Investigating Interference with a Michelson Interferometer

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The Michelson interferometer is an optical device used to study interference, coherence, and fringe contrast. In this experiment, we measured fringe contrast as a function of optical path difference and analyzed its dependence on the coherence properties of a HeNe laser. The results showed a general decrease in contrast with increasing path difference, consistent with coherence-limited decay. However, due to limited resolution, we could not resolve the expected oscillatory behavior of the interference fringes. Experimental imperfections such as misalignment, beam divergence, and polarization effects contributed to deviations from the ideal model.

Introduction

The Michelson interferometer is a fundamental optical instrument that demonstrates interference by splitting and recombining a coherent light source. The traditional version of the device works by splitting the incoming light (typically from a laser) into two perpendicular beams via a 50: 50 beam splitter and recombining them with the help of two mirrors. The superposition of these beams creates an interference pattern that depends on their relative phase difference once they reach the detector.

The phase ϕ of a coherent monochromatic light source at any point is given by

$$\phi = kz - \omega t$$

where z is the position along the direction of wave propagation, ω is the angular frequency of the wave, and t is time. Thus, the spatial dependence of the phase is determined by the kz term. the spatial dependence of the phase is determined by the kz term. As a wave with wavenumber k propagates through a medium, it accumulates phase proportional to the optical path length, which accounts for both the physical length of the path and the refractive index of the medium through which the light propagates. In particular, if a wave travels a physical distance L in a medium with refractive index n, the optical path length is given by nL, and the phase accumulated along that path is

$$\phi = k(nL) = \frac{2\pi nL}{\lambda}$$

In a Michelson interferometer, the optical path difference (OPD) between the two arms is given by:

$$d = 2(L_1 n_1 - L_2 n_2), \tag{1}$$

where L_i are the physical path lengths and n_i are the corresponding refractive indices. The factor of 2 comes from the fact that the beams traverse their respective optical path twice. The phase difference between the two interfering beams at the detector is thus

$$\Delta\phi = \frac{2\pi d}{\lambda} \tag{2}$$

One can see that, upon recombination at the detector, the beams will interfere constructively when d is an integer multiple of λ, and destructively when d is a half-integer multiple of λ, forming an interference pattern.

Since the two beams originate from the same source, they have the same amplitude E_0, and their superposition at the detector can be expressed as

$$E_{\text{tot}} = E_0 e^{i\omega t} + E_0 e^{i(\omega t + \Delta\phi)}$$

Factoring out $E_0 e^{i\omega t}$, we obtain

$$E_{\text{tot}} = E_0 e^{i\omega t}(1 + e^{i\Delta\phi}).$$

Taking the squared magnitude to obtain the intensity

$$I_{\text{tot}} = \frac{nc\epsilon_0}{2}|E_{\text{tot}}|^2 = I_0|1 + e^{i\Delta\phi}|^2.$$

Using the identity

$$1 + e^{i\Delta\phi} = 2\cos\left(\frac{\Delta\phi}{2}\right)e^{i\frac{\Delta\phi}{2}}$$

we get

$$I_{\text{tot}}(d) = 4I_0\cos^2\left(\frac{\pi d}{\lambda}\right). \tag{3}$$

Thus, the intensity follows a $\cos^2$ dependence on the optical path difference. This oscillatory behavior determines the visibility of interference fringes, where constructive interference occurs at $d = m\lambda$ and destructive interference at for an integer m. However, the contrast of these fringes diminishes for large path differences due to the finite coherence of the light source.

Coherence Considerations

The ability of the Michelson interferometer to produce stable interference fringes depends on the coherence properties of the light source. A perfectly monochromatic plane wave has an infinite coherence length, meaning it would produce interference fringes regardless of the optical path difference. However, real light sources have a (nonzero) finite spectral linewidth, which limits their coherence time τ_c and coherence length ℓ_c.

For a laser with a central frequency ν_0 and spectral linewidth $\Delta\nu$, the coherence time is given by [2]

$$\tau_c = \frac{1}{\pi \Delta\nu}. \tag{4}$$

The corresponding coherence length is

$$\ell_c = c\tau_c = \frac{c}{\pi \Delta\nu}. \tag{5}$$

In this experiment, we use a HeNe laser ($\lambda = 632.8$ nm) with a spectral linewidth of about 1.5 GHz, corresponding to a coherence length of approximately 6cm [1]:

$$\tau_c = \frac{1}{\pi \times (1.5 \times 10^9 \text{ Hz})} \approx 0.21\,\text{ns}$$

$$\ell_c = (3.0 \times 10^8 \text{ m/s}) \times (0.21 \times 10^{-9} \text{ s}) \approx 6.3\,\text{cm}.$$

Since the path differences in this experiment are on the order of centimeters, we may observe a gradual reduction in fringe contrast if the optical path difference approaches or exceeds this coherence length.

