Combining Computational Units

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UIC CS 421

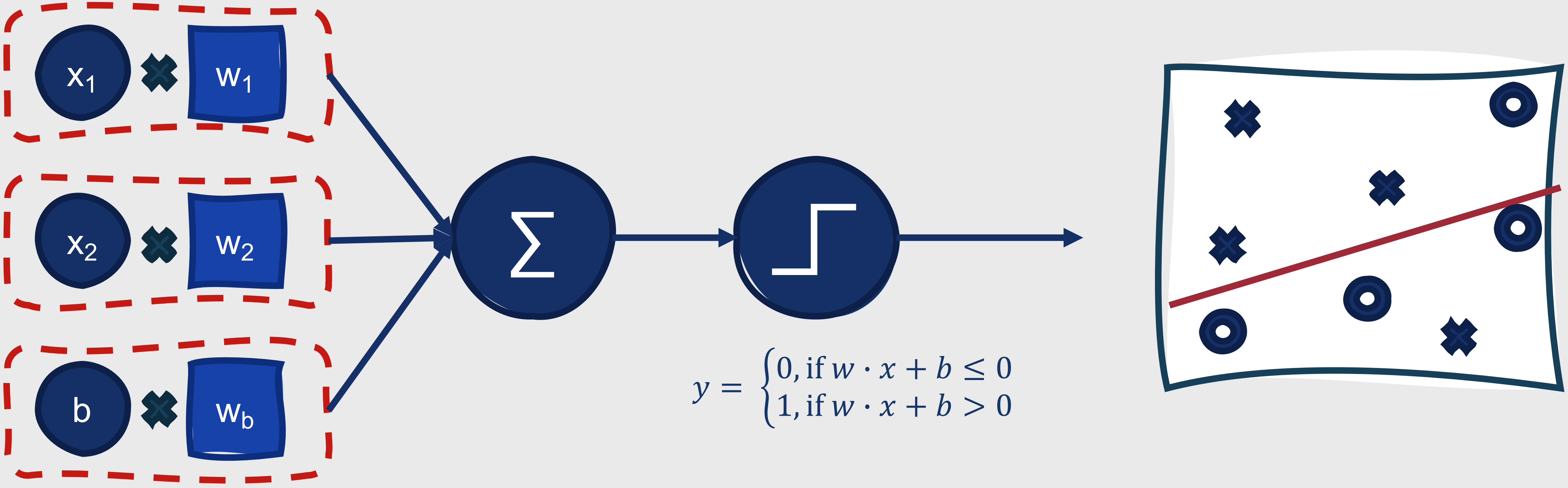
Combining Computational Units

Neural networks are powerful primarily because they are able to **combine multiple computational units into larger networks**

Many problems cannot be solved using a single computational unit

Early example of this: The XOR problem

AND			OR			XOR		
x1	x2	y	x1	x2	y	x1	x2	y
0	0	0	0	0	0	0	0	0
0	1	0	0	1	1	0	1	1
1	0	0	1	0	1	1	0	1
1	1	1	1	1	1	1	1	0

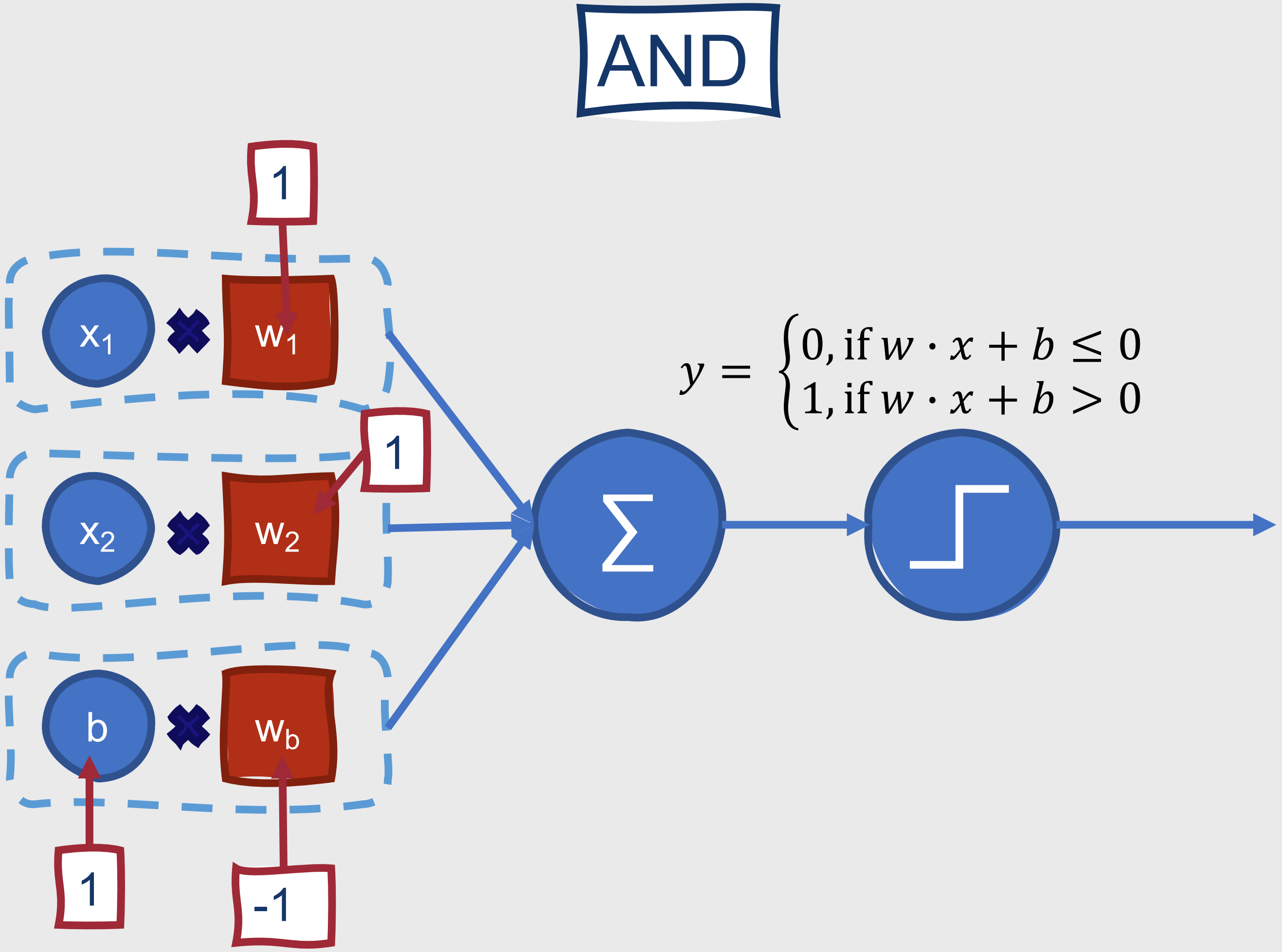


$$y = \begin{cases} 0, \text{if } w \cdot x + b \leq 0 \\ 1, \text{if } w \cdot x + b > 0 \end{cases}$$

AND and OR can both be solved using a single perceptron.

