

Contents

- [QUESTION 1 COMMENTING](#)
- [QUESTION 2: Z-TRANSFORM](#)
- [2 \(a\) FILTER 1](#)
- [2 \(b\) FILTER 2](#)
- [2 \(c\) ANSWER QUESTION](#)
- [2 \(d\) ANSWER QUESTION](#)
- [2 \(e\) ANSWER QUESTION](#)
- [QUESTION 3: IIR FILTERS IN Z-DOMAIN](#)
- [3 \(a\) FILTER 1](#)
- [3 \(b\) ANSWER QUESTIONS](#)
- [3 \(c\) FILTER 2](#)
- [3 \(d\) ANSWER QUESTIONS](#)
- [3 \(e\) FILTER 3](#)
- [3 \(f\) ANSWER QUESTIONS](#)
- [3 \(g\) NOTCH FILTER](#)
- [3 \(h\) NOTCH FILTER APPLIED TO AUDIO](#)
- [3 \(i\) ANSWER QUESTIONS](#)
- [ALL FUNCTIONS SUPPORTING THIS CODE](#) %% % =====

QUESTION 1 COMMENTING

DO NOT REMOVE THE LINE BELOW MAKE SURE 'eel3135_lab08_comment.m' IS IN THE SAME DIRECTORY AS THIS FILE

```
clear; close all; clc;
type('eel3135_lab08_comment.m')
```

```
%% QUESTION #1 COMMENTING
```

```
clear
close all
clc
```

```
%% DEFINE FILTER AND INPUT
```

```
w = -pi:pi/8000:pi-pi/8000; % Frequency range for plotting
N = 100; % Number of Samples
n = 0:(N-1); % Sample indices
```

```
% FILTER 1
```

```
a1 = [1 0 0 0 0 0 -0.9]; % Coefficients for the denominator (poles)
b1 = 1; % Coefficients for the numerator (zeros)
% <-- Answer: Is this an IIR filter or a FIR filter? Why?
% This is an Infinite Impulse Response (IIR) filter because it has poles
% that are in it's structure (not at 0; z = -1, 0.9)
```

```
% <-- Answer: How many poles does this system have? How many zeros?
% This system has 7 poles and 1 zero
```

```
% FILTER 2
```

```
a2 = [1 -0.9]; % Coefficients of the denominator (poles)
b2 = 1; % Coefficients of the numerator (zeros)
% <-- Answer: Is this an IIR filter or a FIR filter? Why?
% This is an IIR filter because it has poles in locations other than zero
% (z = -1, 0.9)
```

```

% <-- Answer: How many poles does this system have? How many zeros?
% The system has 1 pole and 1 zero

% INPUT
x = zeros(N,1); % Initializes input signal
x(1) = 1; % Sets first sample to 1 (impulse signal)

%% DEFINE AND PLOT OUTPUT

% OUTPUT 1
y1 = filter(b1,a1,x); % Applies filter 1 to input signal
y2 = filter(b2,a2,x); % Applies filter 2 to input signal

% OUTPUT 2
H1 = DTFT(y1,w); % Compute the DTFT of the output from filter 1
H2 = DTFT(y2,w); % Compute the DTFT of the output from filter 2

% OUTPUT 3: CASCADE FILTERS
y3 = filter(b2, a2, filter(b1, a1, x)); % Cascade filter 1 and filter 2
H3 = DTFT(y3,w); % Computes the DTFT of the cascaded output
% <-- Express H3(z) as a function of H1(z) and H2(z)
% H3(z) = H1(z) * H2(z), since casecading filters in the time domain
% corresponds to multiplication in the frequency domain, specifically
% H3(z) = z^8/[(z^7 - 0.9)*(z^7 + 0.8)]

% PLOT THE FREQUENCY REPNSE IMPULSE RESPONSE AND DTFT
figure(1)
subplot(3,1,1)
stem(n,y1)
xlabel('Time (Samples)')
ylabel('h[n]')
subplot(3,1,2)
plot(w,abs(H1))
title('Magnitude Response of h1')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(3,1,3)
pzplot(b1,a1)
axis equal

% PLOT THE FREQUENCY REPNSE IMPULSE RESPONSE AND DTFT
figure(2)
subplot(3,1,1)
stem(n,y2)
xlabel('Time (Samples)')
ylabel('h[n]')
subplot(3,1,2)
plot(w,abs(H2))
title('Magnitude Response of h2')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(3,1,3)
pzplot(b2,a2)
axis equal

% PLOT THE FREQUENCY REPNSE IMPULSE RESPONSE AND DTFT
figure(3)
subplot(2,1,1)
stem(n,y3)
xlabel('Time (Samples)')
ylabel('h[n]')
subplot(2,1,2)
plot(w,abs(H3))
title('Magnitude Response of h3')
xlabel('Normalized Frequency (Radians)')

