Full Name: ______________________________

EEL 3135 (Spring 2025) – Lab #07

Question #1: *(Difference Equations and Pole-Zeros Plots)*

Download `EEL3135_lab07_comment.m` from Canvas, replace each of the corresponding comments with the corresponding descriptions. This is designed to show you how to visualize the output impulse response, filter output, pole-zero response for a given transfer function (in the Z-domain).

Note: You should run the code to help you understand how it works and help you write your comments. You will use elements of this MATLAB code for the rest of the lab assignment.

Question #2: *(Z-Transform)*

For the following Z-transforms, plot the corresponding impulse response and the pole-zero plot. Use Lab Question #1 as a guide.

(a) $H(z) = (0.75)z^{-4}$

(b) $H(z) = 1 + z^{-1}$

(c) $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$

(d) $H(z) = \dfrac{1 + 2z^{-2}}{1 - 0.75z^{-1}}$

(e) $H(z) = \dfrac{1 + 2z^{-2}}{1 + 1.75z^{-1}}$

(f) $H(z) = \dfrac{1}{\left(1 - (0.85)e^{j1\pi/3}z^{-2}\right)\left(1 - (0.85)e^{-j1\pi/3}z^{-2}\right)}$

(g) $H(z) = \displaystyle\prod_{m=0}^{2} \left(1 - e^{-j2\pi m/3}\, z^{-1}\right)$

(h) $H(z) = \dfrac{1}{\prod_{m=0}^{2} \left(1 - e^{-j2\pi m/3}\, z^{-1}\right)}$

Question #3: *(More Z-Transform)*

Answer in your comments: Answer the following questions and list one result from Question #2 (i.e, (a) - (h)) that demonstrates this.

(a) For what pole-zero conditions is the impulse response unstable (i.e., goes to ∞ as $n \to \infty$)?

(b) For what pole-zero conditions is the impulse response stable (i.e., goes to zero as $n \to \infty$)?

(c) For what pole-zero conditions is the impulse response critically stable (i.e., steady amplitude as $n \to \infty$)?

(d) For what pole-zero conditions is the impulse response finite in length?

(e) For what pole-zero conditions is the impulse response infinite in length?

(f) For what pole-zero conditions is the impulse response periodic (with a frequency > 0)?

Question #4: *(Loan Difference Equations)*

In this problem, we will study a simplified difference equation model for your college loan payments. Assume the input $x[n]$ is the loan amount when positive. The output $y[n]$ is the value of the loan after n years. The time $n = 0$ corresponds to the time at which you take out your loan for the start of your college career.

For each of these subproblems, let's assume you take out a loan of \$150,000 (i.e., $x[n] = 150000\,\delta[n]$). Also, let the annual interest rate on your loan be 10%, or $\alpha = 0.1$.

(a) Assume you never pay any portion of the loan. Under this condition, the loan's value is

$$y[n] = (1+\alpha)y[n-1] + x[n]$$

Compute and plot y[n] for 40 years, $0 \le n \le 40$. Also, plot the pole-zero plot for the system. Use Lab Question #1 as a guide.

(b) **Answer in your comments:** For the preceding parameters, how many years will it be before you owe 1 million dollars?

(c) Now assume you begin to pay 10% ($\beta = 0.1$) of the **original** loan each year, starting in year $R_0 = 5$. In this condition, the difference equation becomes

$$y[n] = (1+\alpha)y[n-1] + x[n] - \beta \sum_{m=R_0}^{40} x[n-m]$$

Compute and plot y[n] for 40 years, $0 \le n \le 40$. Also, plot the pole-zero plot for the system.

(d) **Answer in your comments:** For the preceding parameters, does the loan amount go zero, level-out, or go toward infinity? If the first, when is the loan first less than 10% of the original loan? If the second, what is the level value? If the last, when will you owe 1 million dollars?

(e) **Answer in your comments:** What value does β need to be for the loan value to level-out? How much more of a percentage must you pay after delaying payments for 4 years instead of paying at the start?

(f) Now assume, instead of the fixed payments made in (c), you pay 10% ($\gamma = 0.1$) of the **current** value of the loan each year after $Q = 5$ years. In this condition, the difference equation becomes[1]

$$y[n] = (1+\alpha)y[n-1] + x[n] - \gamma y[n-1] + \sum_{m=1}^{Q-1} \gamma(1+\alpha)^{m-1} x[n-m]$$

Compute and plot y[n] for 40 years, $0 \le n \le 40$. Also, plot the pole-zero plot for the system.

(g) **Answer in your comments:** For the preceding parameters, does the loan amount go zero, level-out, or go toward infinity? If the first, when is the loan first less than 10% of the original loan? If the second, what is the level value? If the last, when will you owe 1 million dollars?

[1] FYI: This expression has both an additional IIR component and an additional FIR component. The FIR component is negating the effect of the IIR components until year Q.