

Comp790-166: Computational Biology

Lecture 7

January 31, 2023

Today

- Finish up graph-based auto-gating
- Making friends with diffusion
- Imputation for single-cell data + dropout
- Lineage-preserving visualizations of single-cell daya

Do You Remember Questions

- What are the two types of features we can define on either cell-types from manual gating or clustering?
- What are some of the reasons you might do a fully automated analysis rather than a manual one?

All of the Graph Knowledge Finally Applied....

Phenograph starts with a graph representation, sparsifies, and then clusters.

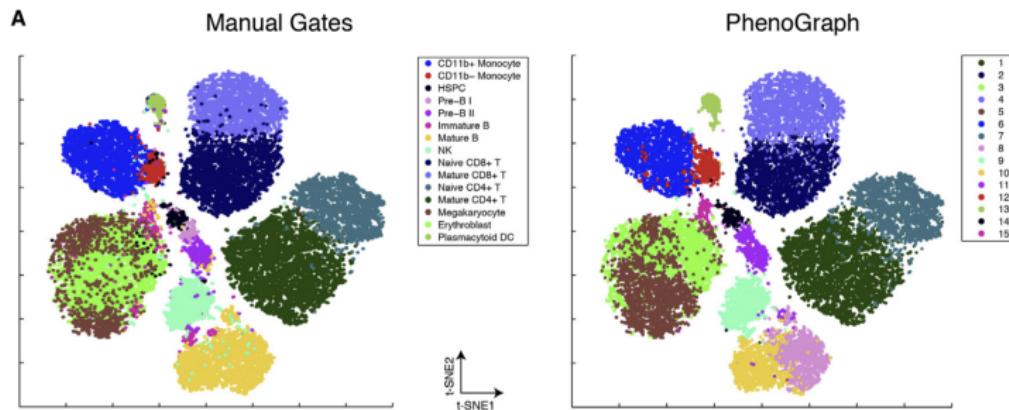


Figure: from Levine *et al.* Cell. 2015

A Direct Application of Louvain

- PhenoGraph is sold as being able to capture cell-populations of various sizes.
- A graph of cells is constructed for each samples
- Edges that exist are then turned into weighted edges, based on shared neighbors (a good contribution)
- The graph for each sample is partitioned with Louvain
- Clusters between samples are mapped with a metaclustering approach.

Communities Map to Cell-Populations

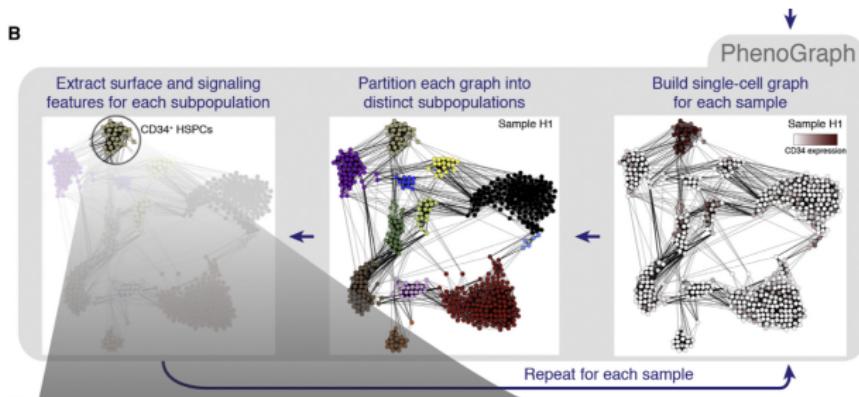


Figure: from Levine et al. Cell. 2015

Refining the k NN graph

Suppose an edge exists between nodes i and j . Then their edgeweight, w_{ij} is defined as the Jaccard Score of shared neighbors as,

$$w_{ij} = \frac{|v(i) \cap v(j)|}{|v(i) \cup v(j)|} \quad (1)$$

Here, $v(i)$ is the set of neighbors of node i . $|\cdot|$ is cardinality (number of neighbors)

Evaluating Similarity to Manual Gates

It's a bit reassuring to know that the performance (in terms of F-measure) is stable, regardless of the choice of k .¹