- **Perceptron:** A function that outputs a binary value based on whether the product of its inputs and associated weights surpasses a threshold
  - Learns this threshold iteratively by trying to find the boundary that is best able to distinguish between data of different categories

It's easy to compute AND and OR using perceptrons.



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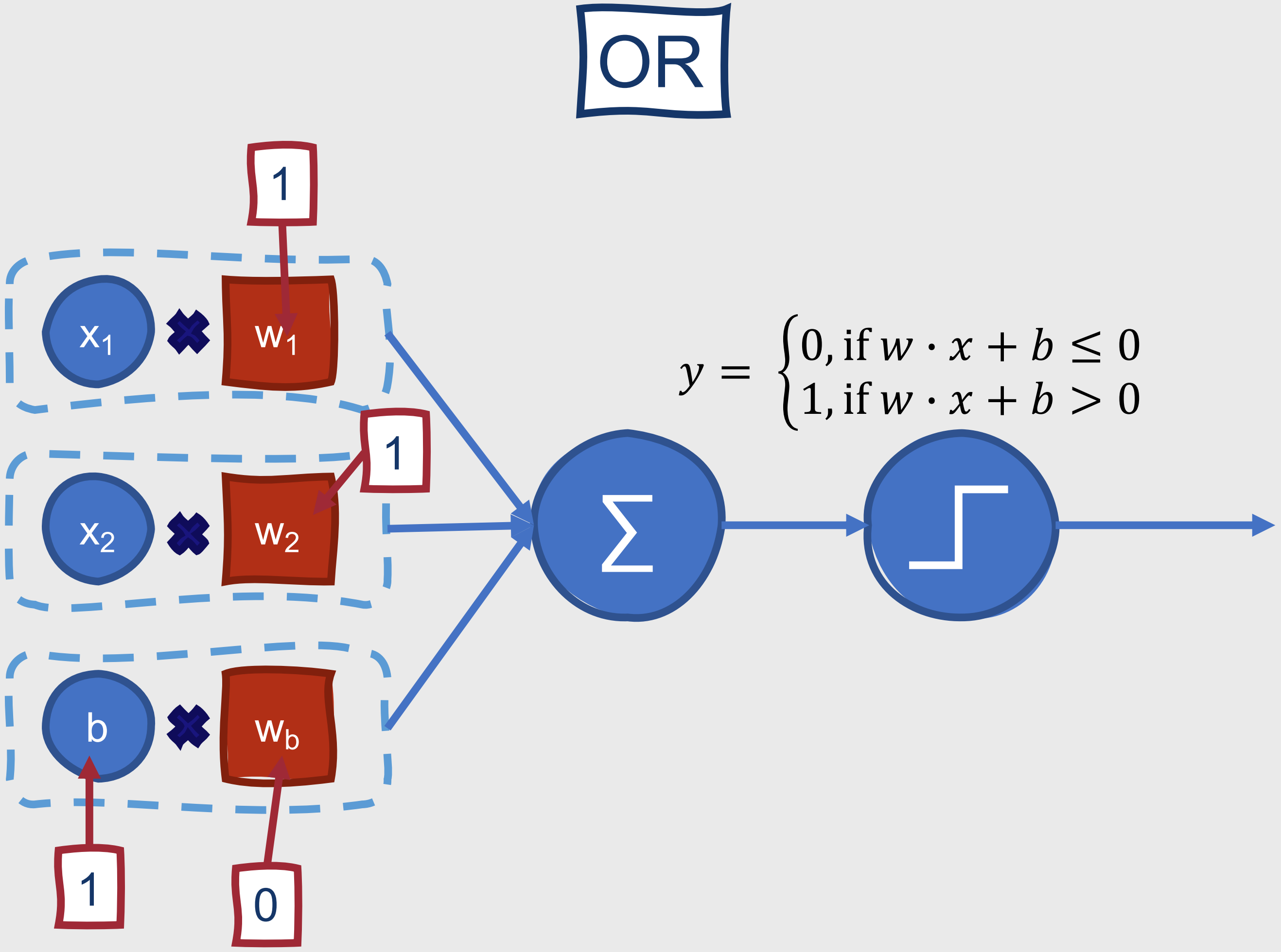
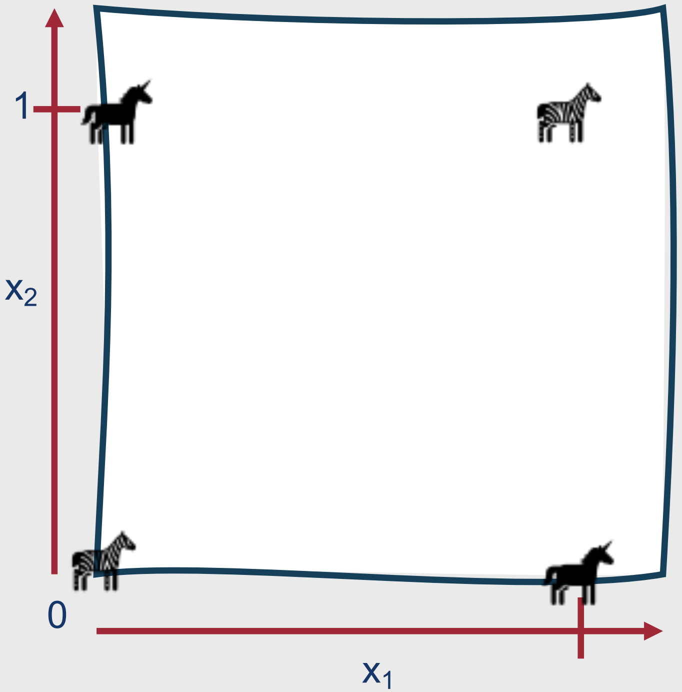


