Informal Lab 10

Traveling Guinea Problem

Problem Definition

Originally the problem was formulated as, a traveling salesperson needs to visit N cities. We'll formulate it as, a traveling guinea pig needs to visit N food bowls

- Can travel in a straight line between any pair of bowls
- We are given the coordinates of the bowls
- Must start and end at the same bowl
- Must visit each bowl at least once

Devise the shortest journey (in terms of total distance travelled).

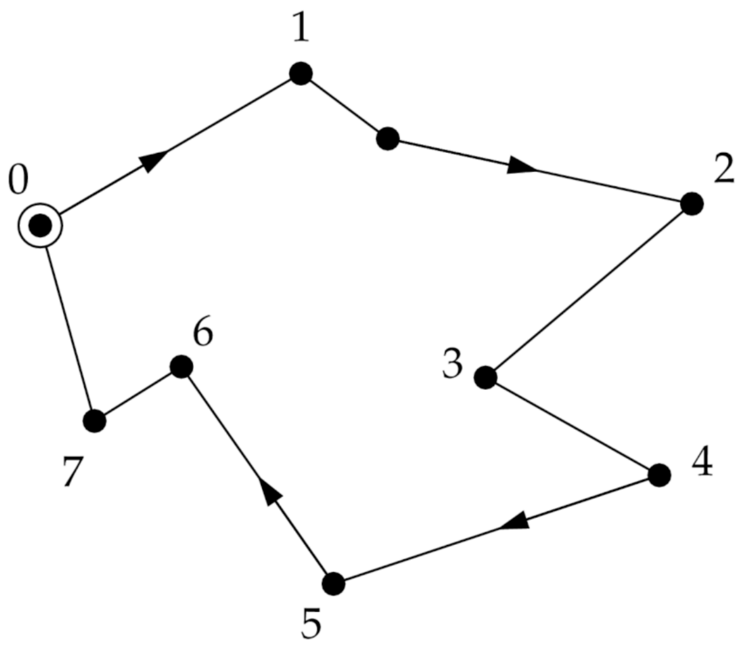
This is a famous NP-hard problem (https://en.wikipedia.org/wiki/NP-hardness). We will make some simplifications:

- Flat 2-dimensional space
- Bowl locations are chosen at random within a square of unit length on each side

Concept for our solution ¶

We'll find the global minimum of *total distance of the route*, using simulated annealing

- Number the bowls in the order in which the guinea visits them
- Consider each bowl to be a single point (i.e. has no spatial extent)
- Denote the position of bowl i by 2D vector $r_i = (x_i, y_i)$ with $r_N = r_0$
- Start with random route
- Try swapping the order of 2 randomly-chosen bowls in the route, use Metropolis to determine whether to keep the swap
    - 'Temperature' T follows exponential cooling schedule, with time constant τ and initial temperature T_0
- Stop when temperature reaches T_f



Exercise 1

Define a function to calculate the total distance travelled by the guinea over the entire journey. This is the quantity we'll be minimizing.

```
import numpy as np

# path -> set of nodes (xi, yi)
def path_length(path):
    N = len(path) # Number of nodes
    total_dist = 0
    for i in range(1, N):
        dx = path[i][0] - path[i - 1][0]
        dy = path[i][1] - path[i - 1][1]
        total_dist += np.sqrt(dx**2 + dy**2)
    return total_dist

test_path = [(0,0), (10,0)]
path_length(test_path)
```

```
10.0
```

Exercise 2

Write a function that randomly sets the bowl locations. It should take N as an argument, and return a 2D array (or, if you prefer, an array of 2-element vectors) containing the x and y positions of the bowls.

```
def create_nodes(N):
    N = 10 # Number of cities/bowls
    nodes = []
    for node in range(N):
        x = np.random.random()
        y = np.random.random()
        nodes.append((x,y))
    return nodes
```

Exercise 3

Write a function that creates a graphical representation of the bowls' positions. It should also draw a line between each bowl i and i+1 (ideally we would have an arrow on each line, but don't bother trying to implement this.)

```
from matplotlib import pyplot as plt

N = 10
bowls = create_nodes(N)
edges = []

def plot_bowls(bowls, title):
    plt.figure()
    # Extract x and y coordinates
    x_coords, y_coords = zip(*bowls)

    # Plot the points and lines
    plt.title(title)
    plt.xlabel("x")
    plt.ylabel("y")
    plt.scatter(x_coords, y_coords, label="Bowls") # Draw the nodes (bowls)
    plt.plot(x_coords, y_coords, linestyle='-', color='orange', label="Connections")  # Add lines (paths)
```