We can quantify the coherence and alignment in the interferometer using the visibility of the interference pattern. Hence, we define the fringe contrast as

$$F = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}} + V_{\text{min}}}, \tag{6}$$

where V_{max} and V_{min} are the maximum and minimum voltages recorded by the photodiode. Equivalently, since $V \propto I$, one can write

$$F = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}, \tag{7}$$

where I_{max} and I_{min} represent the maximum and minimum recorded intensities. In an idealized case where the two interfering beams are perfectly coherent, the total intensity at the detector follows

$$I_{\text{tot}}(d) = 2I_0[1 - \cos(\Delta\phi)], \tag{8}$$

where the phase difference is given by $\Delta\phi = \frac{2\pi d}{\lambda}$. This means that the intensity oscillates between

$$I_{\text{max}} = 2I_0[1 + 1] = 4I_0,$$
$$I_{\text{min}} = 2I_0[1 - 1] = 0.$$

This would yield a perfect fringe contrast of $F = 1$, assuming perfect coherence.

However, real light sources have a finite spectral linewidth, meaning their coherence is limited. This leads to a gradual loss of contrast as the optical path difference increases. This effect can be modeled by introducing a decaying exponential envelope, which accounts for coherence loss:

$$I_{\text{max}}(d) = 2I_0[1 + e^{-|d|/\ell_c}],$$
$$I_{\text{min}}(d) = 2I_0[1 - e^{-|d|/\ell_c}].$$

Substituting these into the fringe contrast formula

$$F(d) = \frac{2I_0[1 + e^{-|d|/\ell_c}] - 2I_0[1 - e^{-|d|/\ell_c}]}{2I_0[21 + e^{-|d|/\ell_c}] + 2I_0[1 - e^{-|d|/\ell_c}]}$$

$$F(d) = \frac{4I_0 e^{-|d|/\ell_c}}{4I_0} = e^{-|d|/\ell_c}.$$

Thus, the contrast follows an exponential decay as a function of path difference. Since the intensity itself oscillates with a $\cos^2(\Delta\phi/2)$ dependence, the full expression for fringe contrast is given by

$$F(d) = e^{-|d|/\ell_c} \cos^2\left(\frac{\pi d}{\lambda}\right). \tag{9}$$

This equation describes how the visibility of the fringes decays exponentially due to the finite coherence of the laser, while still retaining the periodic oscillations due to interference.

However, due to the limited resolution of our measurements, which were taken at millimeter-scale intervals, we were unable to resolve the oscillatory $\cos^2$ dependence. Instead, we expect the data points to be bounded above by the envelope of the exponential decay:

$$F(d) \leq Ae^{-|d|/L_c} \tag{10}$$

where $A < 1$ accounts for additional experimental imperfections, such as alignment errors or beam profile variations, and $\ell_c = 6$ cm is the expected coherence length for our HeNe laser.

Polarization Effects on Interference

If the two arms of the interferometer contain different polarization states, the interference contrast can be reduced or eliminated. If the two beams are fully orthogonal (e.g., one is horizontally polarized and the other is vertically polarized), they cannot interfere at all, and no fringes will be seen. Similarly, if the beams have partially different polarizations, only the common polarization component will interfere, leading to reduced fringe contrast. The introduction of a waveplate in one arm can alter the polarization state and affect fringe contrast.

Experimental Goals

We aim to align a Michelson interferometer and observe interference fringes, measuring how fringe contrast varies as a function of optical path difference. Furthermore, we discuss how deliberately modifying the polarization states of the light in the two arms of the interferometer would change the observed interference pattern.

EXPERIMENTAL SETUP

A schematic of the Michelson interferometer setup is shown in Fig. 1. The position of M1 was varied, keeping M2 fixed, thus modifying the OPD. The laser height was measured to be at 9 cm, and all components were aligned accordingly. A photodiode was used to record intensity fluctuations, and an oscilloscope captured variations in the interference pattern.

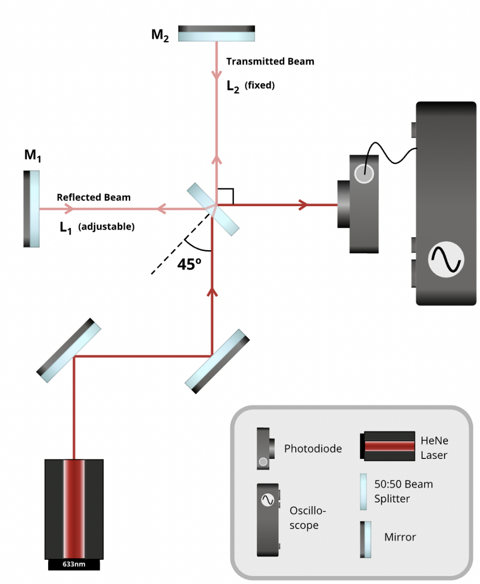


FIG. 1. Experimental setup of the Michelson interferometer.

