CSC384H Tutorial 3

CSP & Backtracking

Summer 2025

CSP Formulation

Map-Coloring Example: The goal is to assign a color to each region, such that all adjacent regions have different colors. Consider the map of figure 1.

Figure 1: A map-coloring problem.

We formulate the CSP as follows:

- Variables: WA, NT, SA, Q, NSW, V, T
- Domains: $D = \{red, green, blue\}$
- Constraints: $WA \neq NT$, $WA \neq SA$, $NT \neq SA$, $NT \neq Q$, $SA \neq Q$, $SA \neq NSW$, $SA \neq V$, $Q \neq NSW$, $NSW \neq V$

A solution to this problem (Figure 1) is the following: $WA = red$, $NT = green$, $SA = blue$, $Q = red$, $NSW = green$, $V = red$, $T = green$.

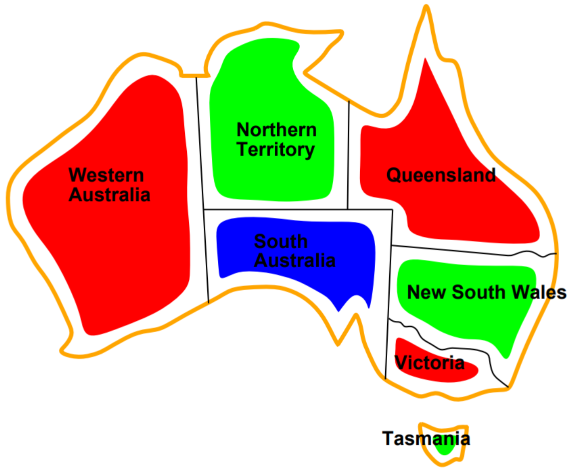

Figure 2: A solution to the Map-Coloring problem.

Backtracking

Consider the following simple CSP:

- Variables: $X,\ Y,\ Z,\ W$
- Domains: $D_X = D_Y = D_Z = \{1, 2, 3\}, D_W = \{1, 2, 3, 4\}$
- Constraints: $C_1 : W = X + Y + Z$

We will find a solution using *plain backtracking*. Notice that we check if constraint C_1 is satisfied only when every variable has been assigned a value.

Current Assignment	Constraint C_1
X = 1	
X = 1, Y = 1	
X = 1, Y = 1, Z = 1	
X = 1, Y = 1, Z = 1, W = 1	False
X = 1, Y = 1, Z = 1, W = 2	False
X = 1, Y = 1, Z = 1, W = 3	True

A satisfying assignment: *(X,Y,Z,W) = (1,1,1,3).*

Questions

1. Execute plain backtracking to find a solution to the Map-Coloring problem of Figure 1. Show all the assigned variables at each step.

2. Consider the following CSP: There are three different musicians: **John, Mark,** and **Sam.** They each come from a different country; one comes from the **United States**, one from **Australia**, and one from **Japan**. They each play a different musical instrument; one plays the **piano**, one the **saxophone**, and one the **violin.** We also have the following information:

- The pianist plays first.
- John plays the saxophone and plays before the Australian.
- Mark comes from the United States and plays before the violinist.

(a) Formulate this problem as a CSP in order the answer the following questions. **1.** What is the order the instruments are played. **2.** Who plays what instrument. **3.** What is the nationality of each player.

(b) Execute plain backtracking to find a solution to this CSP. At each step, you are free to pick any unassigned variable, and you can try values from its domain in any order you want.

3. Consider the *item allocation problem*. We have a group of people $N = \{1, \dots, n\}$, and a group of items $G = \{g_1, \dots, g_m\}$. Each person $i \in N$ has a utility function $u_i : G \to \mathbb{R}^+$. The constraint is that every person is assigned *at most one item*, and each item is assigned to *at most one person*. An allocation simply says which person gets which item (if any).

   In what follows, you *must* use only the binary variables $x_{i,j} \in \{0, 1\}$, where $x_{i,j} = 1$ if person i receives the good g_j, and is 0 otherwise.

   (a) Write out the constraints:

      i. each person receives no more than one item

      ii. each item goes to at most one person

      using only the variables $x_{i,j}$.[1]

   (b) Suppose that people were divided into *disjoint types* $N_1, \dots, N_k$ (e.g. say, genders or ethnicities), whereas items were divided into *disjoint blocks* $G_1, \dots, G_l$. We further require that each N_p only be allowed to take no more than λ_{pq} items from block G_q.

      Write out this constraint using the variables $x_{i,j}$. (Note that each N_i corresponds to the set of people who are of that person-type.)

   (c) We say that player i *envies* player i' if the utility that player i has from their assigned item is strictly lower than the utility that player i has from the item assigned to player i'.

      Write out the constraints that ensure that in the allocation, no player envies any other player. You may assume that the validity constraints from (a) hold.

[1] You may use the simple algebraic functions $+, -, \times, \div$, and numbers.