2.3.2 Regula Falsi (False Position) method

This method, same as the Newton one, is based on approximating f by a quadratic polynomial, but here this polynomial is constructed via two working points with first-order information rather than via a single working point with second-order information. The method, in its most straightforward form, is as follows. Given two latest iterates x_{t-1}, x_{t-2}, along with the values f and f' at these iterates, we approximate $f''(x_{t-1})$ by the finite difference

$$\frac{f'(x_{t-1}) - f'(x_{t-2})}{x_{t-1} - x_{t-2}}$$

and use this approximation to approximate f by a quadratic function

$$p(x) = f(x_{t-1}) + f'(x_{t-1})(x - x_{t-1}) + \frac{1}{2}\frac{f'(x_{t-1}) - f'(x_{t-2})}{x_{t-1} - x_{t-2}}(x - x_{t-1})^2.$$

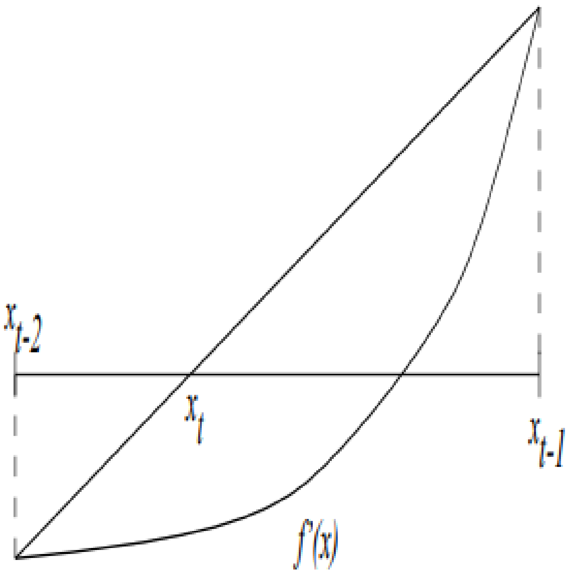
The new iterate is the minimizer of this quadratic function:

$$x_t = x_{t-1} - f'(x_{t-1})\frac{x_{t-1} - x_{t-2}}{f'(x_{t-1}) - f'(x_{t-2})}. \tag{2.3.4}$$

Note that although the polynomial p is chosen in asymmetric with respect to x_{t-1} and x_{t-2} way (it is tangent to f at x_{t-1}, but even not necessarily coincides with f at x_{t-2}), the minimizer x_t of this polynomial is symmetric with respect to the pair of working points; as it is immediately seen, the right hand side of (2.3.4) can be equivalently rewritten as

$$x_t = x_{t-2} - f'(x_{t-2})\frac{x_{t-1} - x_{t-2}}{f'(x_{t-1}) - f'(x_{t-2})}.$$

The geometry of the method is very simple: same as the Newton method, this is the method which actually approximates the zero of $g(x) = f'(x)$ (look: the values of f are not involved into the recurrency (2.3.4)). In the Newton method we, given the value and the derivative of g at x_{t-1}, approximate the graph of g by its tangent at x_{t-1} line $g(x_{t-1}) + g'(x_{t-1})(x - x_{t-1})$ and choose x_t as the point where this tangent line crosses the x-axis. In the Regula Falsi method we, given the values of g at two points x_{t-1} and x_{t-2}, approximate the graph of g by the secant line passing through $(x_{t-1}, g(x_{t-1}))$ and $(x_{t-2}, g(x_{t-2}))$ and choose as x_t the point where this secant line crosses the x-axis.



Regula Falsi method as zero-finding routine

The local rate of convergence of the method is given by the following

References

https://www2.isye.gatech.edu/~nemirovs/Lect_OptII.pdf