3A) 8.10.10

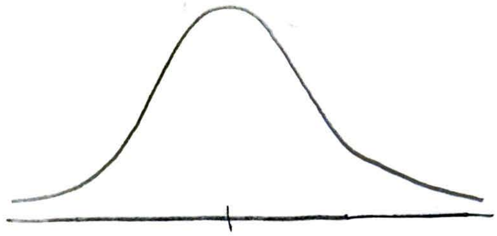
$\hat{\lambda} = \frac{\sum x_i}{n}$ and X_i are independent Poisson

Sum of independent Poisson is Poisson:

$$P(\hat{\lambda} = v) = \frac{(n\lambda_0)^{nv} e^{-\lambda_0 n}}{(nv)!}$$

$$E(\hat{\lambda}) = \lambda_0 \approx \hat{\lambda} = 24.9$$

$$Var(\hat{\lambda}) = \frac{\lambda_0}{n} \approx 1.04^2$$



24.9

Normal approximation:

$$\hat{\lambda} \approx N(24.9, 1.04^2)$$

According to this:

$$P(|\lambda_0 - \hat{\lambda}| > .5) = P\left(\left|\frac{\lambda_0 - \hat{\lambda}}{\sigma_{\hat{\lambda}}}\right| > .5\right) = 2\Phi\left(\frac{-.5}{1.04}\right) = 0.631$$

$$P(|\lambda_0 - \hat{\lambda}| > 1) = 2\Phi\left(\frac{-1}{1.04}\right) = 0.336$$

$$P(|\lambda_0 - \hat{\lambda}| > 1.5) = 0.149$$

$$P(|\lambda_0 - \hat{\lambda}| > 2) = 0.054$$

$$P(|\lambda_0 - \hat{\lambda}| > 2.5) = 0.016$$

3B) $n = 300$, $X_i \sim Pois(\lambda_0)$

$$\hat{\lambda} = \frac{\sum x_i}{n} = \frac{30 + 2(36) + \ldots + 12}{300} = \frac{1168}{300} = 3.893$$

Expected frequency of 0 right turns:

$$P(X=0) \cdot 300 = \frac{(3.893)^0 e^{-3.893}}{0!} \cdot 300 = 6.12$$ (use this formula for each)

n	Observed	Expected
0	14	6.12
1	30	23.8
2	36	46.3
3	68	60.1
4	43	58.5
5	43	45.6
6	30	29.6
7	14	16.4
8	10	8.0
9	6	3.5
10	4	1.3
11	1	.5
12	1	.2
13	0	.05

This fit is not very good. The observed data skews more to the left than the expected distribution

3C

$$E(\hat{\mu}_k) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i^k\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^k\right)$$

$= \frac{1}{n} \sum_{i=1}^{n} E(x_i^k)$ because the x_i are independent

$= \frac{1}{n} E(x_1^k) \cdot n$ because they are indentically distributed

$= E(x_1^k) = \mu_k$ $\square$

3D

1) 1st moment : $\mu_1 = E(X) = \frac{1}{p}$

2) Invert : $p = \frac{1}{\mu_1}$

3) Add hats : $\hat{p}_{MOM} = \frac{1}{\hat{\mu}_1} = \frac{1}{\bar{X}}$ where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

3E

a) $\Gamma(r+1) = \int_0^\infty t^{r+1-1} e^{-t}\, dt$

$u = t^r$ $\quad dV = e^{-t} dt$

$du = r t^{r-1} dt$ $\quad V = -e^{-t}$

$= \int_0^\infty t^r e^{-t}\, dt$

$= -t^r e^{-t} \Big|_0^\infty + r\int_0^\infty t^{r-1} e^{-t}\, dt$

the first term $= 0$ because $e^{-t} \to 0$ faster than $t^r \to \infty$

$= r\Gamma(r)$

b) Chi square density $\rightarrow$ $f_T(t) = \begin{cases} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{t}} e^{-\frac{t}{2}} & t \geq 0 \\ 0 & \text{else} \end{cases}$

From homework 2H:

$$\frac{1}{\sqrt{2\pi}} t^{-1/2} e^{-\frac{1}{2}t} = \frac{(1/2)^{1/2}}{\Gamma(1/2)} t^{-1/2} e^{-\frac{1}{2}t} \leftarrow \text{density of } \text{Gamma}\left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$\downarrow$$

$$\frac{1}{\sqrt{2\pi}} = \frac{1}{\sqrt{2}\,\Gamma(1/2)}$$

$$\downarrow$$

$$\sqrt{\pi} = \Gamma(1/2) \quad \square$$