Implementing Markov Chain with Metropolis Algorithm

Exercise 4

Write a function that decides whether or not to perform a swap. It should take as input: the old distance (before the swap), the new distance (after the swap), and the 'temperature'. It should return something representing reject or accept.

```
57]: def acceptanceProb(T, Di, Df):
         p = np.exp(-(1/T) * (Df - Di))
         if np.random.random() < p:
             return True
         return False
```

Also write a function that chooses, at random, two different bowl numbers (excluding 0 and excluding N). It should take as input how many bowls there are, and return the two chosen numbers.

```
58]: def choose_bowls(N):
         first = None
         second = None
         while first == second:
             first = np.random.randint(1, N-1)
             second = np.random.randint(1, N-1)
         return first, second
```

Then write a function that tries a swap of two randomly-chosen bowls, and keeps or rejects the swap according to the Metropolis algorithm. It should take the array of bowl positions, and T, as input. It should return the new array of bowl positions (which may or may not be different from the input array).

```
54]: def try_swap(bowls, T):
         # Current total path length
         curr_dist = path_length(bowls)

         # Choose two bowls to swap, swap them
         first, second = choose_bowls(len(bowls))
         bowls[first], bowls[second] = bowls[second], bowls[first]

         # Calculate the new path length after the swap
         new_dist = path_length(bowls)

         # Decide whether to keep the swap
         if acceptanceProb(T, curr_dist, new_dist):
             return bowls
         else:
             # Reject the swap: undo the swap
             bowls[first], bowls[second] = bowls[second], bowls[first]
             return bowls
```

Exercise 5

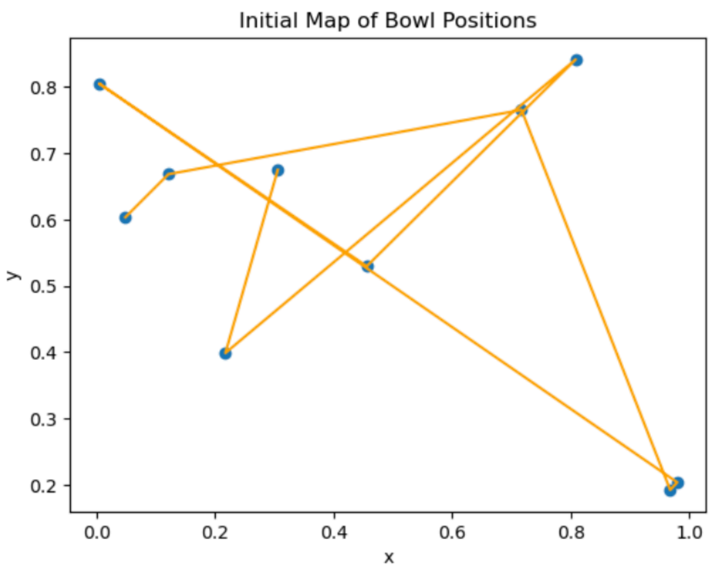
Write code that:

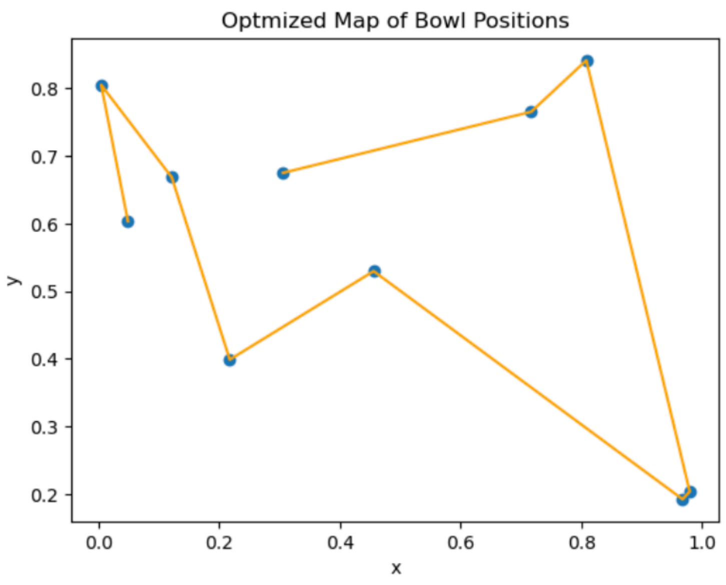
- calls your function to initialize the bowl locations
- calls your function to draw a map of the bowl locations
- prints out the total distance for the initial path
- performs the simulated annealing to minimize the distance along the path
- draws a map of the bowl locations and total distance for the solution

