Lab3_Filter_Design_Problems_2024

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[1]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
```

1 A Notch Digital Filter (6 marks, 3/2/1)

A notch filter is a filter that passes almost all frequencies with unit amplitude, except for a narrow range of frequencies centered on the rejection frequency f_0 (at which frequency nothing is passed).

A simple notch digital filter has a z-transform

$$W(z) = MF(z)F(z)^* = M\frac{z-q}{z-p}\frac{z-q^*}{z-p^*}$$

where M is a normalization factor, $q = e^{-i2\pi f_0/f_s}$, $p = (1+\epsilon)q$, f_s is the sampling rate ($f_s = 1/\Delta$) and ϵ is a small positive number ($0 < \epsilon \ll 1$).

1. What are the poles and zeros of this system? Where are they located with respect to the unit circle? Is this a stable system? Is this filter real?
2. Given $f_s = 12$ cycles/year, $f_0 = 1$ cycle/year, $M = 1.05$ and $\epsilon = 0.05$, plot the power spectrum $|W(f)|^2 = W(f)W(f)^*$ (i.e., square of amplitude spectrum). Sample densely in $[-f_s/2, \ldots, f_s/2]$ (e.g. 1000 points), where $f_s/2$ is the Nyquist frequency.
3. What is the full-width-half-max (FWHM) value f_{fwhm} of this notch filter (based on the plot)? Which parameter(s) should you change to make the notches sharper, i.e., f_{fwhm} smaller?

Hint: For Part 2, $W(f)$ is obtained by $W(z = e^{-i\omega\Delta})$. For Part 3, you don't have to compute the FWHM value analytically (although it can be done); an inspection of the discrete array of $|W(f)|^2$ vector is sufficient. Note here f_{fwhm} is in terms of frequency (1/year), not angular frequency.

2 Implementing the Notch Filter (6 marks, 2/2/1/1)

Notch filter introduced in section 1 can be written out fully as

$$W(z) = \frac{N(z)}{D(z)} = \frac{a + bz + cz^2}{1 + Bz + Cz^2}$$

A filter of the form above can be implemented as two filters in succession: first the 'numerator filter' `[a b c]` as a 3-term direct convolution, then the 'denominator filter' as the inverse filter of the 3-term filter `[1 B C]` by feedback filtering.

1. What are the values of a; b; c; B; C for the notch filter defined in Question 1.2?
2. Write a function for a general rational digital filter with numerator and denominator coefficients N and D which produces the filtered time series y for a given input x, `y = ratFilter(N,D,x)`.
3. Use `ratFilter` function to determine the impulse response of this notch filter (i.e., the output of this filter when the input is a discrete delta function). Define the impulse using $dt = 1/f_s$ and $t = 0$ to $t_{max} = 100$ years (i.e. 1200 samples). Plot the impulse response from 0 to 6 years. Speculate on how the impulse response would change if we halve the f_{fwmh} value.
4. Fourier transform the impulse response to obtain the frequency response $|W(f)|$ of this notch filter. Plot it on top of the magnitude of the theoretical spectrum calculated based on the z-transform, with f ranging from 0 to 6 cycles per year.

3 Global Mean CH_4 Data (6 marks, 1/1/1/1/2)

The file `methane_global.csv` (on Quercus) contains globally averaged methane (CH_4) values (in parts per billion) every month from July 1983 to October 2023. The measurements show clearly a rising trend in atmospheric CH_4. The trend is overlaid with a annual oscillation. Your job is to remove the annual oscillation and display the trend more clearly. There are two possible approaches: (a) you could apply your notch filter to the series to remove this annual variation, or (b) you could Fourier transform it with `fft`, remove the annual variation by setting the spectrum at appropriate frequencies to zero, and transform back to the time domain with `ifft`.

Write code to accomplish the following:

1. Before applying the filters, it is helpful to remove the trend of the signal using numpy function `polyfit`. Fit a straight line to your data and then detrend your data by removing the straight line. Plot both the original data and the detrended data.
2. Apply your notch filter to the detrended data and add back the trend.
3. FT the detrended data into the frequency domain, and plot both its amplitude and phase spectrum. Make another plot that zooms in at $f = [0, 2.5]$ cycles per year. Now set the Fourier spectrum corresponding to frequencies beyond 0.9 cycles per year to zero, which effectively removes the annual oscillation. Transform the spectrum back to the time domain and add back the trend.
4. Now plot the original data, the notch-filtered data from Part 2, and the f-domain filtered data from Part 3 on top of each other with different colors. Which method gives more satisfactory result? Can you think of any advantages/disadvantages in using either method?
5. Now try redo Parts 2, 3, and 4 with the original data, not the detrended data. Of course you don't need to add back the trend after filtering any more. Display your results and comment on the importance of detrending before applying the filters.