```

```

ylabel('Magnitude')

%% ALL FUNCTIONS SUPPORTING THIS CODE %%
% =====
% NOTE: YOU DO NOT NEED TO ADD COMMENTS IN THE CODE BELOW. WE JUST
% NEEDED POLE-ZERO PLOTTING CODE AND THUS WROTE IT.
% =====
function pzplot(b,a)
% PZPLOT(B,A) plots the pole-zero plot for the filter described by
% vectors A and B. The filter is a "Direct Form II Transposed"
% implementation of the standard difference equation:
%
%   a(1)*y(n) = b(1)*x(n) + b(2)*x(n-1) + ... + b(nb+1)*x(n-nb)
%               - a(2)*y(n-1) - ... - a(na+1)*y(n-na)
%
%
% MODIFY THE POLYNOMIALS TO FIND THE ROOTS
b = b(1:find(b,1,'last'));
a = a(1:find(a,1,'last'));
b1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get the right roots
a1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get the right roots
b1(1:length(b)) = b;    % New a with all values
a1(1:length(a)) = a;    % New a with all values

% FIND THE ROOTS OF EACH POLYNOMIAL AND PLOT THE LOCATIONS OF THE ROOTS
h1 = plot(real(roots(a1)), imag(roots(a1)));
hold on;
h2 = plot(real(roots(b1)), imag(roots(b1)));
hold off;

% DRAW THE UNIT CIRCLE
circle(0,0,1)

% MAKE THE POLES AND ZEROS X's AND O's
set(h1, 'LineStyle', 'none', 'Marker', 'x', 'MarkerFaceColor','none', 'linewidth', 1.5, 'markersize', 8);
set(h2, 'LineStyle', 'none', 'Marker', 'o', 'MarkerFaceColor','none', 'linewidth', 1.5, 'markersize', 8);
axis equal;

% DRAW VERTICAL AND HORIZONTAL LINES
xminmax = xlim();
yminmax = ylim();
line([xminmax(1) xminmax(2)],[0 0], 'linestyle', ':', 'linewidth', 0.5, 'color', [1 1 1]*.1)
line([0 0],[yminmax(1) yminmax(2)], 'linestyle', ':', 'linewidth', 0.5, 'color', [1 1 1]*.1)

% ADD LABELS AND TITLE
xlabel('Real Part')
ylabel('Imaginary Part')
title('Pole-Zero Plot')

end

function circle(x,y,r)
% CIRCLE(X,Y,R) draws a circle with horizontal center X, vertical center
% Y, and radius R.
%
%
% ANGLES TO DRAW
ang=0:0.01:2*pi;

% DEFINE LOCATIONS OF CIRCLE
xp=r*cos(ang);
yp=r*sin(ang);

```

```

    % PLOT CIRCLE
    hold on;
    plot(x+xp,y+yp, ':', 'linewidth', 0.5, 'color', [1 1 1]*.1);
    hold off;

end

function H = DTFT(x,w)
% DTFT(X,W) compute the Discrete-time Fourier Transform of signal X
% across frequencies defined by W.

H = zeros(length(w),1);
for nn = 1:length(x)
    H = H + x(nn).*exp(-1j*w.*(nn-1));
end

end

```

QUESTION 2: Z-TRANSFORM

```

% DEFINE AXES/PARAMETERS
w = -pi:pi/500:pi-pi/500; % Frequencies
N = 100; % Number of samples
n = 0:(N-1); % Sample indices
x1 = zeros(N,1);
X1(1) = 1; % generates impulse response

