The list of assumptions is written in the.nlo file and stored in the variable `NLOCT$assumptions` to remind the user that those vertices cannot be used when they are not satisfied. In the complex mass scheme, the expressions do not change depending if the external momentum is above or below the decay threshold and therefore setting `ComplexMass->True` can also reduce the size of the expressions.

The computation of the finite part of the gauge coupling renormalization constant is performed using Eq. (2.22) for the massive modes if the `ZeroMom` option has been used. For example,

```
ZeroMom -> {{aS, {F[7], V[4], -F[7]}}}
```

fixes the finite part of α_S renormalization constant by requiring the finite part of the gluon to the up quark interaction to be zero at zero momentum. In the SM with the number of massless quarks $N_f = 5$, the renormalization of the strong coupling constant in the zero-momentum scheme implies [27]

$$\frac{\delta\alpha_s}{\alpha_s} = \frac{\alpha_s}{2\pi}\left(\frac{N_f}{3} - \frac{11}{2}\right)\frac{1}{\bar{\epsilon}} + \frac{\alpha_s}{6\pi}\left(\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{M_t^2}\right). \tag{3.51}$$

3.3. The output

The vertices computed by the `WriteCT` function are stored in the output file as two lists of vertices in a format similar to the FeynRules one. The only difference is that FeynArts notation is kept for the color matrices since they have the advantage that the summed indices do not appear explicitly. The list with the R_2 terms is called `R2$vertlist` and the one with the UV counterterms `UV$vertlist`. For example, the `R2$vertlist` for the SM with only QCD corrections looks like

```
R2$vertlist = {
{{{anti[u], 1}, {u, 2}}, ((-I/12)*gs^2*
IndexDelta[Index[Colour, Ext[1]], Index[Colour, Ext[2]]]*
IPL[{u, G}]*(TensDot[SlashedP[2], ProjM][Index[Spin, Ext[1]],
Index[Spin, Ext[2]]] + TensDot[SlashedP[2],
ProjP][Index[Spin, Ext[1]], Index[Spin, Ext[2]]]))/Pi^2},
...
}
```

where the dots represent the other vertices. If the option `CTparameters` has been set to `True`, the `FR$CTparam` is a replacement list for the internal parameters used in `UV$vertlist`. The list is empty otherwise. Additionally, all the FeynRules information about the model, the version of the NLOCT package and the date and time of generation appear commented in the header of the file. The FeynArts file names are stored into the variable `CT$Model` and `CT$GenericModel` and their definitions appear with the `NLOCT$assumptions` definition at the beginning of the output file. Finally, a list keeps tack which interactions have been used for the loop corrections. Each interaction is associated with a value one (zero) in `FR$InteractionOrderPerturbativeExpansion` if the R_2 and UV counterterm vertices (do not) contain its contributions. For example, the SM with only QCD correction has

```
FR$InteractionOrderPerturbativeExpansion = {{QCD, 1}, {QED, 0}};
```

The output file can the be loaded using the `Get` MATHEMATICA function in a different kernel after reloading FEYNRULES and the model. The vertices can then be exported in a UFO file using the `UVCounterterms` and `R2Vertices` option of the `WriteUFO` command, i.e.

```
WriteUFO[MyLag, UVCounterterms -> UV$vertlist,
R2Vertices -> R2$vertlist].
```

The running times for the SM, MSSM with flavor conservation and 2HDM do not exceed a few hours on a dual core 2.4 GHz laptop with 4 Gb of RAM.

3.4. Validation

The validation is based mainly on the SM and MSSM for which the R_2 and/or the UV have been published.

3.4.0.1. SM (QCD)

The analytic expressions for the R_2 vertices due to the one-loop corrections from the strong interaction have been found in agreement with [12]. The UV counterterms expressions due to the strong interaction using the on-shell scheme for the two-point functions and the zero-momentum scheme for the strong coupling constant have also been compared to [27]. It should be noted that the expressions remain the same for the complex mass scheme since at most one non-zero mass enters the computation of each wave function or mass renormalization constant. Therefore this mass only appears in the logarithms and the branch cut is never an issue. Finally, the UFO generated automatically by FEYNRULES, NLOCT and FEYNARTS has been used in a recent version of MADGRAPH5_AMC@NLO [5] and found in perfect agreement with the built-in version.

3.4.0.2. SM (EW)

The R_2 vertices from the electroweak corrections have been compared analytically to [13]. The electroweak corrections to the UV counterterms have been validated by comparison with [22]. Again, the on-shell scheme has been used for the two-point functions while the electroweak coupling has been renormalized with the zero-momentum scheme,

$$\frac{\delta e}{e} = \frac{1}{2}\left(\delta Z_{AA} - \frac{s_w}{c_w}\delta Z_{ZA}\right). \tag{3.52}$$

Many other renormalization schemes have been suggested for the electroweak coupling constant. However, only the $\overline{MS}$ and the zero-momentum are implemented so far. The electroweak interaction is responsible for most of the decays of the elementary particles and therefore the complex mass scheme can be used to handle all the widths properly. The UFO with the electroweak corrections included is currently tested in MADGRAPH5_AMC@NLO as its algorithm is currently extended to include the electroweak corrections.

3.4.0.3. MSSM (QCD)

The analytic expressions for R_2 vertices in the MSSM due to the strong interaction (using the FEYNARTS built in model to ease the comparison) from the NLOCT package have been compared to the expressions in the literature [14] and found in perfect agreement. The MSSM allows to check that the Majorana fermions are handled properly. The MSSM is currently tested in MADLOOP using FEYNRULES version of the model to be able to export it in the UFO format. Contrary to the SM, various masses can be relevant for the corrections from the strong interaction to each two-point function due to the presence of the squarks and the gluino.

4. 2HDM

The Two Higgs Doublet Model (2HDM) is one of the simplest extension of the SM. Only one scalar multiplet with the same transformations under the SM gauge symmetries as the Higgs field is added to the field content of the SM. Consequently, the 2HDM Lagrangian can be written as

$$\mathcal{L}_{2HDM} = \left(D_\mu \Phi_1\right)^\dagger D^\mu \Phi_1 + \left(D_\mu \Phi_2\right)^\dagger D^\mu \Phi_2 - \mathcal{V}_{2HDM} + \mathcal{L}^{Yuk}_{2HDM} + \mathcal{L}^{\cancel{Higgs}}_{SM}, \tag{4.53}$$

where Φ_1 and Φ_2 are the two scalar doublets, $\mathcal{L}^{\cancel{Higgs}}_{SM}$ is the SM Lagrangian without all the terms involving the Higgs doublet, $\mathcal{V}_{2HDM}$ is the 2HDM scalar potential and $\mathcal{L}^{Yuk}_{2HDM}$ contains the interactions between the fermions and the scalar fields. The most generic potential reads [28–30]

$$\begin{aligned}\mathcal{V}_{2HDM} = {} & \mu_1 \Phi_1^\dagger \Phi_1 + \mu_2 \Phi_2^\dagger \Phi_2 + \left(\mu_3 \Phi_1^\dagger \Phi_2 + h.c.\right) + \lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 \\ & + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \left(\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + h.c.\right) \\ & + \Phi_1^\dagger \Phi_1 \left(\lambda_6 \left(\Phi_1^\dagger \Phi_2\right) + h.c.\right) + \Phi_2^\dagger \Phi_2 \left(\lambda_7 \left(\Phi_1^\dagger \Phi_2\right) + h.c.\right),\end{aligned} \tag{4.54}$$

where μ_3, λ_5, λ_6 and λ_7 are complex while the other parameters are real. However, not all those parameters are observable since they can be modified by a change of basis,

$$\begin{pmatrix}\Phi_1 \\ \Phi_2\end{pmatrix} \to \begin{pmatrix}\Phi_1' \\ \Phi_2'\end{pmatrix} = U_\Phi \begin{pmatrix}\Phi_1 \\ \Phi_2\end{pmatrix}, \tag{4.55}$$

where U_Φ is a two by two unitary matrix. Contrary to the SM, the vacuum of the 2HDM does not automatically preserve $U(1)_{EM}$. This desirable property is achieved by forcing the two vacuum expectation values of the two scalar doublets to be aligned,

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}. \tag{4.56}$$

For this vacuum, the basis can be chosen such that only one of the doublets has a non vanishing vev,

$$\langle \Phi_1' \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \quad \text{and} \quad \langle \Phi_2' \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{if } U_\Phi = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}, \tag{4.57}$$

where $v^2 = v_1^2 + v_2^2$ and $\tan\beta = v_2/v_1$. This basis is called the Higgs basis. The basis is not entirely fixed yet because the phase of Φ_2 can still be changed. For example, it can be chosen such that either one parameter amongst μ_3, λ_5, λ_6 and λ_7 is real or the CP transformation of the second doublet is given by

$$(\mathcal{CP})\, \Phi_2 \left(\vec{x}, t\right) (\mathcal{CP})^\dagger = \Phi_2^* \left(-\vec{x}, t\right). \tag{4.58}$$

This will be our basis for the following and therefore we will drop the prime in the notation. The following relations have to be satisfied for this vacuum to be the minimum of the potential or equivalently to remove the tadpoles at the tree-level,

$$\begin{aligned} &\mu_1 + v^2\lambda_1 = 0, \\ &\Re[2\mu_3 + v^2\lambda_6] = 0, \\ &\Im[2\mu_3 + v^2\lambda_6] = 0. \end{aligned} \tag{4.59}$$

Therefore, removing the phase of μ_3 or λ_6 is equivalent. In this basis, the Yukawa Lagrangian is

$$\mathcal{L}_{2HDM}^{Yuk} = -\overline{Q}_L \cdot y^d \cdot d_R \Phi_1 - \overline{Q}_L \cdot y^u \cdot u_R \tilde{\Phi}_1 - \overline{L}_L \cdot y^l \cdot l_R \Phi_1 - \overline{Q}_L \cdot G^d \cdot d_R \Phi_2 - \overline{Q}_L \cdot G^u \cdot u_R \tilde{\Phi}_2 - \overline{L}_L \cdot G^l \cdot l_R \Phi_2 + h.c., \tag{4.60}$$

where $\tilde{\Phi}_i = \varepsilon \Phi_i^*$ with $\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, y^f are the Yukawa matrices like in the SM, i.e. diagonal matrices with the diagonal entries equal to $\sqrt{2}\frac{m_f}{v}$ in the physical basis for the fermions while the G^f matrices are free parameters. Since we have not introduced right-handed neutrinos, there are no Yukawa terms for the neutral leptons.

The doublets have to be replaced in the Lagrangian by the component fields,

$$\Phi_1 \equiv \begin{pmatrix} -iG^+ \\ \dfrac{h_1^n + iG_0 + v}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_2 \equiv \begin{pmatrix} H^+ \\ \dfrac{h_2^n + ih_3^n}{\sqrt{2}} \end{pmatrix}, \tag{4.61}$$

where G_0 and G^+ are the Goldstone bosons, to extract the masses. The mass of the physical charged scalar H^+ is

$$m_{H^+}^2 = \mu_2 + \lambda_3 \frac{v^2}{2} \tag{4.62}$$

while the mass matrix of the neutral physical states reads

$$m_0^2 = 2 \begin{pmatrix} \lambda_1 v^2 & \Re\lambda_6 \dfrac{v^2}{2} & -\Im\lambda_6 \dfrac{v^2}{2} \\ \Re\lambda_6 \dfrac{v^2}{2} & \dfrac{1}{4}\left(2m_{H^+}^2 + (\lambda_4 + 2\Re\lambda_5)\, v^2\right) & -\Im\lambda_5 \dfrac{v^2}{2} \\ -\Im\lambda_6 \dfrac{v^2}{2} & -\Im\lambda_5 \dfrac{v^2}{2} & \dfrac{1}{4}\left(2m_{H^+}^2 + (\lambda_4 - 2\Re\lambda_5)\, v^2\right) \end{pmatrix}. \tag{4.63}$$

The mass eigenstates h_i and the corresponding masses m_{h_i} are obtained by the orthogonal transformation

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \equiv T \cdot \begin{pmatrix} h_1^n \\ h_2^n \\ h_3^n \end{pmatrix} \tag{4.64}$$

with

$$T \equiv \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{pmatrix}, \tag{4.65}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. The masses and mixing angles of the physical scalars are functions of the potential parameters such that

$$T^T \cdot \begin{pmatrix} m_{h_1}^2 & 0 & 0 \\ 0 & m_{h_2}^2 & 0 \\ 0 & 0 & m_{h_3}^2 \end{pmatrix} \cdot T = m_0^2. \tag{4.66}$$

Equivalently, Eq. (4.66) has been solved for the parameters of the potential in the FeynRules implementation. As a matter of fact, the masses and mixing angles have been chosen to be external parameters to ease the on-shell renormalization. Additionally, it also ensures

Table 3
List of the external parameters of the 2HDM in the FeynRules implementation.