To measure the intensity of a single fringe at each value of d, we used an iris to isolate it, as seen in Figure 2

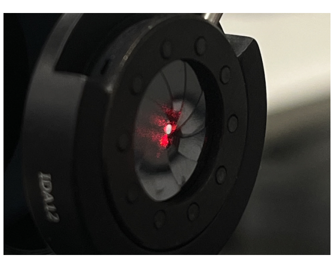


FIG. 2. Using an iris to pick out a single fringe from the interference pattern.

RESULTS

Table I presents the measured values of V_{max}, V_{min}, and the computed fringe contrast F for various path length differences d.

d (cm)	V_{max} **(V)**	V_{min} **(V)**	**Fringe Contrast F**
± 0.05	± 0.3	± 0.3	± 0.4
0.4	8.15	1.30	0.725
0.2	6.17	2.74	0.385
0.0	5.78	2.55	0.388
-0.2	5.73	2.70	0.359
-0.4	5.20	2.84	0.294

TABLE I. Measured fringe contrast values as a function of path length difference.

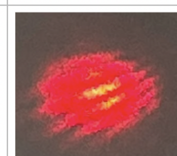
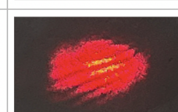
d	**F**	**Fringes**
0.4	0.72	
0.2	0.38	
0.0	0.39	
-0.2	0.36	
-0.4	0.38	

FIG. 3. Observed interference fringes at different path length differences d.

Figure 3 shows the photographed fringes corresponding to each value of d. Due to the fine nature of the fringes and the limited resolution of our phone camera, we were were not able to capture the full detail of the interference pattern. However, the higher fringe visibility is noticeable in the fringe pattern at $d = 0.4$ cm, which is expected given the higher value of F.

ANALYSIS

Figure 4 summarizes the fringe contrast F as a function of path length difference d in our Michelson interferometer. Overall, the measured fringe contrasts lie within the range of 0.30 to 0.40 for most path differences, suggesting a moderate level of coherence that does not vary dramatically over the ± 0.4 cm range. The theoretical curve (green, dashed line) illustrates the expected behavior under an ideal exponential (or cosine) dependence, although our data systematically falls below this idealized model by about 0.3–0.5.

One reason for this significant deviation is that, as discussed in the introduction, the fringe contrast actually oscillates according to a $\cos^2$ dependence, as seen in equation (10). In order for our data points to have matched the expected plot, we would have had to measure the precise center of the fringe (where there was perfect constructive interference) at each d using the iris. However, it is far more likely that we measured some small amount away from the true center.

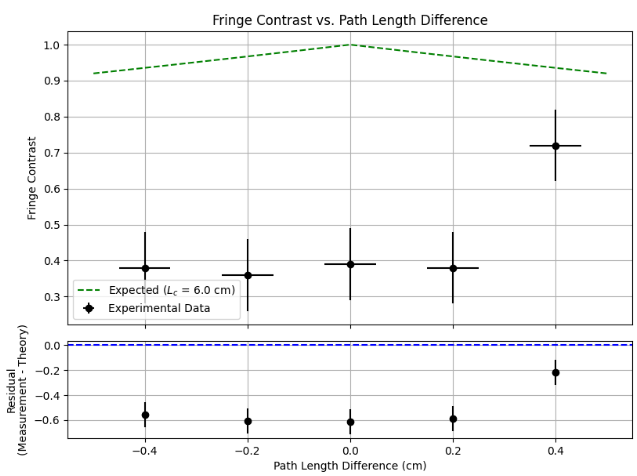


FIG. 4. Fringe contrast as a function of path length difference. The green dashed line represents the expected upper bound based on the coherence length $L_c = 6.0$ cm, while the black data points represent the measured fringe contrast values with error bars. The bottom panel shows the residuals (measured contrast minus expected contrast), indicating systematic deviations from the theoretical model.

We note that the data point at $d = 0.4$ cm exhibits a higher fringe contrast ($F \approx 0.72$), considerably larger than the other points, which can be in part explained by the possibility that we measured closer to the center of the fringe in this instance. However, it could also have been the case that during the $d = 0.4$ cm measurement, the setup may have been less susceptible to vibrations or air currents. A transiently quieter lab environment could allow the photodiode to register a more "ideal" fringe maximum compared to the other points. In addition, lasers can experience momentary changes in output power or mode structure, leading to large swings in $V_{\max}$. If the interference happened to be recorded during such a fluctuation, the resulting contrast could spuriously jump.

