PHY407 Formal Lab Report 3

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1 Introduction

This report covers the analysis of different physical phenomena through the application of Fourier transforms. First, audio filtering techniques were used to smooth out the high-frequency components of an audio file. Then, a stock market analysis was performed on the S&P 500 index, focusing on filtering the data to observe long-term trends. Finally, sea level pressure (SLP) data was analyzed, decomposing it into specific wavenumber components and visualizing the time-longitude variations.

2 Question 1: Audio Filtering

2.1 Part (a)

The following plots show the original audio signal for both channels (Channel 0 and Channel 1) of the audio file `GraviteaTime.wav`. The time is measured in seconds, and the amplitude values are plotted for each channel. Channel 0 is represented in teal, and Channel 1 is represented in orange.

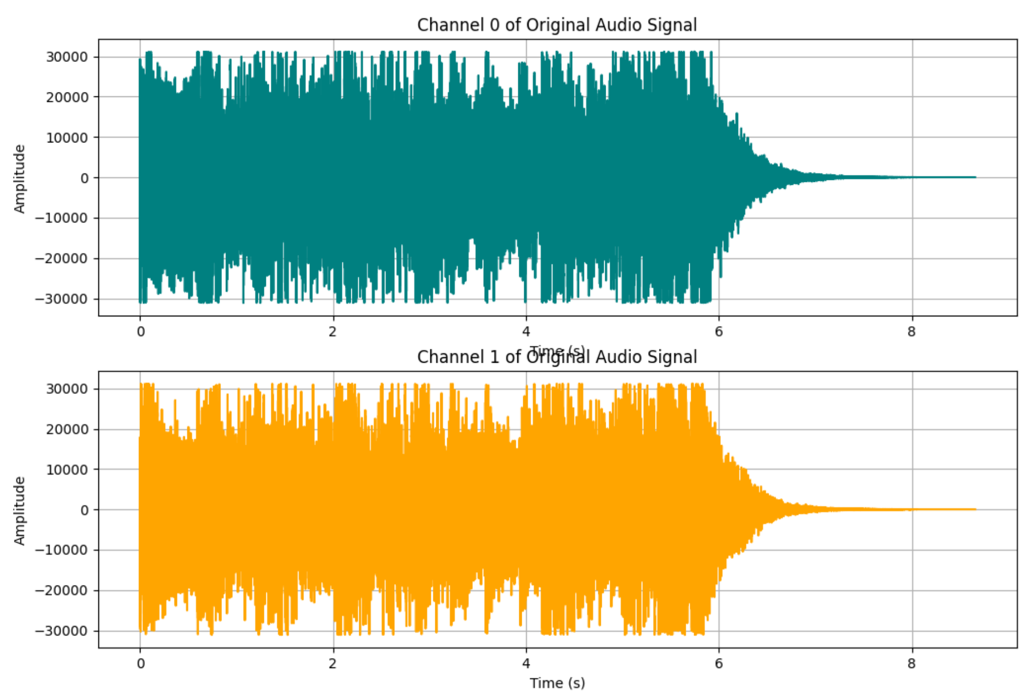

Figure 1: Channels 0 and 1 Amplitudes of Original Audio Signal

2.2 Part (b)

A Fourier transform was performed on both channels of the original signal, filtering out all frequencies greater than 880 Hz. The filtered Fourier coefficients and the corresponding time-domain signals after inverse Fourier transformation are shown below. The plots illustrate the amplitude of the original Fourier coefficients, the filtered Fourier coefficients, and the time-domain signals for both channels.

