#### CKY Algorithm

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#### **CKY Algorithm**

- One of the earliest recognition and parsing algorithms
- Bottom-up dynamic programming
- Standard version can only recognize CFGs in Chomsky Normal Form (CNF)

#### **Chomsky Normal Form**

- Grammars are restricted to production rules of the form:
  - A → B C
  - $A \rightarrow W$
- This means that the righthand side of each rule must expand to either two nonterminals or a single terminal
- Any CFG can be converted to a corresponding CNF grammar that accepts exactly the same set of strings as the original grammar!

#### How does this conversion work?

- Three situations we need to address:
  - 1. Production rules that mix terminals and non-terminals on the righthand side
  - Production rules that have a single non-terminal on the righthand side (unit productions)
  - 3. Production rules that have more than two non-terminals on the righthand side
- Situation #1: Introduce a dummy non-terminal that covers only the original terminal
  - INF-VP → to VP could be replaced with INF-VP → TO VP and TO → to
- Situation #2: Replace the non-terminals with the non-unit production rules to which they
  eventually lead
  - A  $\rightarrow$  B and B  $\rightarrow$  w could be replaced with A  $\rightarrow$  w
- Situation #3: Introduce new non-terminals that spread longer sequences over multiple rules
  - A → B C D could be replaced with A → B X1 and X1 → C D

#### CNF Conversion: Example

- $S \rightarrow NP VP$
- S → Aux NP VP
- $S \rightarrow VP$
- NP → Pronoun
- NP  $\rightarrow$  Proper-Noun
- NP → Det Nominal
- Nominal → Noun
- Nominal → Nominal Noun
- Nominal → Nominal PP
- VP → Verb
- VP → Verb NP
- VP → Verb NP PP
- VP → Verb PP
- $VP \rightarrow VP PP$
- PP → Preposition NP

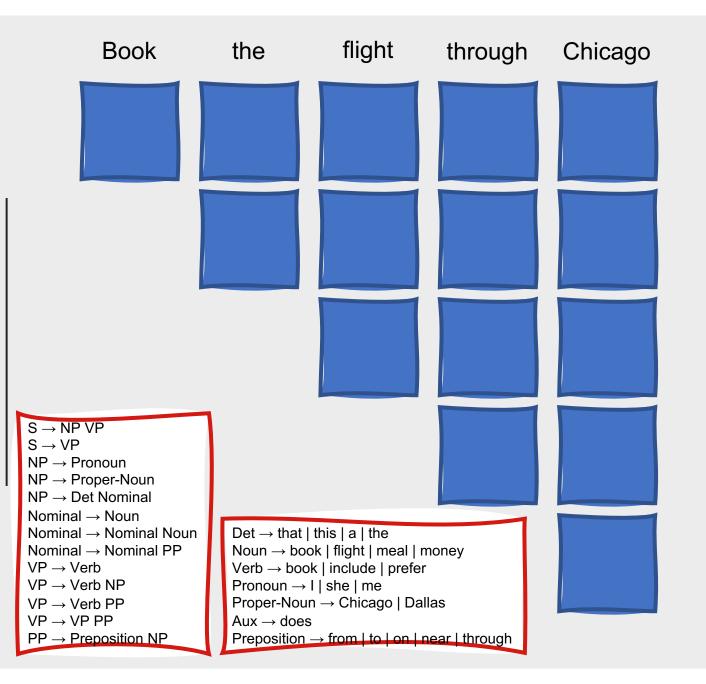
Original	CNF
$S \rightarrow NP VP$	$S \rightarrow NP VP$
$S \rightarrow Aux NP VP$	$S \rightarrow X1 VP$
	X1 → Aux NP
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb NP$
	$S \rightarrow X2 PP$
	$X2 \rightarrow Verb NP$
	$S \rightarrow Verb PP$
	$S \rightarrow VP PP$

#### **CKY Algorithm**

- With the grammar in CNF, each non-terminal node above the POS level of the parse tree will have exactly two children
- Thus, a two-dimensional matrix can be used to encode the tree structure
- For sentence of length n, work with upper-triangular portion of (n+1) x
   (n+1) matrix
- Each cell [i,j] contains a set of non-terminals that represent all constituents spanning positions i through j of the input
  - Cell that represents the entire input resides in position [0,*n*]

#### **CKY Algorithm**

- Non-terminal entries: For each constituent [*i,j*], there is a position, *k*, where the constituent can be split into two parts such that *i* < *k* < *j* 
  - [i,k] must lie to the left of [i,j] somewhere along row i, and [k,j] must lie beneath it along column j
- To fill in the parse table, we proceed in a bottom-up fashion so when we fill a cell [i,j], the cells containing the parts that could contribute to this entry have already been filled



