

Computer Science 384
St. George Campus

Saturday, August 2, 2025
University of Toronto

Homework Assignment #4: KR&R
Due: Friday, August 15, 2025 by 11:00 PM

Late Policy: Please refer to the Course Information sheet posted on Quercus. Please note that grace tokens will be applied **automatically** as long as you submit your assignment during the grace token period.

Total Marks: This assignment represents 5% of the course grade.

Handing in this Assignment

What to hand in on paper: Nothing.

What to hand in electronically: **Submit written answers in a file called** `answers.pdf`. Assignments must be completed **individually**, and you must submit a *single* document. Handwritten submissions are acceptable as long as they are written **neatly** and **legibly** (typed submissions are preferred but **not** required).

IMPORTANT: For each question, please write up detailed answers carefully. Make sure that you use **notation** and **terminology** correctly, and that you explain and **justify** what you are doing. Marks will be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

How to submit: You will submit your assignment using MarkUs. You can submit a new version of any file at any time. For the purposes of determining the number of grace days required for a late submission, the submission time is considered to be the time of your latest submission.

Clarifications: Important corrections (hopefully few or none) and clarifications to the assignment will be announced on Quercus.

You are responsible for monitoring Quercus announcements for any clarifications or corrections.

Help Sessions: There will be help sessions for this assignment. Dates and times for these sessions will be posted on Quercus.

Questions: Questions about the assignment should be posed on Piazza.

1. [15 marks] Consider the blocks world domain (NBW), and suppose the vocabulary of the domain, \mathcal{L}_{NBW} includes the following symbols:

Predicate Symbols:

- $Above(x, y)$ iff x is above y , either directly or indirectly.
- $Under(y, x)$ iff ((x is the **unique block immediately above** block y) or (x is not above any blocks and $y = x$)).
- $Clear(x)$ iff no blocks are above x .
- $OnTable(x)$ iff x is not above any blocks.

Let Φ be a set containing the following sentences:

- $\forall x(\neg Above(x, x))$
- $\forall x \forall y \forall z ((Above(x, y) \wedge Above(x, z) \wedge \neg(y = z)) \rightarrow (Above(z, y) \vee Above(y, z)))$
- $\forall x \forall y \forall z ((Above(x, y) \wedge Above(y, z)) \rightarrow Above(x, z))$

- (a) [5 marks] Construct a model of Φ with size three (i.e., the model includes three elements) which does not satisfy the English description of *Under*.
- (b) [10 marks] Modify Φ , without modifying the vocabulary, so that the models of the resulting set of sentences would be those structures that satisfy the English definition of *Under*, *Clear* and *OnTable*.

2. (10 points) Suppose Φ is a set consisting of the following sentences.

Is Φ satisfiable? **Justify** your answer.

If Φ is satisfiable, provide **two structures** that satisfy Φ .

If it's not, present **two sets** which can be obtained by modifying Φ and are satisfiable. Then **justify** that the two sets you presented are satisfiable.

$$\forall x \forall y \forall z (between(x, y, z) \rightarrow between(z, y, x)) \quad (1)$$

$$\forall x \forall y \forall z ((between(x, y, z) \wedge between(y, x, z)) \rightarrow (x = y)) \quad (2)$$

$$\forall x \forall y \forall z \forall w (between(y, x, z) \rightarrow (between(y, x, w) \vee between(z, x, w))) \quad (3)$$

$$\forall x \forall y \forall z (between(y, x, z) \vee between(z, y, x) \vee between(x, z, y)) \quad (4)$$

$$\forall x \forall y \forall z (between(x, y, z) \rightarrow \neg between(y, x, z)) \quad (5)$$

3. **[10 marks]** For each of the following sentences, give an interpretation that makes that sentence false and the other two sentences true:

(a) $\forall x \forall y \forall z [(P(x, y) \wedge P(y, z)) \rightarrow P(x, z)]$

(b) $\forall x \forall y [(P(x, y) \wedge P(y, x)) \rightarrow (x = y)]$

(c) $\forall x \forall y [P(a, y) \rightarrow P(x, b)]$

Note: a, b are constants.

GOOD LUCK!