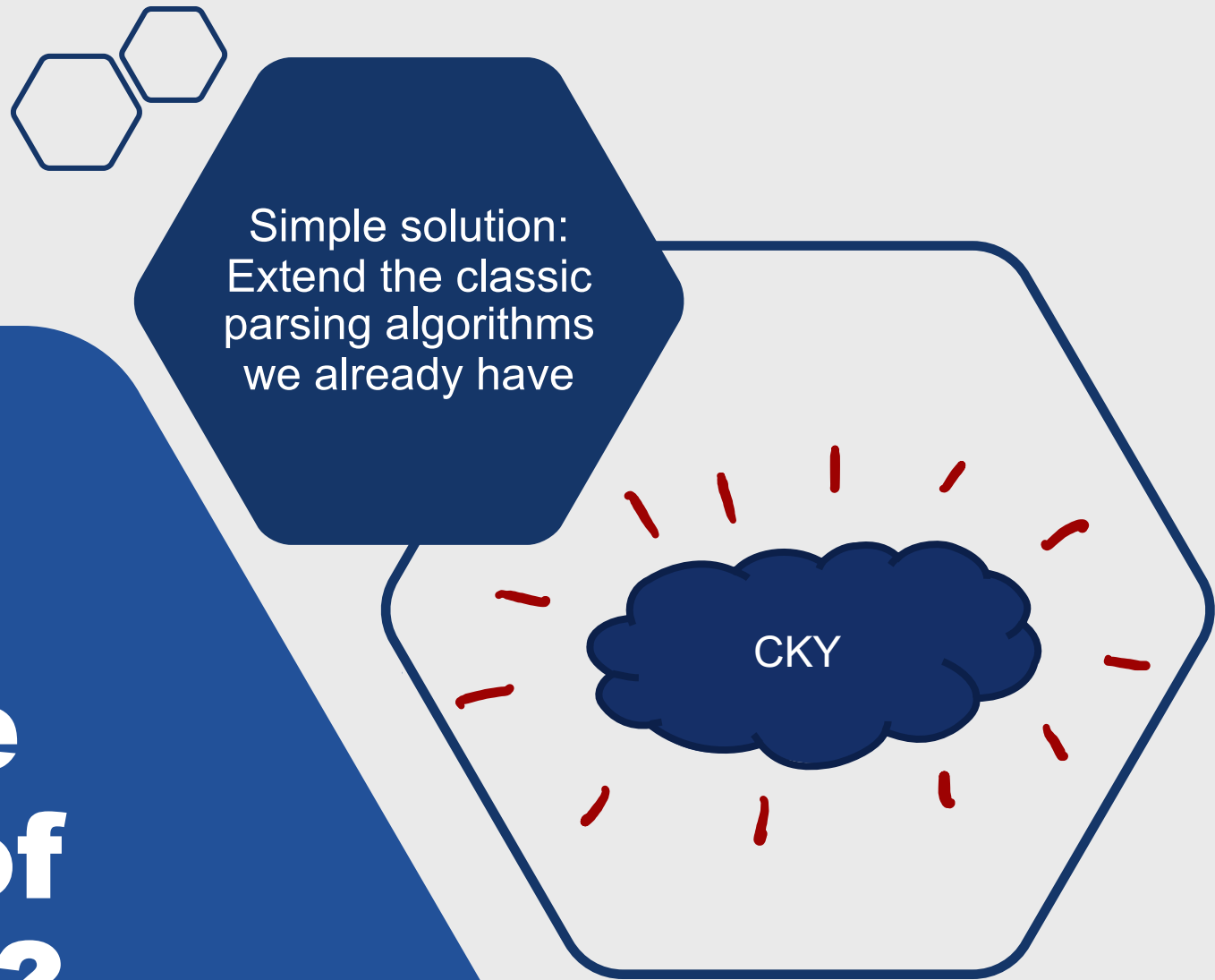


Probabilistic CKY Algorithm

Natalie Parde

UIC CS 421

How do we compute the probability of a parse tree?

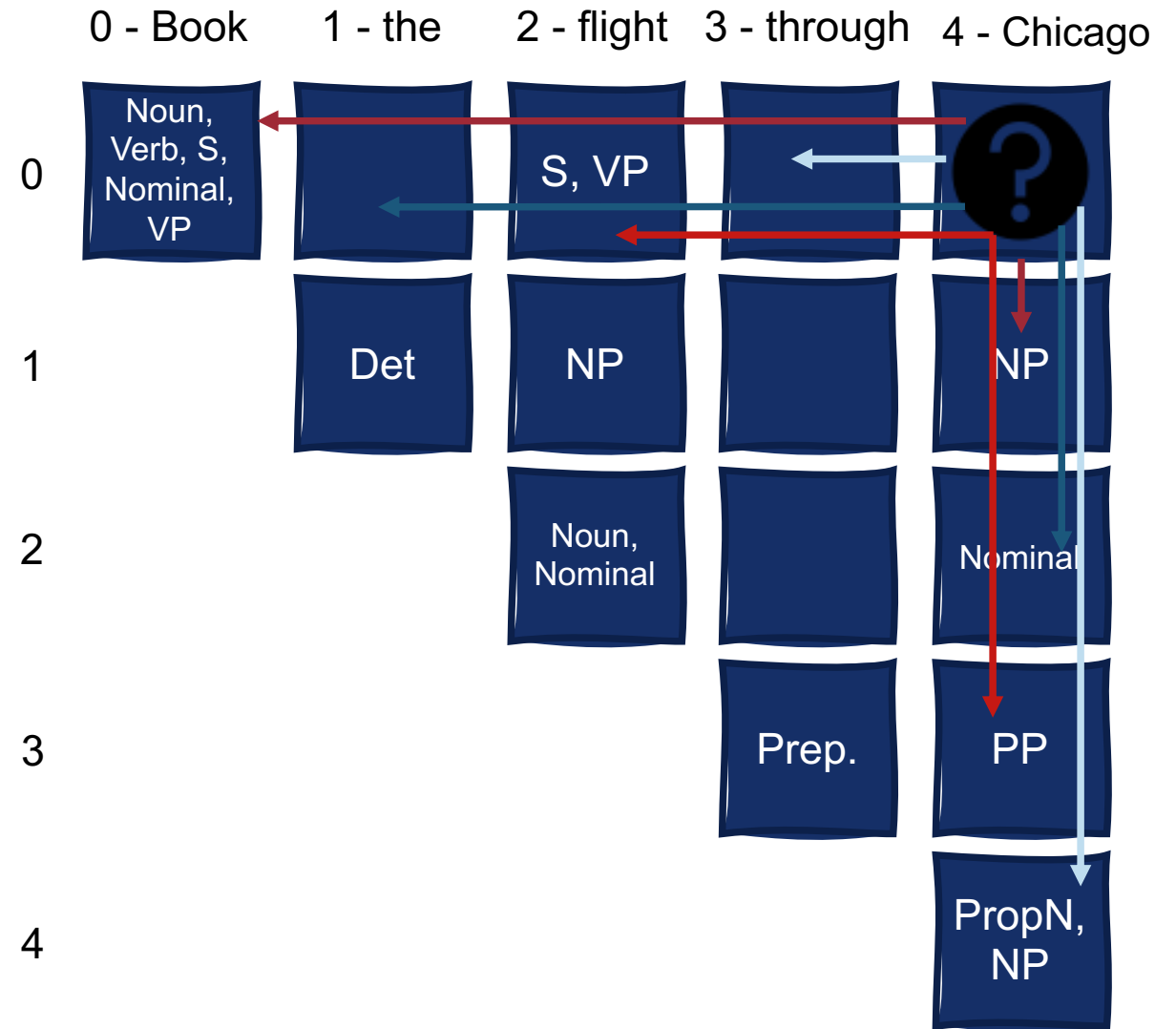


Simple solution:
Extend the classic
parsing algorithms
we already have

CKY

Probabilistic CKY

- Still assume grammar is in Chomsky Normal Form
 - Right-hand side of production rule expands to two non-terminals or one terminal node
 - $A \rightarrow B C$
 - $A \rightarrow w$
- Still work with the upper triangular portion of a matrix