Parameter	ParameterName	Description
α_{EW}^{-1}	aEWM1	Inverse of the electroweak coupling constant
G_F	Gf	Fermi constant
α_S	aS	Strong coupling constant at the Z pole
λ_2	l2	Second quartic coupling
λ_3	l3	Third quartic coupling
$\Re\lambda_7$, $\Im\lambda_7$	l7R,l7I	Real and imaginary parts of the last quartic coupling
$\Re G^u$, $\Im G^u$	GUR, GUI	Real and imaginary parts of the up Yukawa matrix
$\Re G^d$, $\Im G^d$	GDR, GDI	Real and imaginary parts of the down Yukawa matrix
$\Re G^l$, $\Im G^l$	GLR, GLI	Real and imaginary parts of the charged lepton Yukawa matrix
$\theta_1, \theta_2, \theta_3$	mixh,mixh2,mixh3	Neutral scalars mixing angles
M_Z	MZ	Z mass
m_d^y, m_s^y, M_b^y	ymdo, yms, ymb	Down, strange and bottom Yukawa mass
m_u^y, m_c^y, M_t^y	ymup, ymc, ymt	Up, charm and top Yukawa mass
m_e^y, m_μ^y, m_τ^y	yme, ymm, ymtau	Electron, muon and tau Yukawa mass
M_u, M_c, M_t	MU, MC, MT	Up quarks masses
M_d, M_s, M_b	MD, MS, MB	Down quarks masses
M_e, M_μ, M_τ	Me, MMU, MTA	Charged lepton masses
m_{H^+}	mhc	Charged scalar mass
$m_{h_1}, m_{h_2}, m_{h_3}$	mh1, mh2, mh3	Neutral scalar masses

that the potential is positive definite. As a consequence, all the parameters of the potential except λ_2, λ_3 and λ_7 are internal parameters. The internal parameters of the gauge sector like the weak mixing angle are functions of the external parameters M_Z, G_F, α_S, α_{EM}^{-1} as in the SM. The Yukawa matrices y^f are internal parameters fixed by the fermions Yukawa masses.[7] Finally, the real and imaginary parts of the G^f coupling matrices are external parameters (see Table 3). In the type I or II 2HDM, those matrices are related to the masses of the fermions and should be renormalized accordingly. Therefore, they should be set as internal parameters depending on the masses before renormalization.

The Lagrangian (4.53) is already strongly constrained unless some extra symmetries are added. For example, the constraints from flavor changing neutral currents are automatically satisfied if flavor is conserved. Flavor conservation requires all the Yukawa coupling matrices G^f to be diagonal as well as the CKM matrix. Choosing the basis such that the CP transformation of the second doublet are given by Eq. (4.58), CP conservation implies that μ_3, λ_5, λ_6 and λ_7 are real and the G^f matrices are hermitian. Therefore, θ_2 and θ_3 vanish such that only the CP-even scalars can mix. Additionally, custodial symmetry is achieved for the scalar potential either with $\lambda_4 = 2\lambda_5$ or with $\lambda_4 = -2\lambda_5$ and $\lambda_6 = \lambda_7 = 0$. In the first case, the masses of pseudoscalar and the physical charged scalar are equal while in the twisted case, the mass of one of the CP-even scalars is equal to the mass of the charged scalar [33]. While CP and flavor conservations can be imposed before renormalization, the custodial symmetry cannot because it is broken by the Yukawa and by the gauge interactions. Only CP and flavor conservations will be used for the electroweak corrections to shorten the expressions.

5. 2HDM R_2 counterterms

5.1. QCD corrections

For the QCD corrections, we use the generic 2HDM Lagrangian without imposing flavor or CP conservation. Many of the R_2 vertices from the strong interactions of the 2HDM are the same as the SM ones. As a matter of fact, only the vertices between the quarks and a scalar and between two gluons and one or two scalars are modified since the only new tree-level interactions involving colored particles are the quark Yukawa interactions. Only the modified or new vertices are displayed in this section. The R_2 fermion–fermion–scalar vertices are given by,

$$R_2^{\bar{d}uH^-} = \frac{ig_s^2}{3\pi^2}\delta_{i_1 i_2}\left[-\left(V^{CKM\,\dagger}G^u\right)_{f_1 f_2}\gamma_+ + \left(G^{d\,\dagger}V^{CKM\,\dagger}\right)_{f_1 f_2}\gamma_-\right] \tag{5.67}$$

$$R_2^{\bar{u}dH^+} = \frac{ig_s^2}{3\pi^2}\delta_{i_1 i_2}\left[\left(V^{CKM}G^d\right)_{f_1 f_2}\gamma_+ - \left(G^{u\,\dagger}V^{CKM}\right)_{f_1 f_2}\gamma_-\right] \tag{5.68}$$

$$R_2^{\bar{u}uh} = \frac{ig_s^2}{3\sqrt{2}\pi^2}\delta_{i_1 i_2}\left[\left(y^u_{f_1 f_2}T_{1s_3} + G^u_{f_1 f_2}\left(T_{2s_3} - iT_{3s_3}\right)\right)\gamma_+ + \left(y^u_{f_1 f_2}T_{1s_3} + G^{u\,\dagger}_{f_1 f_2}\left(T_{2s_3} + iT_{3s_3}\right)\right)\gamma_-\right] \tag{5.69}$$

$$R_2^{\bar{d}dh} = \frac{ig_s^2}{3\sqrt{2}\pi^2}\delta_{i_1 i_2}\left[\left(y^d_{f_1 f_2}T_{1s_3} + G^d_{f_1 f_2}\left(T_{2s_3} + iT_{3s_3}\right)\right)\gamma_+ + \left(y^d_{f_1 f_2}T_{1s_3} + G^{d\,\dagger}_{f_1 f_2}\left(T_{2s_3} - iT_{3s_3}\right)\right)\gamma_-\right], \tag{5.70}$$

where i_k, f_k and s_k are the color (in the fundamental representation), flavor and scalar indices of the kth particle in the superscript list. The scalar index distinguishes the three neutral physical scalars. If CP is conserved, the scalars h_1 and h_2 couple identically to left and right fermions while the pseudoscalar h_3 vertices are proportional to γ_5 as expected.

[7] The double definition of the fermion masses is kept as in the SM for the same reason[31,32].

The R_2 vertices for the gluon–gluon–scalar can be written as

$$R_2^{GGh} = -\frac{ivg_s^2}{32\pi^2}\delta_{a_1a_2}\eta_{\mu_1\mu_2}\Big(y_b\left(T_{2s_3}-iT_{3s_3}\right)(G_b)^* + y_t\left(T_{2s_3}+iT_{3s_3}\right)(G_t)^* + y_bG_b\left(T_{2s_3}+iT_{3s_3}\right) + y_tG_t\left(T_{2s_3}-iT_{3s_3}\right) + 2\left({y_b}^2+{y_t}^2\right)T_{1s_3}\Big), \tag{5.71}$$

where a_k is the color (in the adjoint representation) index of the kth particle and the masses of the quarks of the lightest two flavors have been put to zero.

Finally, the R_2 vertices between two gluons and two scalars are

$$R_2^{GGH^-G^+} = -\frac{g_s^2}{16\pi^2}\delta_{a_1a_2}\eta_{\mu_1\mu_2}V_{3,3}^{CKM}V_{3,3}^{CKM\,\dagger}\left(y_b\,(G_b)^\dagger + y_tG_t\right) \tag{5.72}$$

$$R_2^{GGH^+G^-} = \frac{g_s^2}{16\pi^2}\delta_{a_1a_2}\eta_{\mu_1\mu_2}V_{3,3}^{CKM}V_{3,3}^{CKM\,\dagger}\left(y_bG_b + y_t\,(G_t)^\dagger\right) \tag{5.73}$$

$$R_2^{GGH^-H^+} = -\frac{ig_s^2}{16\pi^2}\delta_{a_1a_2}\eta_{\mu_1\mu_2}\mathrm{tr}\left(G^{d\dagger}G^d + G^{u\dagger}G^u\right) \tag{5.74}$$

$$R_2^{GGG_0h} = -\frac{g_s^2}{32\pi^2}\delta_{a_1a_2}\eta_{\mu_1\mu_2}\Big(y_bG_b\left(T_{2s_4}+iT_{3s_4}\right) - y_tG_t\left(T_{2s_4}-iT_{3s_4}\right) + y_t\left(T_{2s_4}+iT_{3s_4}\right){G_t}^\dagger - y_b\left(T_{2s_4}-iT_{3s_4}\right){G_b}^\dagger\Big) \tag{5.75}$$

$$R_2^{GGhh} = -\frac{ig_s^2}{32\pi^2}\delta_{a_1a_2}\eta_{\mu_1\mu_2}\Big[2\mathrm{tr}\left(G^uG^{u\dagger}+G^dG^{d\dagger}\right)\left(T_{2s_3}T_{2s_4}+T_{3s_3}T_{3s_4}\right) + \left({G_t}^\dagger y_t + G_by_b\right)\left(\left(T_{2s_3}+iT_{3s_3}\right)T_{1s_4}+T_{1s_3}\left(T_{2s_4}+iT_{3s_4}\right)\right) + \left(G_ty_t+{G_b}^\dagger y_b\right)\left(\left(T_{2s_3}-iT_{3s_3}\right)T_{1s_4}+T_{1s_3}\left(T_{2s_4}-iT_{3s_4}\right)\right) + 2\left((y_t)^2+(y_b)^2\right)T_{1s_3}T_{1s_4}\Big]. \tag{5.76}$$

The vertices GGh_3, $GGG_0h_{1/2}$ and $GGh_3h_{1/2}$ vanish when CP is conserved as it should since they involve only one pseudoscalar.

5.2. EW corrections

The R_2 vertices induced by the electroweak interactions are much longer than the QCD ones. Therefore, CP and flavor conservation are assumed for the EW corrections. The fermion and vector two-point R_2 vertices are not modified compared to the SM. The two-point scalar R_2 vertices are given by

$$R_2^{G_0G_0} = \frac{i}{768\pi^2c_w^2s_w^2}\left[-48c_w^2s_w^2\left(p^2\left({y_b}^2+{y_t}^2\right)-3v^2\left({y_b}^4+{y_t}^4\right)\right)+e^2\left(3m_{h_1}^2+3m_{h_2}^2+3c_{21}\left(m_{h_1}^2-m_{h_2}^2\right)\right. - \left.24M_W^2c_w^2-18M_Z^2+\left(4s_w^2-6\right)p^2\right)\right] \tag{5.77}$$

$$R_2^{G^-G^+} = \frac{i}{768\pi^2c_w^2s_w^2}\left(3c_w^2\left(8s_w^2\left({y_b}^2+{y_t}^2\right)\left(3v^2\left({y_b}^2+{y_t}^2\right)-2p^2\right)+e^2\left(c_{21}\left(m_{h_1}^2-m_{h_2}^2\right)-2\left(M_Z^2\left(2-9s_w^2\right)+5M_W^2+p^2\right)\right.\right. + \left.\left.m_{h_1}^2+m_{h_2}^2\right)\right) - 2e^2s_w^2\left(9M_Z^2s_w^2+45M_W^2+p^2\right)\Big) \tag{5.78}$$

$$R_2^{H^-G^+} = \frac{4\left(y_bG_b+y_tG_t\right)\left(3v^2\left({y_b}^2+{y_t}^2\right)-2p^2\right)-\frac{e^2c_1s_1}{s_w^2}\left(m_{h_1}^2-m_{h_2}^2\right)}{128\pi^2} \tag{5.79}$$

$$R_2^{G^-H^+} = -\frac{4\left(y_bG_b+y_tG_t\right)\left(3v^2\left({y_b}^2+{y_t}^2\right)-2p^2\right)-\frac{e^2c_1s_1}{s_w^2}\left(m_{h_1}^2-m_{h_2}^2\right)}{128\pi^2} \tag{5.80}$$