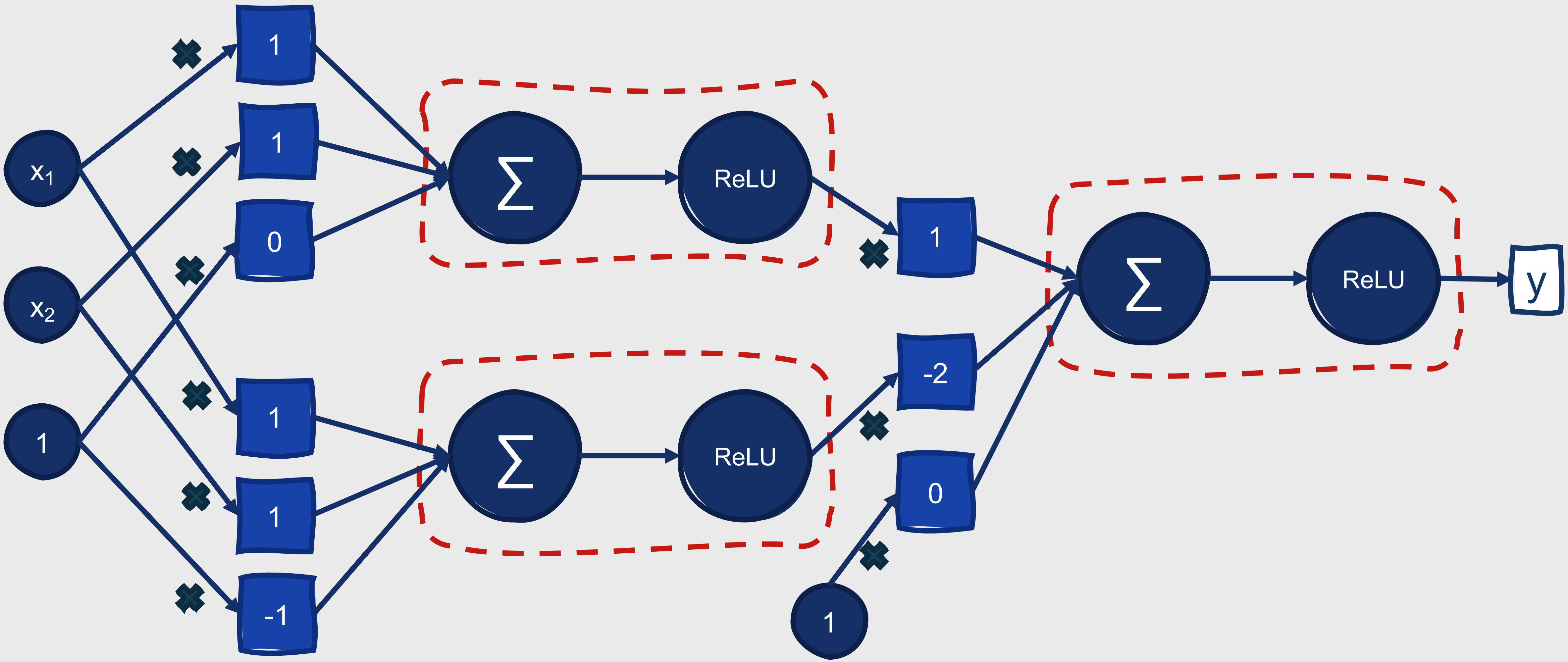


AND			OR			XOR		
x1	x2	y	x1	x2	y	x1	x2	y
0	0	0	0	0	0	0	0	0
0	1	0	0	1	1	0	1	1
1	0	0	1	0	1	1	0	1
1	1	1	1	1	1	1	1	0

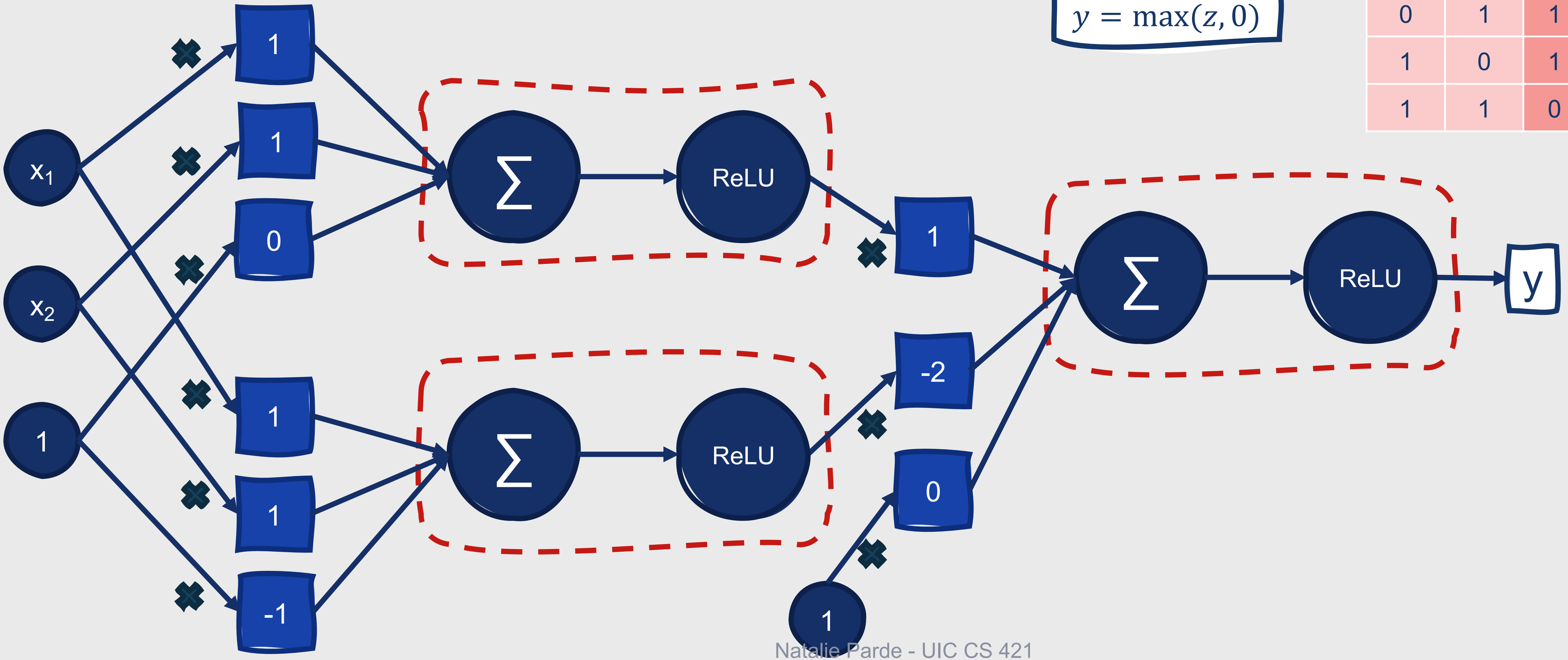
However, it's impossible to compute XOR using a single perceptron.