```

2 (a) FILTER 1

```

H_a = zeros(1000,3);
y1 = zeros(100,3);
a = 0.9;
omega_a = [0 pi/2 pi];
x1 = zeros(N,1);
x1(1) = 1;
b1 = 1;

for i = 1:length(omega_a)
    y1(:,i) = filter(b1, [1 -2*a*cos(omega_a(i)) a^2], x1);
    H_a(:,i) = DTFT(filter(b1, [1 -2*a*cos(omega_a(i)) a^2], x1), w);
end

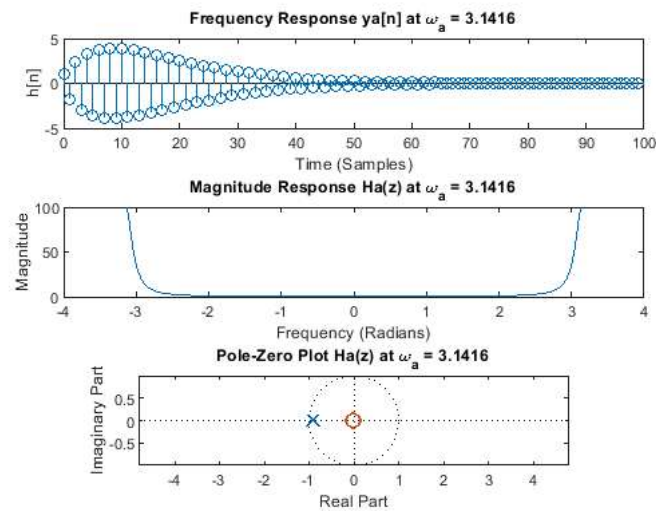
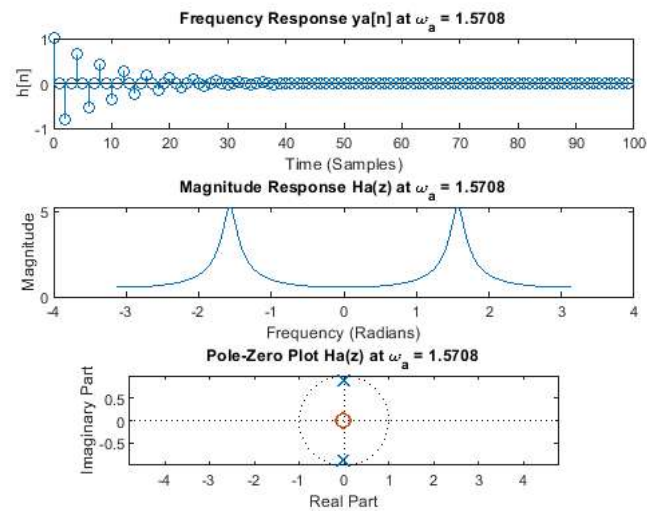
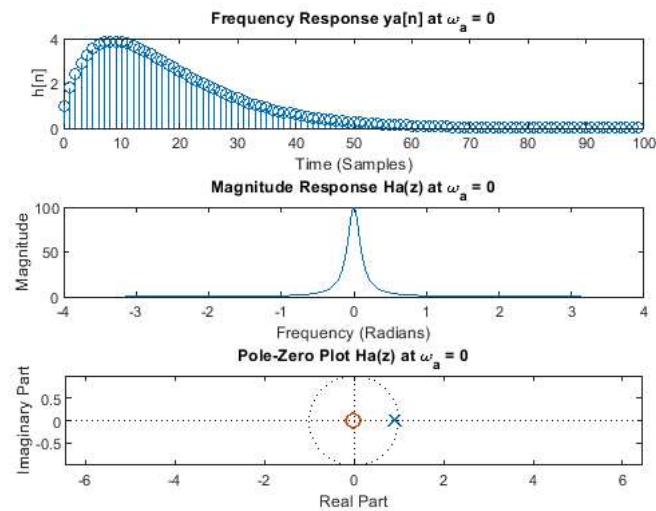
for i = 1:length(omega_a)
    % Frequency response
    figure;
    subplot(3, 1, 1);
    stem(n, y1(:, i));
    title(['Frequency Response ya[n] at \omega_a = ', num2str(omega_a(i))]);
    xlabel('Time (Samples)');
    ylabel('h[n]');

    % Magnitude response
    subplot(3, 1, 2);
    plot(w, abs(H_a(:, i)));
    title(['Magnitude Response Ha(z) at \omega_a = ', num2str(omega_a(i))]);
    xlabel('Frequency (Radians)');
    ylabel('Magnitude');

    % Pole-zero plot
    subplot(3, 1, 3);
    pzplot(b1, [1 -2*a*cos(omega_a(i)) a^2]); % Pole-zero plot
    title(['Pole-Zero Plot Ha(z) at \omega_a = ', num2str(omega_a(i))]);
end