$$R_2^{H^-H^+} = -\frac{i}{384\pi^2}\left(8p^2\left(3R^d+R^l+3R^u\right)+e^2p^2\left(\frac{s_w^2}{c_w^2}+\frac{3}{s_w^2}+1\right)-36v^2\left({y_b}^2+{y_t}^2\right)\left({G_t}^2+{G_b}^2\right)-3e^2m_{H^+}^2\left(\frac{s_w^2}{c_w^2}+\frac{1}{s_w^2}+1\right)\right. - \left.\frac{3e^2}{s_w^2}\left(s_1^2m_{h_1}^2+c_1^2m_{h_2}^2+m_{h_3}^2\right)+9e^2M_Z^2\left(\frac{s_w^2}{c_w^2}+\frac{3}{s_w^2}-5\right)\right) \tag{5.81}$$

$$R_2^{G_0h_2} = \frac{-i\left(y_bG_b\left(p^2-3v^2{y_b}^2\right)+y_tG_t\left(p^2-3v^2{y_t}^2\right)+\frac{e^2c_1s_1}{8c_w^2s_w^2}\left(m_{h_1}^2-m_{h_2}^2\right)\right)}{16\pi^2} \tag{5.82}$$

$$R_2^{h_1h_1} = -\frac{i}{48\pi^2}\Big[s_1^2\left(p^2\left(3R^d+R^l+3R^u\right)-9v^2\left({y_b}^2{G_b}^2+{y_t}^2{G_t}^2\right)\right) - 3s_{21}\left(y_bG_b\left(p^2-3v^2{y_b}^2\right)+y_tG_t\left(p^2-3v^2{y_t}^2\right)\right)+3c_1^2\left(p^2\left({y_b}^2+{y_t}^2\right)-3v^2\left({y_b}^4+{y_t}^4\right)\right)-\frac{e^2}{8c_w^2s_w^2} \times \left(s_1^2\left(6c_w^2m_{H^+}^2+3m_{h_3}^2\right)+p^2\left(2s_w^2-3\right)-3\left(7c_1^2+3\right)M_Z^2\left(2c_w^4+1\right)\right)\Big] \tag{5.83}$$

$$R_2^{h_1h_2} = \frac{i}{96\pi^2}\Big[6c_{21}\left(y_bG_b\left(3v^2{y_b}^2-p^2\right)+y_tG_t\left(3v^2{y_t}^2-p^2\right)\right)$$
$$\times s_{21}\left(-3{y_t}^2\left(3v^2{G_t}^2+p^2\right)-3{y_b}^2\left(3v^2{G_b}^2+p^2\right)+9v^2\left({y_t}^4+{y_b}^4\right)\right.$$
$$\left.+\ 3p^2R^d+p^2R^l+3p^2R^u\right)\Big]-\frac{ie^2s_{21}\left(2c_w^2m_{H^+}^2+m_{h_3}^2+7M_Z^2\left(2c_w^4+1\right)\right)}{256\pi^2c_w^2s_w^2} \tag{5.84}$$

$$R_2^{h_2h_2} = -\frac{i}{48\pi^2}\Big[c_1^2\left(p^2\left(3R^d+R^l+3R^u\right)-9v^2{y_t}^2{G_t}^2-9v^2{y_b}^2{G_b}^2\right)+3\left({y_b}^2s_1^2\left(p^2-3v^2y_b^2\right)+y_t^2s_1^2\left(p^2-3v^2y_t^2\right)\right.$$
$$\left.+\ s_{21}\left(y_bG_b\left(p^2-3v^2{y_b}^2\right)+y_tG_t\left(p^2-3v^2{y_t}^2\right)\right)\right)\Big]+\frac{ie^2}{768\pi^2c_w^2s_w^2}\left(3\left(c_{21}+1\right)\left(m_{h_3}^2+2c_w^2m_{H^+}^2\right)\right.$$
$$\left.+\ 3\left(7c_{21}-13\right)\left(2c_w^4+1\right)M_Z^2-2p^2\left(2c_w^2+1\right)\right) \tag{5.85}$$

$$R_2^{h_3h_3} = \frac{ie^2}{768\pi^2c_w^2s_w^2}\left(2\left(6c_w^2m_{H^+}^2-9M_Z^2\left(2c_w^4+1\right)+p^2\left(2s_w^2-3\right)\right)-3\left(c_{21}-1\right)m_{h_1}^2+3\left(c_{21}+1\right)m_{h_2}^2\right)$$
$$-\frac{i\left(p^2\left(3R^d+R^l+3R^u\right)-9v^2\left(G_b^2y_b^2+G_t^2y_t^2\right)\right)}{48\pi^2}, \tag{5.86}$$

where $c_{21}=\cos 2\theta_1$, $R^f={G_{1,1}^f}^2+{G_{2,2}^f}^2+{G_{3,3}^f}^2$, $y_t=y_{3,3}^u$, $y_b=y_{3,3}^d$, $G_t=G_{3,3}^u$, $G_b=G_{3,3}^d$.

The R_2 vertices for the interactions between two fermions and one scalar are given by

$$R_2^{\bar{b}bG_0} = -\frac{\delta_{i_1i_2}\gamma^5\left(36c_w^2s_w^2y_t\left(G_bG_t+y_by_t\right)+e^2y_b\left(27-34s_w^2\right)\right)}{1152\sqrt{2}\pi^2c_w^2s_w^2} \tag{5.87}$$

$$R_2^{\bar{t}tG_0} = \frac{\delta_{i_1i_2}\gamma_{s_1,s_2}^5\left(y_t\left(e^2\left(14s_w^2+27\right)-36s_w^2c_w^2y_b^2\right)+36G_by_bG_ts_w^2c_w^2\right)}{1152\sqrt{2}\pi^2c_w^2s_w^2} \tag{5.88}$$

$$R_2^{\bar{u}uh_1} = -\frac{i\delta_{i_1i_2}}{1152\sqrt{2}\pi^2c_w^2s_w^2}\left(s_1\left(36c_w^2G_ds_w^2\left(G_dG_u+y_dy_u\right)+e^2G_u\left(14s_w^2+27\right)\right)\right.$$
$$\left.+\ c_1\left(2s_w^2\left(-18c_w^2y_d\left(G_dG_u+y_dy_u\right)-7e^2y_u\right)-27e^2y_u\right)\right) \tag{5.89}$$

$$R_2^{\bar{d}dh_1} = \frac{i\delta_{i_1i_2}}{1152\sqrt{2}\pi^2c_w^2s_w^2}\left(s_1\left(e^2G_d\left(34s_w^2-27\right)-36c_w^2G_us_w^2\left(G_dG_u+y_dy_u\right)\right)\right.$$
$$\left.+\ c_1\left(36c_w^2s_w^2y_u\left(G_dG_u+y_dy_u\right)+e^2y_d\left(27-34s_w^2\right)\right)\right) \tag{5.90}$$

$$R_2^{\bar{l}lh_1} = -\frac{ie^2s_1\left(14s_w^2+3\right)G_l}{128\sqrt{2}\pi^2c_w^2s_w^2} \tag{5.91}$$

$$R_2^{\bar{u}uh_3} = \frac{\delta_{i_1i_2}\gamma^5\left(36c_w^2G_ds_w^2\left(G_dG_u+y_dy_u\right)+e^2G_u\left(14s_w^2+27\right)\right)}{1152\sqrt{2}\pi^2c_w^2s_w^2} \tag{5.92}$$

$$R_2^{\bar{d}dh_3} = -\frac{\delta_{i_1i_2}\gamma^5\left(36c_w^2G_us_w^2\left(G_dG_u+y_dy_u\right)+e^2G_d\left(27-34s_w^2\right)\right)}{1152\sqrt{2}\pi^2c_w^2s_w^2} \tag{5.93}$$

$$R_2^{\bar{l}lh_3} = -\frac{e^2G_l\left(14s_w^2+3\right)\gamma^5}{128\sqrt{2}\pi^2c_w^2s_w^2} \tag{5.94}$$

$$R_2^{\bar{u}dH^+} = -\frac{i\delta_{i_1i_2}}{1152\pi^2c_w^2s_w^2}\left(\gamma_-\left(36c_w^2s_w^2\left(G_uG_d^2+G_dy_dy_u\right)+e^2G_u\left(14s_w^2+27\right)\right)\right.$$
$$\left.+\ \gamma_+\left(e^2G_d\left(34s_w^2-27\right)-36c_w^2s_w^2\left(G_dG_u^2+y_dG_uy_u\right)\right)\right) \tag{5.95}$$

$$R_2^{\bar{\nu}_llH^+} = \frac{ie^2\left(14s_w^2+3\right)G_l\gamma_+}{128\pi^2c_w^2s_w^2} \tag{5.96}$$

$$R_2^{\bar{d}uH^-} = -\frac{i\delta_{i_1i_2}}{1152\pi^2c_w^2s_w^2}\left(\gamma_-\left(e^2G_d\left(34s_w^2-27\right)-36c_w^2s_w^2\left(G_dG_u^2+y_dG_uy_u\right)\right)\right.$$
$$\left.+\ \gamma_+\left(36c_w^2s_w^2\left(G_uG_d^2+G_dy_dy_u\right)+e^2G_u\left(14s_w^2+27\right)\right)\right) \tag{5.97}$$

$$R_2^{\bar{l}\nu_lH^-} = \frac{ie^2\left(14s_w^2+3\right)G_l\gamma_-}{128\pi^2c_w^2s_w^2} \tag{5.98}$$

$$R_2^{\bar{t}bG^+} = \frac{\delta_{i_1 i_2}}{1152\pi^2 c_w^2 s_w^2}\left(\gamma_-\left(2s_w^2\left(y_t\left(-18c_w^2 y_b^2 - 7e^2\right) - 18G_b y_b c_w^2 G_t\right) - 27e^2 y_t\right) + \gamma_+\left(36c_w^2 s_w^2\left(y_b y_t^2 + G_b G_t y_t\right) + e^2 y_b\left(27 - 34s_w^2\right)\right)\right) \tag{5.99}$$

$$R_2^{\bar{b}tG^-} = \frac{\delta_{i_1 i_2}}{1152\pi^2 c_w^2 s_w^2}\left(\gamma_-\left(y_b\left(e^2\left(34s_w^2 - 27\right) - 36c_w^2 s_w^2 y_t^2\right) - 36G_b c_w^2 G_t s_w^2 y_t\right) + \gamma_+\left(y_t\left(e^2\left(14s_w^2 + 27\right) + 36s_w^2 c_w^2 y_b^2\right) + 36G_b y_b G_t s_w^2 c_w^2\right)\right), \tag{5.100}$$

where the top and bottom have been used explicitly when the vertices with the other flavors vanish but otherwise u, d, l and ν_l are generic label that could be replaced by any of the three flavors of up quark, down quark, charged lepton or neutral lepton and y_f and G_f are the corresponding diagonal element of the Yukawa coupling matrices. The vertices with h_2 are obtained by replacing $c_1 \to s_1$ and $s_1 \to -c_1$ in the vertices with h_1. The same trick will be used in the following for all the vertices involving a single h_2 and no h_1.

