1. [Proof of Concept, Hand Calculation] Consider a machined steel circular shaft subjected to a minimum bending moment of 1500 lb.in. and a maximum bending moment of 6000 lb.in.; and a minimum torque of 0 and maximum torque of 2500 lb.in. Consider the safety factor to be 1.5, the yield strength of the materials 58 ksi, and its ultimate strength to be 82 ksi.

Determine the minimum diameter of the shaft for the case where D is 20% larger than d, the fillet radius is 10% of d, and reliability is 99.9%. You may use DE-Gerber formula.

$\frac{D}{d} = 1.2$

Table 6–2
Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

$$k_a = a S_{ut}^b \qquad k_a = (2.70)(82\ \text{ksi})^{-0.265} = 0.8399$$

$$k_b = \begin{cases} (d/0.3)^{-0.107} & \text{for } 0.11 \le d \le 2in \\ 0.91 \cdot d^{-0.157} & \text{for } 2 < d \le 10in \\ (d/7.62)^{-0.107} & \text{for } 2.79 \le d \le 51mm \\ 1.51 \cdot d^{-0.157} & \text{for } 51 < d \le 254mm \end{cases}$$

← involved in iteration

$$S_e = k_a k_b k_c k_d k_e \left(\tfrac{1}{2} S_{ut}\right)$$

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \qquad k_c = 0.59$$

$k_d = 1$ → room temp

$$k_e = 1 - 0.08 z_a$$

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

$k_e = 0.753$

DE-Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\} \quad (7\text{–}9)$$

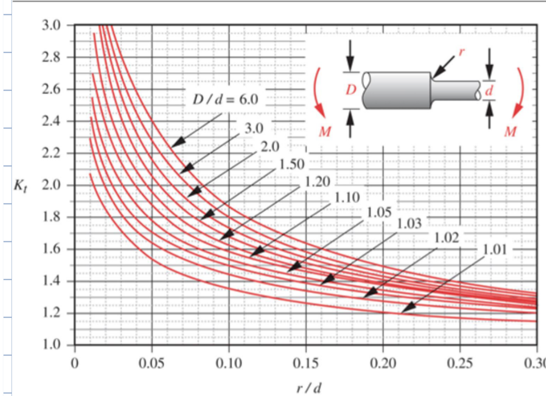
$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3} \quad (7\text{–}10)$$

where

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

Figure C-2 **Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Bending**



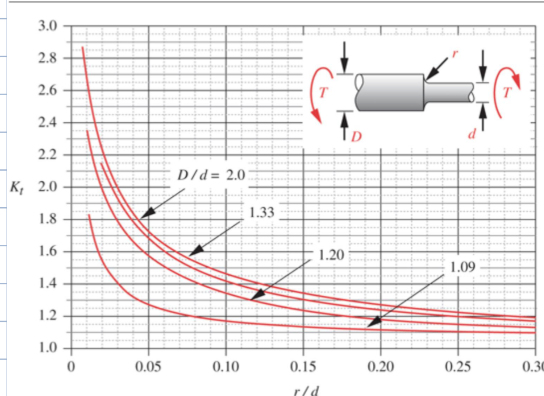
$\frac{r}{d} = 0.1$

$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where:

D/d	A	b
6.00	0.878 68	−0.332 43
3.00	0.893 34	−0.308 60
2.00	0.908 79	−0.285 98
1.50	0.938 36	−0.257 59
1.20	0.970 98	−0.217 96
1.10	0.951 20	−0.237 57
1.07	0.975 27	−0.209 58
1.05	0.981 37	−0.196 53
1.03	0.980 61	−0.183 81
1.02	0.960 48	−0.177 11
1.01	0.919 38	−0.170 32

Figure C-3 **Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Torsion**



$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where:

D/d	A	b
2.00	0.863 31	−0.238 65
1.33	0.848 97	−0.231 61
1.20	0.834 25	−0.216 49
1.09	0.903 37	−0.126 92

$M_a = \frac{6000 - 1500}{2} = 2250\ \text{lb·in}$

$M_m = \frac{6000 + 1500}{2} = 3750\ \text{lb·in}$

$T_a = T_m = 1250\ \text{lb·in}$

from fig. 6-20, 6-21

$K_t = A(0.1)^b = 1.6039 \rightarrow K_f = 1 + 0.85(K_t - 1) = 1.5133$

$K_{ts} = A(0.1)^b = 1.3734 \rightarrow K_{fs} = 1 + 0.88(K_{ts} - 1) = 1.3286$

$A = 7.3924$

$B = 11.7085$

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \qquad q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

Guess 1 → $d = 1$ in eq 7-9

→ $k_b = (1/0.3)^{-0.107} = 0.8791$

→ $S_e = 13.4489$

→ $n = 0.3359$

Guess 2 → $d = 2$ in

→ $k_b = 0.8163$

→ $S_e = 12.4875$

→ $n = 2.5148$

Guess 3 → $d = 1.5$ in

→ $k_b = 0.8418$

→ $S_e = 12.8779$

→ $n = 1.0907$

eq 7-10 → $n = 1.5$, k_b & S_e from $d = 5$

→ $\underline{d = 1.6681}$

→ $k_b = 0.8323$

→ $S_e = 12.7323$

eq 7-10 → $\underline{d = 1.6738}$ → $k_b = 0.8320$ → $S_e = 12.7277$

→ $\underline{d = 1.6740}$ → $S_e = 12.7276$

$d = 1.6740$

$$\sigma_x = K_t \frac{M}{I/c} = K_t \frac{32M}{\pi d^3}$$

$$\tau_{xy} = K_{ts} \frac{Tr}{J} = K_{ts} \frac{16T}{\pi d^3}$$

$\sigma_{\text{von mises}} = \sqrt{\sigma_{\text{Bending}}^2 + 3\tau_{\text{torsion}}^2}$