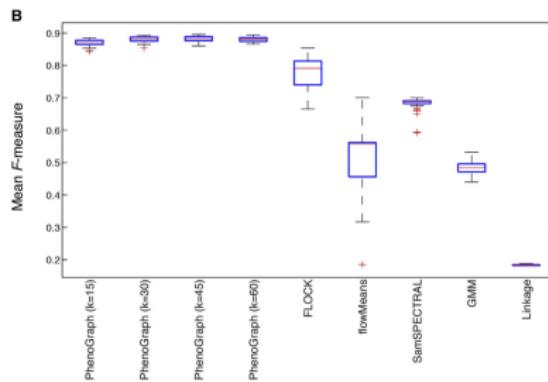


Figure: from Levine *et al.* Cell. 2015

¹discussion of F measure <https://www.nature.com/articles/nmeth.2365.pdf>

Healthy vs AML with Cell Frequencies

Calculate the proportion of each sample's cells assigned to a particular metacluster. As you can see, there are differences in frequency between healthy (first few rows) and AML.

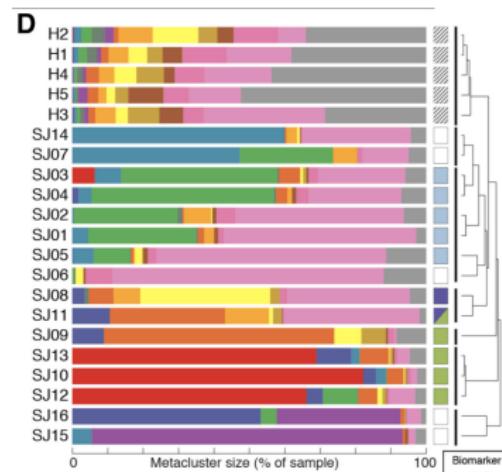


Figure: from Levine *et al.* Cell. 2015

You are ready for your first CyTOF dataset

If the data have signal, cell frequencies are a pretty powerful biologically interpretable feature that can allow us to classify patient samples.

To recap,

- Define per sample clusters
- Define metaclusters representing all samples
- Map cells from individual cells to metaclusters
- Compute frequency features
- You have now build a feature matrix that can be used for classification!

Example of a Powerful Frequency Feature

Frequency differences can be quite prominent in clinical settings. In this example, we are studying differences between patients who received steroid treatment after surgery, versus not.

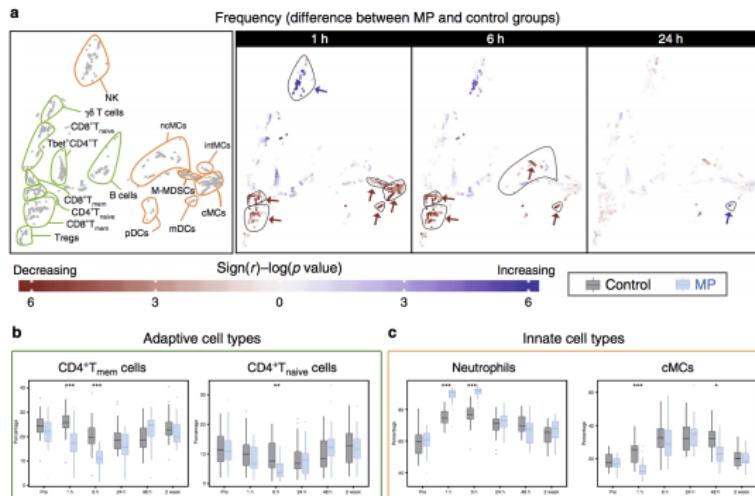


Figure: from Nature Communications. 2019.

Missing Data

- We don't live in a perfect world where we always have all of the data (could be missing features, entire samples, etc.)
- Imputation refers to an approach to 'fill in the blanks' for the missing information, based on the data that is available.
- De-noising is also a form of imputation, where you estimate and remove noise.