```

```
axis equal;
end
```



2 (b) FILTER 2

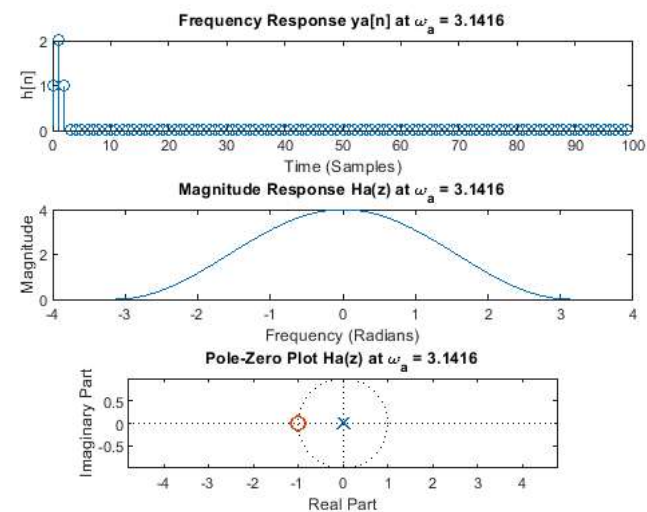
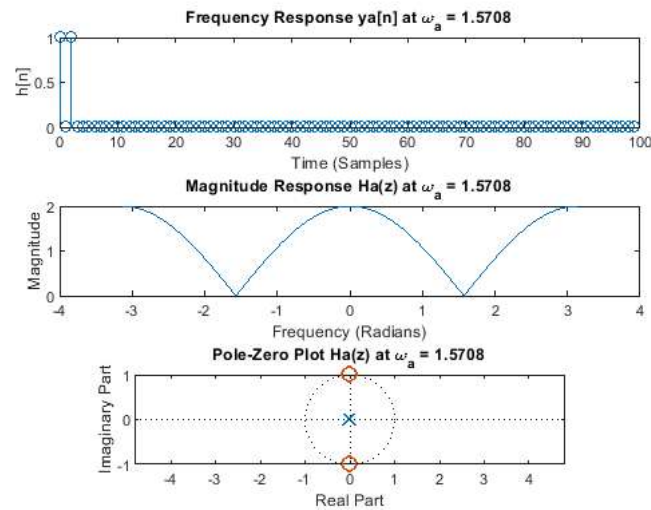
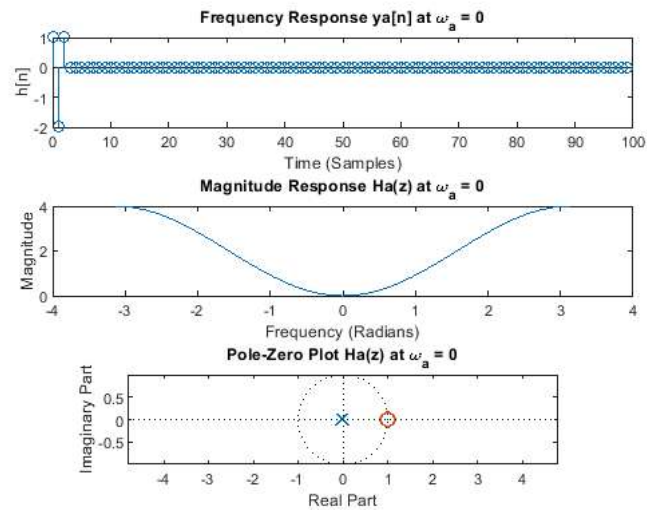
```
H_b = zeros(1000,3);
y1 = zeros(100,3);
b = 1;
omega_a = [0 pi/2 pi];
x1 = zeros(N,1);
x1(1) = 1;
a1 = 1;
```

```
for i = 1:length(omega_a)
    y2(:,i) = filter([1 -2*b*cos(omega_a(i)) b^2],a1, x1);
    H_b(:,i) = DTFT(filter([1 -2*b*cos(omega_a(i)) b^2],a1, x1), w);
end

for i = 1:length(omega_a)
    % Frequency response
    figure;
    subplot(3, 1, 1);
    stem(n, y2(:, i));
    title(['Frequency Response ya[n] at \omega_a = ', num2str(omega_a(i))]);
    xlabel('Time (Samples)');
    ylabel('h[n]');

    % Magnitude response
    subplot(3, 1, 2);
    plot(w, abs(H_b(:, i)));
    title(['Magnitude Response Ha(z) at \omega_a = ', num2str(omega_a(i))]);
    xlabel('Frequency (Radians)');
    ylabel('Magnitude');

    % Pole-zero plot
    subplot(3, 1, 3);
    pzplot([1 -2*b*cos(omega_a(i)) b^2], a1); % Pole-zero plot
    title(['Pole-Zero Plot Ha(z) at \omega_a = ', num2str(omega_a(i))]);
    axis equal;
end
```



2 (c) ANSWER QUESTION

$Ha(z)$ is an IIR filter because it has poles located in places other than the origin. $Hb(z)$ is an FIR filter because its poles are located only at the origin.

