

CSC384H Tutorial 7

First-Order Logic and Resolution

Semantics

1. Consider the following interpretation \mathcal{M} :

$$M = \{A, B, C, D, E\}$$

$$a^{\mathcal{M}} = A, b^{\mathcal{M}} = B, c^{\mathcal{M}} = C, d^{\mathcal{M}} = D, e^{\mathcal{M}} = E$$

$$P^{\mathcal{M}} = \{A, B, C\}$$

$$R^{\mathcal{M}} = \{(A, B), (A, C), (A, D)\}$$

$$Q^{\mathcal{M}} = \{(A, B), (C, D), (B, A), (D, C)\}$$

Which of the following sentences are true in this interpretation \mathcal{M} ?

(a) $\forall x, y Q(x, y) \rightarrow Q(y, x)$

(b) $\forall x, y R(x, y) \rightarrow R(y, x)$

(c) $\forall x, y R(x, y) \rightarrow P(x)$

(d) $\forall x, y R(x, y) \rightarrow P(y)$

(e) $\forall x (\exists y R(x, y) \rightarrow x = a)$

(f) $\forall x, y R(x, y) \rightarrow \exists z Q(z, y)$

2. A **tautology** is a formula that is true in every possible structure. The sentence

$(\forall x)(P(x) \vee \neg P(x))$ is an example of a tautology.

Determine whether or not the following sentence is a tautology. **Justify** your answer.

$$\forall x \left[\left(\forall y (P_1(x, y) \rightarrow (P_2(y) \vee P_3(y))) \right) \rightarrow \left((\forall y (P_1(x, y) \rightarrow P_2(y))) \vee (\forall y (P_1(x, y) \rightarrow P_3(y))) \right) \right]$$

Clausal Form

Convert the following FOL sentences into clausal form:

1. $\forall x (\text{child}(x) \rightarrow \text{loves}(x, \text{santa}))$
2. $\forall x, y (\text{loves}(x, \text{santa}) \wedge \text{reindeer}(y) \rightarrow \text{loves}(x, y))$
3. $\text{reindeer}(\text{rudolph}) \wedge \text{has}(\text{rudolph}, \text{red-nose})$
4. $\forall x (\text{has}(x, \text{red-nose}) \rightarrow (\text{weird}(x) \vee \text{clown}(x)))$
5. $\neg \exists x (\text{reindeer}(x) \wedge \text{clown}(x))$
6. $\neg \exists x (\text{loves}(\text{scrooge}, x) \wedge \text{weird}(x))$
7. $\neg \text{child}(\text{scrooge})$

Resolution

1. Prove statement 7 in the previous question from the previous statements using resolution.
2. Translate the following sentences to first-order logic, and use resolution to show that John will watch MovieA.
 - MovieA is a comedy and will play on TV on Monday.
 - John likes comedies.
 - John doesn't work on Monday.
 - Someone has free time on particular day if they don't work that day.
 - Someone will watch a movie, if that movie plays on a TV on a day where they have free time, and they like that movie.