Imputation for Single-Cell Data

- Especially in scRNAseq, the datasets can be quite sparse, meaning that lowly-expressed genes fail to be detected.
- We can imagine an analog in mass cytometry, where signal can be a bit noisy (sometimes, we call it super staining when an antibody is too sticky)

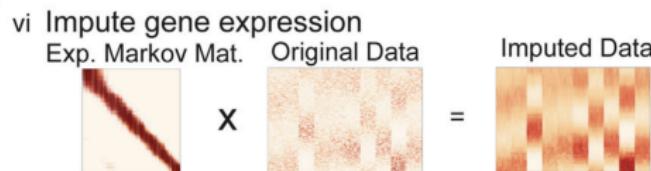


Figure: from van Dijk et al. Cell Systems. 2018

Accurately Representing Gene-Gene or Protein-Protein Interactions

The effect of lowly-expressed genes or proteins that are not adequately detected.

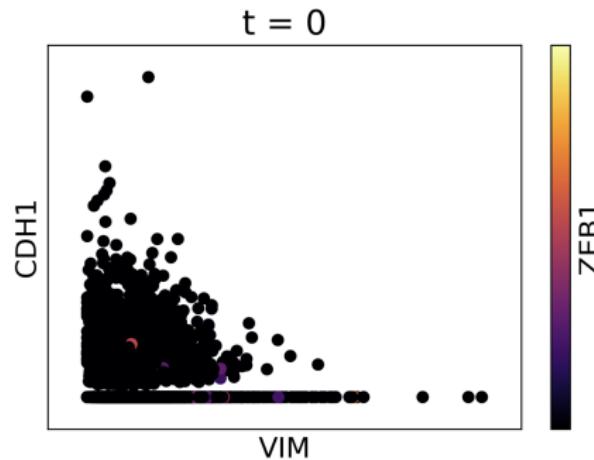
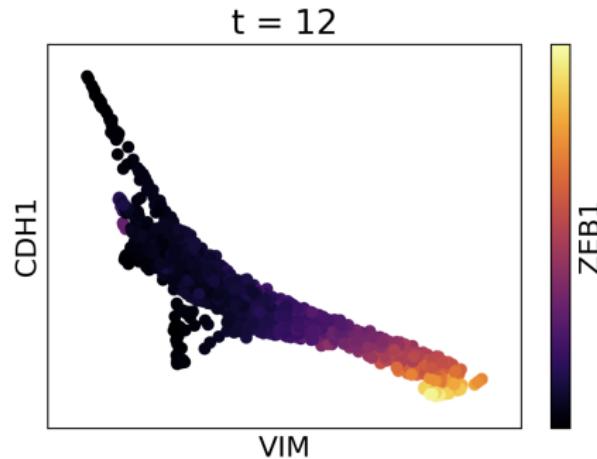


Figure: Note all of the 0 expression for CDH1

Estimating the Dropout, or Artificial Sparsity

If we can consider features of cells (in this case), that are otherwise similar to where dropout seems to have occurred, we can correct for the dropout and restore gene/gene or protein/protein interactions.



MAGIC Imputation for Single Cells

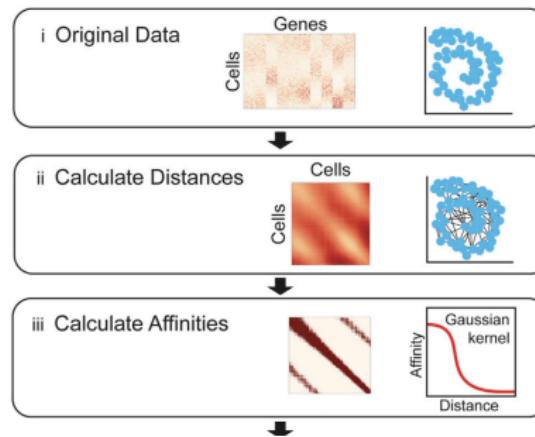
Here is an outline for the main idea.