Probabilistic CKY

- Let n be the length of an input sentence, and V be the number of non-terminals in a grammar
- Consider the constituents *inside* the matrix cells to be part of a third dimension, of maximum length V
- Then, each cell $[i, j, A]$ in the $(n + 1) \times (n + 1) \times V$ matrix corresponds to the probability of constituent A spanning positions i through j of the input

Case Example: Probabilistic CKY

The price includes a facemask

Production Rule	Probability
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$NP \rightarrow Det N$	0.30
$VP \rightarrow V NP$	0.20
$V \rightarrow includes$	0.05
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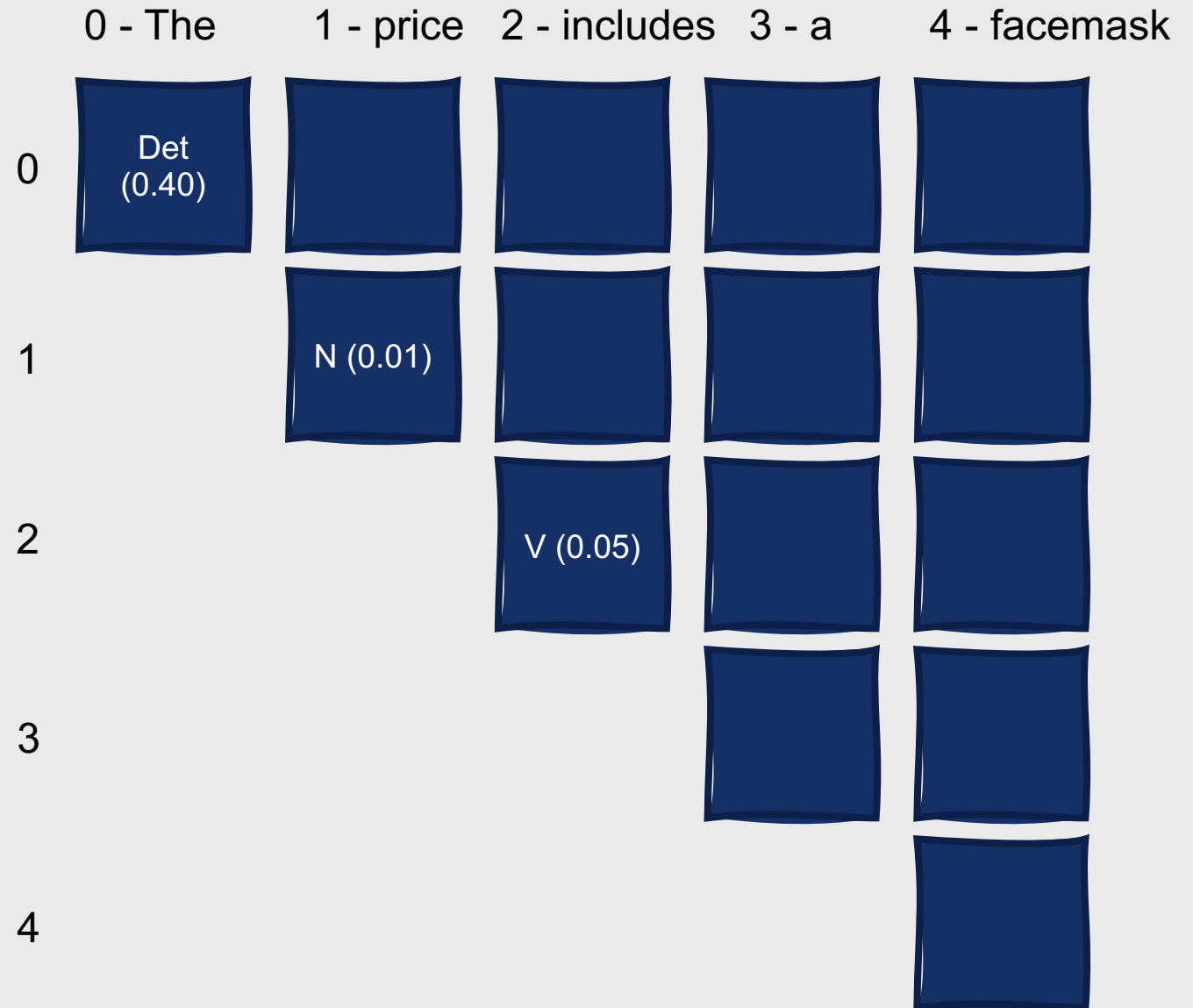
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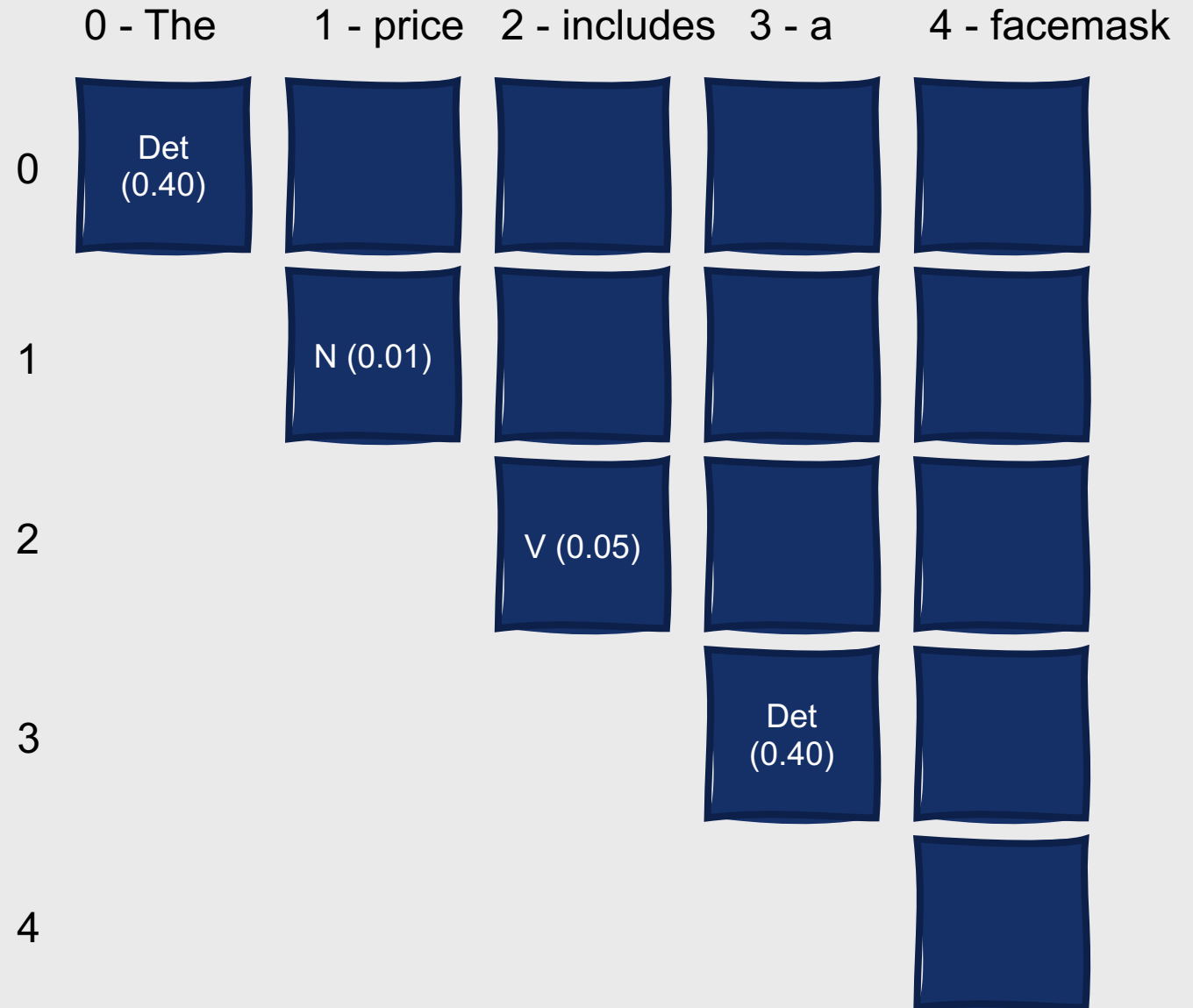