2 (d) ANSWER QUESTION

```
% Changing w0 moves the location of the poles and zeros on the plot. For
% 2(a), changing w0 changed the location of the poles. For 2(b) changing w0
% changed the location of the zeros.
```

2 (e) ANSWER QUESTION

```
% When  $\omega_0 = \pi$ ,  $H_a(z)$  is a highkey high pass filter and  $H_b(z)$  is a lowpass filter
% When  $\omega_0 = \pi/2$ ,  $H_a(z)$  is high pass filter and  $H_b(z)$  is a notch filter
% When  $\omega_0 = 0$ ,  $H_a(z)$  is a lowkey lowpass filter and  $H_b(z)$  is a highpass
% filter

% The filtering response for  $H_a$  and  $H_b$  are opposite because the two filters
% are reciprocal.
```

QUESTION 3: IIR FILTERS IN Z-DOMAIN

```
% DEFINE AXES
w = -pi:pi/500:pi-pi/500;

% DO NOT REMOVE THE THREE LINES BELOW
N = 1000; n = 0:N-1;
x = [cos( (5*pi/8)*n ) zeros(1,N) sin( (2*pi/8)*n ) zeros(1,N) 3*cos( (6*pi/8)*n + 3*pi/4 ) zeros(1,N) ...
     cos( (3*pi/8)*n ) zeros(1,N) 2*cos( (1*pi/8)*n - 2*pi/4 ) zeros(1,N) 0.5*cos( (4*pi/8)*n - pi/8)];
```

3 (a) FILTER 1

```
wa = (5*pi)/8;
wb = (6*pi)/8;
a = 0.999;
b = 1/0.999;

yt = filter([1 -2*b*cos(wa) b^2], [1 -2*a*cos(wa) a^2], x);
yt = filter([1 -2*b*cos(wb) b^2], [1 -2*a*cos(wb) a^2], yt);

Ht = DTFT(yt, w);

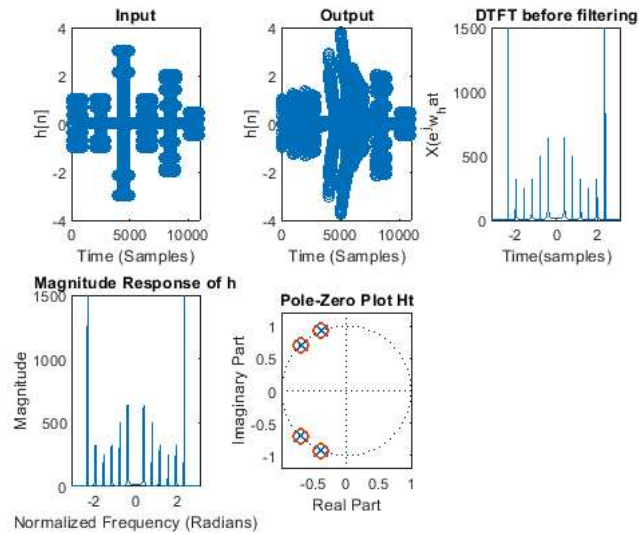
% Frequency response
figure;
subplot(2,3, 1);
stem(x);
title('Input');
xlabel('Time (Samples)');
ylabel('h[n]');

subplot(2, 3, 2);
stem(yt);
title('Output');
xlabel('Time (Samples)');
ylabel('h[n]');

% Magnitude response
subplot(2, 3, 3);
plot(w, abs(DTFT(x,w)));
title('DTFT before filtering');
xlabel('Time(samples)');
ylabel('X(e^jw_hat)');

subplot(2, 3, 4);
plot(w,abs(Ht));
title('Magnitude Response of h');
xlabel('Normalized Frequency (Radians)');
ylabel('Magnitude');

% Pole-zero plot
subplot(2,3, 5);
pzplot(conv([1 -2*b*cos(wa) b^2],[1 -2*a*cos(wb) b^2]), conv([1 -2*a*cos(wa) a^2],[1 -2*a*cos(wb) a^2])); % Pole-zero plot
title('Pole-Zero Plot Ht');
axis equal;
```

3 (b) ANSWER QUESTIONS

% The filter is an all-pass filter because the poles and zeros cancel out.
 % The filter affects the phase or group delay of the input