You should have N, τ, T_0, T_f as parameters; for now, set them to the following values:

4]:
```python
N = 60
tau = 1e4
T0 = 10.0
Tf = 1e-3

def simulated_annealing(bowls, T0, Tf, tau):
    T = T0
    t = 0  # Time step
    path_lengths = [path_length(bowls)]

    while T > Tf:
        bowls = try_swap(bowls, T)
        path_lengths.append(path_length(bowls))
        t += 1
        T = T0 * np.exp(-t / tau)  # Exponential cooling schedule

    return bowls, path_lengths

# Initialize bowls and plot initial configuration
bowls = create_nodes(N)
plot_bowls(bowls, "Initial Map of Bowl Positions")

initial_pathlen = path_length(bowls)

print(f"Initial Total Path Length: {initial_pathlen:.5f}")

# Perform simulated annealing
optimized_bowls, path_lengths = simulated_annealing(bowls, T0, Tf, tau)
print(f"Optimized Total Path Length: {optimized_pathlen:.5f}")

# Plot optimized configuration
plot_bowls(optimized_bowls, "Optmized Map of Bowl Positions")
```

```
Initial Total Path Length: 4.51503
Optimized Total Path Length: 2.68632
```





Exercise 6

Now modify your code to produce, along the way, two 'diagnostic' plots to get some more insight into what the algorithm is doing:

- distance vs t
- T vs t

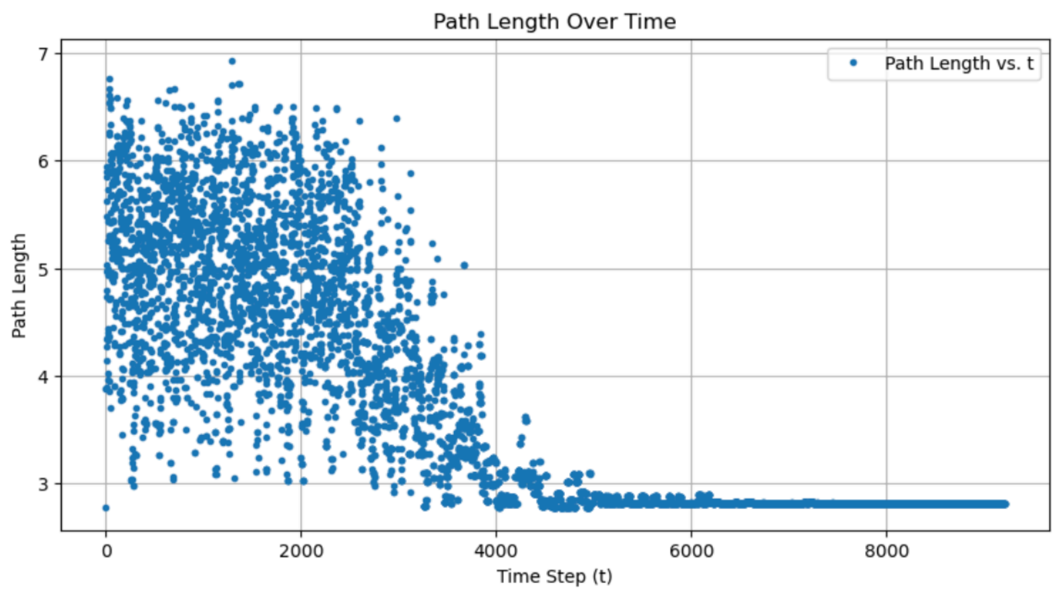
This means your code has to keep track of distance and T at each step.