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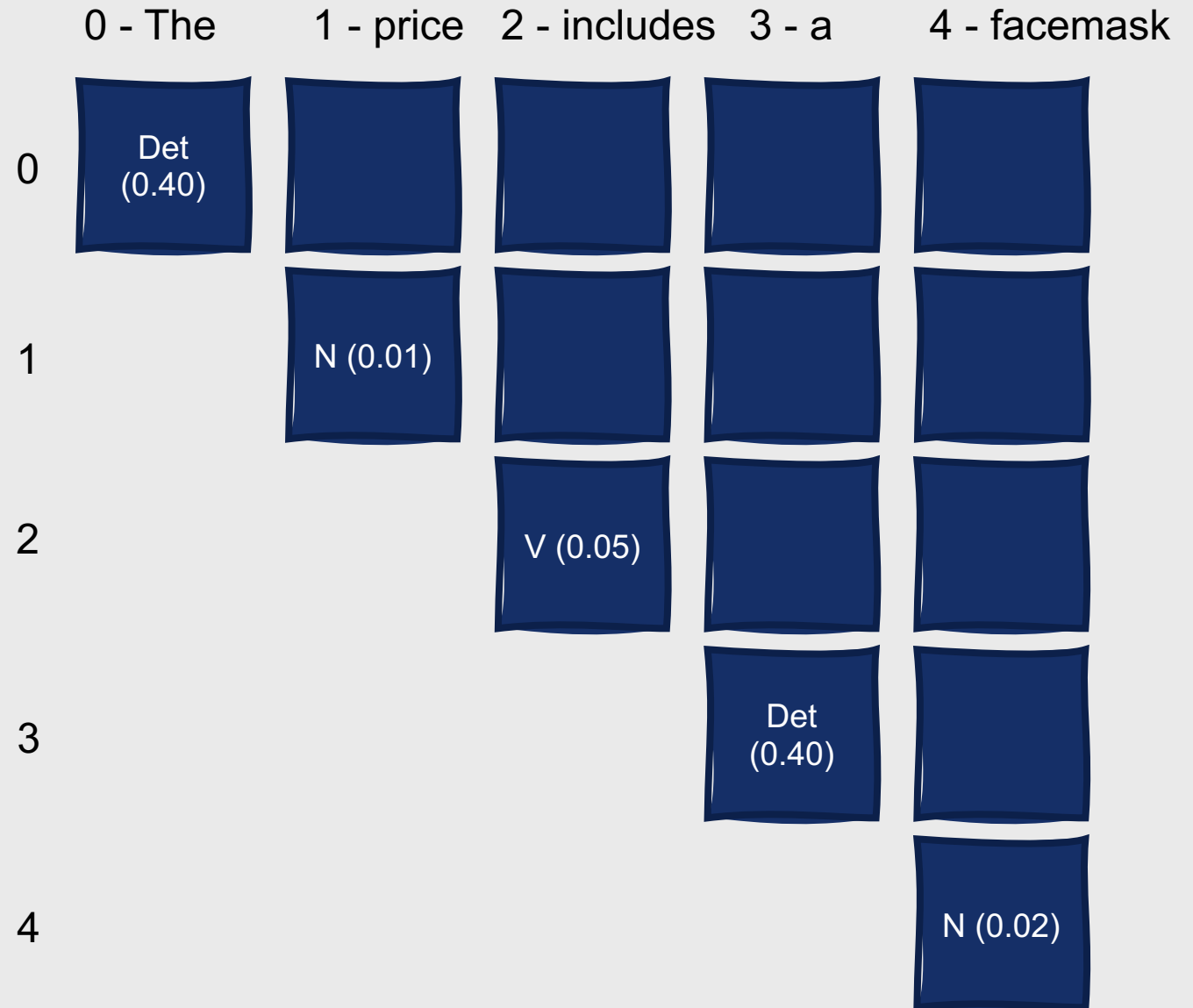
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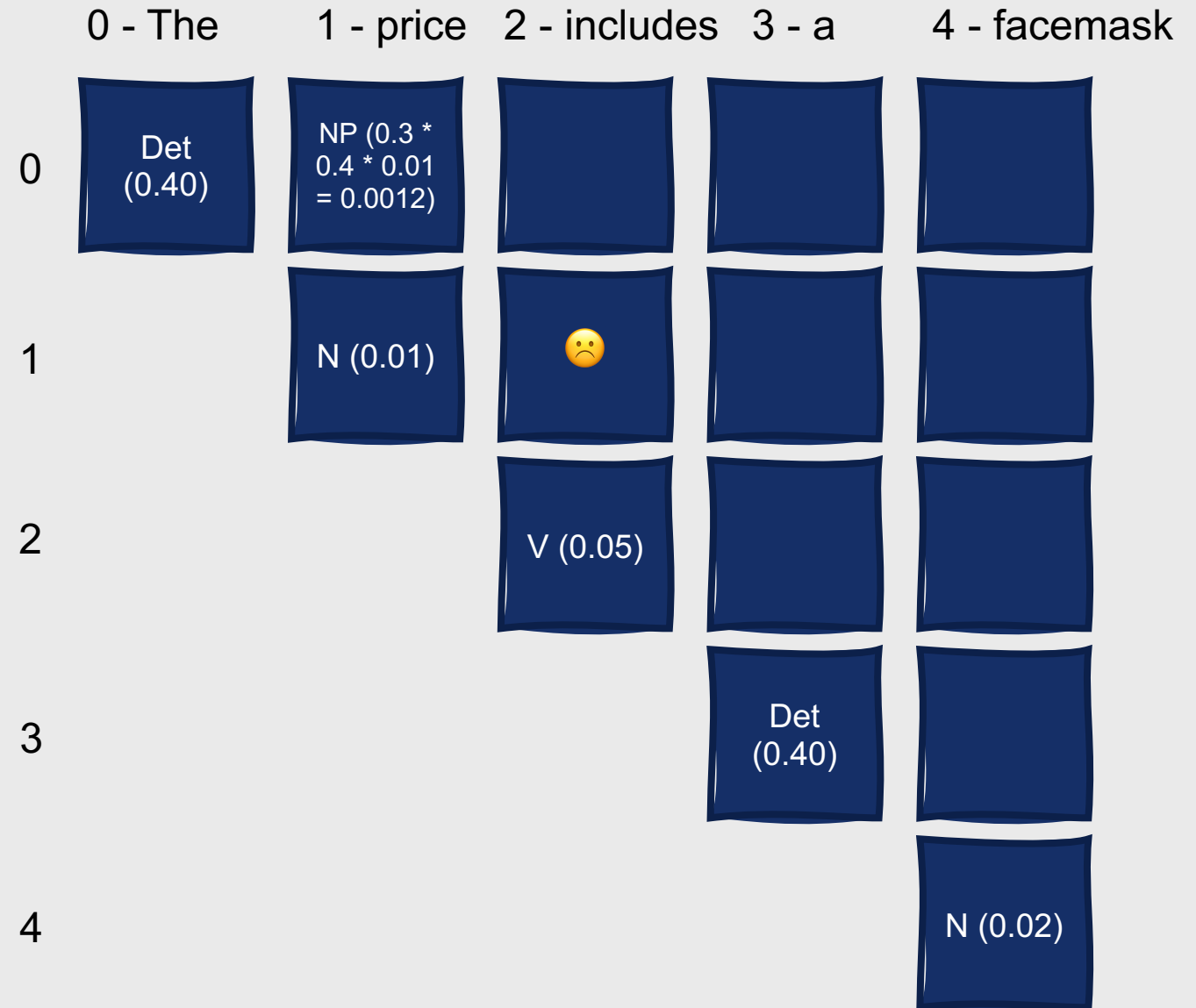
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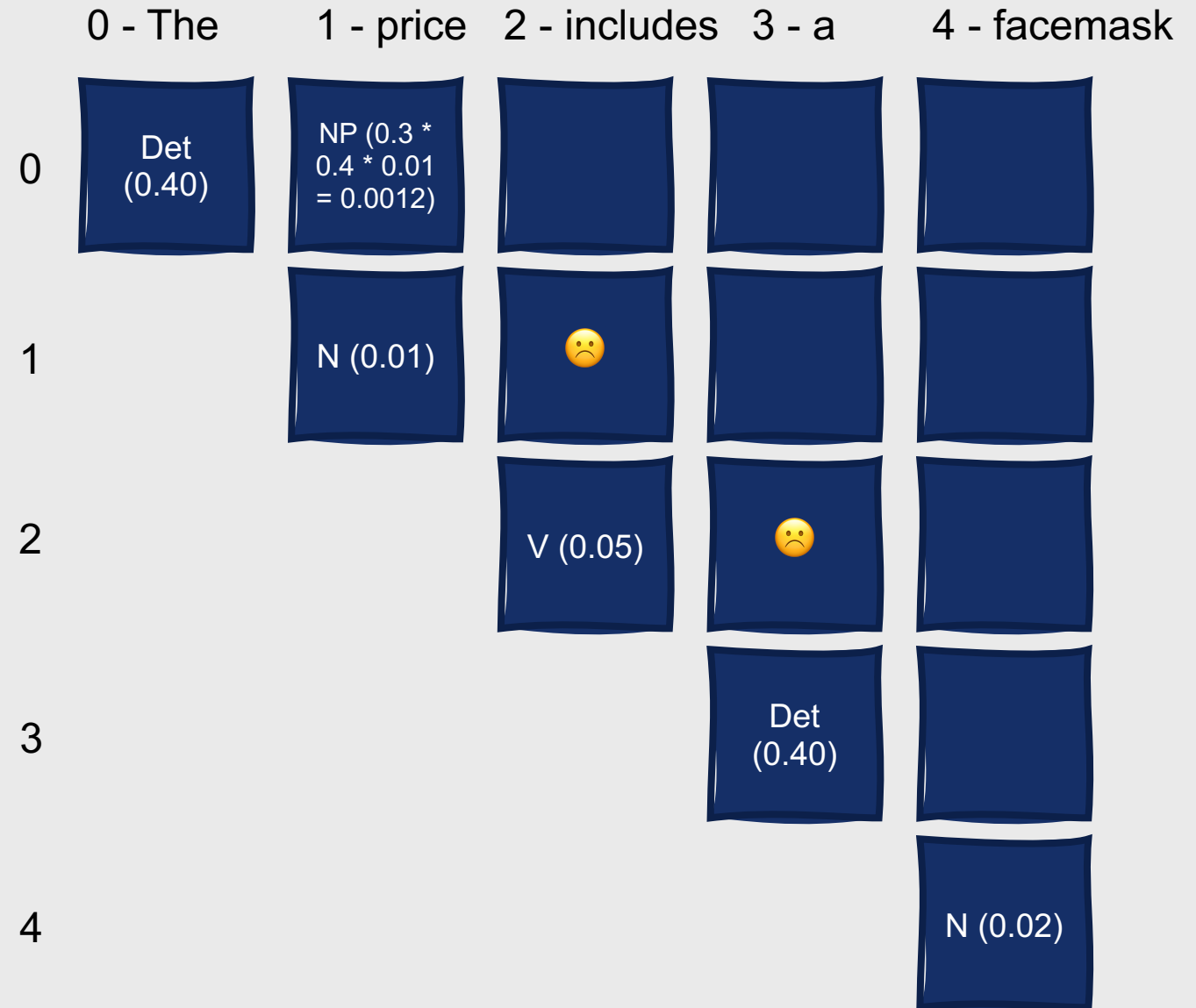
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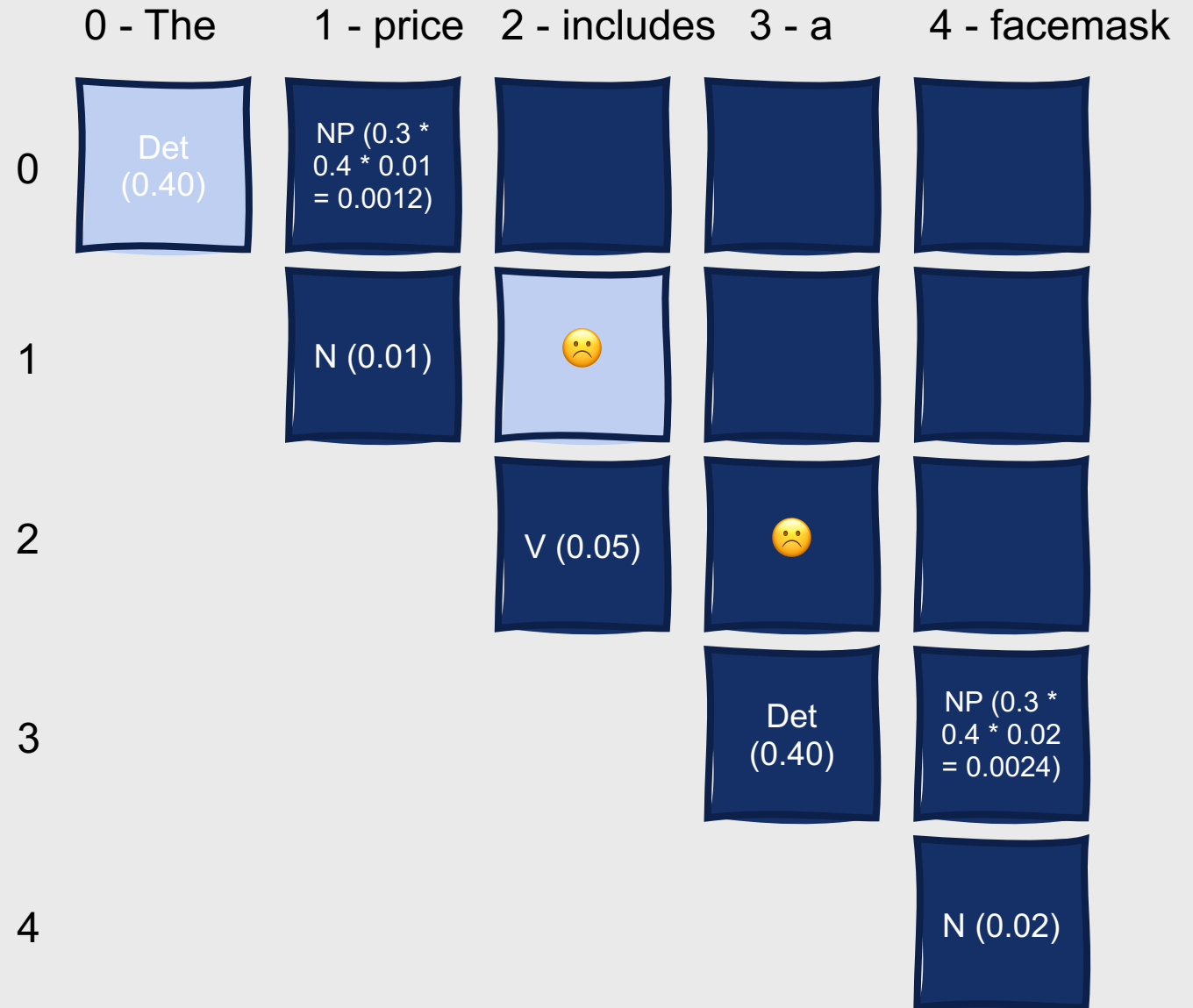
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2			V (0.05)	😞	