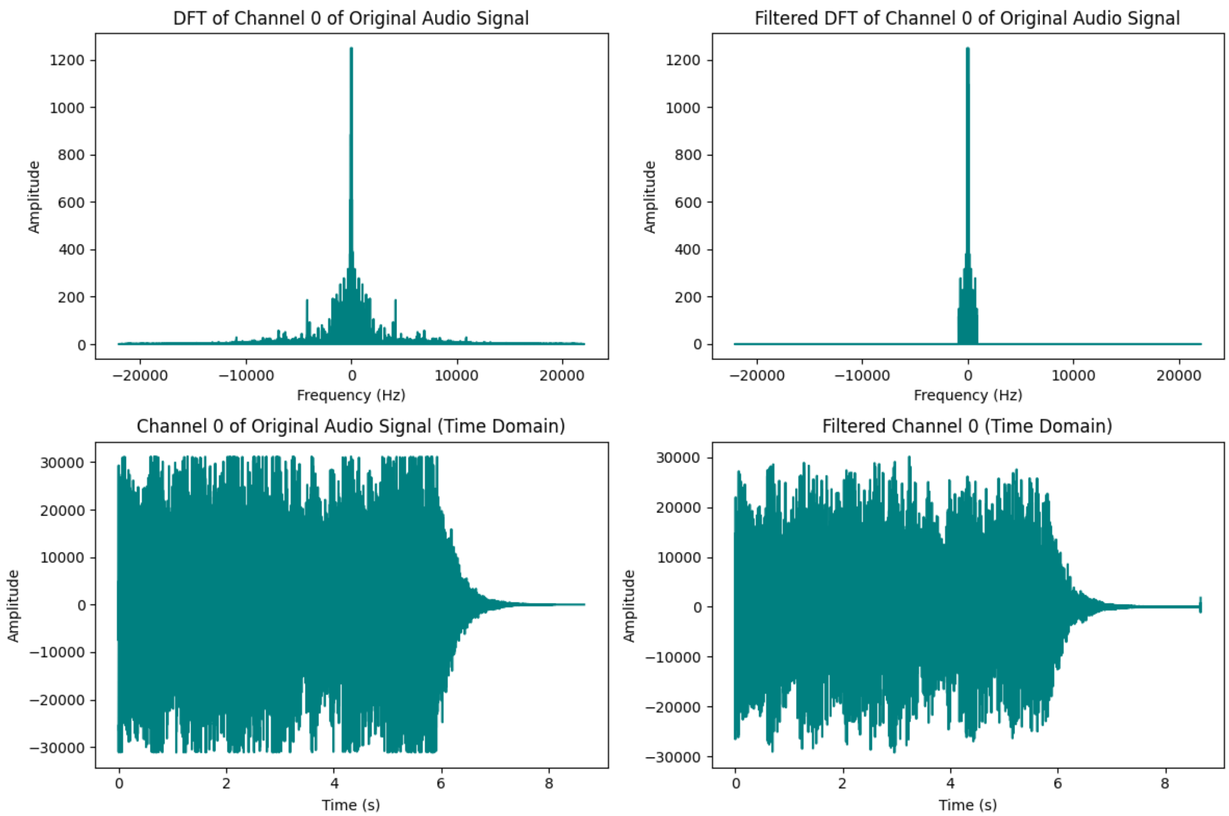

Figure 2: DFT and filtered DFT of Channel 0 of the original audio signal, followed by the original and filtered time-domain signals. The filter removes frequencies above 880 Hz, resulting in a smoother time-domain signal.