The R_2 vertices for the vector interaction with the fermions are

$$R_2^{\bar{l}lA} = \frac{ie\left(\left(c_w^2 G_l^2 s_w^2 + e^2\left(3 - 2s_w^2\right)\right)\gamma^{\mu_3}.\gamma_- + s_w^2\left(c_w^2 G_l^2 + 4e^2\right)\gamma^{\mu_3}.\gamma_+\right)}{32\pi^2 c_w^2 s_w^2} \tag{5.101}$$

$$R_2^{\bar{\nu}_l\nu_l A} = \frac{ieG_l^2\gamma^{\mu_3}.\gamma_-}{32\pi^2} \tag{5.102}$$

$$R_2^{\bar{d}dA} = -\frac{ie\delta_{i_1 i_2}}{864\pi^2 c_w^2}\left(\frac{\left(e^2\left(26s_w^2 - 27\right) - 9c_w^2 s_w^2\left(G_d^2 + y_d^2 - 2G_u^2 - 2y_u^2\right)\right)}{s_w^2}\gamma^{\mu_3}.\gamma_- + \gamma^{\mu_3}.\gamma_+\left(9c_w^2\left(G_d^2 + y_d^2\right) - 4e^2\right)\right) \tag{5.103}$$

$$R_2^{\bar{u}uA} = \frac{ie\delta_{i_1 i_2}}{864\pi^2 c_w^2}\left(\frac{\left(9c_w^2 s_w^2\left(G_d^2 + y_d^2 - 2G_u^2 - 2y_u^2\right) + e^2\left(52s_w^2 - 54\right)\right)}{s_w^2}\gamma^{\mu_3}.\gamma_- - \gamma^{\mu_3}.\gamma_+\left(9c_w^2\left(G_u^2 + y_u^2\right) + 32e^2\right)\right) \tag{5.104}$$

$$R_2^{\bar{l}lZ} = -\frac{ie\left(\gamma^{\mu_3}.\gamma_-\left(2c_w^2 G_l^2 s_w^4 - e^2\left(4c_w^4 - 1\right)\right) + 2s_w^4\left(c_w^2 G_l^2 + 4e^2\right)\gamma^{\mu_3}.\gamma_+\right)}{64\pi^2\left(c_w^2\right)^{3/2} s_w^3} \tag{5.105}$$

$$R_2^{\bar{\nu}_l\nu_l Z} = \frac{ie\gamma^{\mu_3}.\gamma_-\left(e^2\left(2s_w^2 - 3\right) - 2c_w^2 G_l^2 s_w^4\right)}{64\pi^2\left(c_w^2\right)^{3/2} s_w^3} \tag{5.106}$$

$$R_2^{\bar{d}dZ} = \frac{ie\delta_{i_1 i_2}}{1728\pi^2\left(c_w^2\right)^{3/2} s_w^3}\left(2s_w^4\left(9c_w^2\left(G_d^2 + y_d^2\right) - 4e^2\right)\gamma^{\mu_3}.\gamma_+ + \gamma^{\mu_3}.\gamma_- \times\left(e^2\left(52s_w^4 - 132s_w^2 + 81\right) - 18c_w^2 s_w^4\left(G_d^2 + y_d^2 - 2G_u^2 - 2y_u^2\right)\right)\right) \tag{5.107}$$

$$R_2^{\bar{u}uZ} = \frac{ie\delta_{i_1 i_2}}{1728\pi^2\left(c_w^2\right)^{3/2} s_w^3}\left(2s_w^4\left(9c_w^2\left(G_u^2 + y_u^2\right) + 32e^2\right)\gamma^{\mu_3}.\gamma_+ - \gamma^{\mu_3}.\gamma_- \times\left(18c_w^2 s_w^4\left(G_d^2 + y_d^2 - 2G_u^2 - 2y_u^2\right) + e^2\left(104s_w^4 - 186s_w^2 + 81\right)\right)\right) \tag{5.108}$$

$$R_2^{\bar{d}dG} = \frac{ig_s T^{a_3}_{i_1,i_2}}{288\pi^2 c_w^2}\left(\frac{\left(e^2\left(26s_w^2 - 27\right) - 9c_w^2 s_w^2\left(G_d^2 + y_d^2 + G_u^2 + y_u^2\right)\right)}{s_w^2}\gamma^{\mu_3}.\gamma_- - 2\left(9c_w^2\left(G_d^2 + y_d^2\right) + 2e^2\right)\gamma^{\mu_3}.\gamma_+\right) \tag{5.109}$$

$$R_2^{\bar{d}dG} = \frac{ig_s T^{a_3}_{i_1,i_2}}{288\pi^2 c_w^2}\left(\frac{\left(e^2\left(26s_w^2 - 27\right) - 9c_w^2 s_w^2\left(G_d^2 + y_d^2 + G_u^2 + y_u^2\right)\right)}{s_w^2}\gamma^{\mu_3}.\gamma_- - 2\left(9c_w^2\left(G_u^2 + y_u^2\right) + 8e^2\right)\gamma^{\mu_3}.\gamma_+\right), \tag{5.110}$$

where the R_2 vertices with the W-boson are omitted because there are not modified compared to the SM. The three bosons R_2 vertices are also like in the SM.

The vector–vector–scalar R_2 vertices are given by

$$R_2^{AW^\pm H^\mp} = -\frac{ie^2 v\eta_{\mu_1,\mu_2}\left(2G_d y_d + G_u y_u\right)}{32\pi^2 s_w} \tag{5.111}$$

$$R_2^{ZW^\pm H^\mp} = \frac{ie^2 v\eta_{\mu_1,\mu_2}\left(2G_d y_d + G_u y_u\right)}{32\pi^2\sqrt{c_w^2}} \tag{5.112}$$

$$R_2^{AAh_1} = -\frac{ie^2 v\eta_{\mu_1,\mu_2}\left(-4s_1 s_w^2\left(G_d y_d + 4G_u y_u\right) + c_1\left(4s_w^2\left(y_d^2 + 4y_u^2\right) + 3e^2\right)\right)}{96\pi^2 s_w^2} \tag{5.113}$$

$$R_2^{AZh_1} = \frac{ie^2 v \eta_{\mu_1,\mu_2}}{192\pi^2\sqrt{c_w^2 s_w^3}}\Big(-2s_1 s_w^2\left(G_d y_d\left(4s_w^2-3\right)+2G_u\left(8s_w^2-3\right)y_u\right) + c_1\left(2s_w^2\left(y_d^2\left(4s_w^2-3\right)+2\left(8s_w^2-3\right)y_u^2\right)+e^2\left(6s_w^2-9\right)\right)\Big) \tag{5.114}$$

$$R_2^{ZZh_1} = \frac{ie^2 v \eta_{\mu_1,\mu_2}}{96\pi^2 c_w^2 s_w^4}\Big(s_1 s_w^2\left(G_d y_d\left(2s_w^2-3\right)^2+G_u\left(4s_w^2-3\right)^2 y_u\right) - c_1\left(s_w^2\left(y_d^2\left(2s_w^2-3\right)^2+\left(4s_w^2-3\right)^2 y_u^2\right)+3e^2\left(s_w^4-3s_w^2+2\right)\right)\Big) \tag{5.115}$$

$$R_2^{W^-W^+h_1} = \frac{ie^2 v \eta_{\mu_1,\mu_2}\left(3s_1 s_w^2\left(G_d y_d+G_u y_u\right)-c_1\left(3s_w^2\left(y_d^2+y_u^2\right)+2e^2\right)\right)}{32\pi^2 s_w^4} \tag{5.116}$$

while the vertices with the charged Goldstone bosons are given by their SM expressions. The R_2 vertices for the interaction of two Goldstone bosons and a vector boson are SM-like while the other R_2 scalar–scalar–vector vertices are given by

$$R_2^{W^\pm H^\mp G_0} = -\frac{e\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)\left(G_b y_b+G_t y_t\right)}{16\pi^2 s_w} \tag{5.117}$$

$$R_2^{AH^\mp G^\pm} = \frac{e\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)\left(G_b y_b+G_t y_t\right)}{8\pi^2} \tag{5.118}$$

$$R_2^{ZH^\mp G^\pm} = -\frac{e\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)\left(2s_w^2-1\right)\left(G_b y_b+G_t y_t\right)}{16\pi^2\sqrt{c_w^2}s_w} \tag{5.119}$$

$$R_2^{ZH^-H^+} = \frac{ie\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)}{768\pi^2\left(c_w^2\right)^{3/2}s_w^3}\left(16s_w^2\left(2s_w^4-3s_w^2+1\right)\left(3R^d+R^l+3R^u\right)+e^2\left(24\left(s_w^2-2\right)s_w^2+23\right)\right) \tag{5.120}$$

$$R_2^{AH^-H^+} = \frac{ie\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)\left(16c_w^2 s_w^2\left(3R^d+R^l+3R^u\right)+e^2\left(1+12c_w^2\right)\right)}{384\pi^2 c_w^2 s_w^2} \tag{5.121}$$

$$R_2^{W^\pm H^\mp h_1} = \frac{\mp ie\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)}{768\pi^2 c_w^2 s_w^3}\left(e^2 s_1\left(22c_w^2+1\right)+16c_w^2 s_w^2\left(s_1\left(3R^d+R^l+3R^u\right)-3c_1\left(G_b y_b+G_t y_t\right)\right)\right) \tag{5.122}$$

$$R_2^{W^\pm H^\mp h_3} = -\frac{e\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)\left(16c_w^2 s_w^2\left(3R^d+R^l+3R^u\right)+e^2\left(23-22s_w^2\right)\right)}{768\pi^2 c_w^2 s_w^3} \tag{5.123}$$

$$R_2^{W^\pm G^\mp h_1} = \frac{-e\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)}{768\pi^2 c_w^2 s_w^3}\left(c_1\left(48c_w^2 s_w^2\left(y_b^2+y_t^2\right)+e^2\left(1+22c_w^2\right)\right)-48s_1 c_w^2 s_w^2\left(G_b y_b+G_t y_t\right)\right) \tag{5.124}$$

$$R_2^{W^\pm G^\mp h_3} = \frac{\mp ie\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)\left(G_b y_b+G_t y_t\right)}{16\pi^2 s_w} \tag{5.125}$$

$$R_2^{ZG_0h_1} = -\frac{e\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)}{768\pi^2\left(c_w^2\right)^{3/2}s_w^3}\left(-48s_1 c_w^2 s_w^2\left(G_b y_b+G_t y_t\right)+c_1\left(48c_w^2 s_w^2\left(y_b^2+y_t^2\right)+e^2\left(20s_w^4-42s_w^2+23\right)\right)\right) \tag{5.126}$$

$$R_2^{AG_0h_1} = \frac{5c_1 e^3\left(\mathrm{p}_3^{\mu_1}-\mathrm{p}_2^{\mu_1}\right)}{192\pi^2 s_w^2} \tag{5.127}$$

$$R_2^{Ah_1h_3} = \frac{5e^3 s_1\left(\mathrm{p}_3^{\mu_1}-\mathrm{p}_2^{\mu_1}\right)}{192\pi^2 s_w^2} \tag{5.128}$$

$$R_2^{Zh_1h_3} = \frac{e\left(\mathrm{p}_2^{\mu_1}-\mathrm{p}_3^{\mu_1}\right)}{768\pi^2\left(c_w^2\right)^{3/2}s_w^3}\left(e^2 s_1\left(-20s_w^4+42s_w^2-23\right)-16c_w^2 s_w^2\left(s_1\left(3R^d+R^l+3R^u\right)-3c_1\left(G_b y_b+G_t y_t\right)\right)\right), \tag{5.129}$$

where p_k is the momenta of the k^{th} particle and is incoming.

The three scalars R_2 vertices read

$$R_2^{H^-H^+h_1} = -\frac{iv}{256\pi^2 c_w^4 s_w^4}\Big(2s_1 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d^3 y_d+G_u^3 y_u\right)+e^2\lambda_7\left(3-2s_w^2\right)\right) + c_1\left(-48c_w^4 s_w^4\left(G_d^2\left(y_d^2+y_u^2\right)-2G_d y_d G_u y_u+G_u^2\left(y_d^2+y_u^2\right)\right) - 2e^2\lambda_3 s_w^2\left(2s_w^4-5s_w^2+3\right)+3e^4\left(6s_w^4-8s_w^2+3\right)\right)\Big) \tag{5.130}$$

$$R_2^{G^\mp H^\pm h_1} = \frac{\mp v}{256\pi^2 c_w^2 s_w^2}\Big(c_1\left(48c_w^2 s_w^2\left(G_d y_d^3+G_u y_u^3\right)+e^2\lambda_6\left(6-4s_w^2\right)\right) + s_1\left(-24c_w^2 s_w^2\left(G_d^2\left(2y_d^2-y_u^2\right)+2G_d y_d G_u y_u-G_u^2\left(y_d^2-2y_u^2\right)\right)+e^2\left(\lambda_4\left(2s_w^2-3\right)-8\lambda_5 c_w^2\right)+6e^4\right)\Big) \tag{5.131}$$

$$R_2^{G^-G^+h_1} = \frac{iv}{256\pi^2 c_w^4 s_w^4}\left(c_1\left(48c_w^4 s_w^4\left(y_d^4+y_u^4\right)+16e^2\lambda_1 c_w^4 s_w^2-3e^4\left(2s_w^4-4s_w^2+3\right)\right)\right.$$
$$\left.-2s_1 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d y_d^3+G_u y_u^3\right)+e^2\lambda_6\left(3-2s_w^2\right)\right)\right) \tag{5.132}$$

$$R_2^{H^\mp G^\pm h_3} = -\frac{iv\left(24c_w^2 s_w^2\left(y_d G_u - G_d y_u\right)^2+e^2\left(8\lambda_5 c_w^2+\lambda_4\left(2s_w^2-3\right)\right)+6e^4\right)}{256\pi^2 c_w^2 s_w^2} \tag{5.133}$$

$$R_2^{G_0G_0h_1} = \frac{iv}{256\pi^2 c_w^4 s_w^4}\left(c_1\left(48c_w^4 s_w^4\left(y_d^4+y_u^4\right)+4e^2\lambda_1 c_w^2 s_w^2-3e^4\left(2s_w^4-4s_w^2+3\right)\right)\right.$$
$$\left.-2s_1 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d y_d^3+G_u y_u^3\right)+e^2\lambda_6\left(3-2s_w^2\right)\right)\right) \tag{5.134}$$