Det → that | this | a | the

Noun → book | flight | meal | money

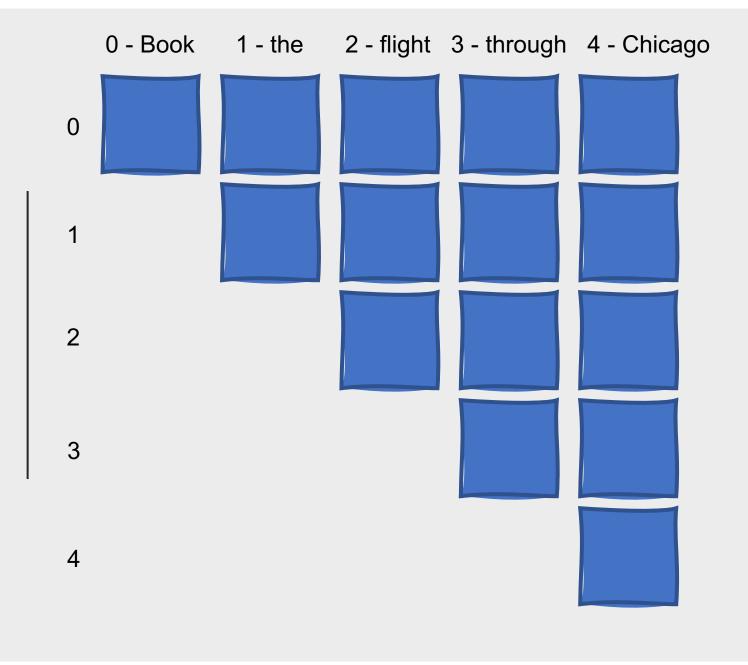
Verb → book | include | prefer

Pronoun → I | she | me

Proper-Noun → Chicago | Dallas

Aux → does

Preposition → from | to | on | near | through



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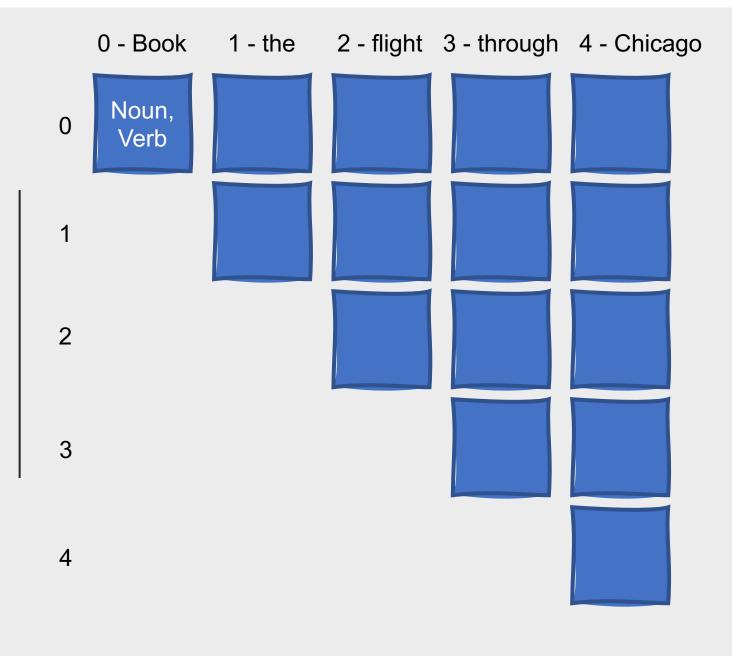
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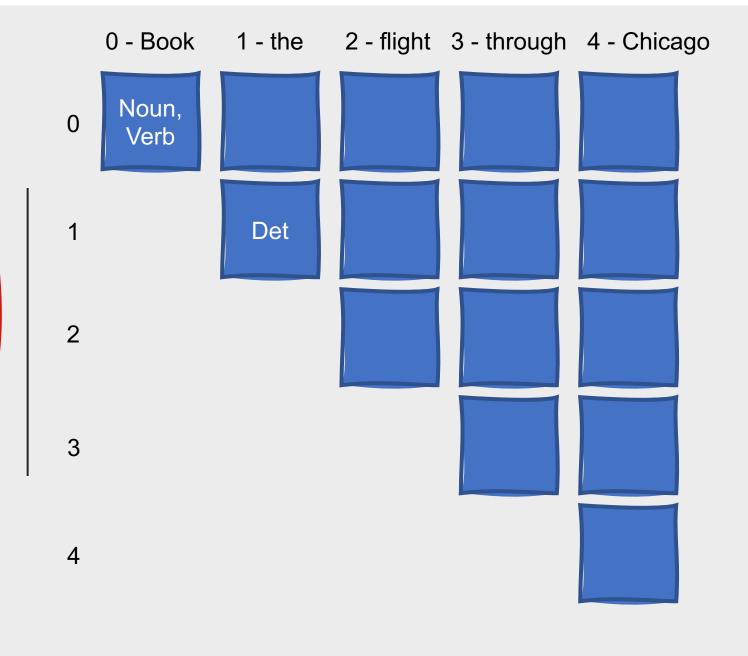
