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COGS 4210, RPI
Spring 2024

Homework 2

Problem 1:

1. TRUE
2. FALSE → The variance of a Wiener process with scale coefficient σ=1 at time *t* is *t.*
3. TRUE
4. TRUE
5. FALSE → The Euler-Maruyama method can be used to simulate both linear and nonlinear stochastic systems represented by stochastic differential equations.
6. FALSE → It is not necessarily true that the prior will always have a smaller variance than the posterior. In some cases, the prior distribution might have a larger variance than the posterior, especially if the data strongly contradict the prior beliefs.
7. TRUE
8. FALSE → The Drift Diffusion Model (DDM) is a special case of the more general Leaky Competing Accumulator (LCA) model.
9. TRUE
10. FALSE → The Monte Carlo Standard Error (MCSE) in the context of MCMC estimation is computed by dividing the standard deviation of the chains by the square root of the effective number of samples. A smaller number of independent samples may carry the same amount of information as the actual number of samples.
11. TRUE
12. TRUE

Problem 2: Solution is in the corresponding Jupyter notebook on the GitHub.

Problem 3: Solution is in the corresponding Jupyter notebook on the GitHub.

Problem 4: $Var[\theta] = \mathbb{E}\big[Var[\theta \mid y]\big] + Var\big[\mathbb{E}[\theta \mid y]\big]$

Note from HW1: $Var[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$$\mathbb{E}\big[Var[\theta \mid y]\big] + Var\big[\mathbb{E}[\theta \mid y]\big] = \mathbb{E}[\mathbb{E}[\theta^2 \mid y] - \mathbb{E}[\theta \mid y]^2] + \mathbb{E}[\mathbb{E}[\theta \mid y]^2] - \mathbb{E}\big[\mathbb{E}[\theta \mid y]\big]^2$$

$$\mathbb{E}\big[Var[\theta \mid y]\big] + Var\big[\mathbb{E}[\theta \mid y]\big] = \mathbb{E}\big[\mathbb{E}[\theta^2 \mid y]\big] - \mathbb{E}[\mathbb{E}[\theta \mid y]^2] + \mathbb{E}[\mathbb{E}[\theta \mid y]^2] - \mathbb{E}[\theta]^2$$

$$\mathbb{E}\big[Var[\theta \mid y]\big] + Var\big[\mathbb{E}[\theta \mid y]\big] = \mathbb{E}[\theta^2] - \mathbb{E}[\theta]^2$$

$$\mathbb{E}\big[Var[\theta \mid y]\big] + Var\big[\mathbb{E}[\theta \mid y]\big] = Var[\theta]$$

Problem 5: Solution is in the corresponding Jupyter notebook on the GitHub.

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Problem 6: Solution is in the corresponding Jupyter notebook on the GitHub.

Problem 7: Solution is in the corresponding Jupyter notebook and diffusion_model.stan on the GitHub.