$$R_2^{h_1h_1h_1} = -\frac{3iv}{256\pi^2 c_w^4 s_w^4}\left(2s_1^3 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d^3 y_d+G_u^3 y_u\right)+e^2\lambda_7\left(3-2s_w^2\right)\right)\right.$$
$$+c_1 s_1^2\left(2e^2 c_w^2 s_w^2\left(-3\lambda_3-3\lambda_4-2\lambda_5+2\left(\lambda_3+\lambda_4\right)s_w^2\right)-144c_w^4 s_w^4\left(G_d^2 y_d^2+G_u^2 y_u^2\right)+3e^4\left(2s_w^4-4s_w^2+3\right)\right)$$
$$+6c_1^2 s_1 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d y_d^3+G_u y_u^3\right)+e^2\lambda_6\left(3-2s_w^2\right)\right)$$
$$\left.+c_1^3\left(-48c_w^4 s_w^4\left(y_d^4+y_u^4\right)-4e^2\lambda_1 c_w^2 s_w^2+3e^4\left(2s_w^4-4s_w^2+3\right)\right)\right) \tag{5.135}$$

$$R_2^{h_1h_1h_2} = \frac{-iv}{512\pi^2 c_w^4 s_w^4}\left(3c_w^2 s_w^2 s_{31}\left(24c_w^2 s_w^2\left(3G_d^2 y_d^2-y_d^4+3G_u^2 y_u^2-y_u^4\right)+e^2\left(-2\lambda_1+3\lambda_3+3\lambda_4+2\lambda_5-2\left(\lambda_3+\lambda_4\right)s_w^2\right)\right)\right.$$
$$+3c_w^2 s_w^2 c_{31}\left(24c_w^2 s_w^2\left(G_d^3 y_d-3G_d y_d^3-3G_u y_u^3+G_u^3 y_u\right)+e^2\left(3\lambda_6-\lambda_7\right)\left(2s_w^2-3\right)\right)+3c_1 c_w^2 s_w^2\left(e^2\left(\lambda_6+\lambda_7\right)\left(2s_w^2-3\right)\right.$$
$$-24c_w^2 s_w^2\left(G_d^3 y_d+G_d y_d^3+G_u y_u\left(G_u^2+y_u^2\right)\right))+s_1\left(-72c_w^4 s_w^4\left(G_d^2 y_d^2+y_d^4+G_u^2 y_u^2+y_u^4\right)\right.$$
$$\left.\left.+e^2 c_w^2 s_w^2\left(-6\lambda_1-3\lambda_3-3\lambda_4-2\lambda_5+2\left(\lambda_3+\lambda_4\right)s_w^2\right)+6e^4\left(2s_w^4-4s_w^2+3\right)\right)\right) \tag{5.136}$$

$$R_2^{h_1h_2h_2} = \frac{-iv}{256\pi^2 c_w^4 s_w^4}\left(c_1^3\left(3e^4\left(2s_w^4-4s_w^2+3\right)-144c_w^4 s_w^4\left(G_d^2 y_d^2+G_u^2 y_u^2\right)\right.\right.$$
$$\left.+2e^2 c_w^2 s_w^2\left(-3\lambda_3-3\lambda_4-2\lambda_5+2\left(\lambda_3+\lambda_4\right)s_w^2\right)\right)+6c_1^2 s_1 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d^3 y_d-2G_d y_d^3-2G_u y_u^3+G_u^3 y_u\right)\right.$$
$$\left.+e^2\left(2\lambda_6-\lambda_7\right)\left(2s_w^2-3\right)\right)+c_1 s_1^2\left(144c_w^4 s_w^4\left(2G_d^2 y_d^2-y_d^4+2G_u^2 y_u^2-y_u^4\right)+4e^2 s_w^2\left(\left(2\lambda_5-3\lambda_1\right)c_w^2\right.\right.$$
$$\left.\left.\left.+\left(\lambda_3+\lambda_4\right)\left(2s_w^4-5s_w^2+3\right)\right)+3e^4\left(2s_w^4-4s_w^2+3\right)\right)+6s_1^3 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d y_d^3+G_u y_u^3\right)+e^2\lambda_6\left(3-2s_w^2\right)\right)\right) \tag{5.137}$$

$$R_2^{h_2h_2h_2} = \frac{3iv}{256\pi^2 c_w^4 s_w^4}\left(2c_1^3 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d^3 y_d+G_u^3 y_u\right)+e^2\lambda_7\left(3-2s_w^2\right)\right)\right.$$
$$+c_1^2 s_1\left(144c_w^4 s_w^4\left(G_d^2 y_d^2+G_u^2 y_u^2\right)-3e^4\left(2s_w^4-4s_w^2+3\right)+2e^2 c_w^2 s_w^2\left(3\lambda_3+3\lambda_4+2\lambda_5-2\left(\lambda_3+\lambda_4\right)s_w^2\right)\right)$$
$$+6c_1 s_1^2 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d y_d^3+G_u y_u^3\right)+e^2\lambda_6\left(3-2s_w^2\right)\right)$$
$$\left.+s_1^3\left(48c_w^4 s_w^4\left(y_d^4+y_u^4\right)+4e^2\lambda_1 c_w^2 s_w^2-3e^4\left(2s_w^4-4s_w^2+3\right)\right)\right) \tag{5.138}$$

$$R_2^{h_1h_3h_3} = \frac{iv}{256\pi^2 c_w^4 s_w^4}\left(c_1\left(48c_w^4 s_w^4\left(G_d^2 y_d^2+G_u^2 y_u^2\right)-3e^4\left(2s_w^4-4s_w^2+3\right)\right.\right.$$
$$\left.\left.-2e^2 c_w^2 s_w^2\left(-3\lambda_3-3\lambda_4+2\lambda_5+2\left(\lambda_3+\lambda_4\right)s_w^2\right)\right)-2s_1 c_w^2 s_w^2\left(24c_w^2 s_w^2\left(G_d^3 y_d+G_u^3 y_u\right)+e^2\lambda_7\left(3-2s_w^2\right)\right)\right) \tag{5.139}$$

$$R_2^{G_0h_1h_3} = \frac{-iv}{128\pi^2 c_w^2 s_w^2}\left(2s_1\left(12c_w^2 s_w^2\left(G_d^2 y_d^2+G_u^2 y_u^2\right)+e^2\lambda_5\right)+c_1\left(e^2\lambda_6\left(2s_w^2-3\right)-24c_w^2 s_w^2\left(G_d y_d^3+G_u y_u^3\right)\right)\right), \tag{5.140}$$

where we remind that the vertices with a single h_2 and no h_1 are obtained by replacing $c_1 \to s_1$ and $s_1 \to -c_1$ in the vertices with a single h_1 and no h_2.

Finally the R_2 vertices between two scalar and two vector fields are

$$R_2^{AW^\pm H^\mp G_0} = \pm\frac{e^2\eta_{\mu_1,\mu_2}\left(3G_d y_d+2G_u y_u\right)}{32\pi^2 s_w} \tag{5.141}$$

$$R_2^{ZW^\pm H^\mp G_0} = \mp\frac{e^2\eta_{\mu_1,\mu_2}\left(3G_d y_d+2G_u y_u\right)}{32\pi^2\sqrt{c_w^2}} \tag{5.142}$$

$$R_2^{AAH^\mp G^\pm} = \mp\frac{5e^2\eta_{\mu_1,\mu_2}\left(G_d y_d+G_u y_u\right)}{12\pi^2} \tag{5.143}$$

$$R_2^{ZAH^\mp G^\pm} = \pm\frac{e^2\eta_{\mu_1,\mu_2}\left(G_d y_d\left(40s_w^2-21\right)+2G_u\left(20s_w^2-9\right)y_u\right)}{96\pi^2\sqrt{c_w^2}s_w} \tag{5.144}$$

$$R_2^{ZZH^\mp G^\pm} = \mp\frac{e^2\eta_{\mu_1,\mu_2}\left(G_d y_d\left(20s_w^4-21s_w^2+6\right)+2G_u\left(10s_w^4-9s_w^2+3\right)y_u\right)}{48\pi^2 c_w^2 s_w^2} \tag{5.145}$$

$$R_2^{W^+W^-H^\mp G^\pm} = \mp\frac{e^2\eta_{\mu_1,\mu_2}\left(G_d y_d + G_u y_u\right)}{8\pi^2 s_w^2} \tag{5.146}$$

$$R_2^{AAH^-H^+} = -\frac{ie^2\eta_{\mu_1,\mu_2}\left(16c_w^2 s_w^2\left(5R^d + 2R^l + 5R^u\right) + e^2\left(22 - 21s_w^2\right)\right)}{192\pi^2 c_w^2 s_w^2} \tag{5.147}$$

$$R_2^{AZH^-H^+} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{384\pi^2\left(c_w^2\right)^{3/2}s_w^3}\left(e^2\left(42s_w^4 - 74s_w^2 + 31\right) + 4c_w^2 s_w^2\left(\left(21 - 40s_w^2\right)R^d + \left(5 - 16s_w^2\right)R^l + 2\left(9 - 20s_w^2\right)R^u\right)\right) \tag{5.148}$$

$$\begin{aligned} R_2^{ZZH^-H^+} = & -\frac{ie^2\eta_{\mu_1,\mu_2}}{768\pi^2 c_w^4 s_w^4}\left(16c_w^2 s_w^2\left(\left(20s_w^4 - 21s_w^2 + 6\right)R^d + \left(8s_w^4 - 5s_w^2 + 2\right)R^l + 2\left(10s_w^4 - 9s_w^2 + 3\right)R^u\right)\right. \\ & \left. + e^2\left(-84s_w^6 + 208s_w^4 - 162s_w^2 + 39\right)\right) \end{aligned} \tag{5.149}$$

$$R_2^{W^+W^-H^-H^+} = -\frac{ie^2\eta_{\mu_1,\mu_2}\left(32c_w^2 s_w^2\left(3R^d + R^l + 3R^u\right) + e^2\left(39 - 38s_w^2\right)\right)}{768\pi^2 c_w^2 s_w^4} \tag{5.150}$$

$$R_2^{AW^\pm H^\mp h_1} = \frac{ie^2\eta_{\mu_1,\mu_2}}{768\pi^2 c_w^2 s_w^3}\left(e^2 s_1\left(1 + 22c_w^2\right) + 8c_w^2 s_w^2\left(s_1\left(9R^d + R^l + 6R^u\right) - 3c_1\left(3G_b y_b + 2G_t y_t\right)\right)\right) \tag{5.151}$$

$$R_2^{ZW^\pm H^\mp h_1} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{768\pi^2\left(c_w^2\right)^{3/2}s_w^2}\left(e^2 s_1\left(22c_w^2 + 1\right) + 8c_w^2 s_w^2\left(s_1\left(9R^d + R^l + 6R^u\right) - 3c_1\left(3G_b y_b + 2G_t y_t\right)\right)\right) \tag{5.152}$$

$$R_2^{AAG_0h_3} = -\frac{ie^2\eta_{\mu_1,\mu_2}\left(G_b y_b + 4G_t y_t\right)}{24\pi^2} \tag{5.153}$$

$$R_2^{ZAG_0h_3} = \frac{ie^2\eta_{\mu_1,\mu_2}\left(G_b y_b\left(4s_w^2 - 3\right) + 2G_t\left(8s_w^2 - 3\right)y_t\right)}{96\pi^2\sqrt{c_w^2}s_w} \tag{5.154}$$

$$R_2^{ZZG_0h_3} = -\frac{ie^2\eta_{\mu_1,\mu_2}\left(G_b y_b\left(2s_w^4 - 3s_w^2 + 6\right) + 2G_t\left(4s_w^4 - 3s_w^2 + 3\right)y_t\right)}{48\pi^2 c_w^2 s_w^2} \tag{5.155}$$

$$R_2^{W^+W^-G_0h_3} = -\frac{ie^2\eta_{\mu_1,\mu_2}\left(G_b y_b + G_t y_t\right)}{8\pi^2 s_w^2} \tag{5.156}$$

$$R_2^{AW^\pm G^\mp h_3} = \frac{ie^2\eta_{\mu_1,\mu_2}\left(3G_b y_b + 2G_t y_t\right)}{32\pi^2 s_w} \tag{5.157}$$

$$R_2^{ZW^\pm G^\mp h_3} = -\frac{ie^2\eta_{\mu_1,\mu_2}\left(3G_b y_b + 2G_t y_t\right)}{32\pi^2\sqrt{c_w^2}} \tag{5.158}$$