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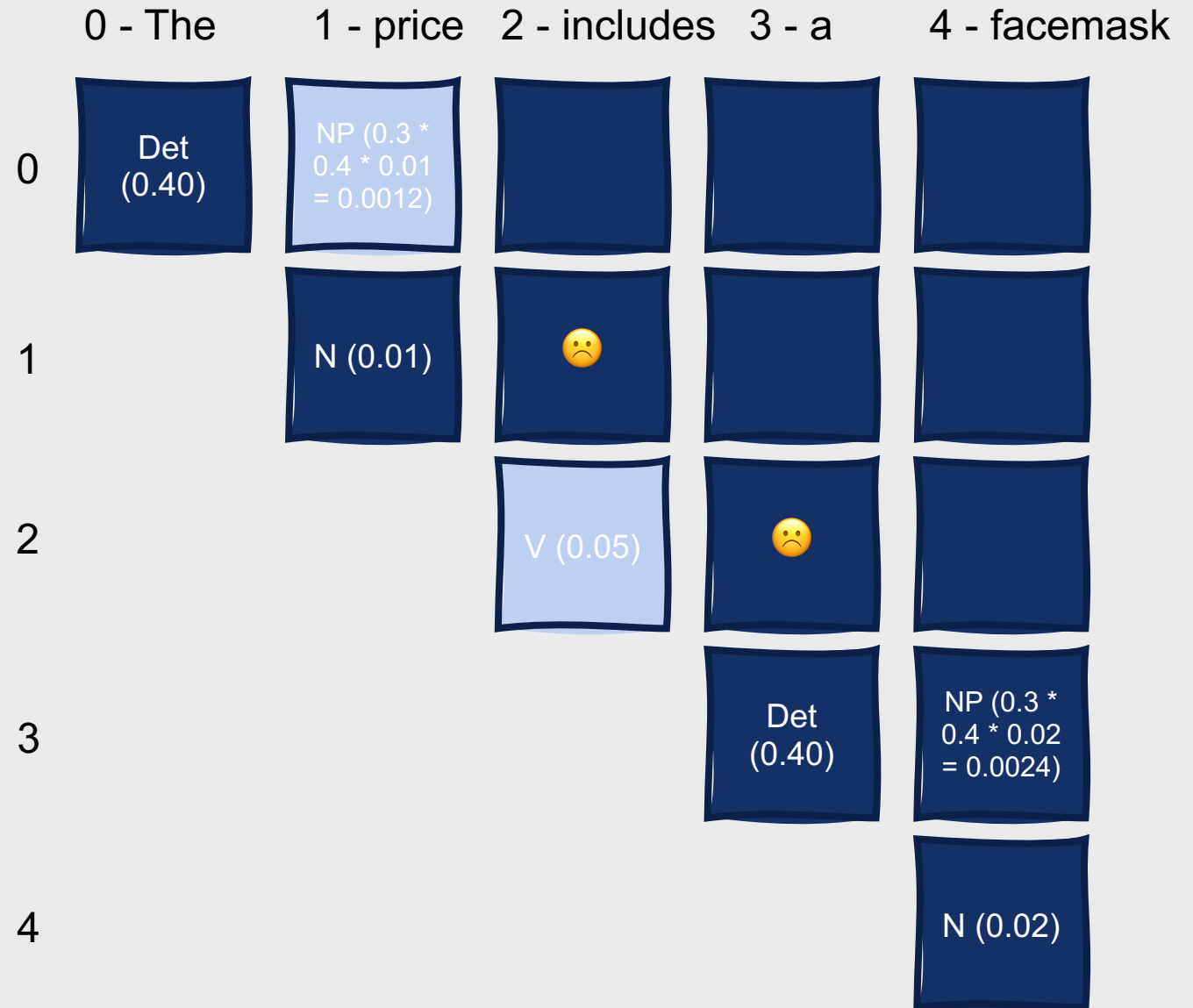
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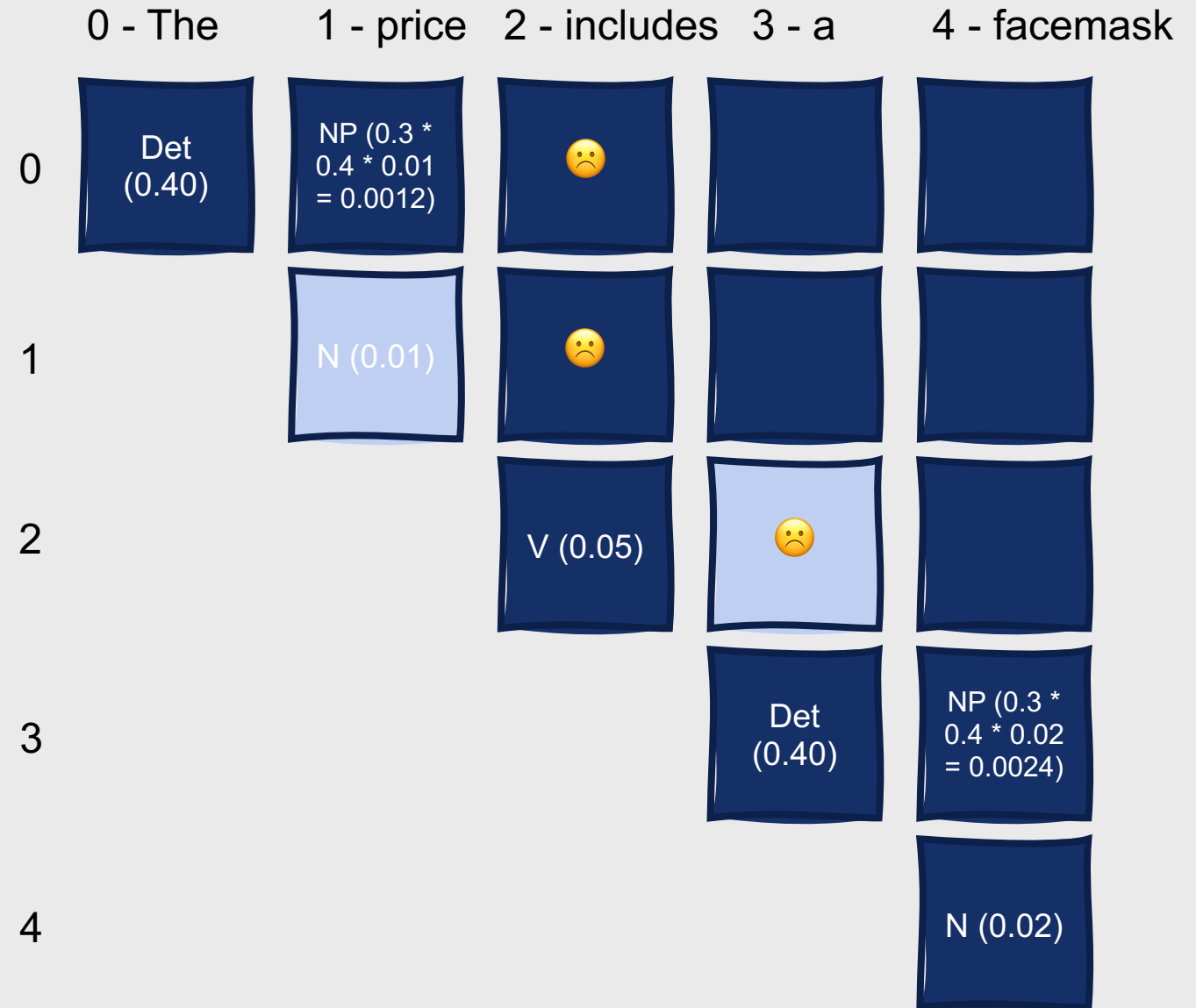
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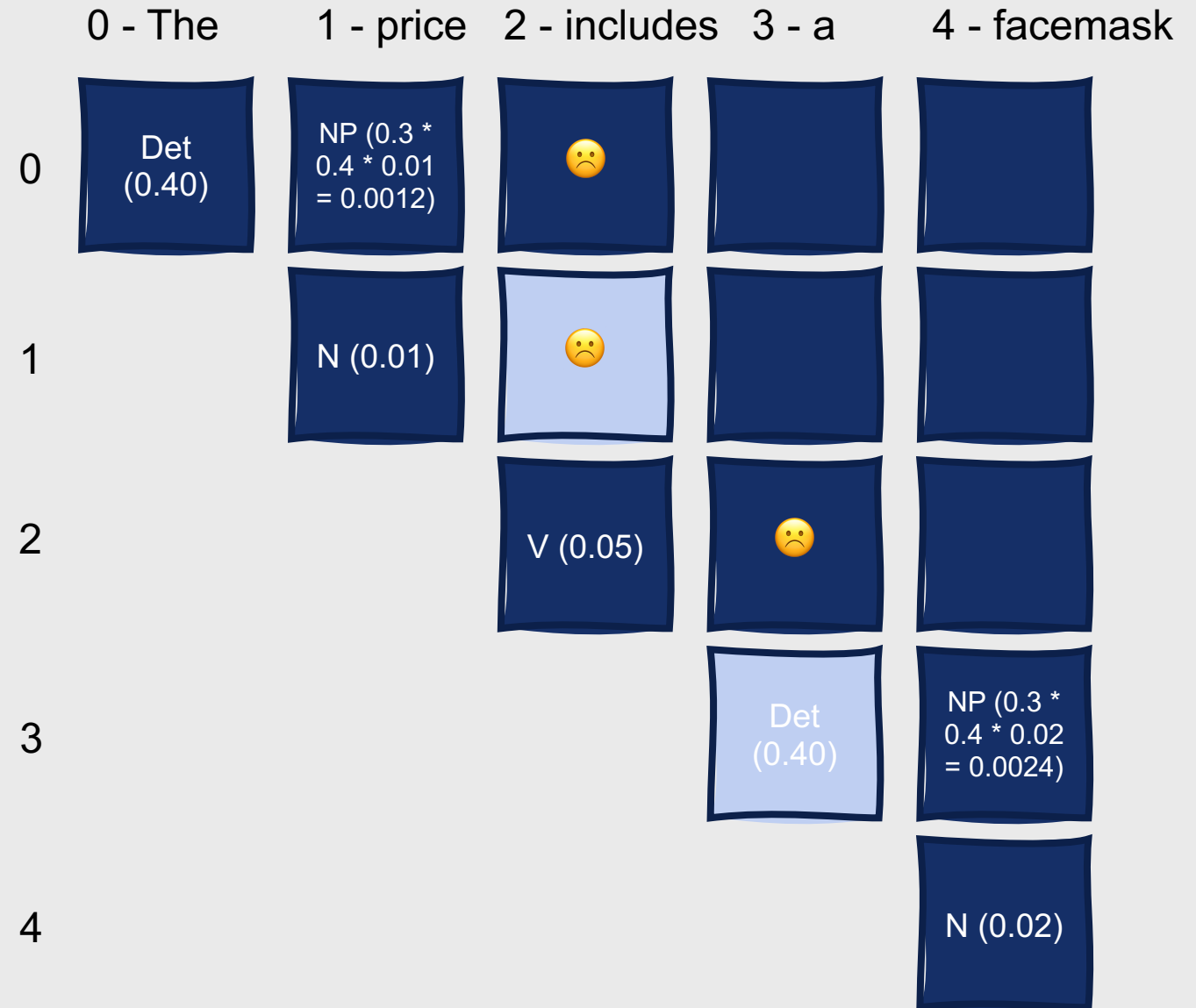
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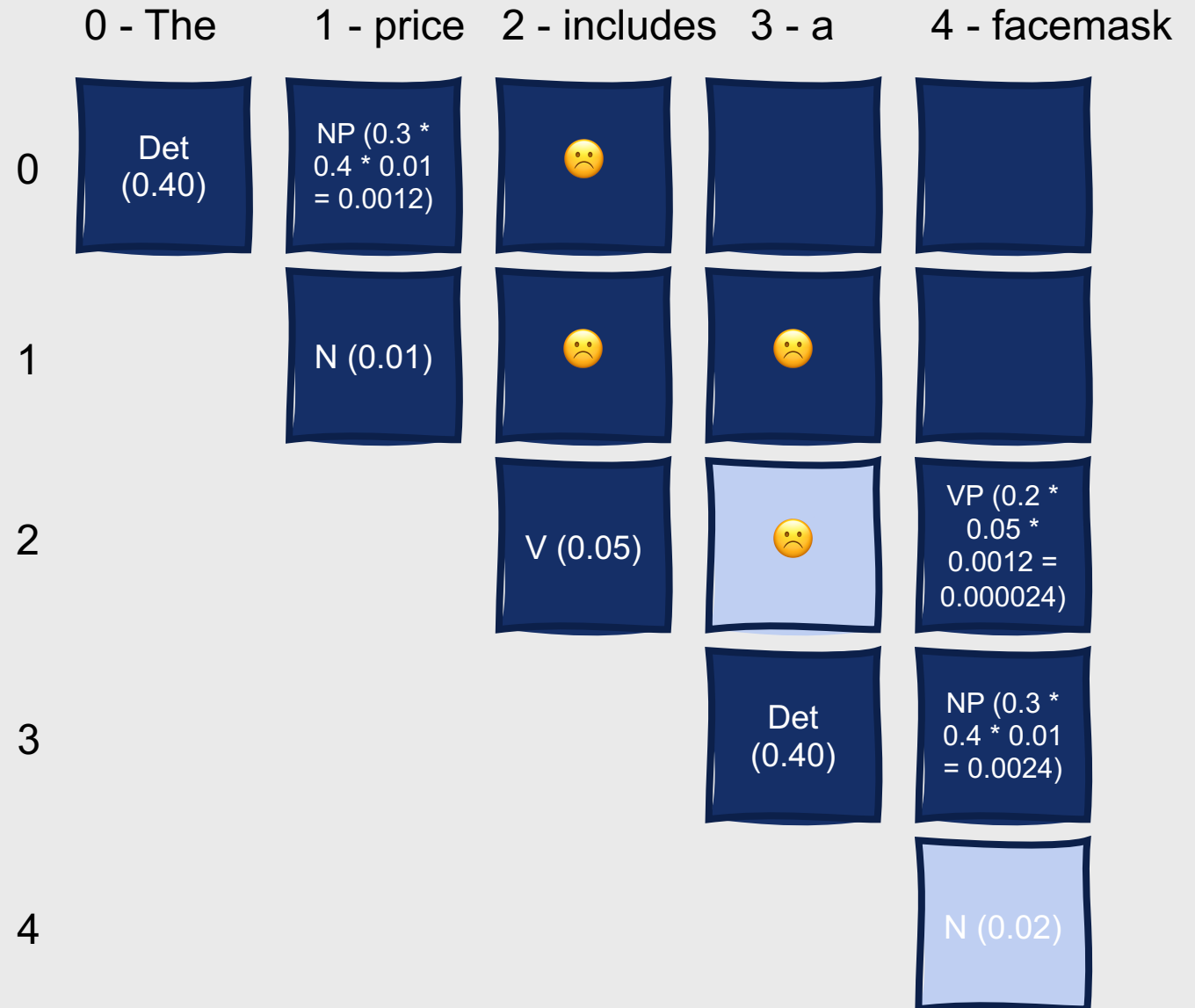
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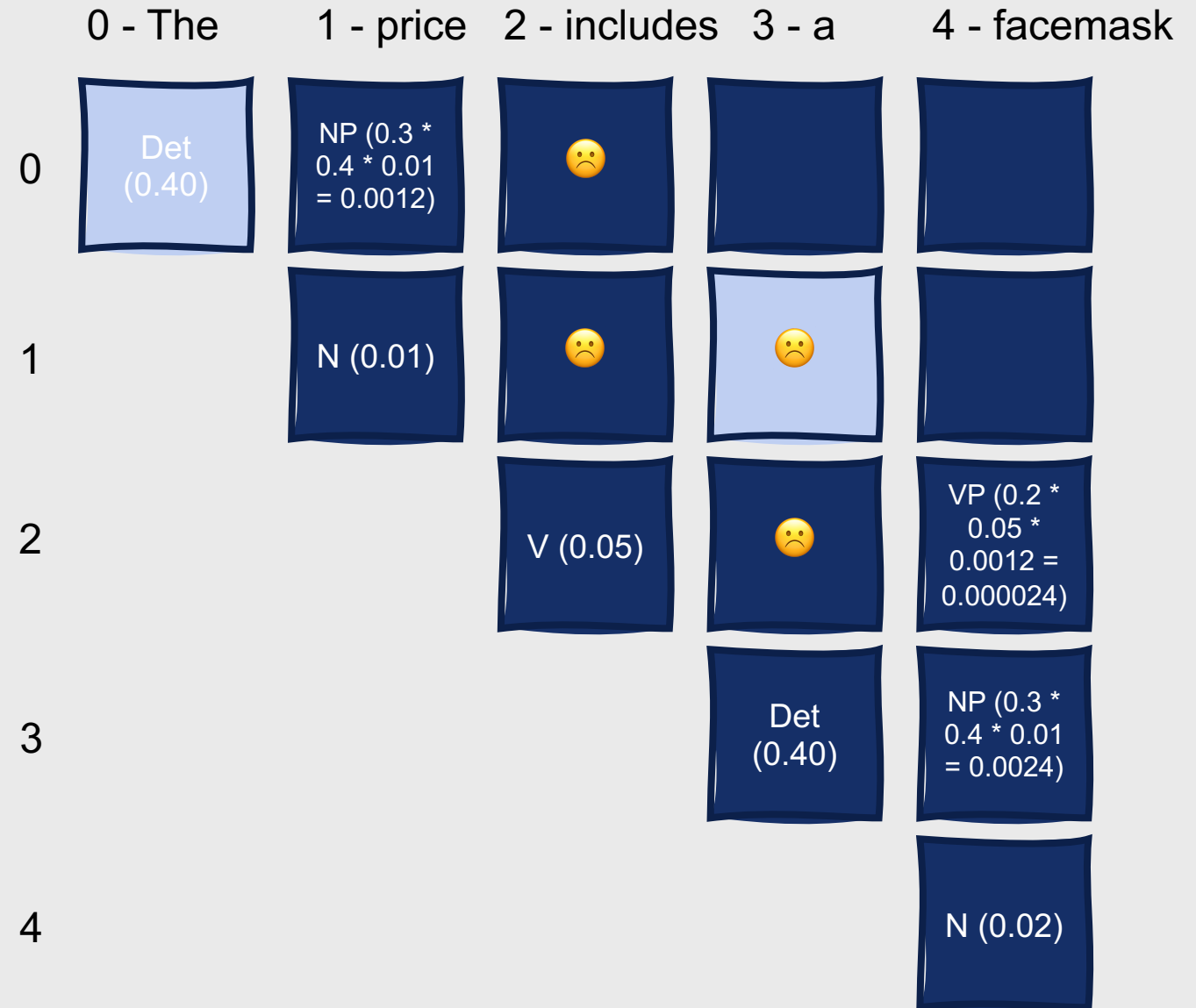