- Why?
  - Perceptrons are **linear classifiers**
  - XOR is not a **linearly separable function**

The only successful way to compute XOR is by combining these smaller units into a larger network.



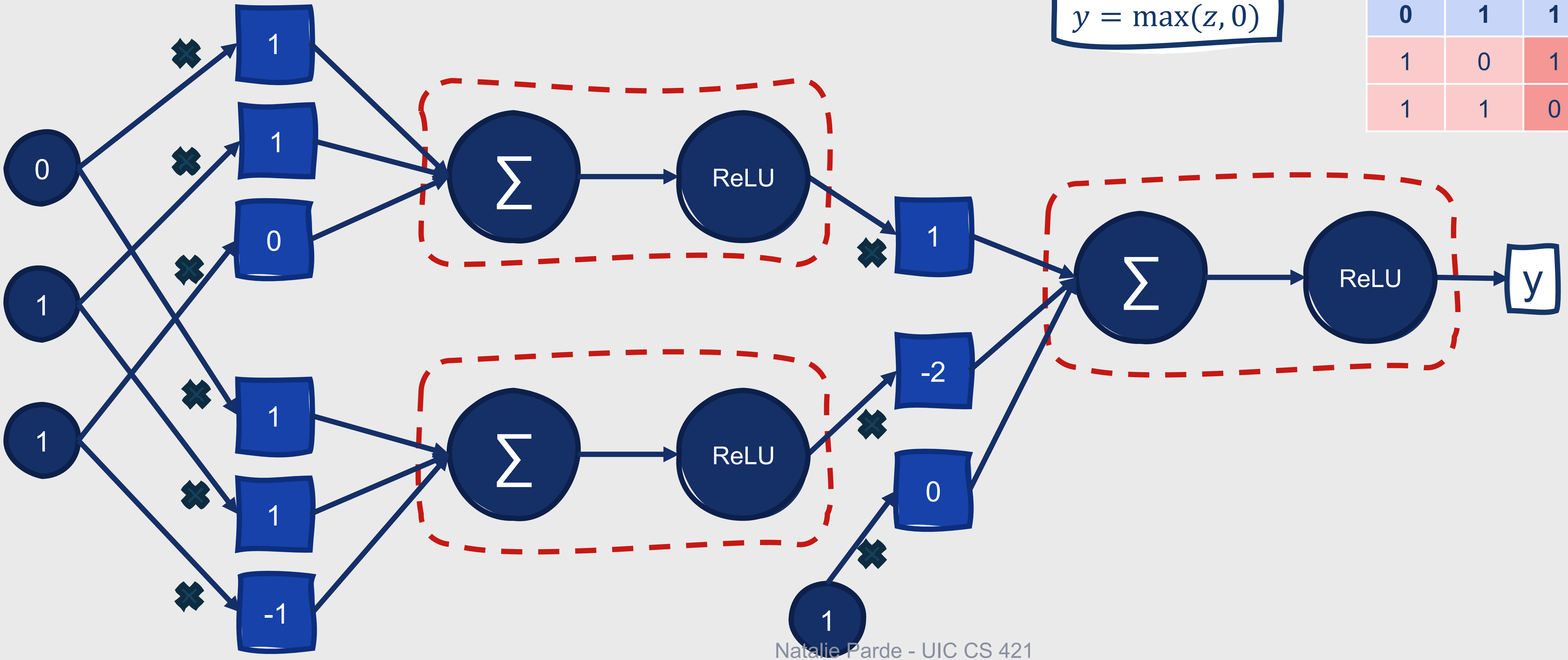
Truth Table Examples: XOR



$y = \max(z, 0)$

XOR		
x1	x2	y
0	0	0
0	1	1
1	0	1
1	1	0

Truth Table Examples: XOR



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XOR		
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0	**1**	**1**
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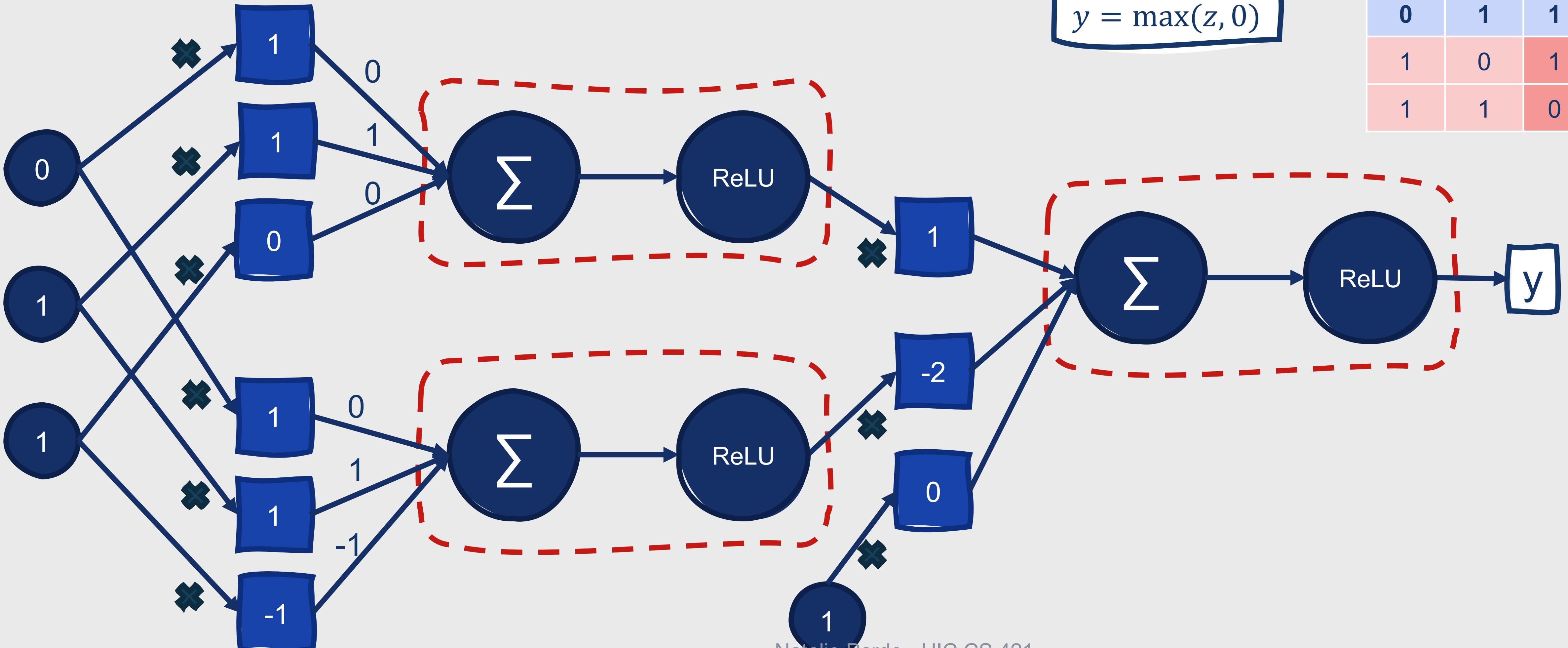