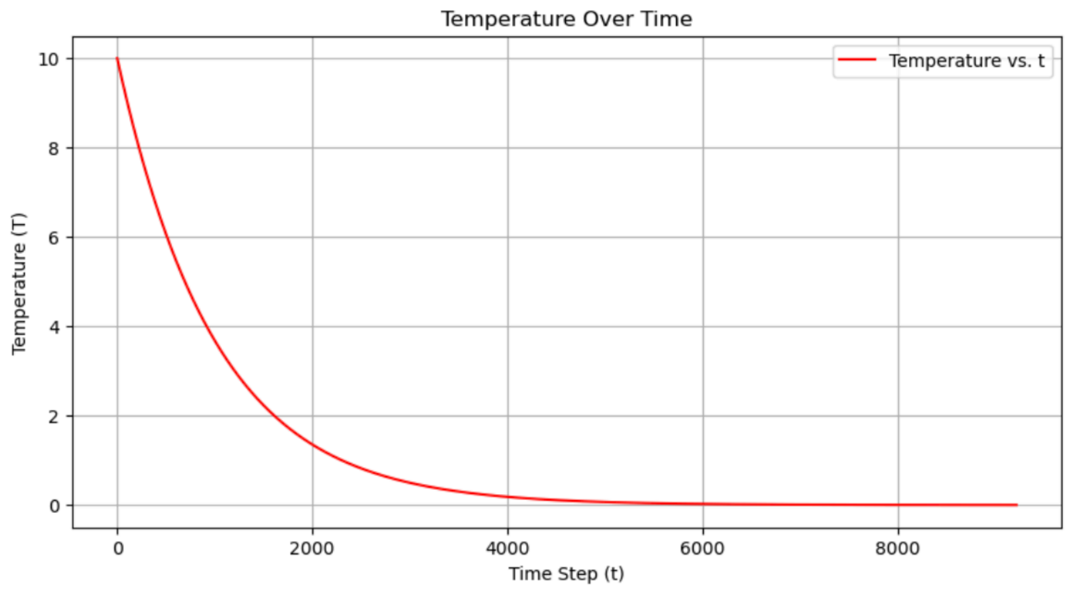
Also make your code time how long the simulated annealing algorithm takes (not including the initialization of the bowls, and not including any plotting).

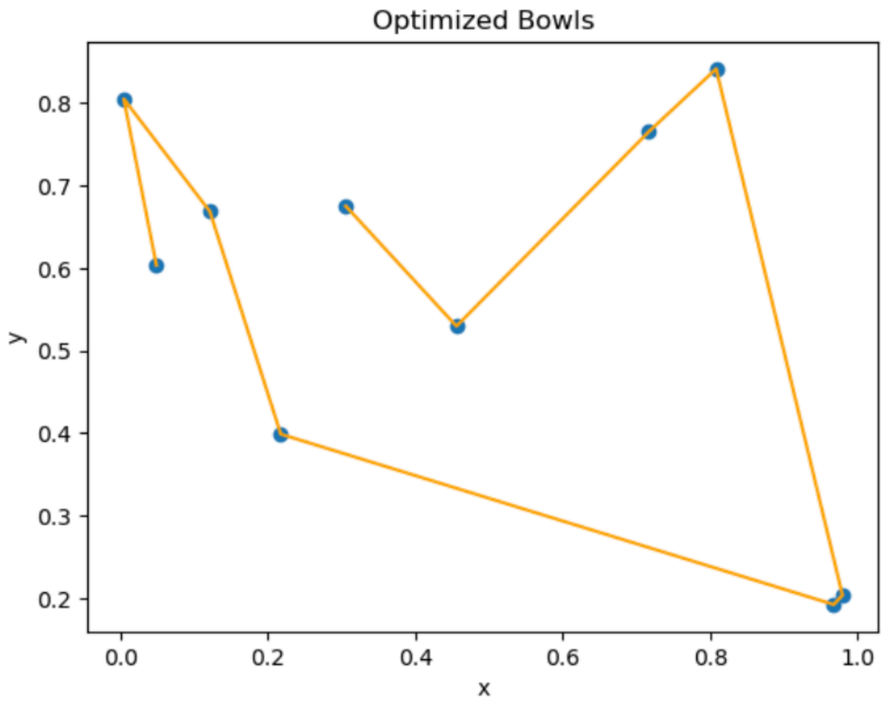
Exercise 7

```
]: port time
port numpy as np
om matplotlib import pyplot as plt

f simulated_annealing_with_diagnostics(bowls, T0, Tf, tau):
  T = T0
  t = 0  # Time step
  path_lengths = [path_length(bowls)]  # Track path lengths
  temperatures = [T]  # Track temperatures

  while T > Tf:
      bowls = try_swap(bowls, T)
      path_lengths.append(path_length(bowls))
      t += 1
      T = T0 * np.exp(-t / tau)  # Exponential cooling schedule
      temperatures.append(T)

  return bowls, path_lengths, temperatures, t


Perform simulated annealing with diagnostics
timized_bowls, path_lengths, temperatures, execution_time = simulated_annealing_with_diagnostics(bowls, T0, Tf, tau)
timized_pathlen = path_length(optimized_bowls)
int(f"Optimized Total Path Length: {optimized_pathlen:.5f}")
int(f"Execution Time: {execution_time:.2f} seconds")

t.figure(figsize=(10, 5))
t.plot(path_lengths,'.', label="Path Length vs. t")
t.xlabel("Time Step (t)")
t.ylabel("Path Length")
t.title("Path Length Over Time")
t.legend()
t.grid()
t.show()

t.figure(figsize=(10, 5))
t.plot(temperatures, label="Temperature vs. t", color="red")
t.xlabel("Time Step (t)")
t.ylabel("Temperature (T)")
t.title("Temperature Over Time")
t.legend()
t.grid()
t.show()

ot_bowls(optimized_bowls, "Optimized Bowls")
```

```
Optimized Total Path Length: 2.80633
Execution Time: 9211.00 seconds
```

