SE 142 HW4

1. 1. Course Reader Problem 5.9 – note: $t_{ply} = 0.127$ mm for both materials.
Use $\Delta T = -150°C$ to estimate the curing stresses in the $[0/90]_s$ laminates of the following two composite materials.

a) AS4/3501-6 graphite/epoxy: $E_1 = 140GPa$, $E_2 = 10GPa$, $G_{12} = 7GPa$, $\nu_{12} = 0.3$, $\alpha_1 = -1 \times 10^{-6}/°C$, $\alpha_2 = 26 \times 10^{-6}/°C$ $\nu_{21} = 0.0214$

b) S glass/epoxy: $E_1 = 45GPa$, $E_2 = 9GPa$, $G_{12} = 4.5GPa$, $\nu_{12} = 0.3$, $\alpha_1 = 5 \times 10^{-6}/°C$, $\alpha_2 = 26 \times 10^{-6}/°C$

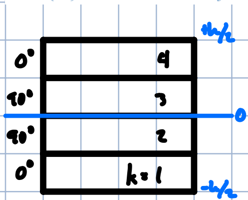
Hint: calc. [A] and also $\{N^T\}$. You'll find $\{M^T\} = 0$. Compute $\{\varepsilon^o\}$ and then ply stresses accounting for the $-\{\alpha_x\}\Delta T$ term. You should find in AS4, σ_{11} = -36.35 MPa. For S glass, σ_{11} = -21,47 MPa.

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1} \rightarrow \nu_{21} = \frac{E_2}{E_1}\nu_{12}$$

$$\alpha = \begin{Bmatrix} -1\cdot10^{-6} \\ 26\cdot10^{-6} \\ 0 \end{Bmatrix} °C^{-1} = \bar{\alpha}_0$$

$$\bar{\alpha}_{90} = \begin{Bmatrix} 26\cdot10^{-6} \\ -1\cdot10^{-6} \\ 0 \end{Bmatrix} °C^{-1}$$

a)



$$A_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (z_k - z_{k-1})$$ with $z_k - z_{k-1} = 0.127$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}, \text{ w/ } Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} \quad Q_{12} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \quad Q_{21} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \quad Q_{66} = G_{12}$$

$$\bar{Q}_0 = \begin{bmatrix} 140.9058 & 3.0194 & 0 \\ 3.0194 & 10.0647 & 0 \\ 0 & 0 & 7 \end{bmatrix} \quad (0°)$$

swap Q_{11} and Q_{22} for 90° $\rightarrow$ $$\bar{Q}_{90} = \begin{bmatrix} 10.0647 & 3.0194 & 0 \\ 3.0194 & 140.9058 & 0 \\ 0 & 0 & 7 \end{bmatrix} \quad (90°)$$

$$A = 2(\underset{m}{0.00127})[\bar{Q}_0 + \bar{Q}_{90}] = \begin{bmatrix} 0.383465 & 0.015339 & 0 \\ 0.015339 & 0.383465 & 0 \\ 0 & 0 & 0.035560 \end{bmatrix}$$

$$\{N^T\} = \Delta T \sum_{k=1}^{n} [\bar{Q}]_k \{\bar{\alpha}\}_k (z_k - z_{k-1})$$

$$\{N^T\} = \Delta T \cdot 2(0.00127m)[\bar{Q}_0\bar{\alpha}_0 + \bar{Q}_{90}\bar{\alpha}_{90}] = \begin{Bmatrix} -0.7478 \\ -0.7478 \\ 0 \end{Bmatrix}$$

$$\{M^T\} = \frac{1}{2}\Delta T \sum_{k=1}^{n} [\bar{Q}]_k \{\bar{\alpha}\}_k (z_k^2 - z_{k-1}^2)$$

$$\{M^T\} = \frac{1}{2}\Delta T[(\cancel{0.00254^2} - \cancel{0.00127^2} + \cancel{0.00127^2} - \cancel{0.00254^2})[\bar{Q}_0]\{\bar{\alpha}_0\} + (\cancel{0.00127^2 - 0.00127^2})[\bar{Q}_{90}]\{\bar{\alpha}_{90}\}] = 0$$

$$\{N^T\} = [A]\{\varepsilon^{oT}\} \rightarrow \{\varepsilon^o\} = [\alpha]\{N^T\}, \text{ where } [\alpha] = [A]^{-1} = \begin{bmatrix} 2.6120 & -0.1045 & 0 \\ -0.1045 & 2.6120 & 0 \\ 0 & 0 & 28.1215 \end{bmatrix}$$

$$\{\varepsilon^o\} = \begin{Bmatrix} -0.1875 \\ -0.1875 \\ 0 \end{Bmatrix}$$

$$\{\sigma\}_k = [\bar{Q}]_k (\{\varepsilon^o\} - \{\bar{\alpha}\}_k \Delta T) = \begin{Bmatrix} -36.3462 \\ 36.3462 \\ 0 \end{Bmatrix} \text{MPa}$$

b) $$\bar{Q}_0 = \begin{bmatrix} 45825 & 2749 & 0 \\ 2749 & 9165 & 0 \\ 0 & 0 & 4500 \end{bmatrix}, \bar{Q}_{90} = \begin{bmatrix} 9165 & 2749 & 0 \\ 2749 & 45825 & 0 \\ 0 & 0 & 4500 \end{bmatrix}, A = \begin{bmatrix} 139.6741 & 13.9674 & 0 \\ 13.9674 & 139.6741 & 0 \\ 0 & 0 & 22.8600 \end{bmatrix}, \{\bar{\alpha}_0\} = \begin{Bmatrix} 5\cdot10^{-6} \\ 2.6\cdot10^{-5} \\ 0 \end{Bmatrix}, \{\bar{\alpha}_{90}\} = \begin{Bmatrix} 26\cdot10^{-6} \\ 5\cdot10^{-6} \\ 0 \end{Bmatrix}$$