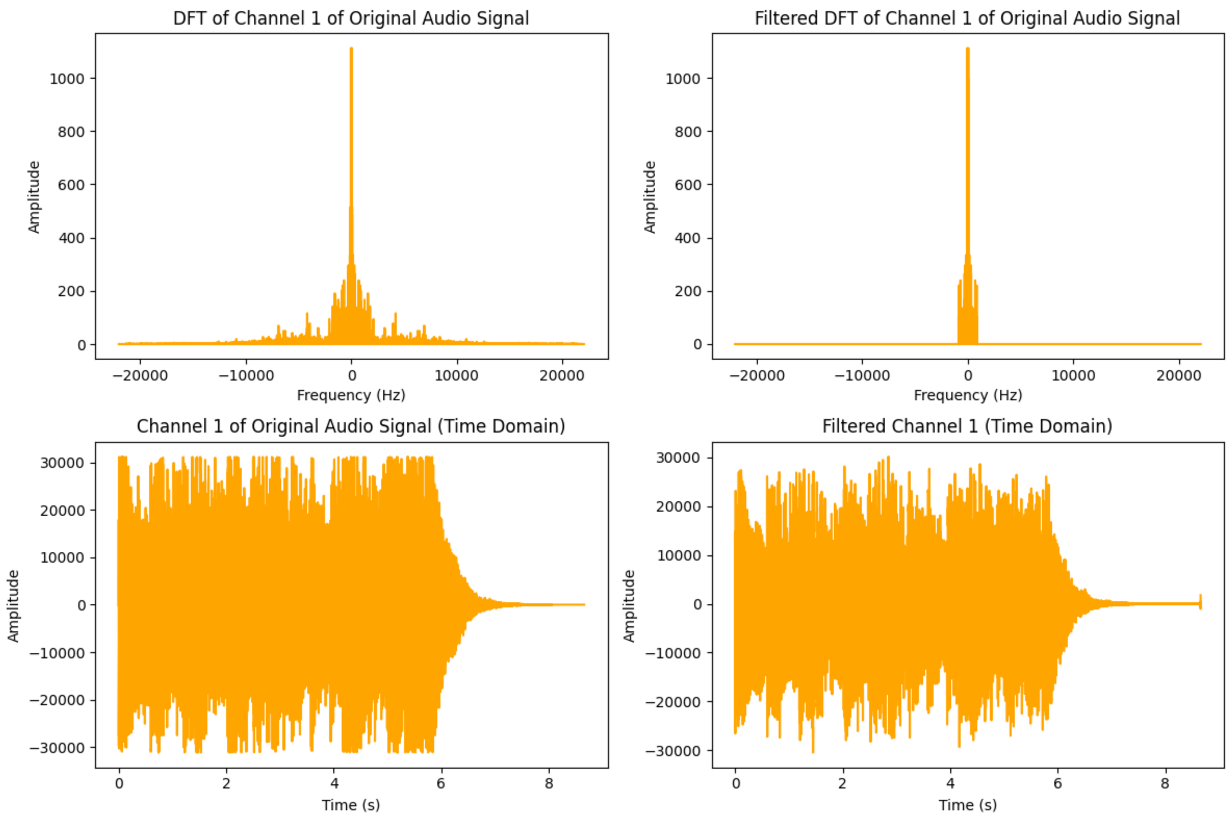

Figure 3: DFT and filtered DFT of Channel 1 of the original audio signal, followed by the original and filtered time-domain signals. The low-pass filter removes frequencies above 880 Hz, leading to a reduction of high-frequency noise in the filtered signal.

2.3 Part (c)

To better show the smoothing impact of the low-pass filtering, a small 30 ms segment of the filtered audio signal for both channels was plotted. This segment reveals how the filtering reduces the high-frequency components, resulting in a smoother signal in the time domain.

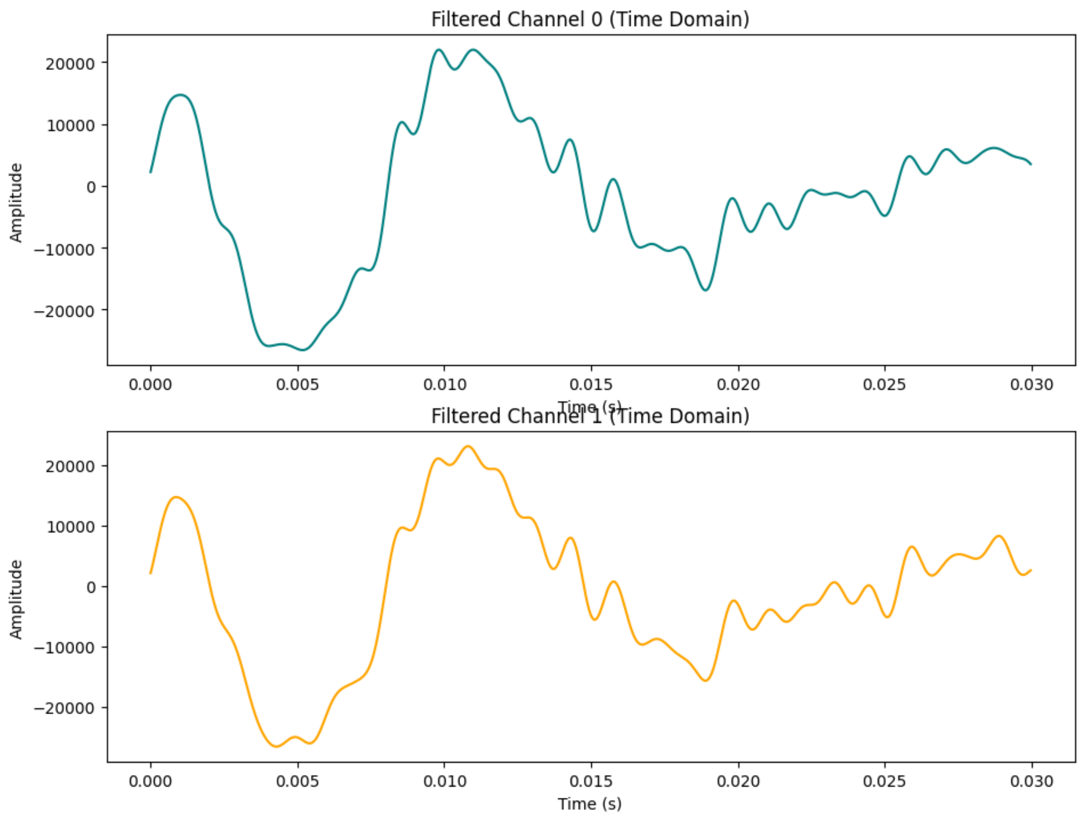

Figure 4: A 30 ms segment of the filtered audio signal for Channel 0 (top) and Channel 1 (bottom) in the time domain, demonstrating the smoothing effect of the low-pass filter by removing high-frequency components.

