王牌陈 讲CFA

高顿教育 GOLDEN EDUCATION

极速串讲

Derivatives

衍生品

Level I

王牌陈 讲CFA

陈一磊，CFA/FRM双证持证人，高顿教育资深总监，首席讲师

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  - 以全优成绩通过CFA三个级别考试；一天时间以全优成绩同时通过FRM两个级别考试
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一 衍生工具的基本概念

1.1 衍生工具的定义

- A **derivative** is a financial instrument that derives its performance from the performance of an **underlying asset**
  - Derivatives usually transform (not simply pass-through) the performance of the underlying asset before paying it out in the derivatives transaction

1.2 衍生工具的应用

- Investors use derivatives to modify investment portfolio cash flows, replicate investment strategy returns in cash markets, and/or create exposures unavailable to cash market participants
  - **Risk management** (eg. hedging) & **Speculating** & **Arbitrage**
- Hedge accounting allows an issuer to offset a hedging instrument (usually a derivative) against a hedged transaction or balance sheet item to reduce financial statement volatility
  - **Cash flow hedging**: absorbing the variable cash flow of floating-rate asset or liability
  - **Fair value hedging**: offsets fluctuation in fair value of an asset or liability
  - **Net investment hedge**: offset the exchange rate risk of the equity of a foreign operation

1.3 使用衍生工具的优势

- Risk allocation, transfer, and management
- Information discovery
  - Price discovery and implied volatility
- Better market efficiency
- Operational advantages
  - Lower transaction cost
  - Greater liquidity
  - Low upfront cash requirement
  - Easy to go short

1.4 使用衍生工具的风险

- Speculative use
- Lack of transparency
- Basis risk
- Liquidity risk
- Counterparty credit risk
- Destabilization and systemic risk

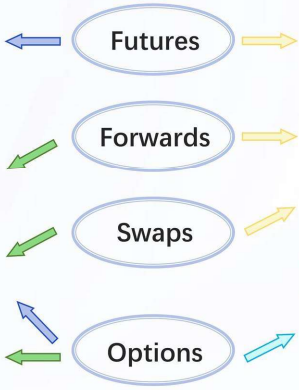
2. 衍生工具的分类

Exchange-traded derivative (ETD) markets

- Standardized
- Lower cost
- More liquid
- No default risk: guaranteed by clearinghouse
- Regulated: transparent

Over-the-counter (OTC) markets

- Customized
- Default risk / counterparty risk
- Less regulated: less transparent



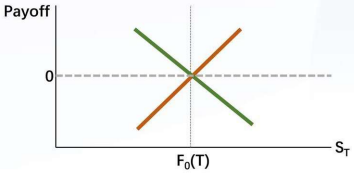
Forward commitment: contracts entered into at one point in time that require both parties to engage in a transaction at a later point in time (the expiration) on terms agreed upon at the start

Contingent claim: derivatives in which one of the counterparties determine whether and when a trade will settle

➢ Central clearing mandate: Central counterparty (CCP) bear the credit risk between counterparties for most OTC derivatives after the 2008 global financial crisis

3. 远期合约的基本概念

- **Forward contract** is an over-the-counter derivative contract, in which two parties agree that one party, the **long position**, will purchase an underlying asset from the other party, the **short position**, at a later date and at a fixed price they agree on when the contract is signed
  - In addition to the **forward price**, the two parties also agree on several other matters, such as the identity and the quantity of the underlying
  - Forward contracts can be structured to create a perfect hedge, providing an assurance that the underlying asset can be bought or sold at a price known when the contract is initiated
- The **long** hopes the price of the underlying will rise above the **forward price**, $F_0(T)$, whereas the **short** hopes the opposite
  - Forward contract provides linear payoff



- Forward contracts can be settled in two ways

1. Delivery of the underlying asset 2. Exchange of cash: non-deliverable forwards (NDFs)

4. 1 期货合约的基本概念

- **Futures contracts** are specialized forward contracts that have been standardized and trade on a futures exchange
  - Futures contracts have specific underlying assets, times to expiration, delivery and settlement conditions, and quantities
  - The exchange offers a facility in the form of a physical location and/or an electronic system as well as liquidity provided by authorized market makers
- **Futures price** is the agree-upon price, just like forward price
- Price limit is a provision limiting price changes, establish a band relative to the previous day's settlement price
  - Settlement price is an average of the a period of final futures trades of the day
- Circuit breaker: pause intraday trading for a period

