

In Problems # 69 - # 71, determine the point of global minimum of the function with the given accuracy by the interval halving method.

#69.  $f(x) = x^4 + 8x^3 - 6x^2 - 72x + 90$ ,  $x \in [1, 5; 2]$ ,  $\varepsilon = 0,05$ .

#70.  $f(x) = x^6 + 3x^2 + 6x - 1$ ,  $x \in [-1; 0]$ ,  $\varepsilon = 0,1$ .

#71.  $f(x) = 3x^4 - 10x^3 + 21x^2 + 12x$ ,  $x \in [0; 0,5]$ ,  $\varepsilon = 0,01$ .

# 72. How many times do we have to evaluate the function  $f$  by the interval halving method to find the point of global minimum with the given accuracy of  $\varepsilon$ ?

# 73. Find the largest value of  $b$  for which the function  $f(x) = -x^2 + 5x - 6$  will be the unimodal function on the closed interval  $[-5, b]$ .

# 74. Find the largest value of  $a$  for which the function  $f(x) = -x^2 - 8x + 9$  will be the unimodal function on the closed interval  $[-10, a]$ .

# 75. Find the largest value of  $c$  for which the function  $f(x) = -3x^2 + 15x + 17$  will be the unimodal function on the closed interval  $[0, c]$ .

# 76. Find the smallest value of  $b$  for which the function  $f(x) = x^2 - 5x + 6$  will be the unimodal function on the closed interval  $[b, 7]$ .

# 77. Suppose  $x_l$  and  $x_r$  are the golden ratio points of the  $[a, b]$  closed interval. Show that  $x_l$  is the right (largest) point of the golden ratio points of the  $[a, x_r]$  closed interval, and  $x_r$  is the left (smallest) point of the golden ratio points of the  $[x_l, b]$  closed interval. Find the lengths of the intervals  $[a, x_r]$  and  $[x_l, b]$ .

# 78. What is the maximum possible accuracy for the golden section method if the values of the objective function have been computed  $N$  times?

# 79. How many times will it be necessary to compute the values of the objective function for the golden ratio method to determine the global minimum point with accuracy of  $\varepsilon$ ?

In the following problems, determine the point of global minimum of the function  $f$  by the golden ratio method, with given accuracy.

#80.  $f(x) = x^4 + 2x^3 + 4x + 1$ ,  $[-1; 0]$ ,  $\varepsilon = 0,1$ .

#81.  $f(x) = (x+1)^4 - 2x^2$ ,  $[-3; -2]$ ,  $\varepsilon = 0,05$ .

#82.  $f(x) = x^5 - 5x^3 + 10x^2 - 5x$ ,  $[-3; -2]$ ,  $\varepsilon = 0,1$

#83. Suppose  $x_l$  and  $x_r$  are the golden ratio points of the  $[a, b]$  closed interval. Prove, that

$$x_l = a + b - x_r \quad \text{and} \quad x_r = a + b - x_l.$$

# 84. Prove, that the golden ratio points of the  $[a, b]$  closed interval are  $x_l = a + \tau(b - a)$  and

$$x_r = b - \tau(b - a).$$