Although it deviates from the broader trend, this high-contrast data point is not necessarily an experimental "mistake." Further measurements at smaller intervals would reveal whether this value reproduces consistently, lending credence to a genuine alignment condition, or if it remains an isolated spike caused by random fluctuation or systematic error.

Experimental Limitations and Sources of Error

Several factors likely contributed to the discrepancy between our measured fringe contrast and the theoretical predictions. First, achieving and maintaining perfect beam overlap in a Michelson interferometer is difficult. Our observed interference included mostly linear (rather than concentric) fringes, indicating partial overlap and a systematic reduction in fringe visibility. In addition, the beam splitter and mirrors may have introduced slight polarization differences between the two arms. Because only the common polarization component interferes, any mismatch can lower the measurable contrast. We did not explicitly control or characterize polarization in each arm, so unaccounted polarization effects likely contributed to the observed data scatter.

Another source of error arises from how we sampled the interference signal. It is possible we did not adjust the iris to allow the optimal amount of light to pass through in order to collect sufficient yet precise measurements. Furthermore, the path-length difference was adjusted in discrete steps, which may fail to capture rapid oscillations of the fringe pattern. A more refined strategy, such as continuously scanning the mirror position and recording the entire photodiode trace, would allow a more accurate measurement of the peak and minimum fringe intensities. This approach would also mitigate the impact of the oscilloscope's voltage uncertainty (± 0.3 V) and our mirror-position uncertainty (± 0.05 cm), both of which introduced random fluctuations into the computed fringe contrast.

Lastly, environmental noise such as vibrations, air currents, and temperature drifts inevitably disturbed mirror alignment on short timescales, causing further variation in the recorded fringe maxima and minima. Enclosing the setup or using a vibration-isolated table could reduce these perturbations significantly. Overall, to improve the fidelity of our results, we recommend implementing a continuous (rather than discrete) scan of the path-length difference, thoroughly stabilizing the optical layout, and carefully verifying the polarization states of the interfering beams.

POLARIZATION EFFECTS IN THE MICHELSON INTERFEROMETER

An important factor influencing interference in a Michelson interferometer is the relative polarization of the two interfering beams. If the two beams have identical polarization states, they will interfere fully, leading to maximum fringe contrast. However, if they are partially or fully orthogonally polarized, the visibility of the interference fringes will decrease accordingly. In the extreme case of complete orthogonality, no interference fringes would be observed.

A waveplate introduces a controlled phase delay between orthogonal polarization components of the light. A quarter-wave plate ($\lambda/4$) converts linearly polarized light into circularly polarized light when oriented at 45° to the incident polarization axis, while a half-wave plate ($\lambda/2$) rotates the polarization direction, modifying the degree of overlap between the two interfering beams.

Expected Results

Since only the common polarization component of the two beams contributes to the interference, introducing a waveplate in one arm of the interferometer would modify the fringe contrast.

When the waveplate is aligned such that it preserves the original polarization state, the fringe contrast should remain unchanged. As the waveplate is rotated, we would expect the contrast to gradually decrease due to the introduction of a phase shift between orthogonal polarization components. At specific angles (e.g., 45° for a quarter-wave plate), the polarization difference would be maximized, leading to a significant reduction in contrast. Moreover, if a linear polarizer is used after the interferometer, it would help study the polarization-induced changes more clearly by filtering out unwanted polarization components.

Although we did not perform this part of the experiment, understanding how polarization affects interference is crucial for applications such as fiber optics, quantum optics, and precision interferometry.

CONCLUSION

In this work, we studied how fringe contrast in a Michelson interferometer depends on path length difference. Our measurements were broadly consistent with an exponential (coherence-limited) envelope superimposed on a $\cos^2$ oscillation, though the finite sampling step and various alignment imperfections obscured the fine oscillatory structure. One data point, at $d = 0.4\,\text{cm}$, yielded a notably higher fringe contrast, suggesting that near-ideal constructive overlap may be sporadically obtained even if the nominal $d = 0$ position is not perfectly aligned. Despite such anomalies, the results qualitatively demonstrate the loss of fringe contrast for path differences approaching the measured coherence length of our HeNe laser. In future work, using a continuously scanning stage, careful polarization control, and improved mechanical isolation would enable more accurate and higher-resolution measurements of the interference pattern.

[1] Thorlabs, Inc., "Helium Neon Lasers," Accessed March 1, 2025. `https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=10776`

[2] RP Photonics Encyclopedia, "Coherence Time," Accessed March 1, 2025. `https://www.rp-photonics.com/coherence_time.html`

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