3 Question 2: Getting Rich on the Stock Market

3.1 Part (a)

A plot was generated showing the opening values of the S&P 500 stock index over business days from late 2014 to 2019. The business days skip weekends and holidays, avoiding gaps in time by using "business day number" instead of calendar days. The plot reveals the general upward trend of the S&P 500 during this period, with apparent dips and peaks.

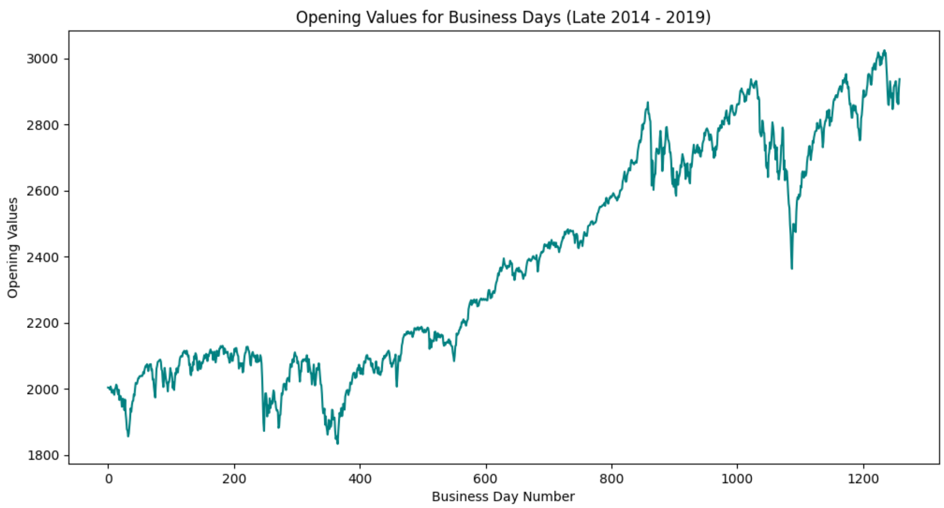

Figure 5: Opening Values of the S&P 500 Stock Index over Business Days from late 2014 to 2019

3.2 Part (b)

Two tests were implemented to confirm that applying the FFT and subsequently the inverse transform retained the original data. First, the original data and the transformed data were plotted on the same graph (Figure 6), showing a nearly 100% overlap, thus providing a visual measure of the similarity between the two datasets. As a more rigorous test, Numpy's `allclose` function was used to compare the two arrays within a tolerance of 1e-08, which confirmed that the inverse transform successfully returned the original data. The code output confirmed the success of this test with the message:

```
The original data and the inverse transformed data match!
```

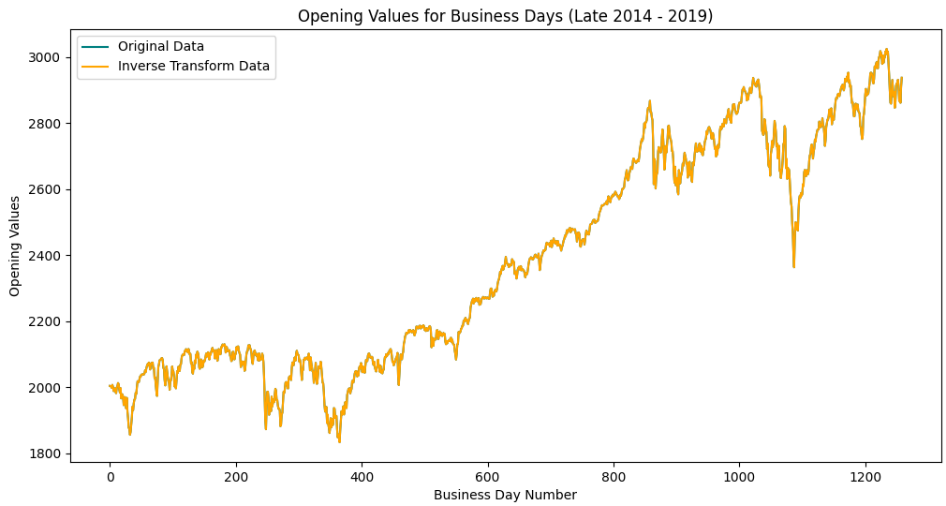

Figure 6: Original Data and Inverse Transformed Data superimposed for visual comparison