4. 2 期货合约的盯市与保证金制度

- **Futures contracts** realize the gain or loss in parts on a day-to-day basis
  - Forward contracts realize the full gain or loss at expiration, thus a loss by one party can be large enough to trigger a default
- **Mark-to-market**: daily settlement of gains and losses to margin account according to the settlement price
- **Initial margin**: both parties deposit a required minimum sum of money when the contract is initiated
- **Maintenance margin**: the amount of money that must maintain in the margin account after the trade is initiated
  - Margin call is a request to deposit enough funds to bring the margin account balance up to the initial margin when it is below the maintenance margin
  - Variation margin is the amount to replenish the margin

5.1 远期合约和期货合约的比较：交易模式

	Forwards	Futures
Trading and Flexibility	Customized	Standardized
Liquidity	Illiquid	Liquid
Counterparty Risk	High	Low
Transaction Costs	High and not easily visible	Low and visible
Timing of Cash Flows	No cash flow, except at maturity	Marked to market daily
Settlement	May settle with physical delivery or cash settlement	Typically settle with cash

5.2 远期合约和期货合约的比较：合约价格

- The time value of money makes **futures price $F_0(T)$** and **forwards price $F_0(T)$** not the same
  - Futures price $F_0(T)$ will be higher than forward price $F_0(T)$ when interest rate and futures price **$F_t(T)$** are positively correlated
  - Futures price $F_0(T)$ will be lower than forward price $F_0(T)$ when interest rate and futures price **$F_t(T)$** are negatively correlated

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6.1 互换合约的基本概念

- **Swaps contract** is an over-the-counter derivative contract in which two parties agree to exchange a series of cash flows, whereby one party pays a variable series that will be determined by an underlying asset or rate, and the other party pays either (1) a variable series determined by a different underlying asset or rate or (2) a fixed series
- A swaps contract is a series of (off-market) forwards contracts
  - Off-market forward is a “forward contract” created with a contract price that gives it an non-zero value at initiation
  - A swaps contract consist of some off-market forwards with positive present value and some off-market forwards with negative present values, so that the sum of their present values equals zero

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6.2 利率互换合约

- **Plain vanilla interest rate swaps**
  - Notional amount is not exchanged at the beginning or end of the swap
  - On settlement dates, interest payments are netted, only the difference is paid by the party owing the greater amount
  - Floating rate payments are typically paid in arrears: payment is made at end-of-period based on beginning-of-period market reference rate (MRR)

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7.1 期权的基本概念

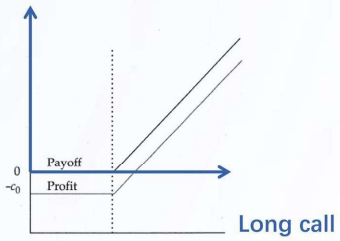
- **Option** is a derivative contract in which one party, the **buyer**, pays a sum of money (**option premium**, c_0, p_0) to the other party, the **seller** or **writer**, and receives the right to either **buy** or **sell** an underlying asset at a fixed price (**exercise price**, **strike price**, X) either on a specific expiration date (**European option**) or at anytime prior to the expiration date (**American option**)
  - An option is a right, but not an obligation

Position	What to do
long call	right to buy
short call	obligation to sell
long put	right to sell
short put	obligation to buy

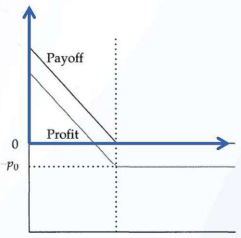
7.2 期权投资的到期收益和整体利润

- **Moneyness** refers to whether immediate exercise would generate a positive payoff

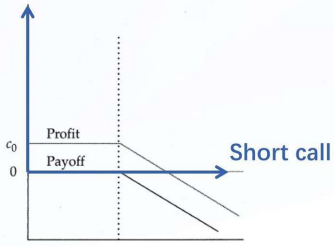
1. **In the money**
   - Call: S > X
   - Put: S < X
2. **At the money**
   - S = X
3. **Out of the money**
   - Call: S < X
   - Put: S > X