$$\{N^T\} = \begin{Bmatrix} -0.2106 \\ -0.2106 \\ 0 \end{Bmatrix}, [\alpha] = \begin{bmatrix} 0.0072 & -0.0007 & 0 \\ -0.0007 & 0.0072 & 0 \\ 0 & 0 & 0.0437 \end{bmatrix}, \{\varepsilon_0\} = \begin{Bmatrix} -0.0014 \\ -0.0014 \\ 0 \end{Bmatrix}, \{\sigma\} = \begin{Bmatrix} -21.4773 \\ 21.4773 \\ 0 \end{Bmatrix} \text{MPa}$$

②

2. Course Reader Problem 5.13 – use AS4 properties* from Problem 5.9 with $t_{ply} = 0.127$ mm. Due to the presence of curing stresses, the [0/90] laminate would warp after curing. If the laminate is constrained so the $\kappa_y = \kappa_{xy} = 0$ but $\kappa_x \neq 0$, find the curvature κ_x after curing. Assume $\Delta T = -150°$C. The composite is AS4/3501-6.

Hint: In this problem, observe that only κ_y and κ_{xy} are constrained = 0. κ_x and the in-plane midplane strains are unconstrained (i.e., free to be whatever value results from the loading). You need to calc N_{Tot} and M_{Tot} as sum of mechanical and thermal N and M. For the constraints stated, note particularly that M_y and M_{xy} mechanical are generally not 0. Other mechanical N_i and M_i are 0 since there are no constraints on the related strains or curvatures. Find: κ_x to have magnitude 13.04 1/m.

Additional solving hints:

You'll find the following set of six equations, incorporating BC information, and including which A B D matrix terms are zero (note: you'll get a sparsely populated B matrix).

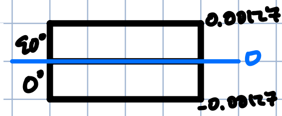
$$\begin{Bmatrix} N_x^T \\ N_y^T \\ 0 \\ M_x^T \\ M_y + M_y^T \\ \cancel{M_{xy}} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & -B_{11} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ B_{11} & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & -B_{11} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \cancel{\gamma_{xy}^o}\ 0 \\ \kappa_x \\ 0 \\ 0 \end{Bmatrix}$$

$\rightarrow M_{xy} = 0$

There are six unknowns (circled quantities) and six eqns.

You can solve for the six unknowns now, mainly seeking kappa_x.

It is suggested that you 1st inspect above eqns and you can immediately see a couple quantities are zero.



$$A = (0.00127)\left[\bar{Q}_0 + \bar{Q}_{90}\right] = \begin{bmatrix} 191.7326 & 7.6693 & 0 \\ 7.6693 & 191.7326 & 0 \\ 0 & 0 & 17.7800 \end{bmatrix}$$

$$B = \frac{1}{2}\left(-(0.00127)^2[\bar{Q}_0] + (0.00127)^2[\bar{Q}_{90}]\right)$$

$$D = \frac{1}{3}(0.00127)^3\left[\bar{Q}_0 + \bar{Q}_{90}\right] = \begin{bmatrix} 0.1031 & 0.0041 & 0 \\ 0.0041 & 0.1031 & 0 \\ 0 & 0 & 0.0096 \end{bmatrix}$$

$$B = B^T = \begin{bmatrix} -0.1055 & 0 & 0 \\ 0 & 0.1055 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\{N^T\} = \Delta T \cdot (0.00127\,m)\left[\bar{Q}_0\bar{\alpha}_0 + \bar{Q}_{90}\bar{\alpha}_{90}\right] = \begin{Bmatrix} -0.0374 \\ -0.0374 \\ 0 \end{Bmatrix}$$

$$\{M^T\} = \frac{1}{2}\Delta T\left[-(0.00127)^2([\bar{Q}_0]\{\bar{\alpha}_0\}) + (0.00127)^2([\bar{Q}_{90}]\{\bar{\alpha}_{90}\})\right] = \begin{Bmatrix} -3.884\cdot 10^{-5} \\ 3.884\cdot 10^{-5} \\ 0 \end{Bmatrix}$$

$$M_y + M_y^T = -B_{11}(\varepsilon_y^o) + D_{12}(\kappa_x)$$

$$N_y^T = A_{12}(\varepsilon_x^o) + A_{22}(\varepsilon_y^o)$$

$$N_x^T = A_{11}(\varepsilon_x^o) + A_{12}(\varepsilon_y^o) + B_{11}(\kappa_x)$$

$$M_x^T = B_{11}(\varepsilon_x^o) + D_{11}(\kappa_x)$$

$$\{\varepsilon^o\} = [\alpha]\{N^T\}$$

$$[\alpha] = [A]^{-1} + [A]^{-1}[B][\delta][B][A]^{-1} = \begin{bmatrix} 0.0120 & -0.0005 & 0 \\ -0.0005 & 0.0120 & 0 \\ 0 & 0 & 0.0562 \end{bmatrix}$$

$$[\delta] = \left([D] - [B][A]^{-1}[B]\right)^{-1}$$

$$\{\varepsilon^o\} = \begin{Bmatrix} -4.303\cdot 10^{-4} \\ 4.303\cdot 10^{-4} \\ 0 \end{Bmatrix}$$

$$\kappa_x = \frac{M_x^T - B_{11}(\varepsilon_x^o)}{D_{11}} = 13.04\ m^{-1}$$