Problem 1:

Mark all statements which are FALSE.

1. Direct K-fold cross-validation requires K model re-fits, which may be computationally demanding, especially when inverse inference is costly. ***TRUE***
2. Bayes factors (BFs) are relative measures, that is, they cannot differentiate between “equally good” and “equally bad” models. ***TRUE***
3. Marginal likelihoods and, by extension, Bayes factors (BFs) cannot be used to compare models with different likelihoods. ***FALSE***
4. Both the Binomial and the Dirichlet distribution can be formulated as special cases of the Multinomial distribution. ***TRUE***
5. Bayesian leave-one-out cross-validation (LOO-CV) relies on the posterior predictive distribution of left-out data points. ***TRUE***
6. The Akaike Information Criterion (AIC) penalizes model complexity indirectly through the variance of a model’s marginal likelihood. ***FALSE***
7. The log-predictive density (LPD) is a relative metric of model complexity. ***FALSE***
8. The LPD can be approximated by evaluating the likelihood of each posterior draw (e.g., as provided by an MCMC sampler) and taking the average of all resulting likelihood values. ***TRUE***
9. Bayes factors do not depend on the prior odds, that is, the ratio of prior model probabilities p(M1)/p(M2). ***TRUE***
10. You should always prefer information criteria to cross-validation in terms of estimation predictive performance. ***FALSE***

Problem 2: Solution is in the corresponding Jupyter notebook on the GitHub.

Problem 3: Solution is in the corresponding Jupyter notebook on the GitHub.

Problem 4: Derive the analytic posterior for the conjugate Dirichlet-Multinomial model.

Assume we have categories. Thus, we have that and such that .

From , we know the prior:

is the multivariate Beta function.

From , we know the likelihood:

From Bayes’ rule proportionality, we can get the posterior:

Drop the constants:

We see that this is just another Dirichlet distribution. Thus, we have the following posterior:

We see that and such that .

Problem 5: Solution is in the corresponding Jupyter notebook on the GitHub.

Problem 6: Solution is in the corresponding Jupyter notebook on the GitHub.