CSC384H Tutorial 6

Logic

Overview of Logic

Given any variables p, q, and r, we have the following:

1	Commutative law	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2	Associative law	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3	Distributive law	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4	Identity law	$p \wedge \mathbf{true} \equiv p$	$p \vee \mathbf{false} \equiv p$
5	Universal bound law	$p \vee \mathbf{true} \equiv \mathbf{true}$	$p \wedge \mathbf{false} \equiv \mathbf{false}$
6	Idempotent law	$p \wedge p \equiv p$	$p \vee p \equiv p$
7	Negation law	$p \vee \neg p \equiv \mathbf{true}$	$p \wedge \neg p \equiv \mathbf{false}$
8	Double negation law	$\neg(\neg p) \equiv p$	
9	de Morgan's law	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
10	Absorption law	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11	Implication law	$p \rightarrow q \equiv \neg p \vee q$	

Propositional Logic

1. Using the table provided above, verify the following logical equivalences.

   (a) $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$

   (b) $(p \wedge \neg(\neg p \vee q)) \vee (p \wedge q) \equiv p$

2. Suppose we are given the following knowledge base:

   - $P_1 : p \rightarrow q$
   - $P_2 : r \rightarrow p$
   - $P_3 : \neg q$
   - $P_4 : r \vee p \vee s$

   Show that this knowledge base entails s.

3. Show that the following statement is unsatisfiable.

   $$(p \vee q) \wedge (\neg p \vee r) \wedge (\neg p \vee \neg r) \wedge (p \vee \neg q)$$

First-Order Logic

1. Translate the following sentences from English to first-order-logic.
   (a) Nobody likes taxes.
   (b) Some people like anchovies.
   (c) Emma is a Doberman pincher and a good dog.
   (d) All hounds howl at night.
   (e) Anyone who has any cats will not have any mice.
   (f) Light sleepers do not have anything which howls at night.
   (g) There is only one ring that rules them all.
   (h) Ham and Enos are the only chimpanzees that have been to space.