- For a particular cell, find other cells that are most similar
- Construct a weighted affinity matrix between cells to more accurately capture between-cell similarities within a neighborhood
- Use affinities, and cell-cell neighborhood structure to correct the data

Step 1: Compute a Markov Affinity Matrix

Given the cell \times feature matrix, \mathbf{X} , compute the cell-to-cell affinity matrix, \mathbf{A} as follows. In particular, we compute affinities between each pair of cells (e.g. cell i and cell j).

$$A_{ij} = \exp(-(\text{Dist}(i, j)/\sigma)^2) \quad (2)$$



Notes on the σ parameter

$$\ln A_{ij} = \exp(-(\text{Dist}(i,j)/\sigma)^2)$$

- If σ is too small, the graph can become too disconnected.
- If σ is too large, cell-type resolution will be lost in the data and cell-types will be merged together.
- The solution is to adapt σ for each cell (σ_i) to take into account the density of its neighborhood. This prevents denser cell-types from dominating the imputation.
- In particular, σ_i is the distance between i and its k th nearest neighbor (default $k = 5$) ².

²<https://github.com/KrishnaswamyLab/MAGIC/blob/master/python/magic/magic.py>

//github.com/KrishnaswamyLab/MAGIC/blob/master/python/magic/magic.py

Question for you

Consider two cells, i and j , with corresponding feature vectors, \mathbf{x}_i and \mathbf{x}_j . What problems do you see if the dataset dimension is large (let's say 20K features)? Also, what could be done to deal with this?

Affinity Matrix Cleaning

- Due to the adaptive σ_i , the affinity matrix is not symmetric!
- This can quickly be made symmetric with,

$$\mathbf{A} = \mathbf{A} + \mathbf{A}^T \tag{3}$$

- The next step is in row-stochastic normalization that renders the affinity matrix into a Markov transition matrix, \mathbf{M}

$$M_{ij} = \frac{A_{ij}}{\sum_k A_{ik}} \tag{4}$$

Markov Affinity Based Graph Diffusion : Exponentiating \mathbf{M}

- By powering the matrix (\mathbf{M}^t), you can refine between-cell similarities related to the neighborhood structure. In other words, increase weight between pairs that have many common neighbors.
- M_{ij}^t represents the probability that a random walk of length t will reach node j after starting from node i .
- As t increases, false similarity between cells that was based strongly on noise gets filtered out. So, spurious neighbors will be down-weighted.

Try it Out

```
>>> A = np.matrix([[1/3,1/3,1/3],[0,0,1],[2/3,1/3,0]]) represents  
>>> A at cell i will reach cell j, thus we call t the "diffusion time."  
matrix([[0.33333333, 0.33333333, 0.33333333],  
       [0.66666667, 0.33333333, 0.33333333],  
       [0.22222222, 0.22222222, 0.55555556]]) paths and thus weight  
>>> matrix_power(A,2) main with the highest probability: (i) The probability of a pa  
matrix([[0.33333333, 0.22222222, 0.44444444],  
       [0.66666667, 0.33333333, 0.33333333],  
       [0.22222222, 0.22222222, 0.55555556]])  
>>> matrix_power(A,3)  
matrix([[0.40740741, 0.25925926, 0.33333333],  
       [0.22222222, 0.22222222, 0.55555556],  
       [0.44444444, 0.25925926, 0.29629633]])  
>>> matrix_power(A,4)  
matrix([[0.35802469, 0.24691358, 0.39506173],  
       [0.44444444, 0.25925926, 0.29629633],  
       [0.34567901, 0.24691358, 0.40740741]])  
>>> matrix_power(A,5)  
matrix([[0.38271605, 0.25102881, 0.36625514],  
       [0.34567901, 0.24691358, 0.40740741],  
       [0.38683128, 0.25102881, 0.36213992]])
```



VizQu

Visualizing what is happening....

As t increases, dense areas of the data result in more possible paths and weights are then concentrated in this area (e.g. closest neighbors remain with high probability). Also, helps to filter out noise.