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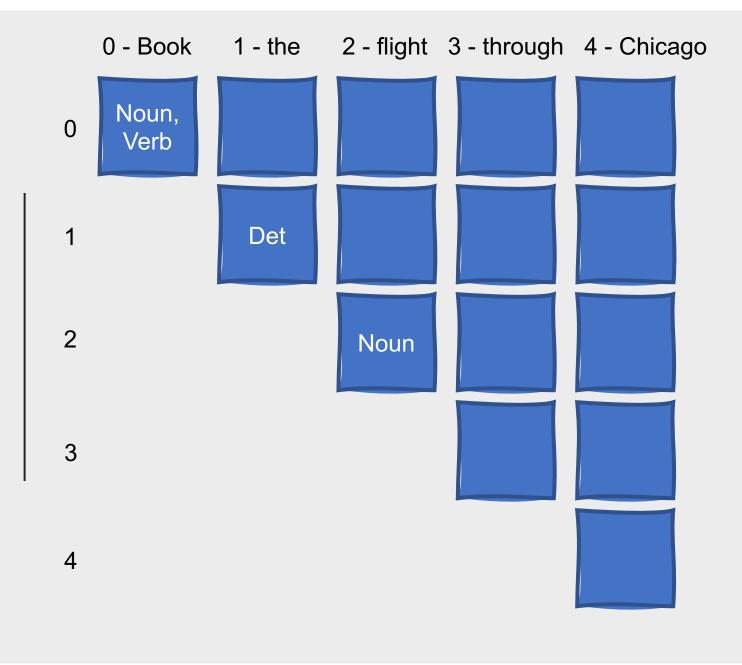
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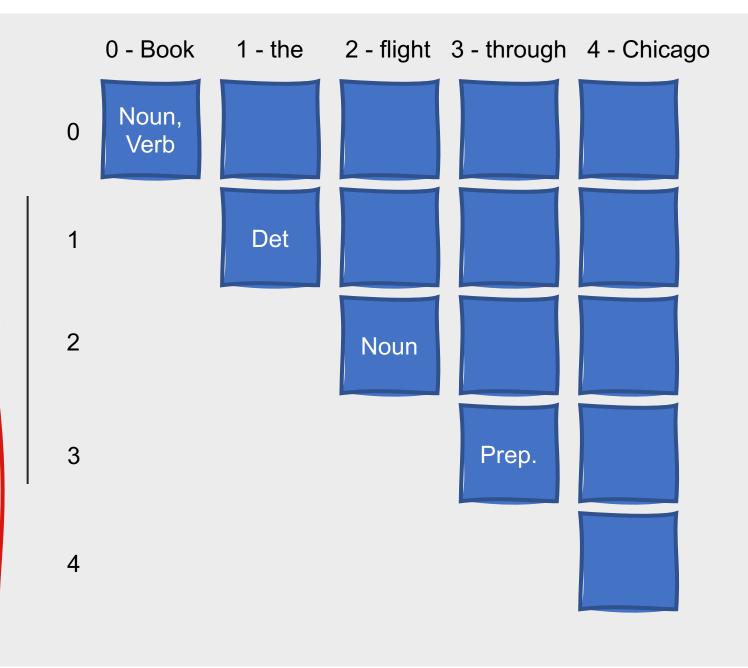
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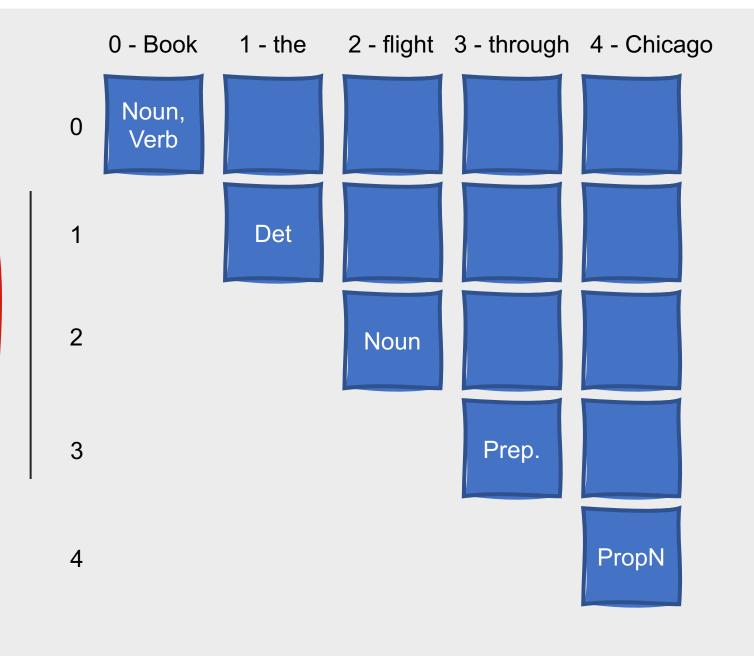
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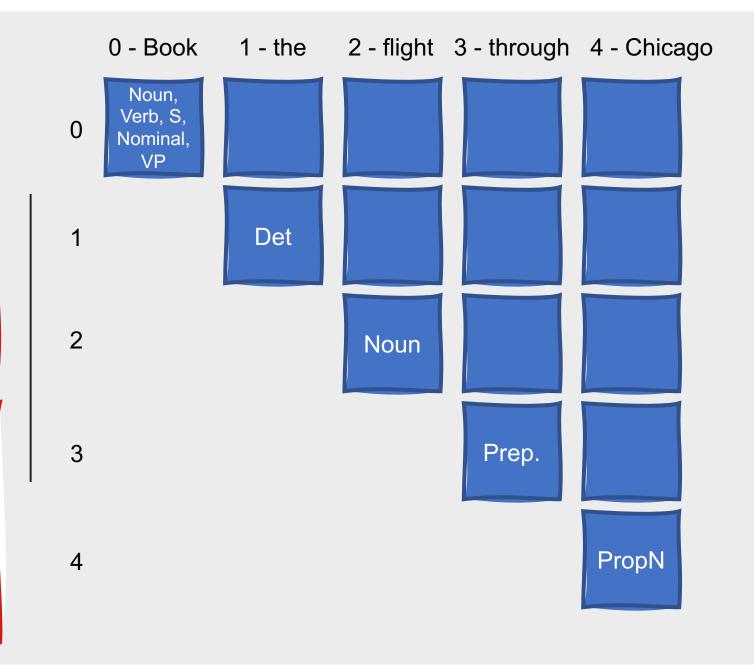