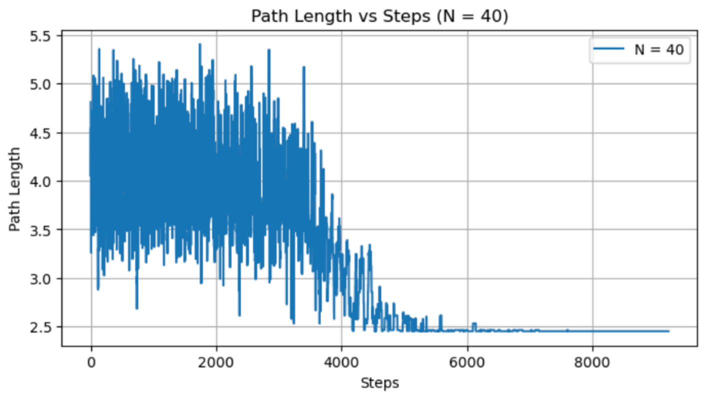

Path Length Over Time
Path Length vs. t
Path Length
Time Step (t)
0
2000
4000
6000
8000
3
4
5
6
7

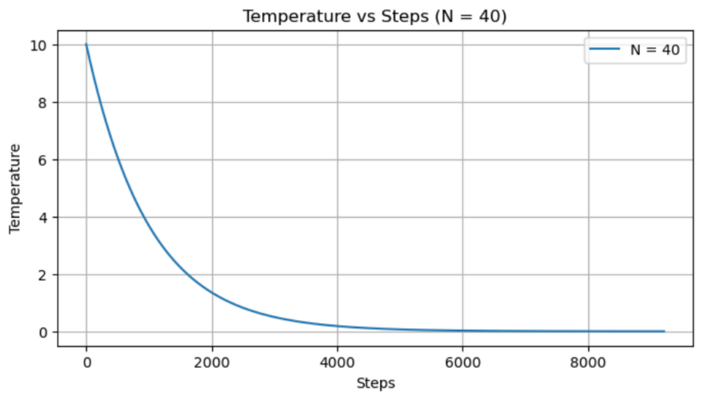

Temperature Over Time
Temperature vs. t
Temperature (T)
Time Step (t)
0
2000
4000
6000
8000
0
2
4
6
8
10