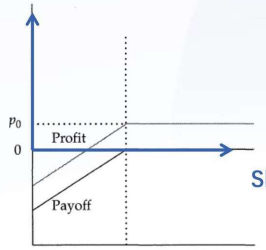
Long call



Long put



Short call

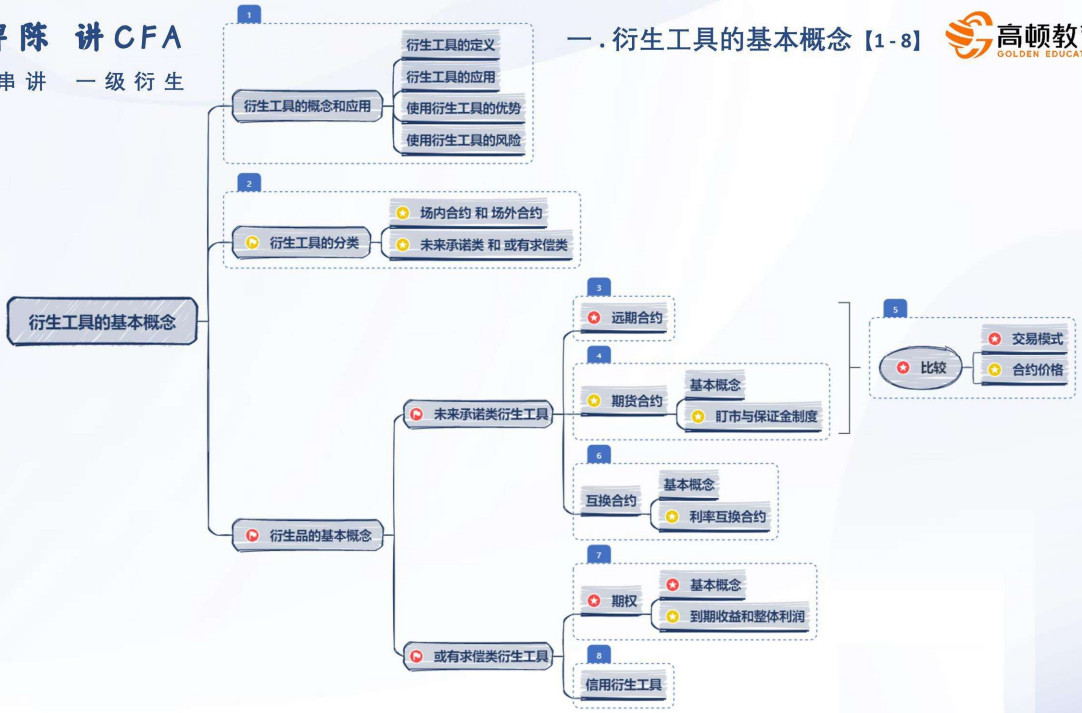


Short put

8. 信用衍生工具

- **Credit derivative** is a class of derivative contracts between two parties, a credit protection buyer and a credit protection seller, in which the latter provides protection against a specific credit loss
- **Credit default swap (CDS)**: the buyer makes a series of cash payments to the seller based on a CDS credit spread and receives a contingent payment upon credit event
  - A protection buyer can use CDS to hedge the existing credit risk of a third-party issuer, as a CDS is essentially an insurance contract against default
  - If the credit spread of the underlying issuer increases, the protection buyer earn a MTM gain from the CDS contract, while the protection seller suffer a MTM loss

一. 衍生工具的基本概念【1-8】



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二

远期，期货，互换的定价和估值

9. 衍生工具的定价原理

- **Law of one price**: assets that produce identical future cash flows, regardless of future events, should have the same price
- **No arbitrage pricing**: determine the price of a derivative by assuming that there are no arbitrage opportunities
  - Replication: an asset and a hedging position of derivative on the asset can be combined to produce a position equivalent to a risk-free asset
    - risky asset + derivative = risk free asset
    - risky asset – risk free asset = – derivative
- **Risk-neutral pricing**: the investor can be assumed to be risk neutral
  - Discount the expected payoff of the derivative at the risk-free rate
  - The price of a derivative can then be inferred from the characteristics of the underlying of the derivative, and the risk-free rate

10. 未来承诺类衍生工具的价格和价值

- **Price** of forward, futures, and swap is the fixed price or rate at which the underlying will be purchased at a later date
  - The price is determined at the initiation when the contract is signed, and it will not change as the (expected) price of the underlying asset changes
  - The principle to determine the price is the valuation for both parts is zero at the initiation
- **Value** of forward, futures, and swap is the difference of "with the position" from "without the position"
  - The value may increase or decrease as the (expected) price of the underlying asset changes
  - It is always the zero-sum game at any time