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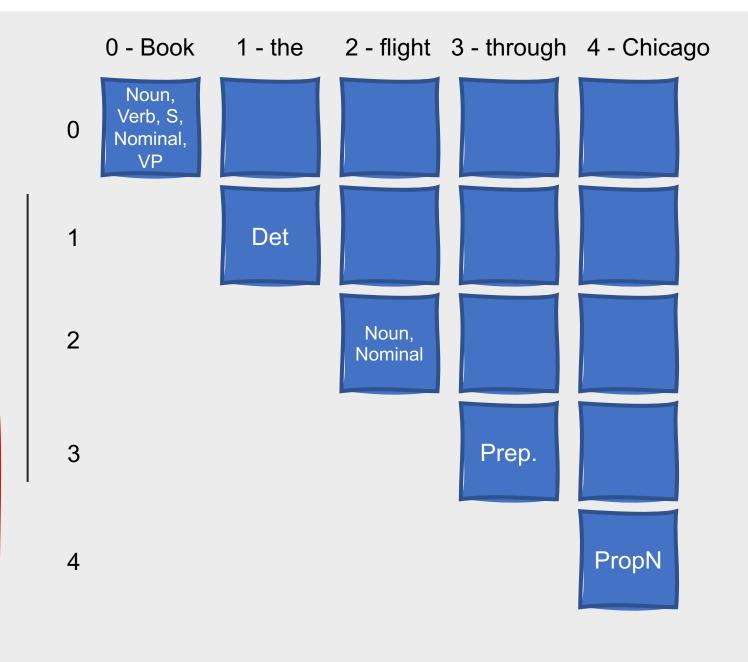
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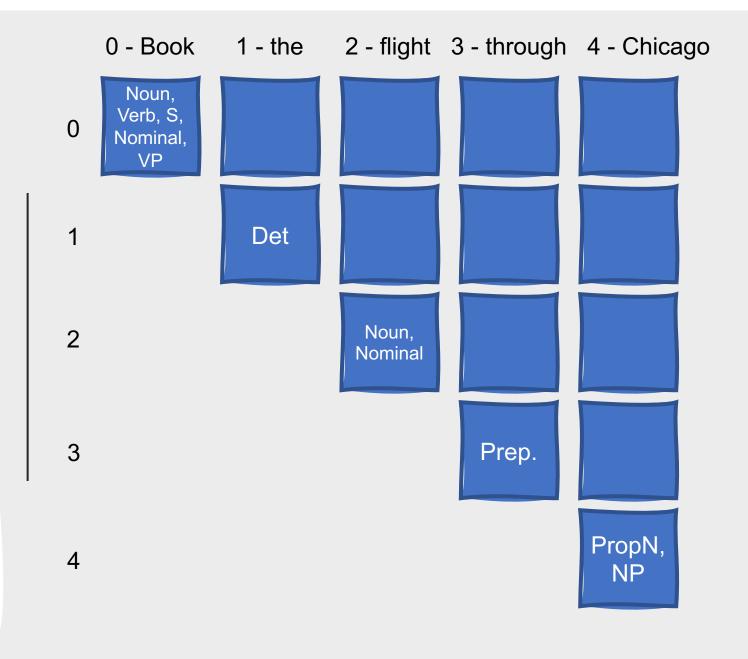
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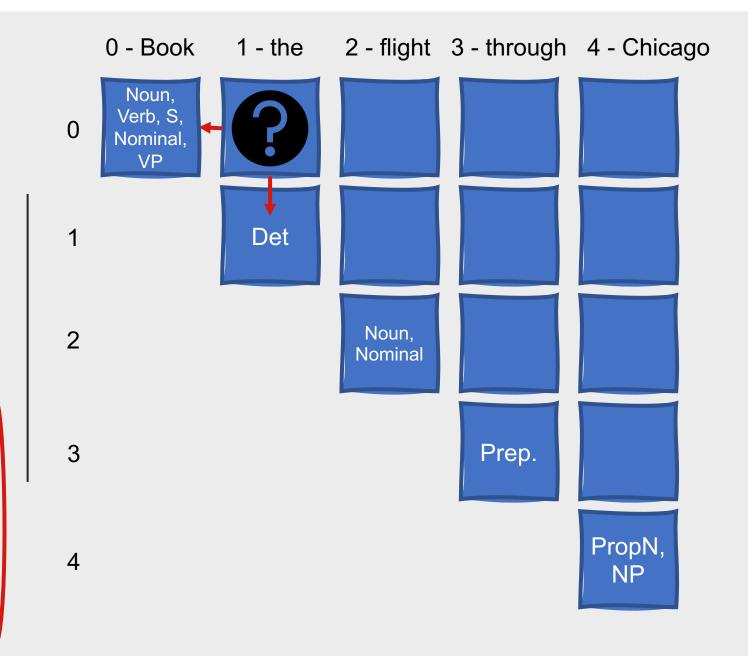
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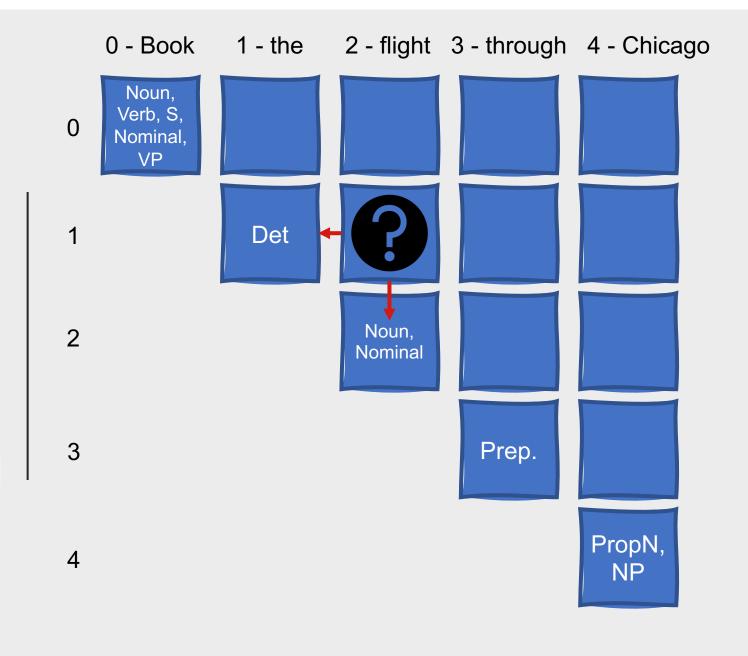
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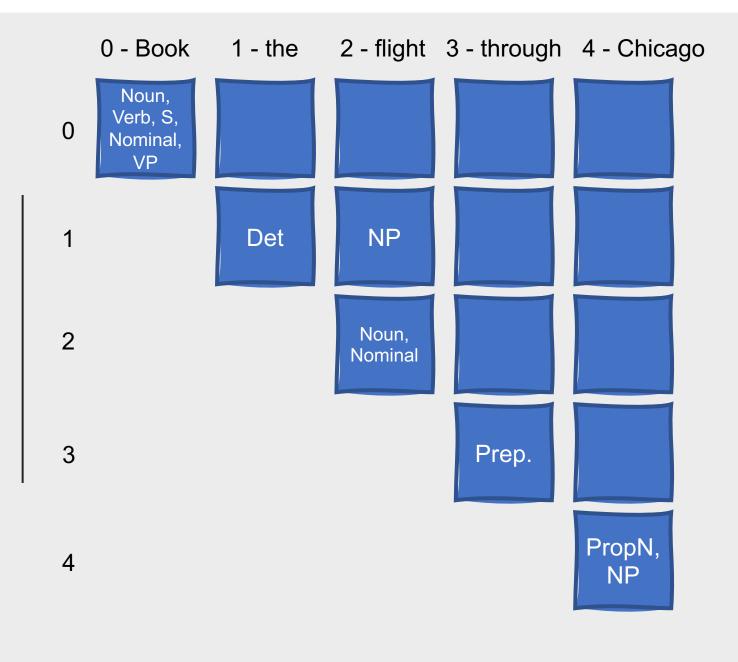