$$R_2^{AW^\pm G^\mp h_3} = \pm\frac{e^2\eta_{\mu_1,\mu_2}\left(8c_w^2 s_w^2\left(9R^d + R^l + 6R^u\right) + e^2\left(23 - 22s_w^2\right)\right)}{768\pi^2 c_w^2 s_w^3} \tag{5.159}$$

$$R_2^{ZW^\pm G^\mp h_3} = \mp\frac{e^2\eta_{\mu_1,\mu_2}\left(8c_w^2 s_w^2\left(9R^d + R^l + 6R^u\right) + e^2\left(23 - 22s_w^2\right)\right)}{768\pi^2\left(c_w^2\right)^{3/2}s_w^2} \tag{5.160}$$

$$R_2^{AAh_3h_3} = \frac{ie^2\eta_{\mu_1,\mu_2}\left(e^2 - 8s_w^2\left(R^d + 3R^l + 4R^u\right)\right)}{192\pi^2 s_w^2} \tag{5.161}$$

$$R_2^{ZAh_3h_3} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{192\pi^2\sqrt{c_w^2}s_w^3}\left(e^2\left(s_w^2 + 4\right) + 2s_w^2\left(\left(3 - 4s_w^2\right)R^d + \left(3 - 12s_w^2\right)R^l + 2\left(3 - 8s_w^2\right)R^u\right)\right) \tag{5.162}$$

$$\begin{aligned} R_2^{ZZh_3h_3} = & -\frac{ie^2\eta_{\mu_1,\mu_2}}{768\pi^2 c_w^4 s_w^4}\left(16c_w^2 s_w^2\left(\left(2s_w^4 - 3s_w^2 + 6\right)R^d + \left(6s_w^4 - 3s_w^2 + 2\right)R^l + 2\left(4s_w^4 - 3s_w^2 + 3\right)R^u\right)\right. \\ & \left. + e^2\left(4s_w^6 + 28s_w^4 - 70s_w^2 + 39\right)\right) \end{aligned} \tag{5.163}$$

$$R_2^{W^+W^-h_3h_3} = -\frac{ie^2\eta_{\mu_1,\mu_2}\left(32c_w^2 s_w^2\left(3R^d + R^l + 3R^u\right) + e^2\left(39 - 38s_w^2\right)\right)}{768\pi^2 c_w^2 s_w^4} \tag{5.164}$$

$$R_2^{W^\mp AG^\pm h_1} = \pm\frac{e^2\eta_{\mu_1,\mu_2}}{768\pi^2 c_w^2 s_w^3}\left(c_1\left(e^2\left(22s_w^2 - 23\right) - 24c_w^2 s_w^2\left(3y_b^2 + 2y_t^2\right)\right) + 24s_1 c_w^2 s_w^2\left(3G_b y_b + 2G_t y_t\right)\right) \tag{5.165}$$

$$R_2^{W^\mp ZG^\pm h_1} = \pm\frac{e^2\eta_{\mu_1,\mu_2}}{768\pi^2\left(c_w^2\right)^{3/2}s_w^2}\left(c_1\left(24c_w^2 s_w^2\left(3y_b^2 + 2y_t^2\right) + e^2\left(23 - 22s_w^2\right)\right) - 24s_1 c_w^2 s_w^2\left(3G_b y_b + 2G_t y_t\right)\right) \tag{5.166}$$

$$\begin{aligned} R_2^{W^+W^-h_1h_1} = & \frac{ie^2\eta_{\mu_1,\mu_2}}{3072\pi^2 c_w^2 s_w^4}\left(-128c_w^2 s_w^2 c_1\left(s_1^2\left(3R^d + R^l + 3R^u\right)\right.\right. \\ & \left.\left. + 3\left(-2s_1 G_b y_b + c_1 y_b^2 - 2s_1 G_t y_t + c_1 y_t^2\right)\right) + e^2\left(152s_w^2 - 156\right)\right) \end{aligned} \tag{5.167}$$

$$R_2^{ZZh_1h_1} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{768\pi^2c_w^4s_w^4}\left(16s_w^2\left(c_1y_b\left(2s_w^4-3s_w^2+6\right)\left(c_1y_bc_w^2-2s_1G_bc_w^2\right)\right.\right.$$
$$+\ s_1^2c_w^2\left(2s_w^4-3s_w^2+6\right)R^d+s_1^2c_w^2\left(6s_w^4-3s_w^2+2\right)R^l+2s_1^2c_w^2\left(4s_w^4-3s_w^2+3\right)R^u+2c_1^2c_w^2\left(4s_w^4-3s_w^2+3\right)y_t^2$$
$$\left.\left.+\ 2s_{21}G_t\left(4s_w^6-7s_w^4+6s_w^2-3\right)y_t\right)+e^2\left(4s_w^6+28s_w^4-70s_w^2+39\right)\right) \quad (5.168)$$

$$R_2^{ZAh_1h_1} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{192\pi^2\sqrt{c_w^2}s_w^3}\left(2s_w^2\left(2s_1^2\left(3-8s_w^2\right)R^u+s_1^2\left(3-4s_w^2\right)R^d+3s_1^2\left(1-4s_w^2\right)R^l+c_1y_b\left(4s_w^2-3\right)\left(2s_1G_b-c_1y_b\right)\right.\right.$$
$$\left.\left.+\ 2c_1\left(3-8s_w^2\right)y_t\left(c_1y_t-2s_1G_t\right)\right)+e^2\left(s_w^2+4\right)\right) \quad (5.169)$$

$$R_2^{AAh_1h_1} = \frac{ie^2\eta_{\mu_1,\mu_2}}{192\pi^2s_w^2}\left(e^2-8s_w^2\left(c_1^2y_b^2-s_{21}G_by_b+4c_1^2y_t^2-4s_{21}G_ty_t+s_1^2\left(R^d+3R^l+4R^u\right)\right)\right) \quad (5.170)$$

$$R_2^{W^+W^-h_1h_2} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{48\pi^2s_w^2}\left(6c_{21}\left(G_by_b+G_ty_t\right)-s_{21}\left(3R^d+R^l+3R^u-3y_b^2-3y_t^2\right)\right) \quad (5.171)$$

$$R_2^{ZAh_1h_2} = \frac{ie^2\eta_{\mu_1,\mu_2}}{192\pi^2\sqrt{c_w^2}s_w}\left(2c_{21}\left(G_by_b\left(4s_w^2-3\right)+2G_t\left(8s_w^2-3\right)y_t\right)+s_{21}\left(4s_w^2\left(y_b^2-3R^l-4R^u+4y_t^2-R^d\right)\right.\right.$$
$$\left.\left.+\ 3\left(R^d-y_b^2+R^l+2R^u-2y_t^2\right)\right)\right) \quad (5.172)$$

$$R_2^{ZZh_1h_2} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{96\pi^2c_w^2s_w^2}\left(s_{21}\left(y_b^2\left(2s_w^4-3s_w^2+6\right)+2\left(4s_w^4-3s_w^2+3\right)y_t^2+\left(6s_w^2-6-8s_w^4\right)R^u+\left(-2s_w^4+3s_w^2-6\right)R^d\right.\right.$$
$$\left.\left.+\ \left(-6s_w^4+3s_w^2-2\right)R^l\right)+2c_{21}\left(G_by_b\left(2s_w^4-3s_w^2+6\right)+2G_ty_t\left(4s_w^4-3s_w^2+3\right)\right)\right) \quad (5.173)$$

$$R_2^{AAh_1h_2} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{48\pi^2}\left(2c_{21}\left(G_by_b+4G_ty_t\right)+s_{21}\left(y_b^2-R^d-3R^l-4R^u+4y_t^2\right)\right) \quad (5.174)$$

$$R_2^{W^+W^-h_2h_2} = \frac{ie^2\eta_{\mu_1,\mu_2}}{3072\pi^2c_w^2s_w^4}\left(e^2\left(152s_w^2-156\right)-128c_w^2s_w^2\left(c_1^2\left(3R^d+R^l\right.\right.\right.$$
$$\left.\left.\left.+\ 3R^u\right)+3s_1\left(2c_1\left(G_by_b+G_ty_t\right)+s_1\left(y_b^2+y_t^2\right)\right)\right)\right) \quad (5.175)$$

$$R_2^{ZZh_2h_2} = -\frac{ie^2\eta_{\mu_1,\mu_2}}{768\pi^2c_w^4s_w^4}\left(16s_w^2\left(2s_{21}G_t\left(-4s_w^6+7s_w^4-6s_w^2+3\right)y_t-s_1y_b\left(2s_w^4-3s_w^2+6\right)\left(-2c_1G_bc_w^2-s_1y_bc_w^2\right)\right.\right.$$
$$+\ 2s_1^2c_w^2\left(4s_w^4-3s_w^2+3\right)y_t^2\Big)+16c_1^2c_w^2s_w^2\left[\left(2s_w^4-3s_w^2+6\right)R^d+\left(6s_w^4-3s_w^2+2\right)R^l+2\left(4s_w^4-3s_w^2+3\right)R^u\right]$$
$$\left.+e^2\left(4s_w^6+28s_w^4-70s_w^2+39\right)\right) \quad (5.176)$$

$$R_2^{ZAh_2h_2} = \frac{-ie^2\eta_{\mu_1,\mu_2}}{192\pi^2c_ws_w^3}\left(2s_w^2\left(s_1\left(y_b\left(3-4s_w^2\right)\left(2c_1G_b+s_1y_b\right)+2y_t\left(2c_1G_t+s_1y_t\right)\left(3-8s_w^2\right)\right)\right)+e^2\left(s_w^2+4\right)\right.$$
$$\left.+2c_1^2s_w^2\left[\left(3-4s_w^2\right)R^d+3\left(1-4s_w^2\right)R^l+2\left(3-8s_w^2\right)R^u\right]\right) \quad (5.177)$$

$$R_2^{AAh_2h_2} = \frac{ie^2\eta_{\mu_1,\mu_2}}{192\pi^2s_w^2}\left(e^2-8s_w^2\left(s_1\left(2c_1\left(G_by_b+4G_ty_t\right)+s_1\left(y_b^2+4y_t^2\right)\right)+c_1^2\left(R^d+3R^l+4R^u\right)\right)\right), \quad (5.178)$$

where the vertices with two Goldstone bosons have been omitted since they are identical to the SM ones. The four scalars R_2 vertices have not been computed due to their size and their low phenomenological relevance.

6. 2HDM UV counterterms

6.1. QCD corrections

As for the R_2 terms, only the UV vertices which differ from the SM are shown in this section. The first two fermion flavors have been assumed again to be massless. Only the interactions between the quarks and the scalars have new UV divergences,

$$UV^{\bar{d}uH^-} = \frac{ig_s^2}{4\pi^2\bar{\epsilon}}\delta_{i_1i_2}\left[\left(G^{d\dagger}V^{CKM\dagger}\right)_{f_1f_2}\gamma_- - \left(V^{CKM\dagger}G^u\right)_{f_1f_2}\gamma_+\right] \quad (6.179)$$

$$UV^{\bar{b}uH^-} = \frac{ig_s^2}{24\pi^2}\delta_{i_1i_2}\left(\frac{9}{\bar{\epsilon}}+4-6\log\frac{M_b}{\mu}\right)\left[\left(G^{d\dagger}V^{CKM\dagger}\right)_{3f_2}\gamma_- - \left(V^{CKM\dagger}G^u\right)_{3f_2}\gamma_+\right] \quad (6.180)$$

$$UV^{\bar{d}tH^-} = \frac{ig_s^2}{24\pi^2}\delta_{i_1i_2}\left(\frac{9}{\bar{\epsilon}}+4-6\log\frac{M_t}{\mu}\right)\left[\left(G^{d\dagger}V^{CKM\dagger}\right)_{f_13}\gamma_- - \left(V^{CKM\dagger}G^u\right)_{f_13}\gamma_+\right] \quad (6.181)$$

$$UV^{\bar{b}tH^-} = \frac{ig_s^2}{12\pi^2}\delta_{i_1i_2}\left(\frac{6}{\bar{\epsilon}}+4-3\log\frac{M_b}{\mu}-3\log\frac{M_t}{\mu}\right)\left[\left(G^{d\dagger}V^{CKM\dagger}\right)_{3,3}\gamma_- - \left(V^{CKM\dagger}G^u\right)_{3,3}\gamma_+\right] \quad (6.182)$$