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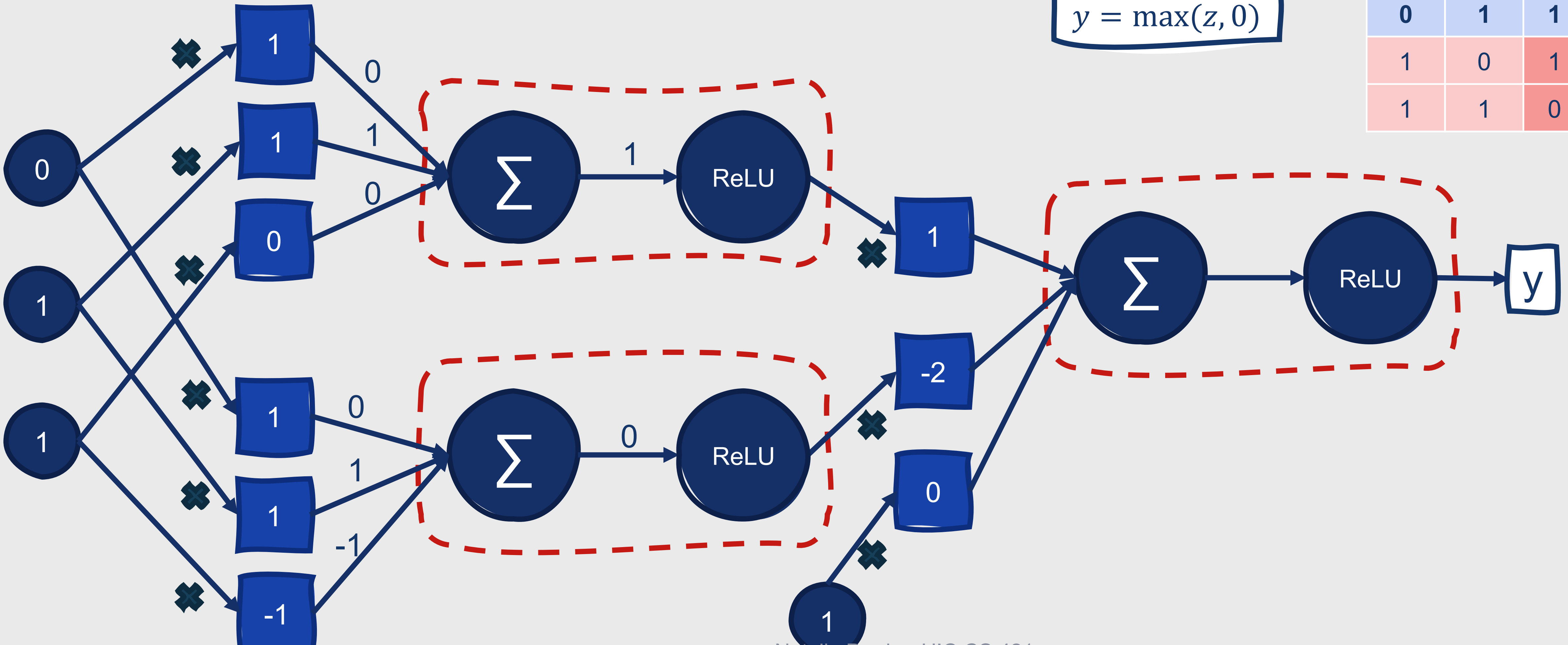
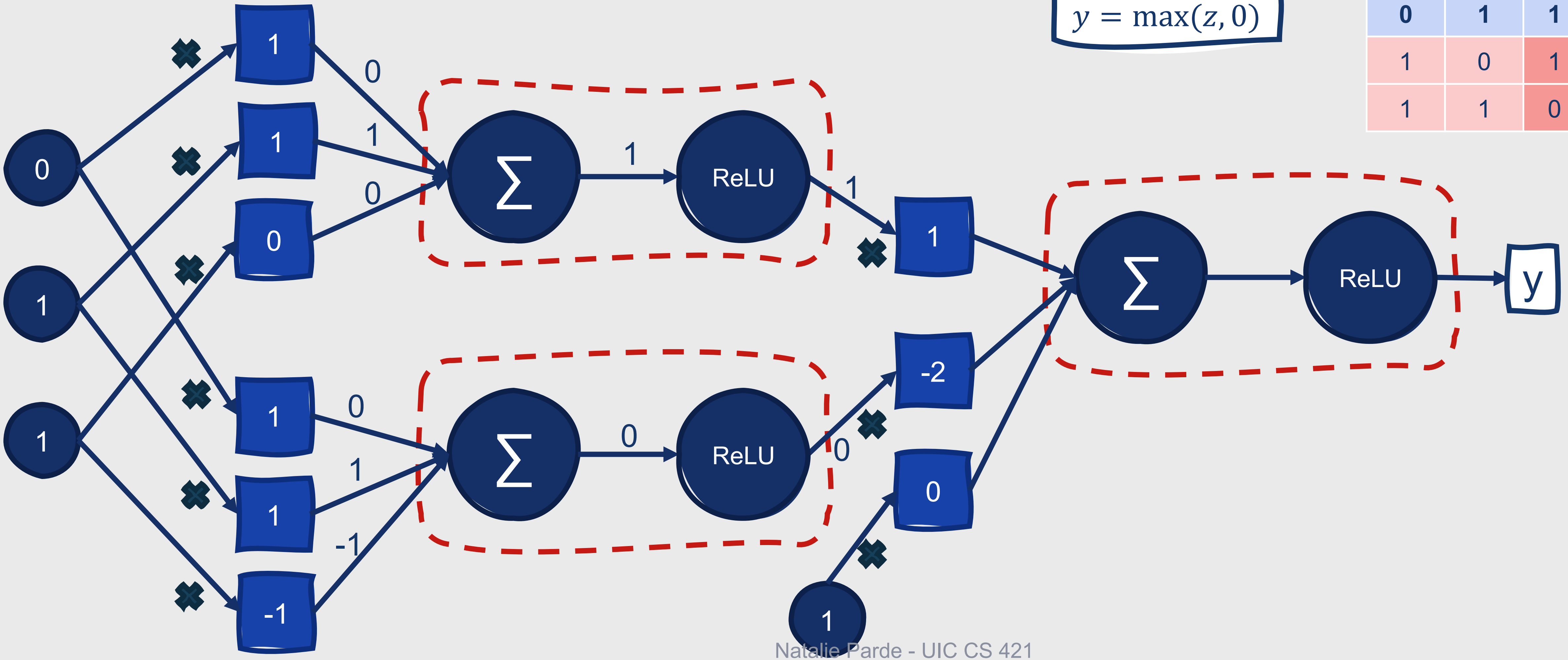