3 (c) FILTER 2

```
a = 0.8;
b = 1;

yt = filter(1, [1 -2*a*cos(pi/2) a^2], x);
yt = filter([1 -2*b*cos(0) b^2], 1, yt);
yt = filter([1 -2*b*cos(pi/4) b^2], 1, yt);
yt = filter([1 -2*b*cos((3*pi)/8) b^2], 1, yt);
yt = filter([1 -2*b*cos((5*pi)/8) b^2], 1, yt);
yt = filter([1 -2*b*cos((6*pi)/8) b^2], 1, yt);
yt = filter([1 -2*b*cos(pi) b^2], 1, yt);

Ht = DTFT(yt, w);

% Frequency response
figure;
subplot(2,3, 1);
stem(x);
title('Input');
xlabel('Time (Samples)');
ylabel('h[n]');

subplot(2, 3, 2);
stem(yt);
title('Output');
xlabel('Time (Samples)');
ylabel('h[n]');

% Magnitude response
subplot(2, 3, 3);
plot(w, abs(DTFT(x,w)));
title('DTFT before filtering');
```

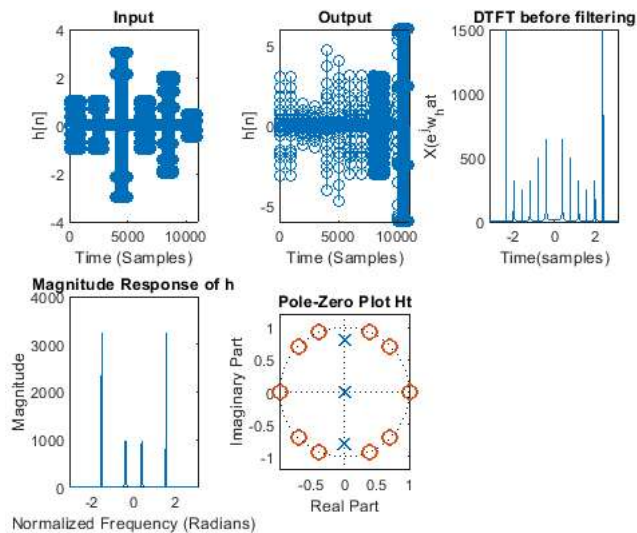
```

xlabel('Time(samples)');
ylabel('X(e^jw_hat)');

subplot(2, 3, 4);
plot(w,abs(Ht));
title('Magnitude Response of h');
xlabel('Normalized Frequency (Radians)');
ylabel('Magnitude');

% Pole-zero plot
subplot(2,3, 5);
pzplot(conv(conv(conv(conv([1 -2*b*cos(0) b^2], [1 -2*b*cos(pi/4) b^2]), [1 -2*b*cos((3*pi)/8) b^2]), [1 -2*b*cos((5*pi)/8) b^2]), [1 -2*b*cos((6*pi)/8) b^2]), [1 -2*b*cos(pi) b^2]), [1 -2*a*cos(pi/2) a^2]); % Pole
title('Pole-Zero Plot Ht');
axis equal;

```



3 (d) ANSWER QUESTIONS

```

% The filter is a bandpass filter. Ha(z) provides the nonzero poles of the
% filter, if removed if removed the magnitude would be higher since poles
% pull up the magnitude response.

```

3 (e) FILTER 3

```

b = 1;
a = 0.66;

yt = filter([1 -2*b*cos(pi) b^2], 1, x);
yt = filter(1, [1 -2*a*cos(pi/8) a^2], yt);
yt = filter(1, [1 -2*a*cos(pi/8) a^2], yt);

Ht = DTFT(yt, w);

% Frequency response
figure;
subplot(2,3, 1);
stem(x);
title('Input');

```

```

xlabel('Time (Samples)');
ylabel('h[n]');

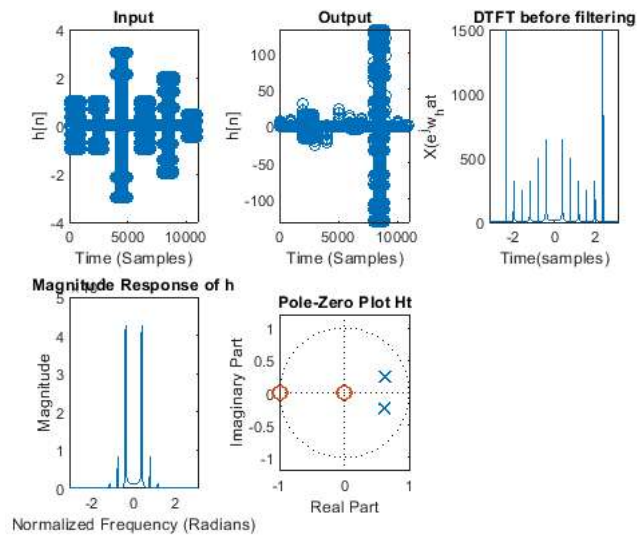
subplot(2, 3, 2);
stem(yt);
title('Output');
xlabel('Time (Samples)');
ylabel('h[n]');

% Magnitude response
subplot(2, 3, 3);
plot(w, abs(DTFT(x,w)));
title('DTFT before filtering');
xlabel('Time(samples)');
ylabel('X(e^jw_hat)');

subplot(2, 3, 4);
plot(w,abs(Ht));
title('Magnitude Response of h');
xlabel('Normalized Frequency (Radians)');
ylabel('Magnitude');

% Pole-zero plot
subplot(2,3, 5);
pzplot([1 -2*b*cos(pi) b^2], conv([1 -2*a*cos(pi/8) a^2],[1 -2*a*cos(pi/8) a^2])); % Pole-zero plot
title('Pole-Zero Plot Ht');
axis equal;