11.1 远期合约的定价

- Simplified formula
  - $F_0(T) = S_0 \times (1 + r_f)^T$ 【$F_0(T) = S_0 \times e^{R_f T}$】
- Complete formula
  - $F_0(T) = [S_0 - PV(I) + PV(C)] \times (1 + r_f)^T = S_0 \times (1 + r_f)^T - FV(I) + FV(C)$

    【$F_0(T) = S_0 \times e^{(R_f + c - i) \times T}$】
    - **Costs** of carry underlying assets
      - ① opportunity cost, r_f
      - ② costs of ownership, **c**: storage, transportation, insurance, spoilage costs
    - **Benefits** of carry underlying assets, **i**: stock dividends, bond coupons, convenience yield
- Pricing for foreign exchange forward
  - $F_{A/B} = S_{A/B} \times (1 + r_A)^T / (1 + r_B)^T$

    【$F_{A/B} = S_{A/B} \times e^{(R_A - R_B)T}$】

11.2 远期合约的估值

- Simplified formula
  - T = 0：$V_{L,0} = 0$ $V_{S,0} = 0$
  - T = T：$V_{L,T} = S_T - F_0(T)$ $V_{S,T} = -V_{L,T} = F_0(T) - S_T$
  - T = t：$V_{L,t} = S_t - F_0(T) / (1 + r_f)^{T-t}$ 【$V_{L,T} = S_t - F_0(T) \times e^{-R_f \times (T-t)}$】 $V_{S,t} = -V_{L,t}$
- Complete formula
  - T = 0：$V_{L,0} = 0$ $V_{S,0} = 0$
  - T = T：$V_{L,T} = S_T - F_0(T)$ $V_{S,T} = -V_{L,T} = F_0(T) - S_T$
  - T = t：$V_{L,t} = [S_t - PV_t(I) + PV_t(C)] - [F_0(T) / (1 + r_f)^{T-t}]$ $V_{S,t} = -V_{L,t}$
- Valuation for foreign exchange forward (A/B)
  - T = 0：$V_{L,0} = 0$ $V_{S,0} = 0$
  - T = T：$V_{L,T} = S_T - F_0(T)$ $V_{S,T} = -V_{L,T} = F_0(T) - S_T$
  - T = t：$V_{L,t} = S_t - F_0(T) \times e^{-(r_A - r_B) \times (T-t)}$ $V_{S,t} = -V_{L,t}$

12. 1 远期利率协议的定义和应用

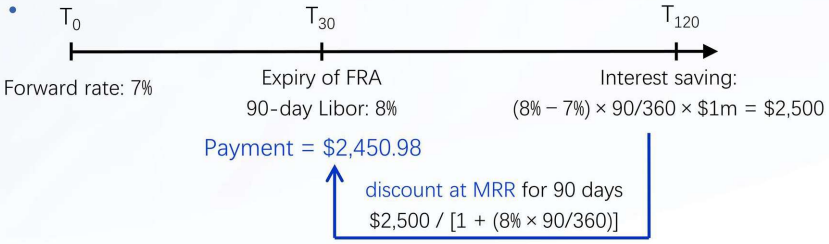
- **Forward rate agreement (FRA)** is the forwards contract, in which the underlying is an interest rate
  - **Long position** can be viewed as the obligation to take a (hypothetical) loan at the contract rate (i.e., borrow at the fixed rate); gains when reference rate increase
  - **Short position** can be viewed as the obligation to make a (hypothetical) loan at the contract rate (i.e., lend at the contract rate); gains when reference rate decrease
- The notation of FRA is typically "**a×b FRA**"
  - **a** is the number of months until the contract expires
  - **b** is the number of months until the underlying loan is settled
- The uses of FRA
  - Lock the interest rate or hedge the risk of borrowing or lending at some future date
  - Speculate that the interest rate may rise or go down

12. 2 远期利率协议的结算

- One party will pay the other party the difference (based on notional value) between the interest rate specified in the FRA and the market interest rate at contract settlement

"1×4 FRA", notional amount is $1 million, underlying rate is 90-day MRR, forward rate = 7%