Truth Table Examples: XOR



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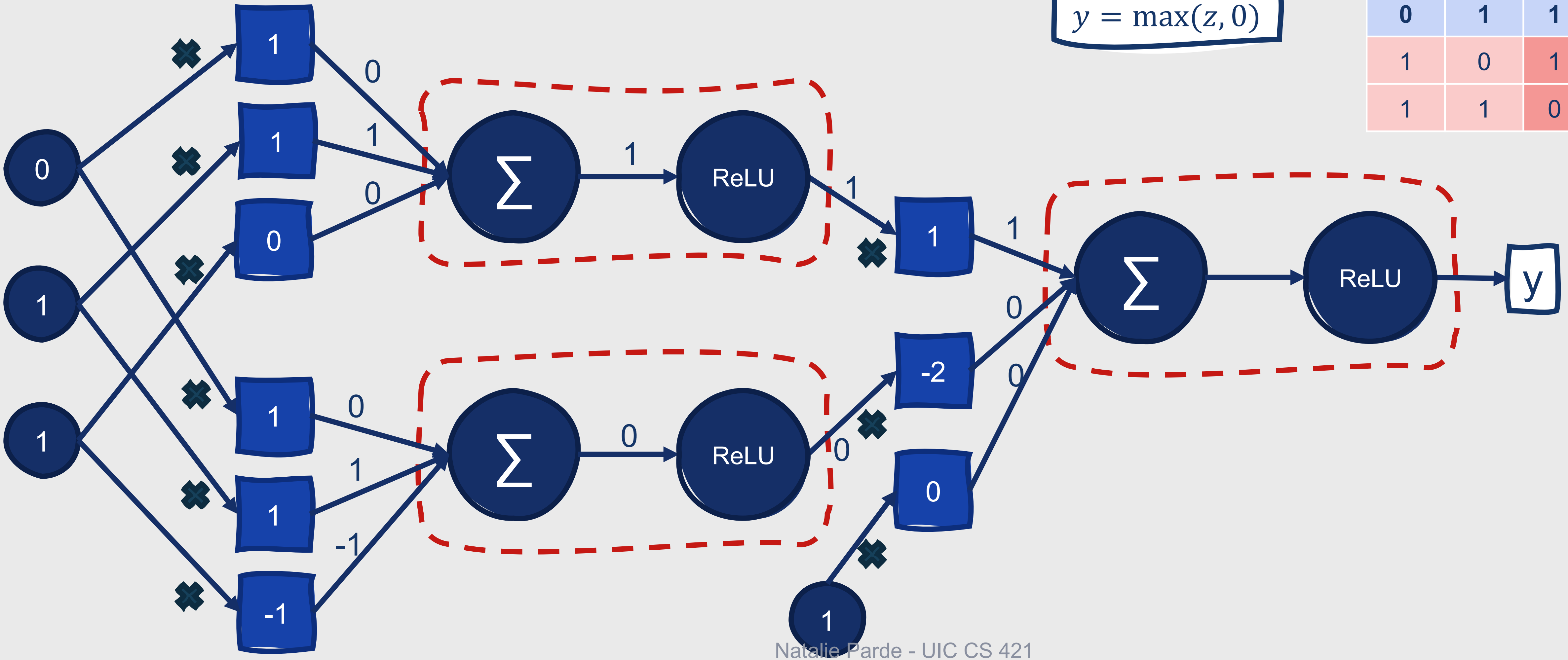
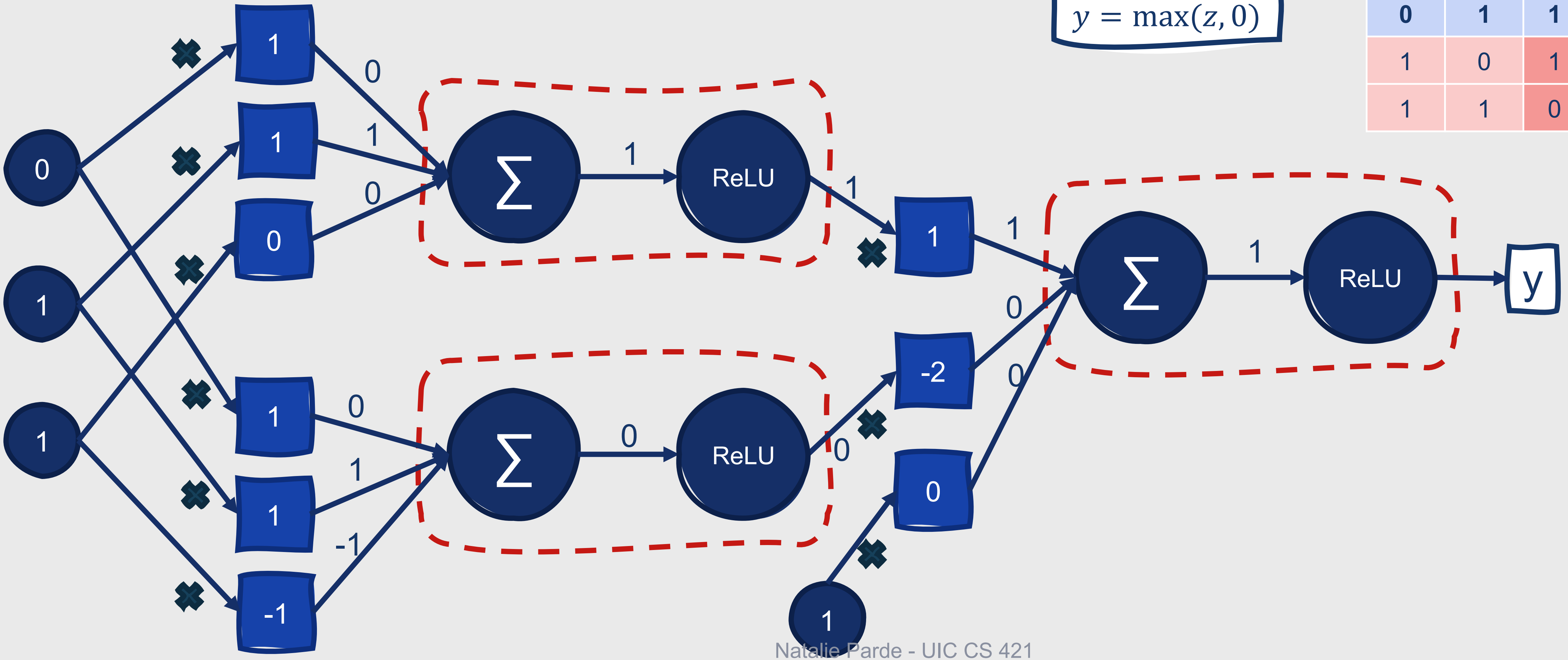