```



3 (f) ANSWER QUESTIONS

```

% This filter is lowpass since there is a zero at pi, so the higher
% frequencies are attenuated. The zero at 0 also means there is a positive
% time shift

```

3 (g) NOTCH FILTER

```
b = 1;
a = 0.95;

yt = filter([1 -2*b*cos(pi/4) b^2], [1 -2*a*cos(pi/4) a^2], x);

Ht = DTFT(yt, w);

% Frequency response
figure;
subplot(2,3, 1);
stem(x);
title('Input');
xlabel('Time (Samples)');
ylabel('h[n]');

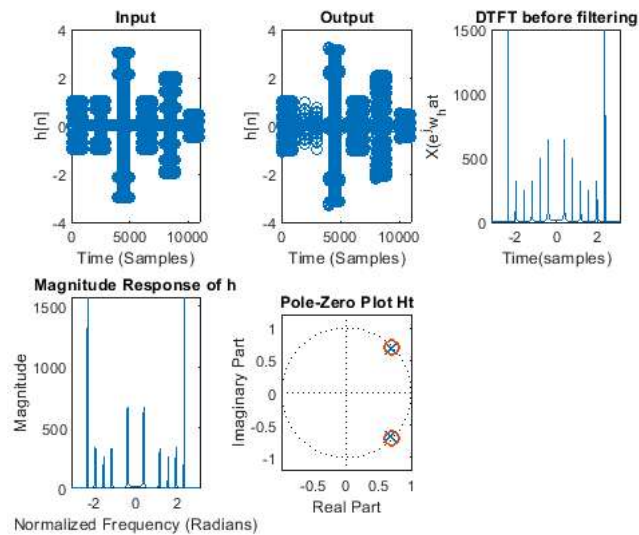
subplot(2, 3, 2);
stem(yt);
title('Output');
xlabel('Time (Samples)');
ylabel('h[n]');

% Magnitude response
subplot(2, 3, 3);
plot(w, abs(DTFT(x,w)));
title('DTFT before filtering');
xlabel('Time(samples)');
ylabel('X(e^jw_hat)');

subplot(2, 3, 4);
plot(w,abs(Ht));
title('Magnitude Response of h');
xlabel('Normalized Frequency (Radians)');
ylabel('Magnitude');

% Pole-zero plot
subplot(2,3, 5);
pzplot([1 -2*b*cos(pi/4) b^2], [1 -2*a*cos(pi/4) a^2]); % Pole-zero plot
title('Pole-Zero Plot Ht');
axis equal;

sound(yt, 2000);
```



3 (h) NOTCH FILTER APPLIED TO AUDIO

```
% DO NOT REMOVE THE TWO LINES BELOW
% MAKE SURE 'noisy2.wav' IS IN THE SAME DIRECTORY AS THIS FILE
[x2,fs] = audioread('noisy2.wav');

w = -pi:pi/5000:pi;
Hn = DTFT(x2, w);

a = 0.95;
b = 1;
y = 1;
Ht = 1;

[~, max_n] = maxk(abs(Hn), 5);
max2 = unique(w(max_n), 'stable');

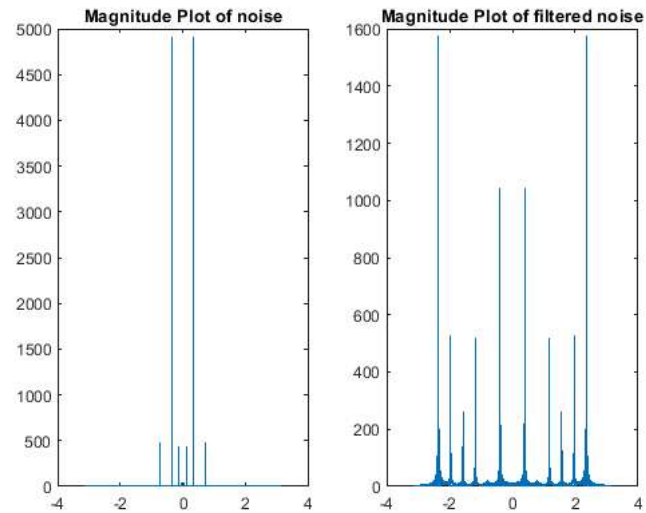
y = x2;

for i = 1:length(max2)
    y = filter(1, [1 -2*a*cos(max2(i)) a^2], filter([1 -2*b*cos(max2(i)) b^2], 1, y));
end

Ht = DTFT(yt, w);

figure;
subplot(1, 2, 1);
plot(w, abs(Hn));
title('Magnitude Plot of noise');
subplot(1, 2, 2);
plot(w, abs(Ht));
title('Magnitude Plot of filtered noise');

audiowrite('filtered_noisy2.wav', y/max(abs(y)), fs);
```