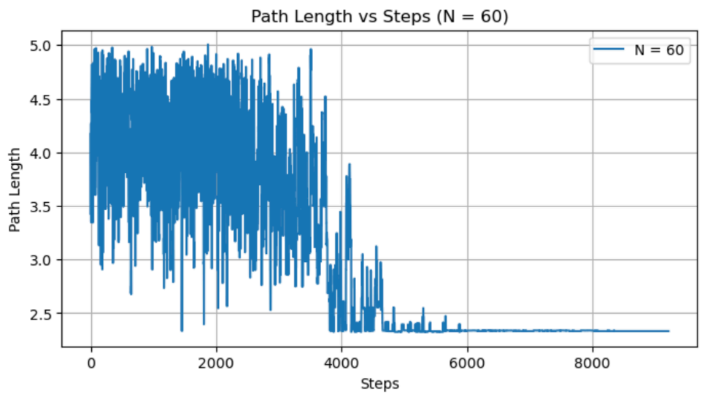

Optimized Bowls
y
x
0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8

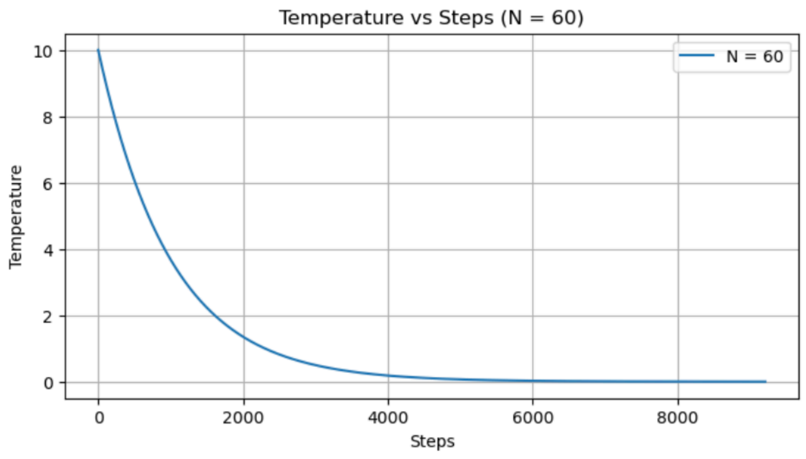
Try your code from the previous exercise with $N = 40$, then $N = 60$. As you increase N, do you notice the computation time going up? What about the number of steps required to reach a solution?

```
]: import matplotlib.pyplot as plt
   from time import time  # Importing time for measuring execution duration

   def run_with_different_N(N_values, T0, Tf, tau):
       results = []
       for N in N_values:
           # Generate initial bowl positions
           bowls = create_nodes(N)
           initial_pathlen = path_length(bowls)

           start_time = time()
           final_bowls, path_lengths, temperatures, total_steps = simulated_annealing_with_diagnostics(
               bowls, T0, Tf, tau)

           execution_time = time() - start_time

           final_pathlen = path_lengths[-1]

           result = {
               "N": N,
               "Initial Path Length": initial_pathlen,
               "Final Path Length": final_pathlen,
               "Steps": total_steps,
               "Execution Time (s)": execution_time,
               "Path Lengths": path_lengths,
               "Temperatures": temperatures,
           }
           results.append(result)

           # Plot Path Length vs Steps
           plt.figure(figsize=(8, 4))
           plt.plot(path_lengths, label=f'N = {N}')
           plt.xlabel("Steps")
           plt.ylabel("Path Length")
           plt.title(f"Path Length vs Steps (N = {N})")
           plt.legend()
           plt.grid()
           plt.show()

           # Plot Temperature vs Steps
           plt.figure(figsize=(8, 4))
           plt.plot(temperatures, label=f'N = {N}')
           plt.xlabel("Steps")
           plt.ylabel("Temperature")
           plt.title(f"Temperature vs Steps (N = {N})")
           plt.legend()
           plt.grid()
           plt.show()

       return results

   results = run_with_different_N(N_values, T0, Tf, tau)

   for result in results:
       print(f"N = {result['N']}, Initial Path Length: {result['Initial Path Length']:.2f}, "
             f"Final Path Length: {result['Final Path Length']:.2f}, Steps: {result['Steps']}, "
             f"Execution Time: {result['Execution Time (s)']:.2f} seconds")
```









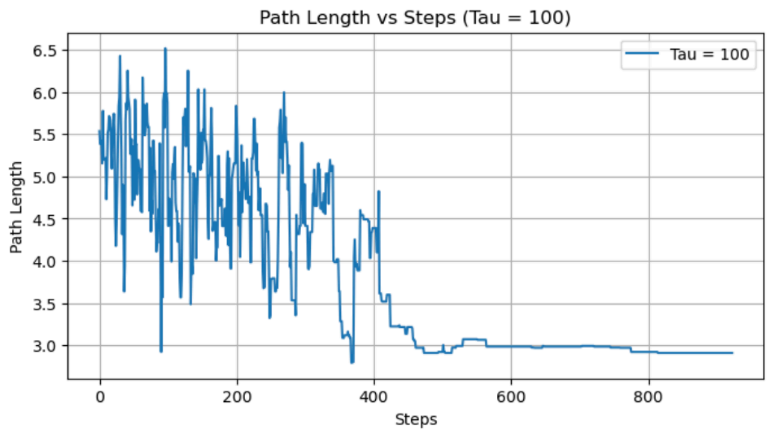
```
N = 40, Initial Path Length: 4.81, Final Path Length: 2.45, Steps: 9211, Execution Time: 0.32 seconds
N = 60, Initial Path Length: 4.17, Final Path Length: 2.33, Steps: 9211, Execution Time: 0.30 seconds
```