Truth Table Examples: XOR



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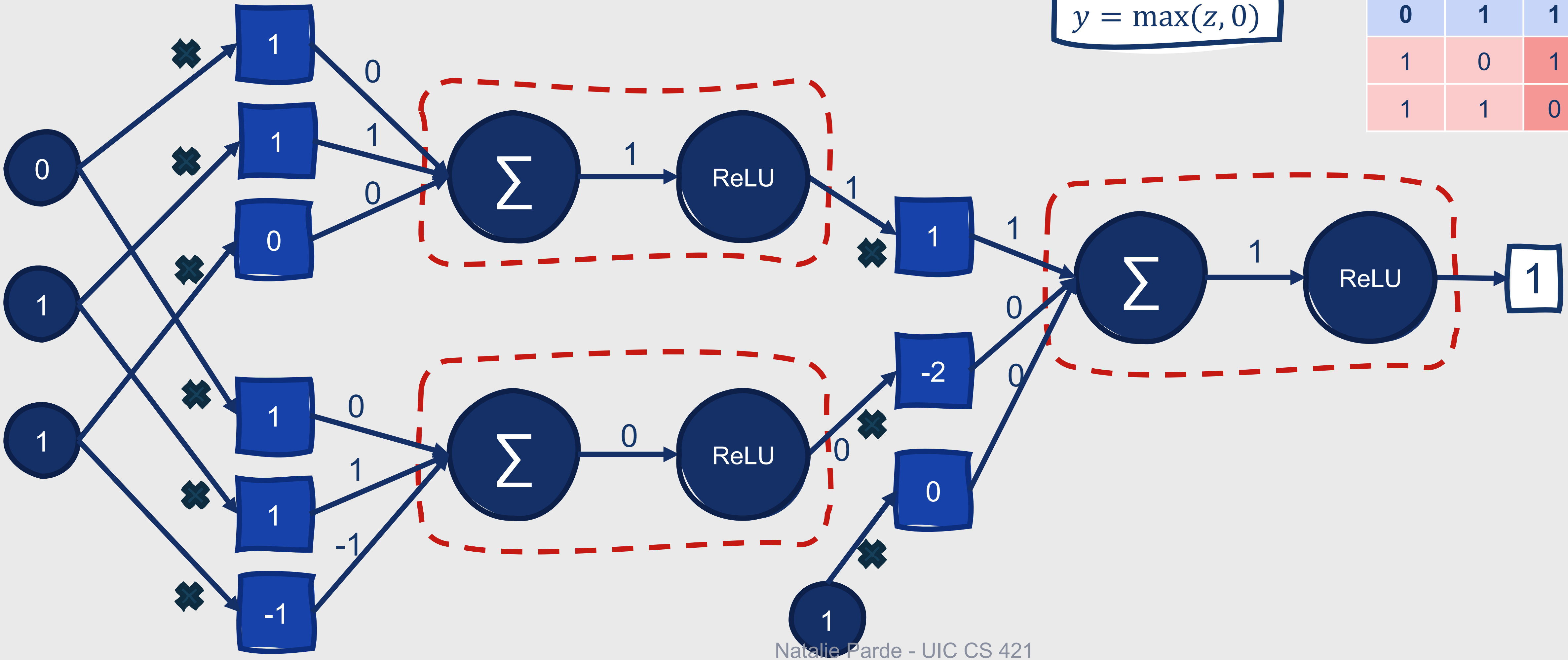