3 (i) ANSWER QUESTIONS

```
% The difference between the results from lab 6 and the new results are
% according to the chart there are much more of a magnitude response with
% the filtered noise, since we canceled out the noisy frequency it was
% easier to hear the sound. When comparing what was heard, the original
% noisy2 sounded like a singular noise slowly increasing to be louder and then
% decreasing to be lower. However, while the filtered_noise2 follows this
% pattern, there is much more of a dynamic noise that it makes as it is
% rising and falling.
```

ALL FUNCTIONS SUPPORTING THIS CODE % % % =====

NOTE: YOU DO NOT NEED TO ADD COMMENTS IN THE CODE BELOW. WE JUST NEEDED POLE-ZERO PLOTTING CODE AND THUS WROTE IT. =====

```
function pzplot(b,a)
% PZPLOT(B,A) plots the pole-zero plot for the filter described by
% vectors A and B. The filter is a "Direct Form II Transposed"
% implementation of the standard difference equation:
%
% a(1)*y(n) = b(1)*x(n) + b(2)*x(n-1) + ... + b(nb+1)*x(n-nb)
%             - a(2)*y(n-1) - ... - a(na+1)*y(n-na)
%
%
% MODIFY THE POLYNOMIALS TO FIND THE ROOTS
b1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get the right roots
a1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get the right roots
b1(1:length(b)) = b; % New a with all values
a1(1:length(a)) = a; % New a with all values

% FIND THE ROOTS OF EACH POLYNOMIAL AND PLOT THE LOCATIONS OF THE ROOTS
h1 = plot(real(roots(a1)), imag(roots(a1)));
hold on;
h2 = plot(real(roots(b1)), imag(roots(b1)));
hold off;

% DRAW THE UNIT CIRCLE
circle(0,0,1)
```

```

% MAKE THE POLES AND ZEROS X's AND O's
set(h1, 'LineStyle', 'none', 'Marker', 'x', 'MarkerFaceColor', 'none', 'linewidth', 1.5, 'markersize', 8);
set(h2, 'LineStyle', 'none', 'Marker', 'o', 'MarkerFaceColor', 'none', 'linewidth', 1.5, 'markersize', 8);
axis equal;

% DRAW VERTICAL AND HORIZONTAL LINES
xminmax = xlim();
yminmax = ylim();
line([xminmax(1) xminmax(2)], [0 0], 'linestyle', ':', 'linewidth', 0.5, 'color', [1 1 1]*.1)
line([0 0], [yminmax(1) yminmax(2)], 'linestyle', ':', 'linewidth', 0.5, 'color', [1 1 1]*.1)

% ADD LABELS AND TITLE
xlabel('Real Part')
ylabel('Imaginary Part')
title('Pole-Zero Plot')

end

function circle(x,y,r)
% CIRCLE(X,Y,R) draws a circle with horizontal center X, vertical center
% Y, and radius R.
%
% ANGLES TO DRAW
ang=0:0.01:2*pi;

% DEFINE LOCATIONS OF CIRCLE
xp=r*cos(ang);
yp=r*sin(ang);

% PLOT CIRCLE
hold on;
plot(x+xp,y+yp, ':', 'linewidth', 0.5, 'color', [1 1 1]*.1);
hold off;

end

function H = DTFT(x,w)
% DTFT(X,W) compute the Discrete-time Fourier Transform of signal X
% across frequencies defined by W.

H = zeros(length(w),1);
for nn = 1:length(x)
    H = H + x(nn).*exp(-1j*w.*(nn-1));
end

end

```