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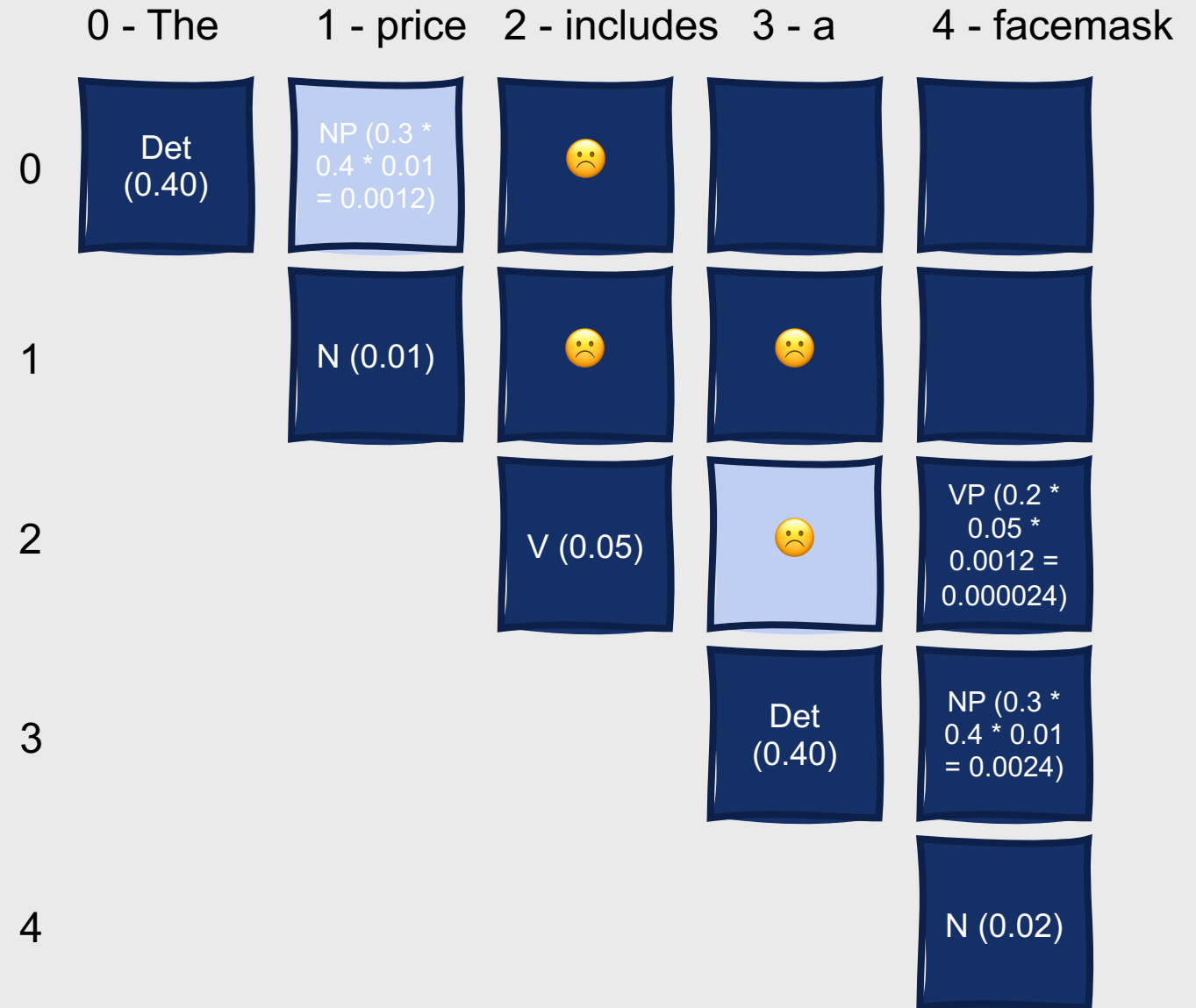
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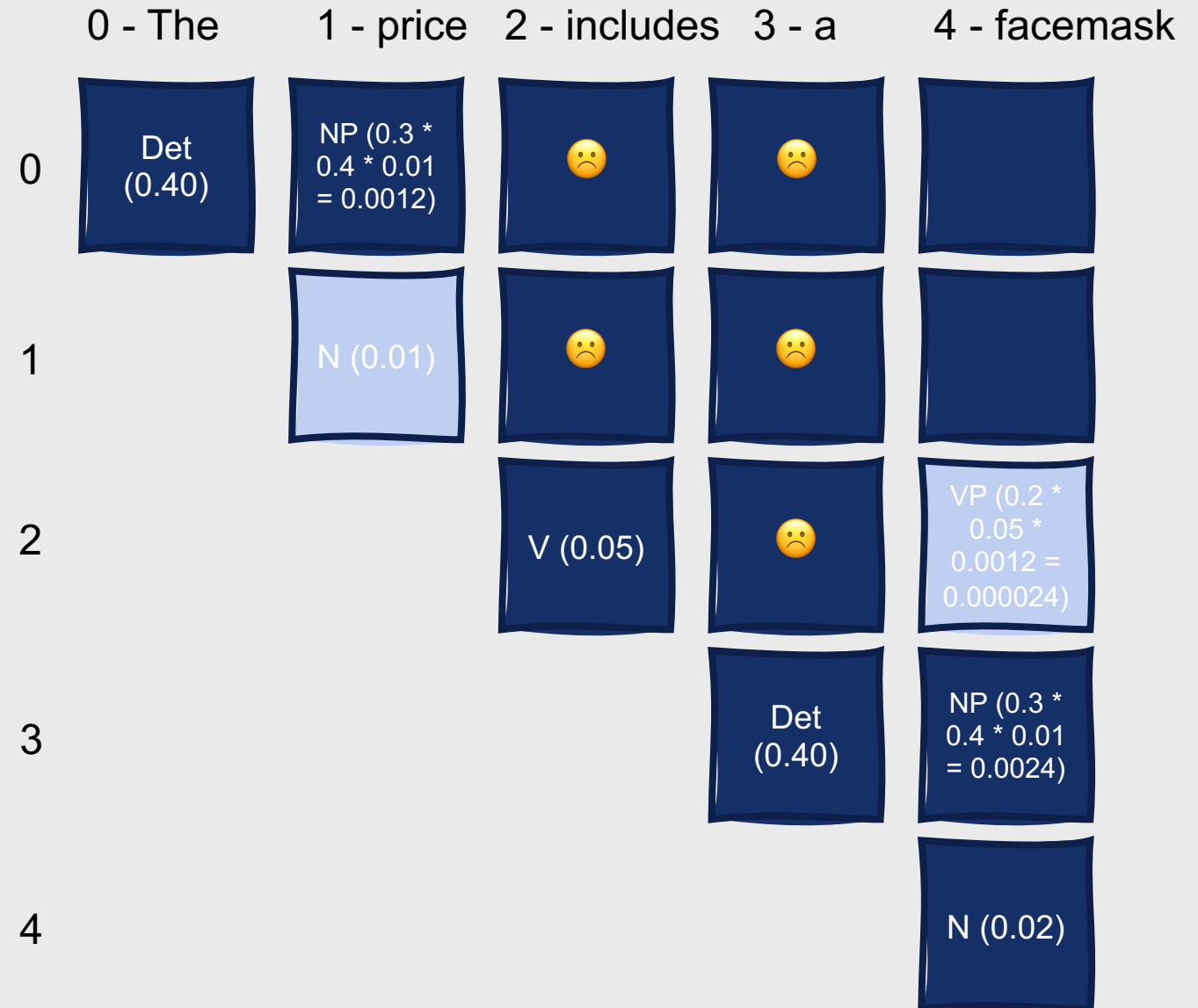
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$N \rightarrow \text{facemask}$	0.02

	0 - The	1 - price	2 - includes	3 - a	4 - facemask
0	Det (0.40)	NP ($0.3 * 0.4 * 0.01 = 0.0012$)	☹️	☹️	S ($0.8 * 0.0012 * 0.000024 = 2.304 * 10^{-8}$)
1		N (0.01)	☹️	☹️	☹️
2			V (0.05)	☹️	VP ($0.2 * 0.05 * 0.0012 = 0.000024$)
3				Det (0.40)	NP ($0.3 * 0.4 * 0.01 = 0.0024$)
4					N (0.02)



Where did these probabilities come from?

- Often, a corpus
 - $P(\alpha \rightarrow \beta | \alpha) = \frac{\text{Count}(\alpha \rightarrow \beta)}{\sum_{\gamma} \text{Count}(\alpha \rightarrow \gamma)} = \frac{\text{Count}(\alpha \rightarrow \beta)}{\text{Count}(\alpha)}$
- Or, if we don't have a labeled corpus, we can apply a generalization of the forward-backward algorithm called the **inside-out algorithm**
 - Start with equal probabilities for each rule
 - Parse the input
 - Compute a probability for each parse
 - Weight the counts based on these probabilities
 - Re-estimate the probabilities accordingly
 - Repeat until convergence