$$UV^{\bar{u}dH^+} = \frac{ig_s^2}{4\pi^2\bar{\epsilon}}\delta_{i_1 i_2}\left[\left(V^{CKM}G^d\right)_{f_1 f_2}\gamma_+ - \left(G^{u\dagger}V^{CKM}\right)_{f_1 f_2}\gamma_-\right] \tag{6.183}$$

$$UV^{\bar{u}uh} = \frac{ig_s^2}{4\sqrt{2}\pi^2\bar{\epsilon}}\delta_{i_1 i_2}\left(G^u_{f_1 f_2}\left(T_{2s_3} - iT_{3s_3}\right)\gamma_+ + {G^u_{f_1 f_2}}^{\dagger}\left(T_{2s_3} + iT_{3s_3}\right)\gamma_-\right) \tag{6.184}$$

$$\begin{aligned} UV^{\bar{t}th} = {} & \frac{ig_s^2}{6\sqrt{2}\pi^2}\delta_{i_1 i_2}\Big[y_t T_{1s_3}\left(\gamma_+ + \gamma_-\right)\left(\frac{3}{\bar{\epsilon}} + 2 - 3\log\frac{M_t}{\mu}\right) \\ & + \left(G_t\left(T_{2s_3} - iT_{3s_3}\right)\gamma_+ + {G_t}^{\dagger}\left(T_{2s_3} + iT_{3s_3}\right)\gamma_-\right)\left(\frac{3}{\bar{\epsilon}} + 4 - 6\log\frac{M_t}{\mu}\right)\Big] \end{aligned} \tag{6.185}$$

$$UV^{\bar{d}dh} = \frac{ig_s^2}{4\sqrt{2}\pi^2\bar{\epsilon}}\delta_{i_1 i_2}\left(G^d_{f_1 f_2}\left(T_{2s_3} + iT_{3s_3}\right)\gamma_+ + G^{d\dagger}_{f_1 f_2}\left(T_{2s_3} - iT_{3s_3}\right)\gamma_-\right) \tag{6.186}$$

$$\begin{aligned} UV^{\bar{b}bh} = {} & \frac{ig_s^2}{6\sqrt{2}\pi^2}\delta_{i_1 i_2}\Big[y_b T_{1s_3}\left(\gamma_+ + \gamma_-\right)\left(\frac{3}{\bar{\epsilon}} + 2 - 3\log\frac{M_b}{\mu}\right) \\ & + \left(G_b\left(T_{2s_3} + iT_{3s_3}\right)\gamma_+ + G_b^{\dagger}\left(T_{2s_3} - iT_{3s_3}\right)\gamma_-\right)\left(\frac{3}{\bar{\epsilon}} + 4 - 6\log\frac{M_b}{\mu}\right)\Big], \end{aligned} \tag{6.187}$$

where u and d are here generic label for the massless quarks. The same factors should be applied in front of the square brackets to the vertices with the positively charged scalar as for the negatively charged scalar when one or both fermions are massive. The same factor should also be applied to the vertices with the neutral scalar as for the negatively charged scalar when one fermion is massive. The extra parts of these factors come from the renormalization of the wave function of the massive fermions in the on-shell scheme [27]

$$\delta Z^{L/R}_{t/b} = -\frac{g_s^2}{12\pi^2}\left(\frac{1}{\bar{\epsilon}_{UV}} + \frac{2}{\bar{\epsilon}_{IR}} + 4 - 6\log\frac{M_{t/b}}{\mu}\right). \tag{6.188}$$

On the contrary no such factor appear for the massless fermions since the UV and IR poles cancel for the massless fermions [27]

$$\delta Z^{L/R}_{q} = -\frac{g_s^2}{12\pi^2}\left(\frac{1}{\bar{\epsilon}_{UV}} - \frac{1}{\bar{\epsilon}_{IR}}\right) \tag{6.189}$$

and therefore their wave function renormalization constants vanish. In both expressions, we have add the subscript *UV* and *IR* to emphasize that the on-shell scheme does not only change the finite part of the counterterms compared to the $\overline{MS}$ but also affects the poles due to the infrared divergences. The remaining part of the poles for the quark–quark–scalar vertices are purely of UV origin. For the interaction between the neutral scalars and two massive quarks, the renormalization of the quark masses changes also the renormalization of the Yukawa couplings y_f since they are directly related by

$$\delta y_{t/b} = \sqrt{2}\frac{\delta M_{t/b}}{v} \tag{6.190}$$

with [27]

$$\delta M_{t/b} = -\frac{g_s^2}{12\pi^2}M_{t/b}\left(\frac{3}{\bar{\epsilon}_{UV}} + 4 - 6\log\frac{M_{t/b}}{\mu}\right). \tag{6.191}$$

6.2. EW corrections

The electroweak contributions to the UV counterterm vertices have been computed assuming CP and flavor symmetries as for the R_2 vertices. Furthermore, we have assumed that none of the masses are accidentally equal or vanishes and the complex mass scheme is used. The electroweak corrections involve many more particles with different masses than the QCD corrections. As a result, the finite parts of the two-point functions probe almost all the mass hierarchies and therefore all the expressions of the b_0 functions. For example, the top mass renormalization constant gets new contributions with the charged scalar and the bottom quark in the loop as well as with all the new neutral scalars and the top quark itself,

$$\begin{aligned} \delta M_t = {} & \delta M_t^{SM/H} + \frac{1}{64\pi^2 M_t^3}\Bigg[\Bigg\{(s_1 G_t - c_1 y_t)^2\Bigg(\left(4M_t^2 - m_{h_1}^2\right)\mathrm{l}\left(M_t, m_{h_1}, M_t\right) \\ & + m_{h_1}^4\log\frac{m_{h_1}}{M_t} + M_t^4\left(4\log\frac{m_{h_1}}{M_t} - 4\log\frac{m_{h_1}M_t}{\mu^2} + 2\log\frac{M_t}{\mu} + \frac{3}{\bar{\epsilon}} + 7\right) \\ & - m_{h_1}^2 M_t^2\left(7\log\frac{m_{h_1}}{M_t} - \log\frac{m_{h_1}M_t}{\mu^2} + 2\log\frac{M_t}{\mu} + 1\right)\Bigg) + h_1 \to h_2, c_1 \to s_1, s_1 \to -c_1\Bigg\} \\ & + G_t^2\Bigg(m_{h_3}^4\log\frac{m_{h_3}}{M_t} - m_{h_3}^2\mathrm{l}\left(M_t, m_{h_3}, M_t\right) - M_t^4\left(\frac{1}{\bar{\epsilon}} - 2\log\frac{M_t}{\mu} + 1\right) \\ & - m_{h_3}^2 M_t^2\left(3\log\frac{m_{h_3}}{M_t} - \log\frac{m_{h_3}M_t}{\mu^2} + 2\log\frac{M_t}{\mu} + 1\right)\Bigg) \end{aligned}$$

$$
\begin{aligned}
&- 4G_bM_bG_tM_t\left(\mathrm{l}\left(M_b, m_{H^+}, M_t\right) + \left(M_b^2 - m_{H^+}^2\right)\log\frac{m_{H^+}}{M_b} + M_t^2\left(\frac{1}{\bar{\epsilon}} + 2 - \log\frac{M_b m_{H^+}}{\mu^2}\right)\right)\\
&+ \left(G_b^2 + G_t^2\right)\left(\left(-m_{H^+}^2 + M_b^2 + M_t^2\right)\mathrm{l}\left(M_b, m_{H^+}, M_t\right) + M_t^4\left(\frac{1}{\bar{\epsilon}} - \log\frac{M_b m_{H^+}}{\mu^2} + 2\right) + \left(m_{H^+}^2 - M_b^2\right)^2\log\frac{m_{H^+}}{M_b}\right.\\
&\left.+ M_t^2M_b^2\left(1 - \log\frac{M_b m_{H^+}}{\mu^2} + \log\frac{m_{H^+}}{\mu} + \log\frac{M_b}{\mu}\right) - M_t^2m_{H^+}^2\left(1 - \log\frac{M_b m_{H^+}}{\mu^2} + 3\log\frac{m_{H^+}}{M_b} + 2\log\frac{M_b}{\mu}\right)\right)\Bigg], \qquad (6.192)
\end{aligned}
$$

where $\delta M_t^{SM/H}$ is the SM renormalization constant without the physical Higgs contribution[8] but keeping those of the Goldstone bosons, and

$$
\begin{aligned}
\mathrm{l}\left(m_1, m_2, m_3\right) &= \sqrt{\left(m_1^4 + \left(m_2^2 - m_3^2\right)^2 - 2m_1^2\left(m_2^2 + m_3^2\right)\right)}\\
&\times \log\left(\frac{m_1^2 + m_2^2 - m_3^2 + \sqrt{\left(m_1^4 + \left(m_2^2 - m_3^2\right)^2 - 2m_1^2\left(m_2^2 + m_3^2\right)\right)}}{2m_1m_2}\right). \qquad (6.193)
\end{aligned}
$$

Only the real part of this l function should be taken if the on-shell scheme was used instead since the other terms are real when the masses are real. Moreover, the new fields two-point functions are corrected only by the electroweak interactions. For example, the wave function renormalization constant for the physical charged scalar is given by

$$
\begin{aligned}
\delta Z_{H^+H^+} &= \frac{1}{16\pi^2}\Bigg[-\frac{e^2c_{2w}^2}{4c_w^2s_w^2m_{H^+}^4}\left(2m_{H^+}^4\left(\log\frac{M_Z m_{H^+}}{\mu^2} - \frac{1}{\bar{\epsilon}}\right) - m_{H^+}^2M_Z^2\right.\\
&\left.+ \left(m_{H^+}^2 - M_Z^2\right)\left(\mathrm{l}\left(m_{H^+}, M_Z, m_{H^+}\right) + \left(2m_{H^+}^2 - M_Z^2\right)\log\frac{M_Z}{m_{H^+}}\right)\right)\\
&+ \left\{\frac{e^2s_1^2}{4s_w{}^2m_{H^+}^4}\left(\frac{\mathrm{l}\left(M_W, m_{h_1}, m_{H^+}\right)}{\left(\left(m_{h_1} - M_W\right)^2 - m_{H^+}^2\right)\left(\left(m_{h_1} + M_W\right)^2 - m_{H^+}^2\right)}\right.\right.\\
&\times \left(M_W^2\left(m_{h_1}^2m_{H^+}^2 - 2m_{H^+}^4 + 5m_{h_1}^4\right) - 2\left(m_{h_1}^2 - m_{H^+}^2\right)\left(m_{H^+}^4 + m_{h_1}^4\right)\right.\\
&+ M_W^6 - M_W^4\left(m_{H^+}^2 + 4m_{h_1}^2\right)) + 2m_{H^+}^4\left(\frac{1}{\bar{\epsilon}} + 1 - \log\frac{M_W m_{h_1}}{\mu^2}\right)\\
&+ m_{H^+}^2\left(M_W^2 - 2m_{h_1}^2\right) + \log\frac{m_{h_1}}{M_W}\left(M_W^4 - 3m_{h_1}^2M_W^2 + 2m_{h_1}^4\right)\Bigg) + \frac{v^2\left(\lambda_4s_1 - 2c_1\lambda_6\right)^2}{4m_{H^+}^4}\left(\left(\left(m_{h_1}^2 - M_W^2\right)\log\frac{M_W}{m_{h_1}} + m_{H^+}^2\right)\right.\\
&\left.\left.+ \frac{M_W^4 - m_{h_1}^2\left(m_{H^+}^2 + 2M_W^2\right) - M_W^2m_{H^+}^2 + m_{h_1}^4}{m_{h_1}^4 - 2m_{h_1}^2\left(m_{H^+}^2 + M_W^2\right) + \left(m_{H^+}^2 - M_W^2\right)^2}\mathrm{l}\left(M_W, m_{h_1}, m_{H^+}\right)\right)\right.\\
&\left.+ h_1 \to h_2, c_1 \to s_1, s_1 \to -c_1 + h_1 \to h_3, c_1 \to 0, s_1 \to 1\right\}\\
&+ \left\{\frac{v^2\left(c_1\lambda_3 - \lambda_7s_1\right)^2}{m_{H^+}^4}\left(\left(\left(m_{H^+}^2 - m_{h_1}^2\right)\log\frac{m_{h_1}}{m_{H^+}} - m_{H^+}^2\right)\right.\right.\\
&\left.\left.- \frac{m_{h_1}^2 - 3m_{H^+}^2}{4m_{H^+}^2 - m_{h_1}^2}\mathrm{l}\left(m_{H^+}, m_{h_1}, m_{H^+}\right)\right) + h_1 \to h_2, c_1 \to s_1, s_1 \to -c_1\right\}\\
&- \sum_l G_l^2\left(-\log\left(\frac{m_{H^+}^2}{\mu^2}\right) + \frac{1}{\bar{\epsilon}} + i\pi + 1\right) - 3\sum_{light}\left(G_d^2 + G_u^2\right)\left(-\log\left(\frac{m_{H^+}^2}{\mu^2}\right) + \frac{1}{\bar{\epsilon}} + i\pi + 1\right)\\
&- \frac{12G_bG_tM_bM_t}{m_{H^+}^4}\left(\frac{M_b^2\left(m_{H^+}^2 + 2M_t^2\right) + M_t^2m_{H^+}^2 - M_b^4 - M_t^4}{-2M_b^2\left(m_{H^+}^2 + M_t^2\right) + \left(m_{H^+}^2 - M_t^2\right)^2 + M_b^4}\mathrm{l}\left(M_t, M_b, m_{H^+}\right) - \left(m_{H^+}^2 - \left(M_b^2 - M_t^2\right)\log\frac{M_b}{M_t}\right)\right)\\
&- \frac{3\left(G_b^2 + G_t^2\right)}{m_{H^+}^4}\left(\frac{\mathrm{l}\left(M_t, M_b, m_{H^+}\right)}{\left(M_b^2 - M_t^2\right)^2 - 2m_{H^+}^2\left(M_b^2 + M_t^2\right) + m_{H^+}^4}\right.\\
&\times \left(\left(M_b^2 + M_t^2\right)\left(\left(M_t^2 - M_b^2\right)^2 - m_{H^+}^2\left(M_b^2 + M_t^2\right) - m_{H^+}^4\right) + m_{H^+}^6\right)\\
&\left.+ \left(M_b^2 + M_t^2\right)\left(m_{H^+}^2 + \left(M_b^2 - M_t^2\right)\log\frac{M_t}{M_b}\right) + m_{H^+}^4\left(\frac{1}{\bar{\epsilon}} + 1 - \log\left(\frac{M_bM_t}{\mu^2}\right)\right)\right)\Bigg]. \qquad (6.194)
\end{aligned}
$$