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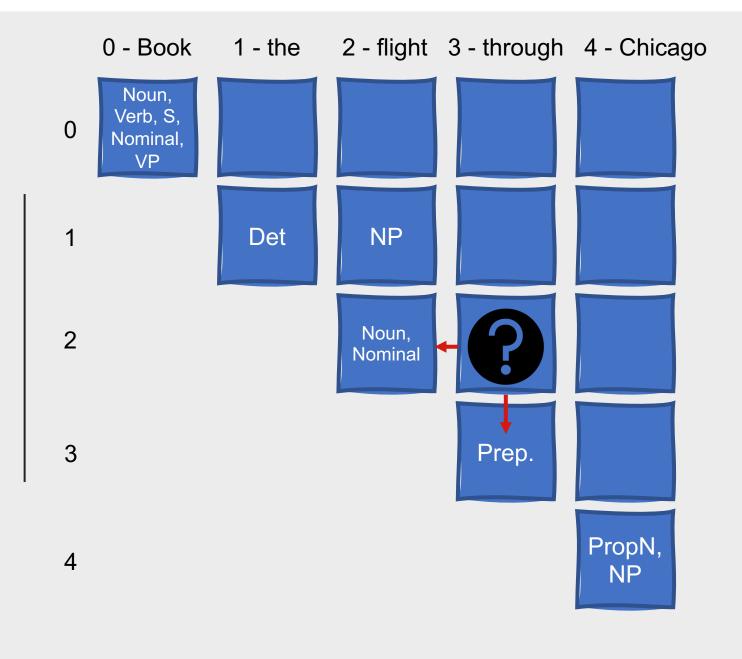
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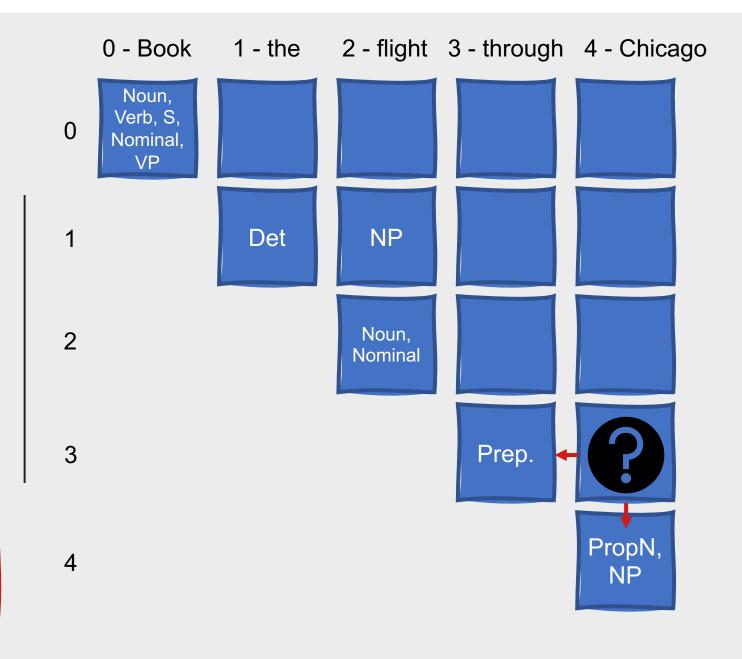
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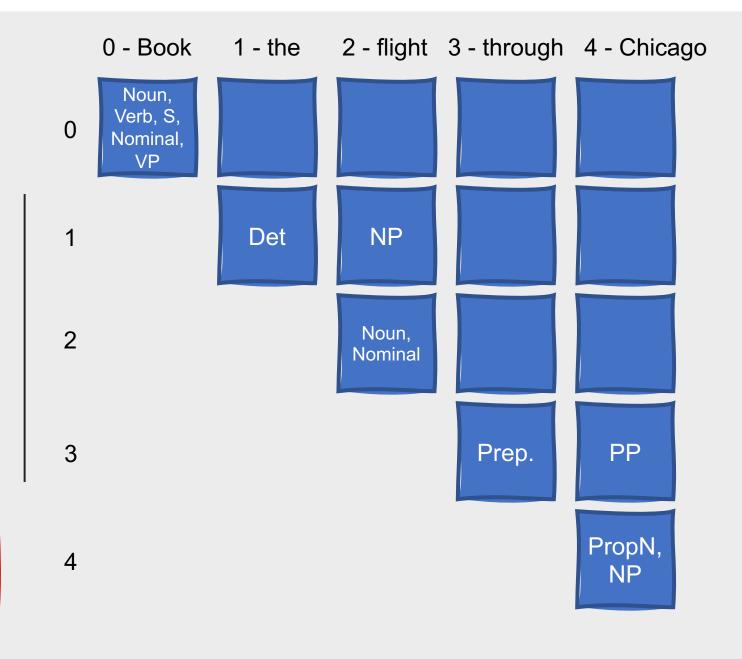
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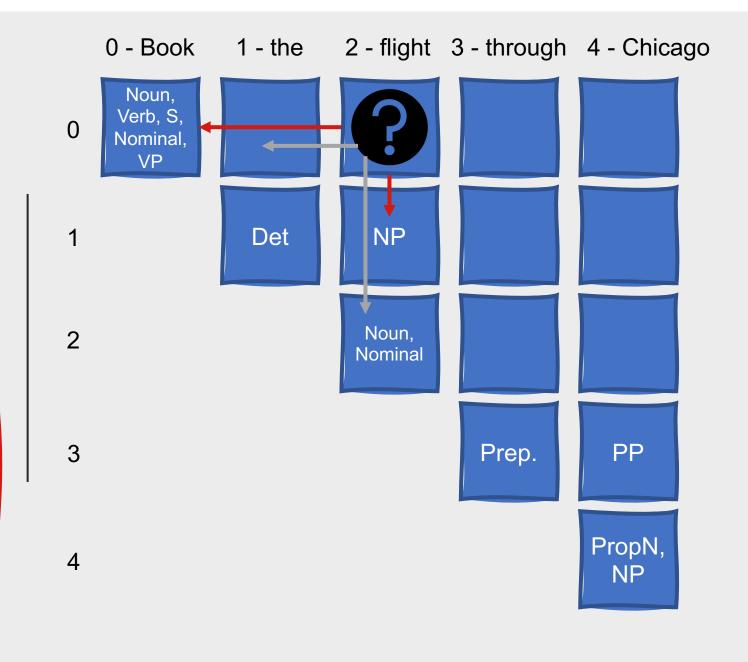
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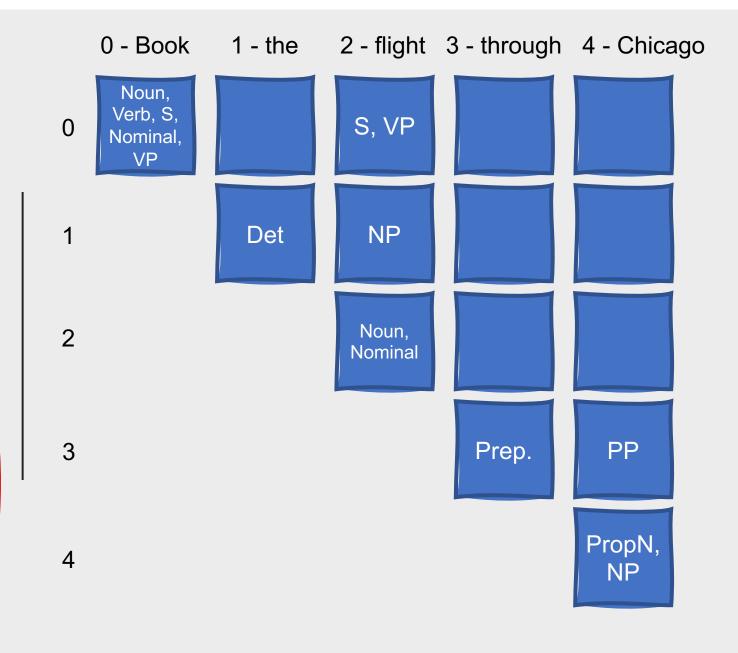