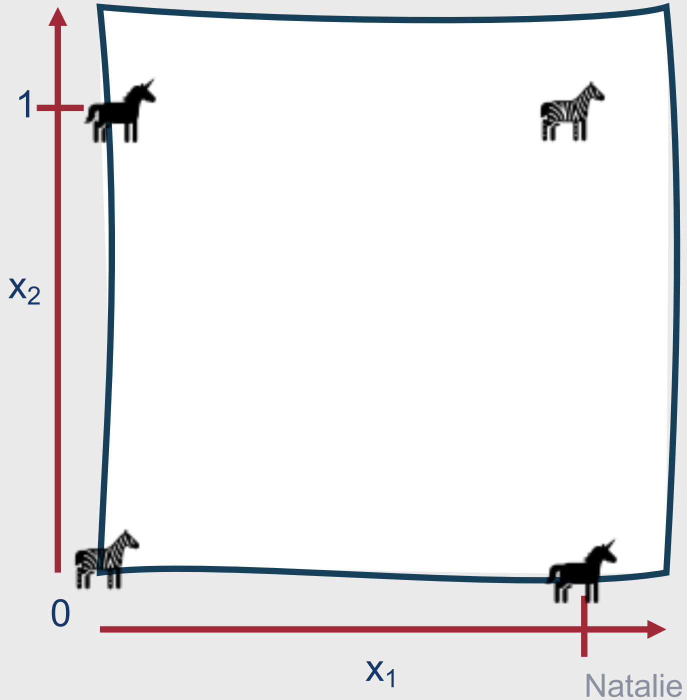
$$y = \max(z, 0)$$

XOR		
x1	**x2**	**y**
0	0	0
0	**1**	**1**
1	0	1
1	1	0

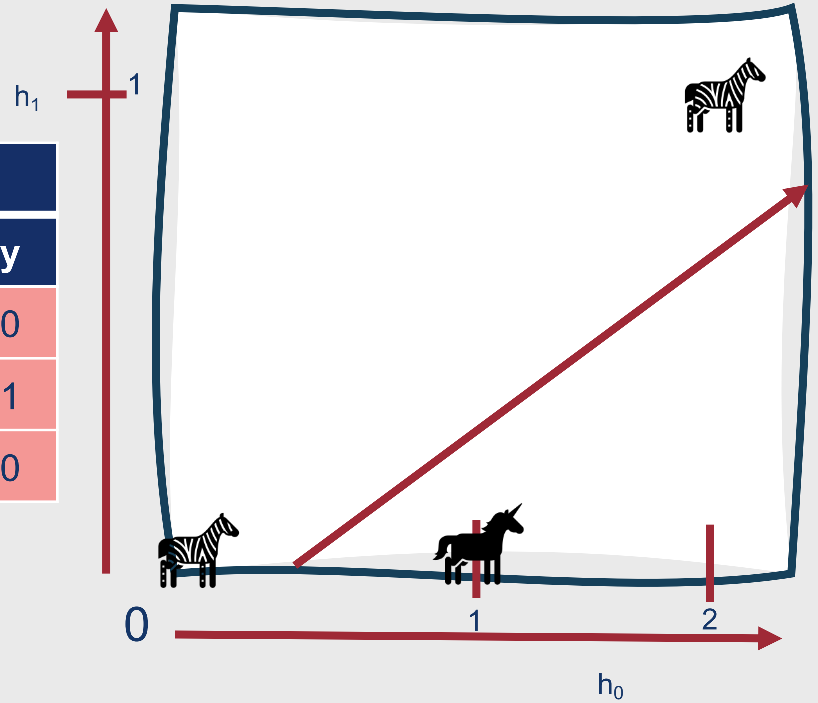
Why does this work?

- When computational units are combined, the outputs from each successive layer provide **new representations** for the input
- These new representations are **linearly separable**

XOR		
x1	**x2**	**y**
0	0	0
0	1	1
1	0	1
1	1	0



XOR		
h0	**h1**	**y**
0	0	0
1	0	1
2	1	0



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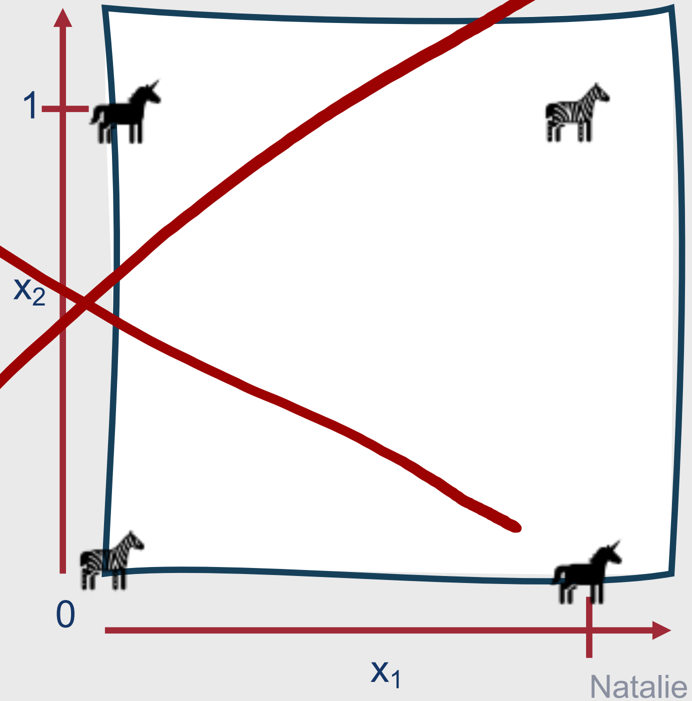

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h0	h1	y
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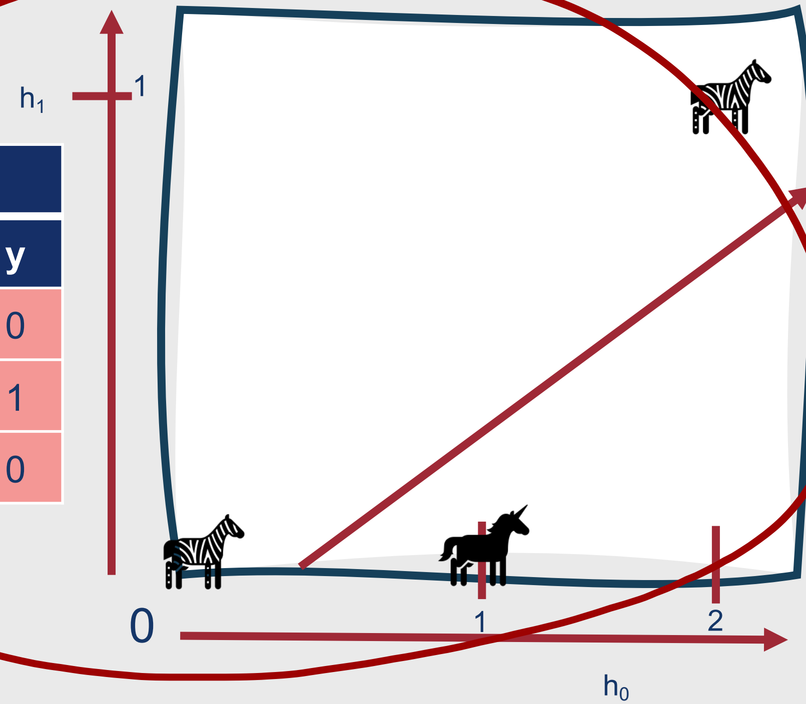

Combining Computational Units

- In our XOR example, we manually assigned weights to each unit
- In real-world examples, these weights are learned automatically using a **backpropagation** algorithm
- Thus, the network is able to learn a useful representation of the input training data on its own
  - Key advantage of neural networks