[8] The Higgs contribution in the SM is obtained by replacing $\theta_1 \to 0$ and $m_{h_1} \to m_H$ in the contribution of h_1.

Finally, electroweak corrections mix different fields. The photon-Z mixing receives an extra contribution from the physical charged scalar

$$\delta Z_{AZ} = \delta Z_{AZ}^{SM} - \frac{e^2 c_{2w}}{288\pi^2 c_w M_Z^2 s_w}\left(3\left(4m_{H^+}^2 - M_Z^2\right)\mathrm{l}\left(m_{H^+}, m_{H^+}, M_Z\right) + 24M_Z^2 m_{H^+}^2 - M_Z^4\left(\frac{3}{\bar{\epsilon}} - 6\log\frac{m_{H^+}}{\mu} + 8\right)\right). \quad (6.195)$$

Contrary to the *UV* QCD vertices, only those few illustrative examples of electroweak UV counterterms are displayed due to the size of the expressions. The full list can be found on the FeynRules 2HDM web page [34].

7. Conclusion

The counterterm vertices for any model with a renormalizable Lagrangian, namely with all the operators of dimension four or less, can be obtained automatically using three packages. At this stage, the renormalizability of the model is assumed by the packages and is not checked. The model should be available in FeynRules and the Feynman gauge should be implemented. The renormalization is performed into FeynRules while the UV and R_2 counterterm vertices are computed in the new NLOCT package using FeynArts. Those vertices can be loaded afterwards into FeynRules and exported in the UFO format to MadGraph5_aMC@NLO, GoSam or any other one-loop tool. Consequently, NLO computations are now fully automated for any renormalizable BSM model.

The code has been validated by comparing analytically the R_2 and UV counterterm vertices for the SM induced both by the strong and electroweak corrections to the expressions found in the literature. The R_2 vertices due to QCD for the MSSM have also been checked. Finally, the SM UFO with the QCD counterterms has been compared to the built in version of MadGraph5_aMC@NLO and found in perfect agreement.

We have presented the full QCD R_2 and UV counterterms for the generic 2HDM. The full list of R_2 terms but the four scalars vertices due to the electroweak interactions have also been given for the 2HDM with CP and flavor conservations. Only a few representative examples of the electroweak UV counterterms have been displayed due to the size of the expressions. However, the full electroweak UV counterterms have been obtained automatically using the method and the packages described in the previous sections and are publicly available on the FeynRules web site.

The extension of the package NLOCT to handle effective Lagrangian is in progress. This extension will additionally allow to compute the counterterm vertices in any gauge.

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References

[1] J. Butterworth, G. Dissertori, S. Dittmaier, D. de Florian, N. Glover, et al. Les Houches 2013: Physics at TeV Colliders: Standard Model Working Group Report. arXiv:1405.1067.
[2] G. Ossola, C.G. Papadopoulos, R. Pittau, Reducing full one-loop amplitudes to scalar integrals at the integrand level, Nucl.Phys. B 763 (2007) 147–169. http://dx.doi.org/10.1016/j.nuclphysb.2006.11.012. arXiv:hep-ph/0609007.
[3] R. Britto, F. Cachazo, B. Feng, Generalized unitarity and one-loop amplitudes in $N = 4$ super-Yang-Mills, Nucl.Phys. B 725 (2005) 275–305. http://dx.doi.org/10.1016/j.nuclphysb.2005.07.014. arXiv:hep-th/0412103.
[4] V. Hirschi, R. Frederix, S. Frixione, M.V. Garzelli, F. Maltoni, et al., Automation of one-loop QCD corrections, J. High Energy Phys. 1105 (2011) 044. http://dx.doi.org/10.1007/JHEP05(2011)044. arXiv:1103.0621.
[5] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, et al. The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations. arXiv:1405.0301.
[6] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, T. Stelzer, MadGraph 5 : Going Beyond, J. High Energy Phys. 1106 (2011) 128. http://dx.doi.org/10.1007/JHEP06(2011)128. arXiv:1106.0522.
[7] G. Ossola, C.G. Papadopoulos, R. Pittau, On the rational terms of the one-loop amplitudes, J. High Energy Phys. 0805 (2008) 004. http://dx.doi.org/10.1088/1126-6708/2008/05/004. arXiv:0802.1876.
[8] G. t Hooft, M. Veltman, Regularization and renormalization of gauge fields, Nucl.Phys. B 44 (1972) 189–213. http://dx.doi.org/10.1016/0550-3213(72)90279-9.
[9] D. Kreimer, The Role of gamma(5) in dimensional regularization. arXiv:hep-ph/9401354.
[10] J. Korner, D. Kreimer, K. Schilcher, A Practicable gamma(5) scheme in dimensional regularization, Z.Phys. C 54 (1992) 503–512. http://dx.doi.org/10.1007/BF01559471.
[11] D. Kreimer, The $\gamma(5)$ problem and anomalies: a clifford algebra approach, Phys.Lett. B 237 (1990) 59. http://dx.doi.org/10.1016/0370-2693(90)90461-E.
[12] P. Draggiotis, M. Garzelli, C. Papadopoulos, R. Pittau, Feynman rules for the rational part of the QCD 1-loop amplitudes, J. High Energy Phys. 0904 (2009) 072. http://dx.doi.org/10.1088/1126-6708/2009/04/072. arXiv:0903.0356.
[13] M. Garzelli, I. Malamos, R. Pittau, Feynman rules for the rational part of the electroweak 1-loop amplitudes, J. High Energy Phys. 1001 (2010) 040. http://dx.doi.org/10.1007/JHEP10(2010)097, http://dx.doi.org/10.1007/JHEP01(2010)040, arXiv:0910.3130.
[14] H.-S. Shao, Y.-J. Zhang, Feynman rules for the rational part of one-loop QCD corrections in the MSSM, J. High Energy Phys. 1206 (2012) 112. http://dx.doi.org/10.1007/s13130-012-4240-2. arXiv:1205.1273.
[15] M. Garzelli, I. Malamos, R2SM: A package for the analytic computation of the R_2 Rational terms in the standard model of the electroweak interactions, Eur.Phys.J. C 71 (2011) 1605. http://dx.doi.org/10.1140/epjc/s10052-011-1605-6. arXiv:1010.1248.
[16] A. Alloul, N.D. Christensen, C. Degrande, C. Duhr, B. Fuks, FeynRules 2.0—A complete toolbox for tree-level phenomenology. arXiv:1310.1921.
[17] T. Hahn, Generating Feynman diagrams and amplitudes with FeynArts 3, Comput. Phys. Commun. 140 (2001) 418–431. http://dx.doi.org/10.1016/S0010-4655(01)00290-9. arXiv:hep-ph/0012260.
[18] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, et al., UFO—the universal FeynRules output, Comput. Phys. Commun. 183 (2012) 1201–1214. http://dx.doi.org/10.1016/j.cpc.2012.01.022. arXiv:1108.2040.
[19] G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, et al., Automated one-loop calculations with GoSam, Eur. Phys. J. C 72 (2012) 1889. http://dx.doi.org/10.1140/epjc/s10052-012-1889-1. arXiv:1111.2034.
[20] A. Denner, S. Dittmaier, M. Roth, L. Wieders, Electroweak corrections to charged-current e^+e^- to 4 fermion processes: Technical details and further results, Nucl. Phys. B 724 (2005) 247–294. http://dx.doi.org/10.1016/j.nuclphysb.2011.09.001, http://dx.doi.org/10.1016/j.nuclphysb.2005.06.033, arXiv:hep-ph/0505042.
[21] http://feynrules.irmp.ucl.ac.be/wiki/StandardModel.
[22] A. Denner, Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200, Fortschr. Phys. 41 (1993) 307–420. arXiv:0709.1075.

[23] M. Bohm, A. Denner, H. Joos, Gauge theories of the strong and electroweak interaction.
[24] H. Burkhardt, B. Pietrzyk, Update of the hadronic contribution to the QED vacuum polarization, Phys. Lett. B 356 (1995) 398–403. http://dx.doi.org/10.1016/0370-2693(95)00820-B.
[25] S. Dittmaier, Kramer Michael, Electroweak radiative corrections to W boson production at hadron colliders, Phys. Rev. D 65 (2002) 073007. http://dx.doi.org/10.1103/PhysRevD.65.073007. arXiv:hep-ph/0109062.
[26] S. Eidelman, F. Jegerlehner, Hadronic contributions to g-2 of the leptons and to the effective fine structure constant alpha ($M(z) * *2$), Z. Phys. C 67 (1995) 585–602. http://dx.doi.org/10.1007/BF01553984. arXiv:hep-ph/9502298.
[27] W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper, M. Spira, et al., NLO QCD corrections to t anti-t H production in hadron collisions, Nucl. Phys. B 653 (2003) 151–203. http://dx.doi.org/10.1016/S0550-3213(03)00044-0. arXiv:hep-ph/0211352.
[28] G.C. Branco, L. Lavoura, J.P. Silva, CP violation, Int. Ser. Monogr. Phys. 103 (1999) 1–536.
[29] H.E. Haber, D. O'Neil, Basis-independent methods for the two-Higgs-doublet model. II. The Significance of tan beta, Phys. Rev. D 74 (2006) 015018. http://dx.doi.org/10.1103/PhysRevD.74.015018. arXiv:hep-ph/0602242.
[30] D. Eriksson, J. Rathsman, O. Stal, 2HDMC: two-higgs-doublet model calculator physics and manual, Comput. Phys. Commun. 181 (2010) 189–205. http://dx.doi.org/10.1016/j.cpc.2009.09.011. arXiv:0902.0851.
[31] T. Plehn, Charged Higgs boson production in bottom gluon fusion, Phys. Rev. D 67 (2003) http://dx.doi.org/10.1103/PhysRevD.67.014018. arXiv:hep-ph/0206121.
[32] E.L. Berger, T. Han, J. Jiang, T. Plehn, Associated production of a top quark and a charged Higgs boson, Phys. Rev. D 71 (2005) 115012. http://dx.doi.org/10.1103/PhysRevD.71.115012. arXiv:hep-ph/0312286.
[33] J.-M. Gerard, M. Herquet, A twisted custodial symmetry in the two-Higgs-doublet model, Phys. Rev. Lett. 98 (2007) 251802. http://dx.doi.org/10.1103/PhysRevLett.98.251802. arXiv:hep-ph/0703051.
[34] http://feynrules.irmp.ucl.ac.be/wiki/2HDM.