3.3 Part (c)

The resulting filtered data is significantly smoother than the original data, with the smaller fluctuations no longer present in the graph. Several peaks and dips which were prominent in the unfiltered/original data (such as those around business day number 850 and 1100) are not nearly as magnified in the filtered counterpart. However, for long-term trend analysis, the smoothened graph provides a good representation of the oscillating yet typically increasing trend over time.

While the loss of some short-term fluctuations may make certain details less clear, the goal is to identify the broader trend. By filtering out the noise, the graph emphasizes the long-term behavior of the stock market, illustrating its general rise over the selected period.

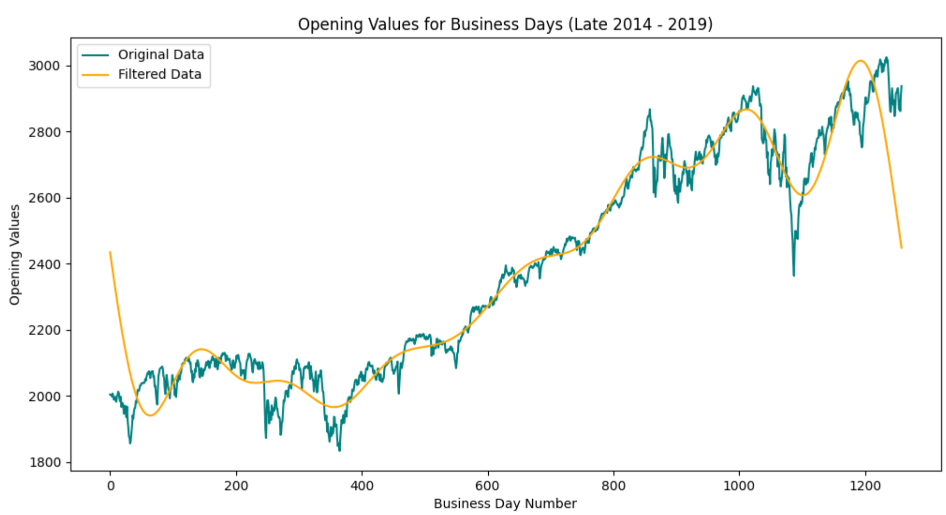

Figure 7: A comparison of the original S&P 500 opening values (teal) with the inverse Fourier transformed data after applying a low-pass filter (orange)

4 Question 3: Analysis of sea level pressure

The sea level pressure (SLP) dataset at 50°S was analyzed by decomposing the SLP in the longitudinal direction using Fourier transforms. The SLP data was broken down into components corresponding to wavenumbers $m = 3$ and $m = 5$, and the time-longitude domain for these components was plotted.

4.1 Part (a)

The SLP components corresponding to Fourier wavenumbers $m = 3$ and $m = 5$ were extracted, and filled contour plots were created in the time-longitude domain for each component.