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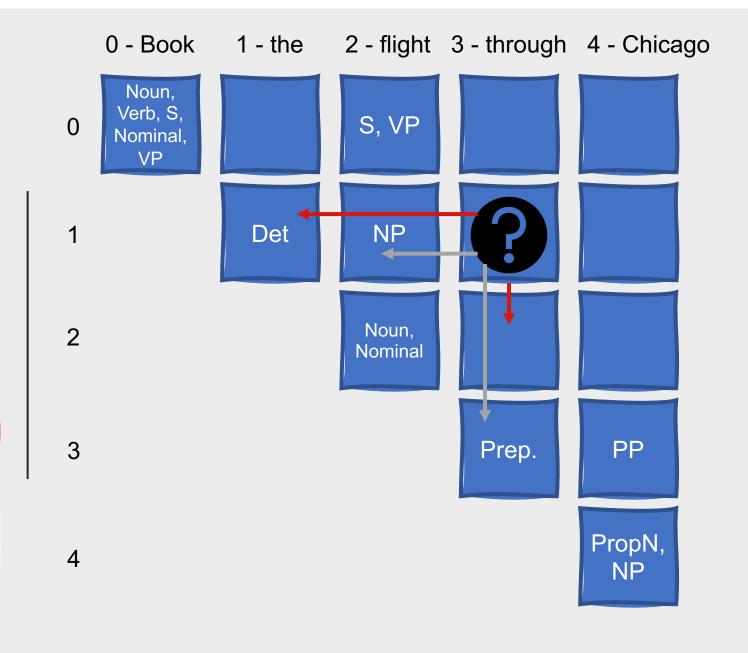
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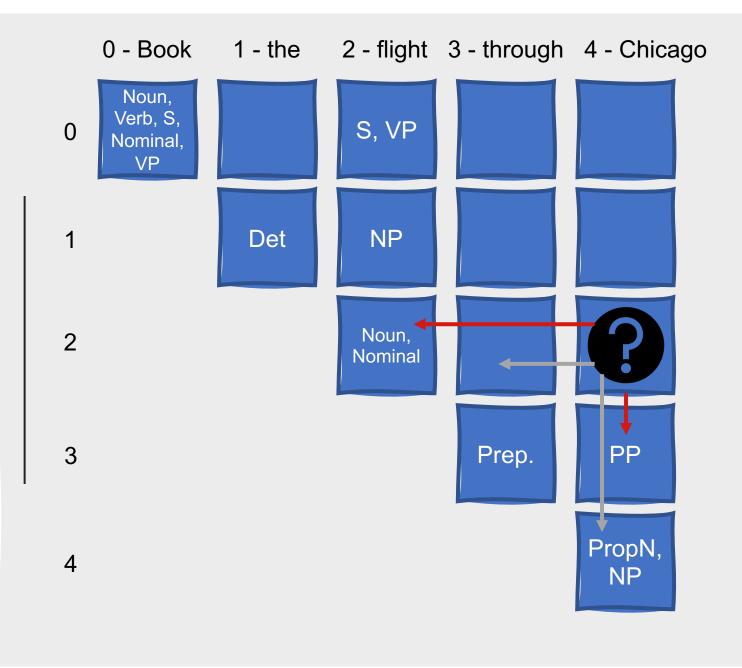
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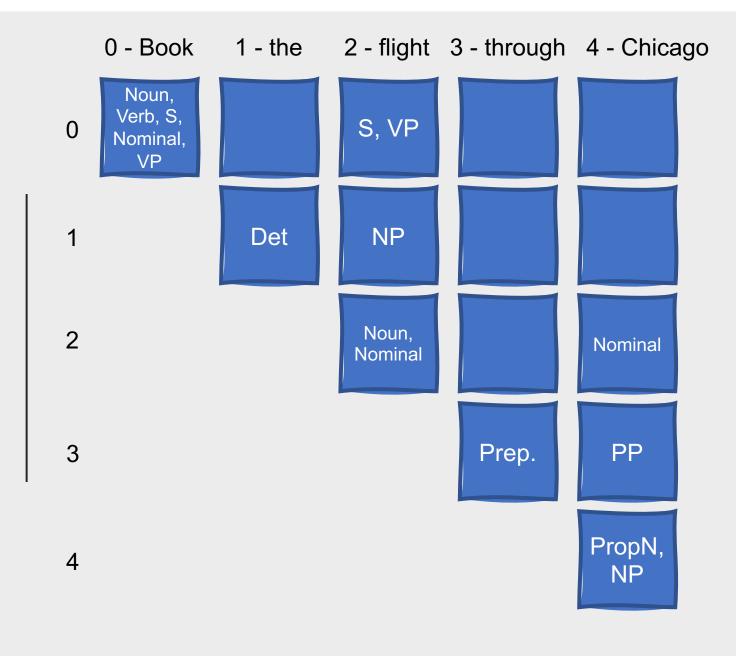
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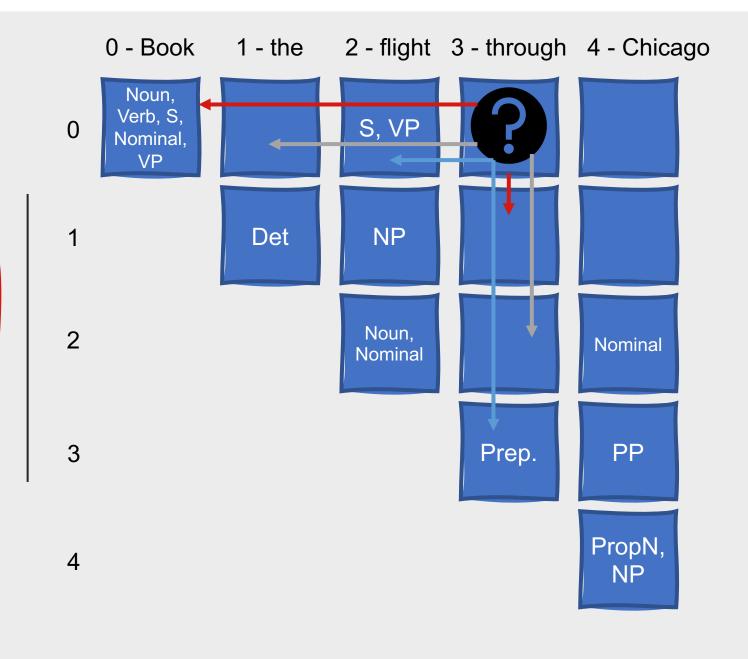
