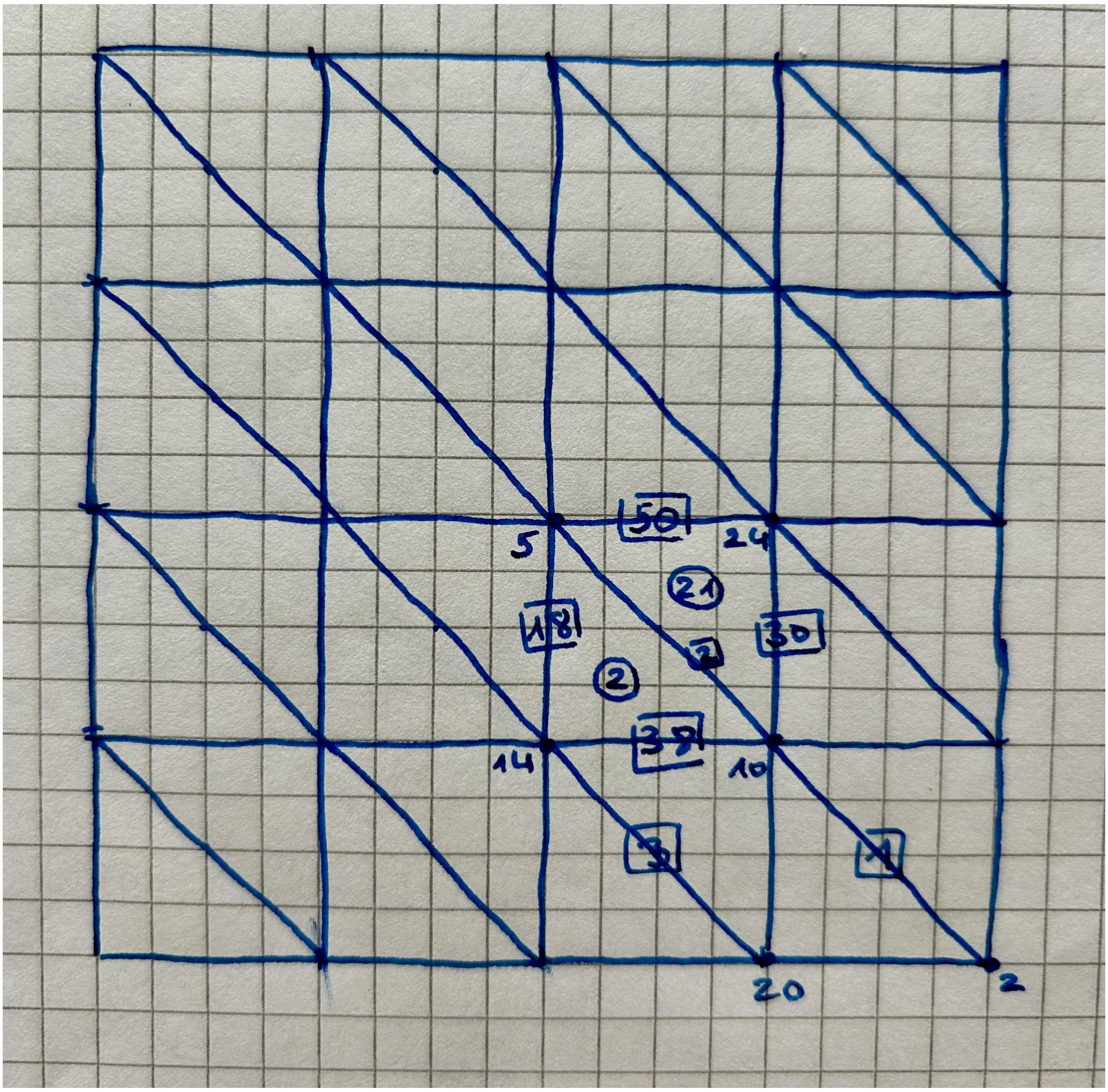
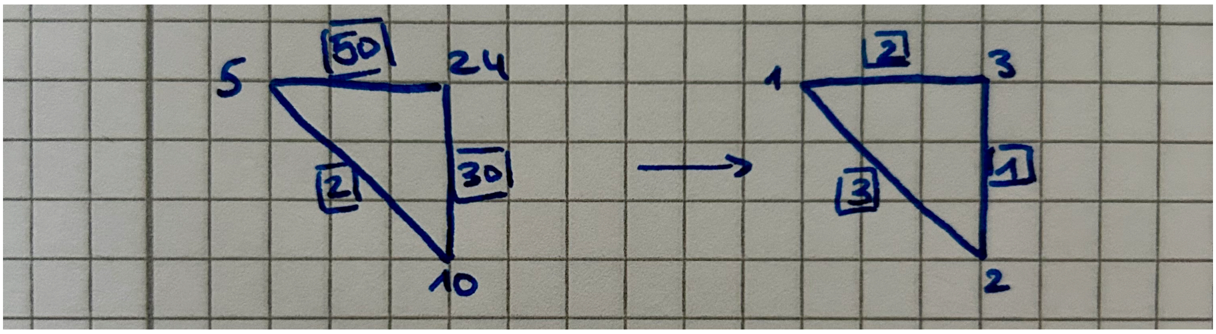


computing the volume-bubble-coefficient for the triangle number 21 and node number 24 given the Crouzeix-Raviart vector: $v_{\text{CR}} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$ that is one only for the edge number 2.

