Goal: compute the coefficients $\int_T (v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}})\Phi_{\mathrm{T,P}}$ for every triangle T and vertex P.

Given:

$$b_F := 6 \prod_{P \in \mathcal{V}(F)} \lambda_P$$

$$\Phi_{\mathrm{T,P}} := \sqrt{\frac{20}{27|T|}}\left(\left(\sqrt{7}+1\right) - 3\sqrt{7}\lambda_P\right) =: \alpha - \beta\lambda_P$$

$$\begin{aligned} v_{\mathrm{CR}} &= x_1\psi_1 + x_2\psi_2 + x_3\psi_3 \\ &= x_1(1-2\lambda_1) + x_2(1-2\lambda_2) + x_3(1-2\lambda_3) \\ &= \underbrace{(x_1+x_2+x_3)}_{=:\chi} - 2(x_1\lambda_1 + x_2\lambda_2 + x_3\lambda_3) \end{aligned}$$

$$\begin{aligned} J_2 v_{\mathrm{CR}} &= J_1 v_{\mathrm{CR}} + \sum_{F \in \mathcal{F}(\Omega)} \fint_F (v_{\mathrm{CR}} - J_1 v_{\mathrm{CR}}) b_F \\ &= a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3 + 6(b_1\lambda_2\lambda_3 + b_2\lambda_1\lambda_3 + b_3\lambda_1\lambda_2) \end{aligned}$$

Calculate:

$$\begin{aligned} v_{\mathrm{CR}}\Phi_{\mathrm{T,P}} &= [\chi - 2(x_1\lambda_1 + x_2\lambda_2 + x_3\lambda_3)][\alpha - \beta\lambda_P] \\ &= \chi\alpha - \chi\beta\lambda_P - 2\alpha(x_1\lambda_1 + x_2\lambda_2 + x_3\lambda_3) + 2\beta(x_1\lambda_1\lambda_P + x_2\lambda_2\lambda_P + x_3\lambda_3\lambda_P) \end{aligned}$$

$$\begin{aligned} J_2 v_{\mathrm{CR}}\Phi_{\mathrm{T,P}} &= [a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3 + 6(b_1\lambda_2\lambda_3 + b_2\lambda_1\lambda_3 + b_3\lambda_1\lambda_2)][\alpha - \beta\lambda_P] \\ &= \alpha(a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3 + 6(b_1\lambda_2\lambda_3 + b_2\lambda_1\lambda_3 + b_3\lambda_1\lambda_2)) \\ &\quad -\beta(a_1\lambda_1\lambda_P + a_2\lambda_2\lambda_P + a_3\lambda_3\lambda_P) \\ &\quad -6\beta(b_1\lambda_2\lambda_3\lambda_P + b_2\lambda_1\lambda_3\lambda_P + b_3\lambda_1\lambda_2\lambda_P) \end{aligned}$$

using $\int_T \lambda_1^\alpha \lambda_2^\beta \lambda_3^\gamma = 2|T|\frac{\alpha!\beta!\gamma!}{(2+\alpha+\beta+\gamma)!}$ we get the following cases:
P0: $\int_T 1 = |T|$
P1: $\int_T \lambda_j = \frac{1}{3}|T|$
P2: $\int_T = \lambda_i\lambda_j = |T|\begin{cases} \frac{1}{12} \text{ for } i \neq j \\ \frac{1}{6} \text{ for } i = j \end{cases}$
P3: $\int_T \lambda_i\lambda_j\lambda_k = |T|\begin{cases} \frac{1}{60} \text{ for } i \neq j \neq k \\ \frac{1}{30} \text{ for } i \neq j = k \\ \frac{1}{10} \text{ for } i = j = k \end{cases}$

collect the coefficients of $v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}}$:
P0: $\chi\alpha$
P1: $\chi(-\beta - 2\alpha) - \alpha(a_1 + a_2 + a_3)$
P2 case 1: $2\beta(x_{P+1} + x_{P+2}) - 6\alpha(b_1 + b_2 + b_3) + \beta(a_{P+1} + a_{P+2})$
P2 case 2: $\beta(2x_P + a_P)$
P3 case 1: $6\beta b_P$
P3 case 2: $6\beta(b_{P+1} + b_{P+2})$
P3 case 3: 0

combine for the final formula:

$$
\begin{aligned}
\int_T (v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}})\Phi_{\mathrm{T,P}} = |T|\Big[&\chi\alpha \\
&-\frac{1}{3}(\chi(\beta+2\alpha)+\alpha a_{\mathrm{sum}}) \\
&+\frac{1}{12}(2\beta x_{\mathrm{other}} - 6\alpha b_{\mathrm{sum}} + \beta a_{\mathrm{other}}) \\
&+\frac{1}{6}\beta(2x_P + a_P) \\
&+\frac{1}{60}6\beta b_P \\
&+\frac{1}{30}6\beta b_{\mathrm{other}}\Big]
\end{aligned}
$$

$$
\begin{aligned}
= |T|\Big[&\frac{1}{3}(\chi(\alpha-\beta)-\alpha a_{\mathrm{sum}}) + \frac{1}{12}(\beta(2x_{\mathrm{other}}+a_{\mathrm{other}}) - 6\alpha b_{\mathrm{sum}}) \\
&+\frac{1}{6}\beta(2x_P + a_P) + \frac{1}{10}\beta b_P + \frac{1}{5}\beta b_{\mathrm{other}}\Big]
\end{aligned}
$$