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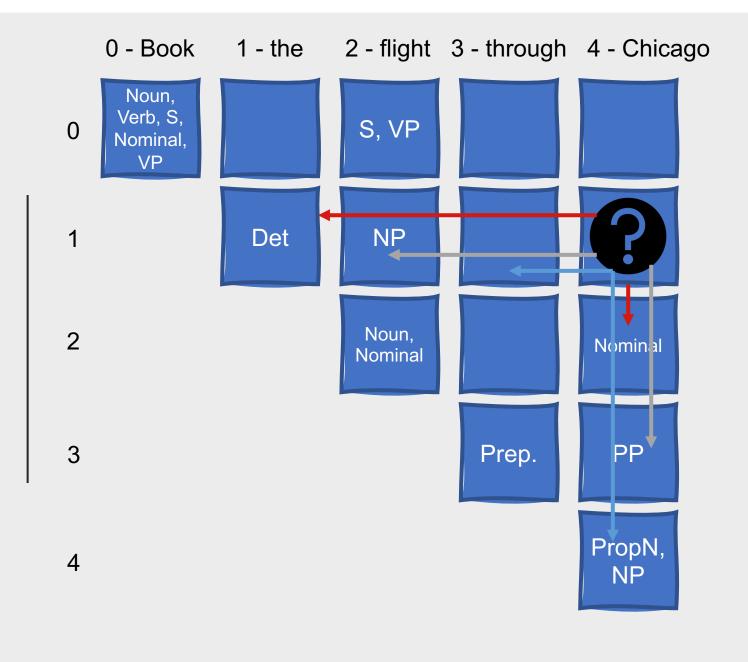
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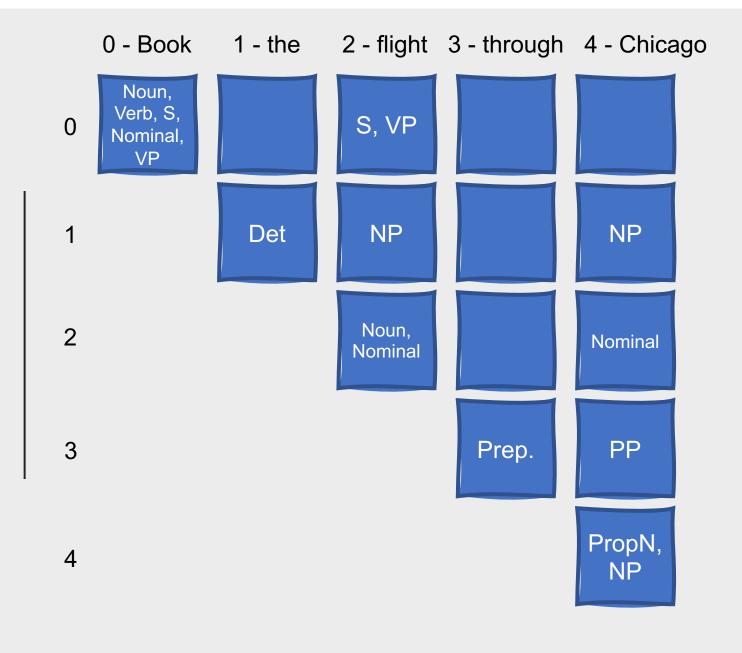
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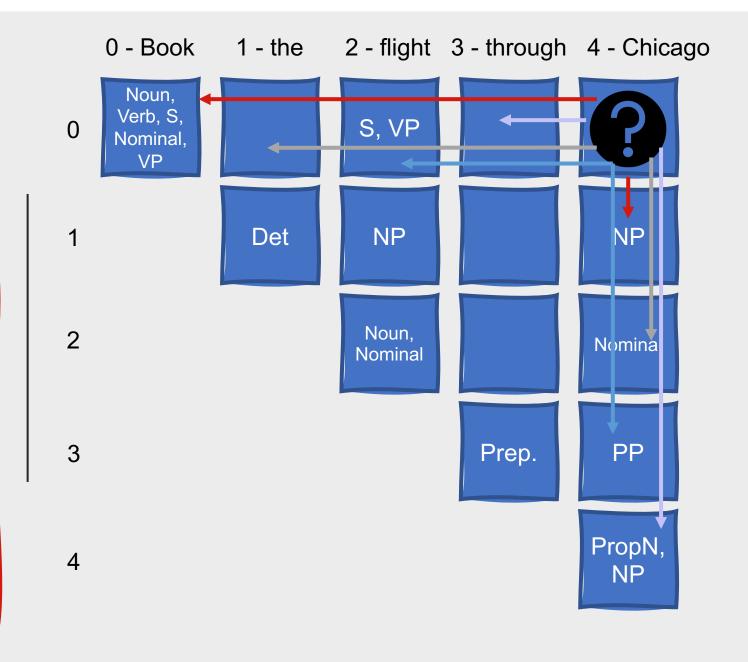
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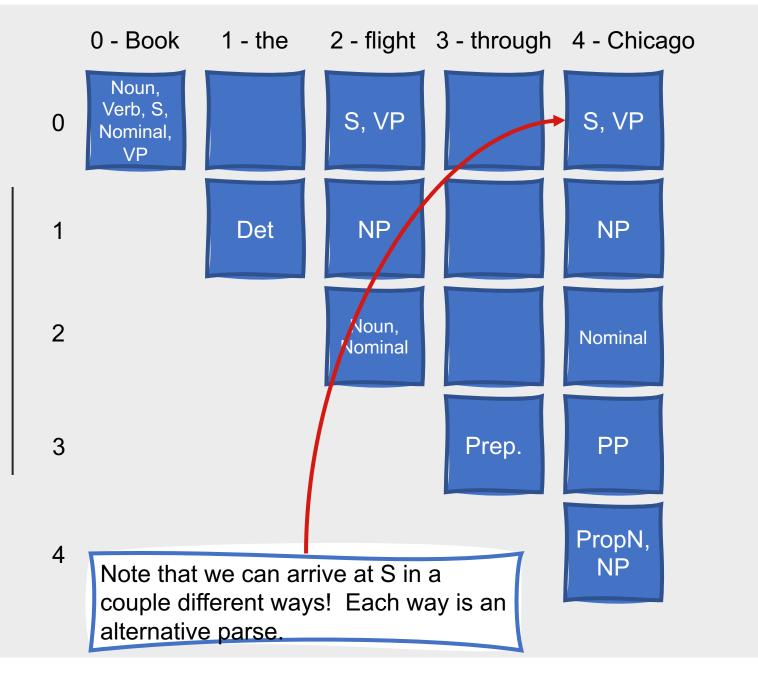
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#### **CKY Algorithm**