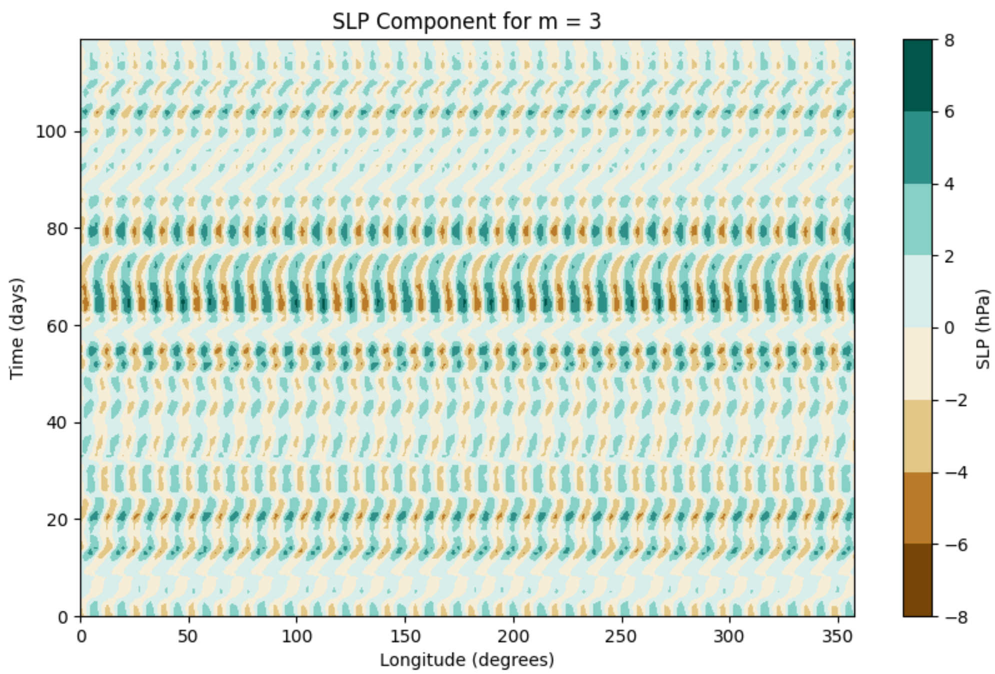

Figure 8: Contour plot of the SLP component for wavenumber m=3 as a function of longitude and time

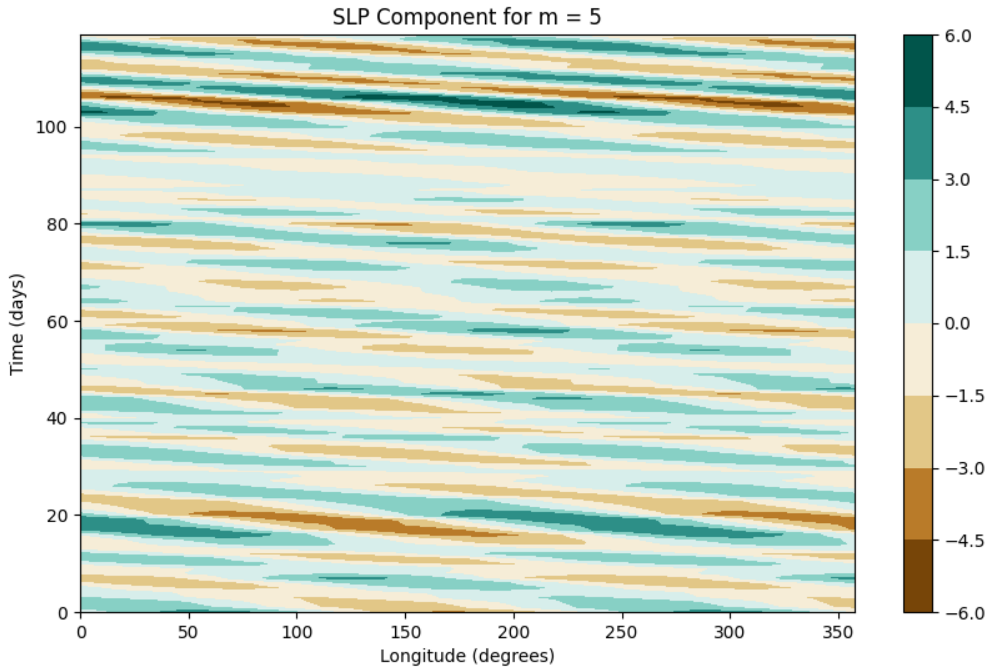

Figure 9: Contour plot of the SLP component for wavenumber m=5 as a function of longitude and time

4.2 Part (b)

The theory of atmospheric wave propagation suggests that wave disturbances propagate at a speed that is proportional to the wavenumber. The plots for the SLP components corresponding to wavenumbers 3 and 5 have distinct qualitative qualities, demonstrating how there is a much higher dispersion (and thus propagation speed) for the wave disturbances corresponding to wavenumber m $= 5$ in comparison to the m $= 3$ data.

In the m=3 plot, one can see small, somewhat round patches representing variations in sea level pressure across time and longitude. The patches corresponding to uniform SLP levels are somewhat stretched out in the time direction, indicating that the SLP remains constant for several days at a time. Furthermore, these patches are very narrow in the longitudinal direction, indicating that they have a low propagation speed longitudinally. In contrast, the m=5 plot shows very elongated patches of uniform SLP stretched out in the longitude direction while relatively thin in the time direction. This suggests that, over a short span of time, the same pressure level is sustained over longer distances longitudinally, indicating that shorter-wavelength disturbances propagate quickly across longitude.