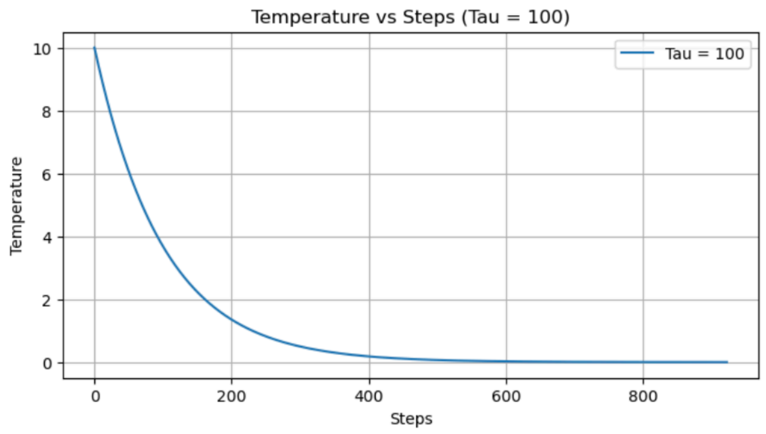
Exercise 8

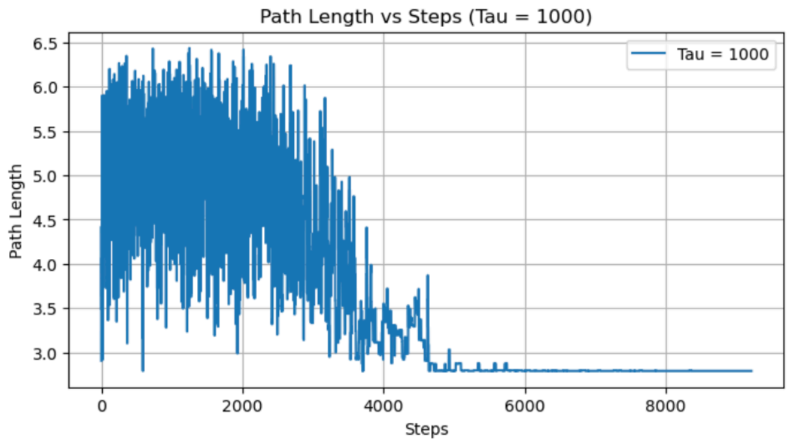
With $N = 60$, now try a few different values of $\tau : \ 100, \ 1000, \ 10000$, with the same random seed each time. Does the solution itself change? What about the execution time? Can you see the trade-off between computational speed and possibility of getting stuck in local minimum?

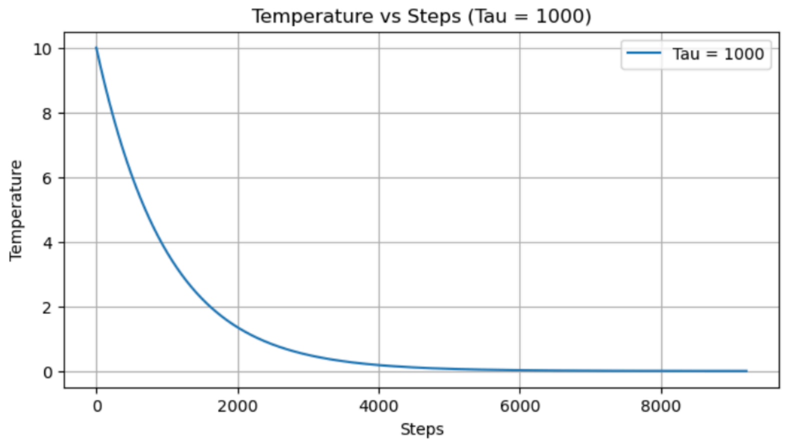
```
]: def run_with_different_tau(tau_values, N, T0, Tf):
       np.random.seed(42)  # Set a fixed random seed for consistent results
       results = []
       bowls = create_nodes(N)  # Create the bowl positions

       for tau in tau_values:
           start_time = time()
           final_bowls, path_lengths, temperatures, total_steps = simulated_annealing_with_diagnostics(
               bowls, T0, Tf, tau
           )
           execution_time = time() - start_time

           result = {
               "Tau": tau,
               "Initial Path Length": path_length(bowls),
               "Final Path Length": path_lengths[-1],
               "Steps": total_steps,
               "Execution Time": execution_time,
               "Path Lengths": path_lengths,
               "Temperatures": temperatures,
           }
           results.append(result)

       return results

   results = run_with_different_tau(tau_values, N, T0, Tf)

   for result in results:
       print(f"Tau = {result['Tau']}, Initial Path Length: {result['Initial Path Length']:.2f}, "
             f"Final Path Length: {result['Final Path Length']:.2f}, Steps: {result['Steps']}, "
             f"Execution Time: {result['Execution Time']:.2f} seconds")


   for result in results:
       # Plot Path Length vs Steps
       plt.figure(figsize=(8, 4))
       plt.plot(result['Path Lengths'], label=f'Tau = {result["Tau"]}')
       plt.xlabel("Steps")
       plt.ylabel("Path Length")
       plt.title(f"Path Length vs Steps (Tau = {result['Tau']})")
       plt.legend()
       plt.grid()
       plt.show()

       # Plot Temperature vs Steps
       plt.figure(figsize=(8, 4))
       plt.plot(result['Temperatures'], label=f'Tau = {result["Tau"]}')
       plt.xlabel("Steps")
       plt.ylabel("Temperature")
       plt.title(f"Temperature vs Steps (Tau = {result['Tau']})")
       plt.legend()
       plt.grid()
       plt.show()
```