- The example we just saw functions as a recognizer ...for it to succeed (i.e., find a valid sentence according to this grammar), is simply needs to find an S in cell [0,n]
- To return all possible parses, we need to make two changes to the algorithm:
  - Pair each non-terminal with pointers to the table entries from which it was derived
  - Permit multiple versions of the same non-terminal to be entered into the table
- Then, we can choose an S from cell [0,n] and recursively retrieve its component constituents from the table

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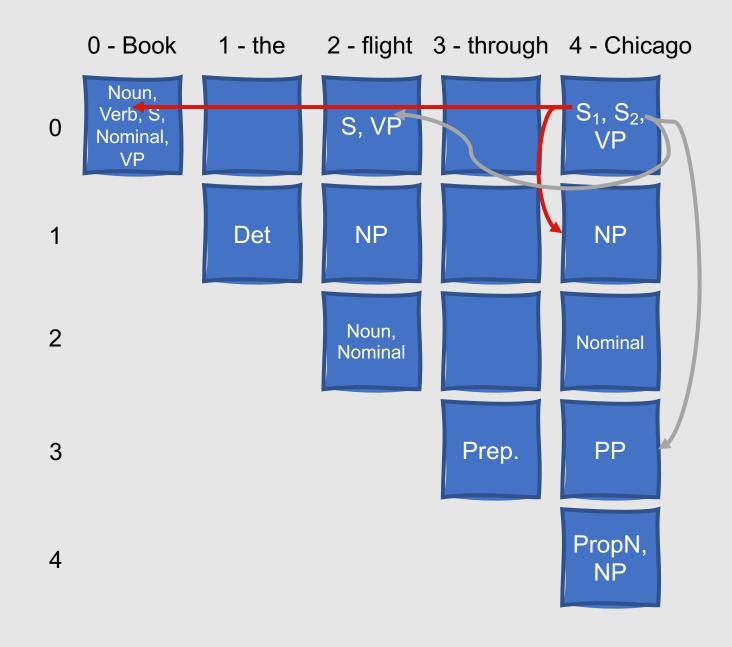
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Time Complexity: O(n<sup>3</sup>)

Space Complexity: O(n<sup>2</sup>)