Step 1: compute a_{CR}

The nodal averages are $\frac{1}{3}$ for the nodes 5 and 10 and $-\frac{1}{6}$ for the nodes 14 and 24. Therefore the vector looks like: $a_{\text{CR}} = \frac{1}{6} \begin{pmatrix} \vdots \\ 2 \\ \vdots \\ 2 \\ \vdots \\ -1 \\ \vdots \\ -1 \end{pmatrix}$ where all everything else is 0.



Step 2: compute $J_2 v_{\mathrm{CR}}$
With the bubble function defined as: $b_F := 6\lambda_{F+1}\lambda_{F+2}$ for the local ennumeration of the edges 1,2,3; we get:

$$\begin{aligned}
J_2 v_{\mathrm{CR}} &:= a_{\mathrm{CR}} + \sum_{F=1}^{3} \left(\fint_F v_{\mathrm{CR}} - a_{\mathrm{CR}} \right) b_F \\
&= \frac{1}{3}\lambda_1 + \frac{1}{3}\lambda_2 - \frac{1}{6}\lambda_3 + \left(0 - \frac{1}{12}\right) \cdot 6\lambda_2\lambda_3 + \left(0 - \frac{1}{12}\right) \cdot 6\lambda_1\lambda_3 + \left(1 - \frac{1}{3}\right) \cdot 6\lambda_1\lambda_2 \\
&= \frac{1}{3}\lambda_1 + \frac{1}{3}\lambda_2 - \frac{1}{6}\lambda_3 - \frac{1}{2}\lambda_2\lambda_3 - \frac{1}{2}\lambda_1\lambda_3 + 4\lambda_1\lambda_2
\end{aligned}$$

Step 3: compute $v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}}$

$$\begin{aligned}
v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}} &= 1 - 2\lambda_3 - \left[\frac{1}{3}\lambda_1 + \frac{1}{3}\lambda_2 - \frac{1}{6}\lambda_3 - \frac{1}{2}\lambda_2\lambda_3 - \frac{1}{2}\lambda_1\lambda_3 + 4\lambda_1\lambda_2 \right] \\
&= 1 - \frac{1}{3}\lambda_1 - \frac{1}{3}\lambda_2 - \frac{11}{6}\lambda_3 + \frac{1}{2}\lambda_2\lambda_3 + \frac{1}{2}\lambda_1\lambda_3 - 4\lambda_1\lambda_2
\end{aligned}$$

Step 4: compute $(v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}})\Phi_3$
With the definition of the correction function: $\Phi = \alpha - \beta\lambda_P$ and the given node $P = 3$, we get:

$$\begin{aligned}
(v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}})\Phi_3 &= \left(1 - \frac{1}{3}\lambda_1 - \frac{1}{3}\lambda_2 - \frac{11}{6}\lambda_3 + \frac{1}{2}\lambda_2\lambda_3 + \frac{1}{2}\lambda_1\lambda_3 - 4\lambda_1\lambda_2\right) \cdot (\alpha - \beta\lambda_3) \\
&= \alpha - \frac{1}{3}\alpha\lambda_1 - \frac{1}{3}\alpha\lambda_2 - \frac{11}{6}\alpha\lambda_3 + \frac{1}{2}\alpha\lambda_2\lambda_3 + \frac{1}{2}\alpha\lambda_1\lambda_3 - 4\alpha\lambda_1\lambda_2) \\
&-\beta\lambda_3 + \frac{1}{3}\beta\lambda_1\lambda_3 + \frac{1}{3}\beta\lambda_2\lambda_3 + \frac{11}{6}\beta\lambda_3^2 - \frac{1}{2}\beta\lambda_2\lambda_3^2 - \frac{1}{2}\beta\lambda_1\lambda_3^2 + 4\beta\lambda_1\lambda_2\lambda_3
\end{aligned}$$

Step 5: compute the integral $\int_T (v_{\mathrm{CR}} - J_2 v_{\mathrm{CR}})\Phi_3$

$$
\begin{gathered}
\int_T (v_{\mathrm{CR}} - J_2 v\mathrm{CR})\Phi_3 = |T|\Big\{\alpha \\
+\frac{1}{3}\cdot\left(-\frac{1}{3}\alpha - \frac{1}{3}\alpha - \frac{11}{6}\alpha - \beta\right) \\
+\frac{1}{12}\cdot\left(\frac{1}{2}\alpha + \frac{1}{2}\alpha - 4\alpha + \frac{1}{3}\beta + \frac{1}{3}\beta\right) \\
+\frac{1}{6}\cdot\frac{11}{6}\beta \\
+\frac{1}{30}\cdot\left(-\frac{1}{2}\beta - \frac{1}{2}\beta\right) \\
+\frac{1}{60}\cdot 4\beta\Big\} \\
= |T|\Big\{\frac{1}{6}\alpha - \frac{1}{3}\beta - \frac{1}{4}\alpha + \frac{1}{18}\beta + \frac{11}{36}\beta - \frac{1}{30}\beta + \frac{1}{15}\beta\Big\} \\
= |T|\left(-\frac{1}{12}\alpha + \frac{11}{180}\beta\right)
\end{gathered}
$$