At t = 30 day, 90-day MRR = 8%

- Underlying floating rate > fixed rate, so long position receives payment
- 

13. 利率期货合约

- **Interest rate futures** is traded on a price basis:
  - $f_{A,B-A} = 100 - 100 \times MRR_{A,B-A}$
    - $f_{A,B-A}$ is the futures price for the underlying MRR
    - A: MRR starts, B: MRR ends, B − A: MRR tenor
- FRA vs. interest rate futures
  - Difference in price changes result from convexity bias
  -

when MRR increases	Long Position	Short Position
Forward Rate Agreements	value increases	value decreases
Interest Rate Futures	value decreases	value increases

- Interest rate futures is daily settled on price changes by basis point value (BPV)
  - **BPV = notional principal × 0.01% × period**

14.1 利率互换合约的定价

- The price of a **interest rate swap** is the fixed rate F (a.k.a. swap rate)
  - Principle of pricing the swap: the fixed rate (swap rate) in a plain vanilla swaps contract should makes the contract value zero at initiation

① ∑PV(fixed payments) = ∑PV(floating payments)

- The expected floating cash flows depend on the MRRs
  - The first MRR is known at T = 0
  - Remaining MRRs are unknown but expected to equal implied forward rates (IFRs)

② ∑PV(fixed payments) + Par value = ∑PV(floating payments) + Par value

- T= 0：$PV_{\text{fix-coupon bond}} = PV_{\text{floating-coupon bond}}$ = Par value
- **$F = (1 - D_N) / (D_1 + D_2 + \cdots\cdots + D_N)$**
  - D_i is the discount factor or PV factor, $D_i = 1 / (1 + S_i)^i$

14.2 利率互换合约的估值

- At initiation, a swaps contract has zero value
- The swap value on any settlement date = settlement value + MTM value
  - The periodic settlement value for the fixed-rate payer is:
    (MRR − F) × notional amount × period
  - The MTM value for the fixed-rate payer is:
    ∑PV(remaining floating payments) − ∑PV(remaining fixed payments)
    - An increase in expected future rates will produce a MTM gain value for the fixed-price payer
- The value of a swap is the difference of value between the floating-rate bond and the fixed-rate bond at any time during the life of the swap
  - For fixed-rate payer (floating-rate receiver)
    $V_t = PV_{\text{floating-rate bond}} - PV_{\text{fixed-rate bond}}$

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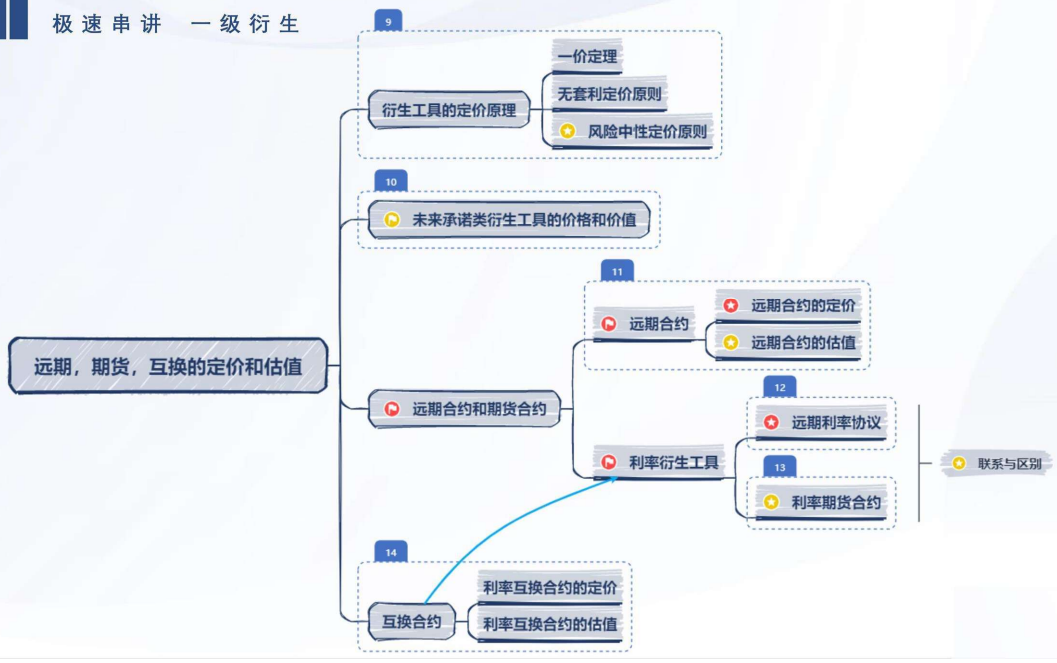
二.远期，期货，互换的定价和估值【9-14】