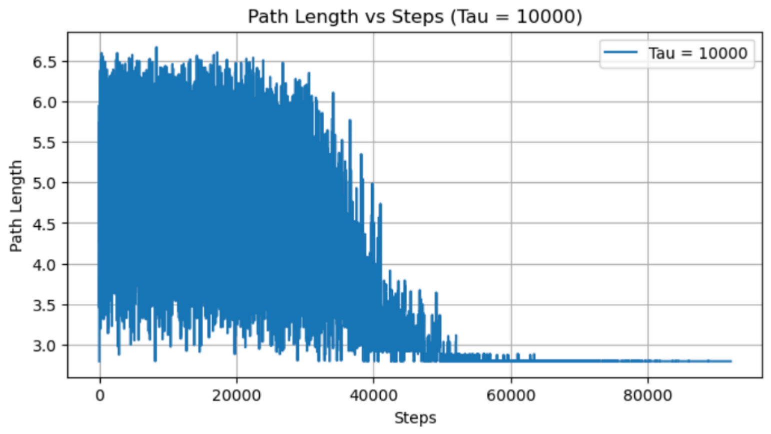
```
Tau = 100, Initial Path Length: 2.91, Final Path Length: 2.91, Steps: 922, Execution Time: 0.03 seconds
Tau = 1000, Initial Path Length: 2.79, Final Path Length: 2.79, Steps: 9211, Execution Time: 0.30 seconds
Tau = 10000, Initial Path Length: 2.79, Final Path Length: 2.79, Steps: 92104, Execution Time: 3.04 seconds
```

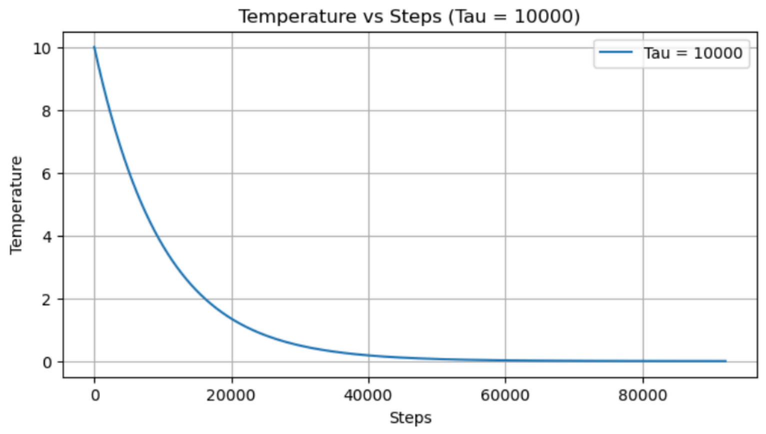

Path Length vs Steps (Tau = 100)
Tau = 100
Path Length
Steps


Temperature vs Steps (Tau = 100)
Tau = 100
Temperature
Steps


Path Length vs Steps (Tau = 1000)
Tau = 1000
Path Length
Steps


Temperature vs Steps (Tau = 1000)
Tau = 1000
Temperature
Steps


Path Length vs Steps (Tau = 10000)
Tau = 10000
Path Length
Steps


Temperature vs Steps (Tau = 10000)
Tau = 10000
Temperature
Steps