三

期权价值

15. 期权价值的组成

- **Total value of option = exercise value + time value**
  - If the market is efficient, the option price (option premium) reflects the option value

1. The **exercise value** (**intrinsic value**) is the maximum of zero and the amount that the option is in the money
   - Exercise value for call option = $\max(0, S_t - X/(1+r_f)^{T-t})$
   - Exercise value for put option = $\max(0, X/(1+r_f)^{T-t} - S_t)$
2. The **time value** of an option is always positive but declines to zero at maturity
   - At expiration, the time value is zero, and the option value is its intrinsic value

16. 期权价值的上下限

- Call option: $\max(S_t - X/(1+r_f)^{T-t}, 0) < c_t \leq S_t$
- Put option: $\max(X/(1+r_f)^{T-t} - S_t, 0) < p_t \leq X$

17. 期权价值的影响因素

	Call option	Put option
Price of the underlying asset (S_t)	+	–
Exercise price (X)	–	+
Risk-free rate (R_f)	+	–
Volatility of the underlying (δ)	+	+
Time to expiration (T – t)	+	+ with exception
Costs (θ) of holding the asset	+	–
Benefits (γ) of holding the asset	–	+

- For some European put options, longer time to expiration may decrease its value
  - The deeper a put option is in the money
  - The higher risk-free rate

18. 期权平价公式

- If the following requirements are met: call and put are European-style with same exercise price (X) and time to expiration (T) and the maturity of the bond should be the same (T), the payoff on a fiduciary call $[c_0 + X/(1+r_f)^T]$ is the same as the payoff on a protective put $[p_0 + S_0]$
  - **Fiduciary call** $[\mathbf{c_0 + X/(1+r_f)^T}]$ is the combination of a European call option with exercise price of X and a pure-discount, riskless bond with face value of X
  - **Protective put** $[\mathbf{p_0 + S_0}]$ is a share of stock together with a put option on the stock with exercise price of X
- **Put-call parity**: $\mathbf{c_0 + X/(1+r_f)^T = p_0 + S_0}$
  - **Put-call forward parity**: $\mathbf{c_0 + X/(1+r_f)^T = p_0 + F_0(T)/(1+r_f)^T}$
- **Synthetic** call / put / stock / bond

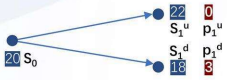
19. 期权价值的应用和类比

- Model the firm value to describe the interests and financial claims of shareholders and debtholders: **firm value** $\mathbf{V_0 = c_0 + PV(D) - p_0}$
  - For shareholders: a call option on the firm's assets $\mathbf{E_T = \max(V_T - D, 0)}$
  - For debtholders
    1. $D_T = V_T - \max(V_T - D, 0)$
    2. A combination of the risk-free debt D, and short a put option $\mathbf{D_T = D - \max(D - V_T, 0)}$

20. 单期二叉树模型

- **One-period binomial model** is based on the idea that, over the next period, the value will change to one of two possible values
  - To construct a binomial model, we need to know the beginning asset value (S_0), the size of the two possible changes (R^u, R^d)
  - When t = 1, the underlying value will changes to S_1^u or S_1^d
- For the binomial model, we assume the probability of underlying price increases is π^u, the probability of underlying price decreases is $\pi^d = 1 - \pi^u$
  - π is the **risk-neutral probability** which makes the discounted weighted sum of S_1^u and S_1^d equals to S_0
  - $\pi^u = [(1+r_f)^t - R^d] / (R^u - R^d)$ $\quad \pi^d = 1 - \pi^u = [R^u - (1+r_f)^t] / (R^u - R^d)$

20. 单期二叉树模型



- A put option on a non-dividend-paying stock has an exercise price, X, of £21 and six months left to maturity.
- The current stock price, S_0, is £20, and an investor believes that the stock's price in six months' time will be either 10% higher or 10% lower.
- Assume a risk-free rate is 4%.
  - $S_1^u = S_0 \times R^u = 20 \times 1.1 = £22$ $S_1^d = S_0 \times R^d = 20 \times 0.9 = £18$
  - $p_1^u = \max(X - S_1^u, 0) = \max(21 - 22, 0) = 0$
    $p_1^d = \max(X - S_1^d, 0) = \max(21 - 18, 0) = £3$
  - $\pi^d = [R^u - (1 + r_f)^t] / (R^u - R^d) = (1.1 - 1.04^{0.5}) / (1.1 - 0.9) = 40.10\%$
    $\pi^u = 1 - \pi^d = 59.90\%$
  - $E(p_1) = \pi^u \times p_1^u + \pi^d \times p_1^d = 59.90\% \times 0 + 40.10\% \times 3 = £1.203$
  - $p_0 = E(p_1) / (1 + r_f)^t = 1.203 / 1.04^{0.5} = £1.18$

21. 套期保值投资组合和套期保值比率

Hedged portfolio

Long (or short) **h underlying assets**, **short** (or long) **1 call option**

- $V_{portfolio} = h \times S - c$

① $V_1^u = h \times S_1^u - c_1^u$ $V_1^d = h \times S_1^d - c_1^d$

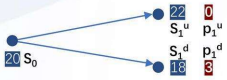
② $V_1^u = V_1^d$ ⇨ **hedge ratio** $h = (c_1^u - c_1^d) / (S_1^u - S_1^d)$ $0 < h < 1$

③ $V_0 = h \times S_0 - c_0 = V_1 / (1 + r_f)^t$ ⇨ c_0

Long (or short) **|h| underlying assets**, **long** (or short) **1 put option**

- $V_{portfolio} = |h| \times S + p$
- **hedge ratio** $h = (p_1^u - p_1^d) / (S_1^u - S_1^d)$ $-1 < h < 0$

21. 套期保值投资组合和套期保值比率



- A put option on a non-dividend-paying stock has an exercise price, X, of £21 and six months left to maturity.
- The current stock price, S_0, is £20, and an investor believes that the stock's price in six months' time will be either 10% higher or 10% lower.
- Assume a risk-free rate is 4%.
  - $h = (p_1^u - p_1^d) / (S_1^u - S_1^d) = (0 - 3) / (22 - 18) = -0.75$
  - The hedged portfolio can be constructed as:
    long 0.75 share stocks, and long 1 put option
  - $V_1^u = |h| \times S_1^u + p_1^u = 0.75 \times 22 + 0 = £16.5 = V_1^d = 0.75 \times 18 + 3$
  - $V_0 = V_1 / (1 + r_f)^t = 16.5 / 1.04^{0.5} = 16.18$
  - $V_0 = |h| \times S_0 + p_0 = 0.75 \times 20 + p_0$
    $p_0 = 16.18 - 0.75 \times 20 = £1.18$

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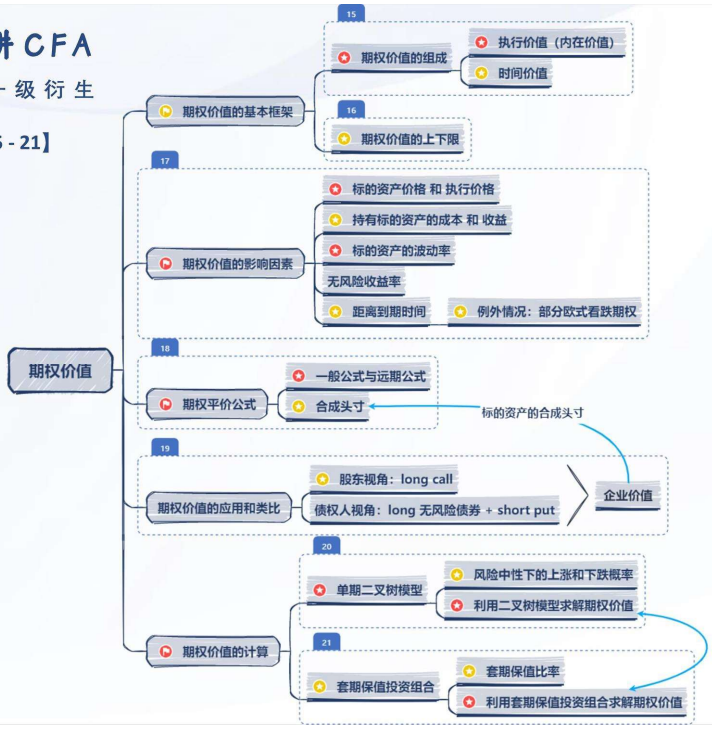
三.期权价值【15-21】



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