- v Exponentiate markov matrix

$$[\quad]^t$$

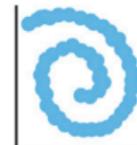
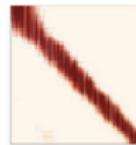


Figure: from van Dijk et al. Cell Systems. 2018

Imputation After Graph Diffusion

Finally, the original data \mathbf{X} (of cells \times features) is transformed as,

$$\mathbf{X}_{\text{impute}} = \mathbf{M}^t \mathbf{X} \quad (5)$$

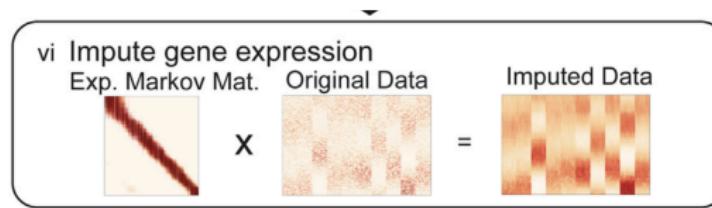


Figure: from van Dijk *et al.* Cell Systems. 2018

On a Practical Note, t

The t , or how you power the matrix is important to the results.

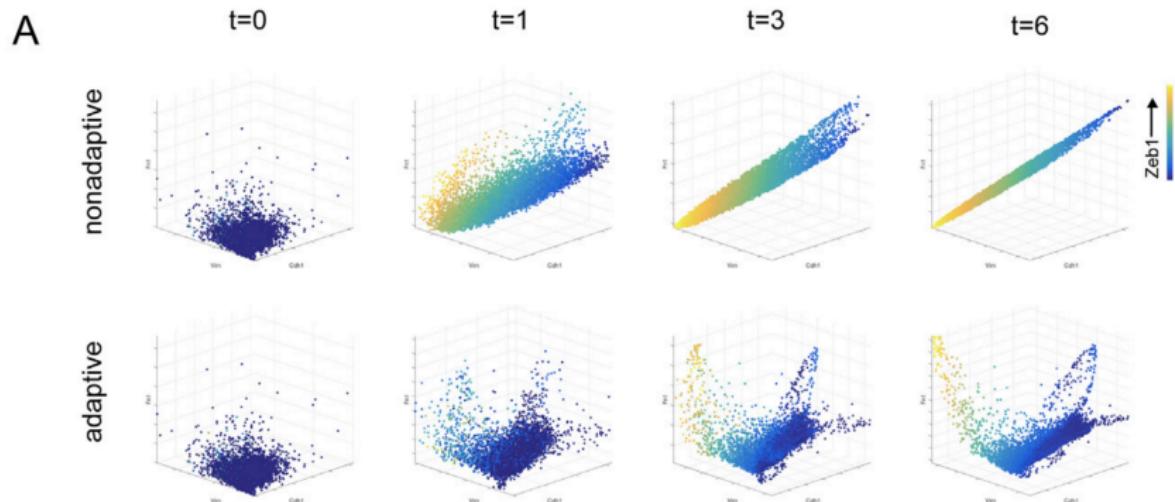


Figure: from van Dijk *et al.* Cell Systems. 2018. Adaptive means that you chose a σ_i for a particular cell, i .

Convergence of $\mathbf{X}_{\text{impute}}$ for different t

C

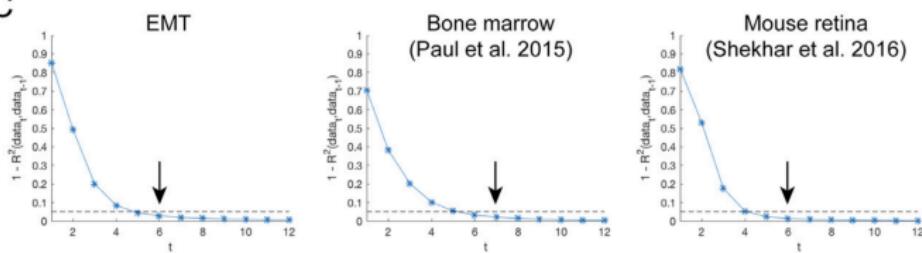


Figure: from van Dijk *et al.* Convergence of $\mathbf{X}_{\text{impute}}$ between t and $t - 1$.

Effectiveness in Noise Reduction

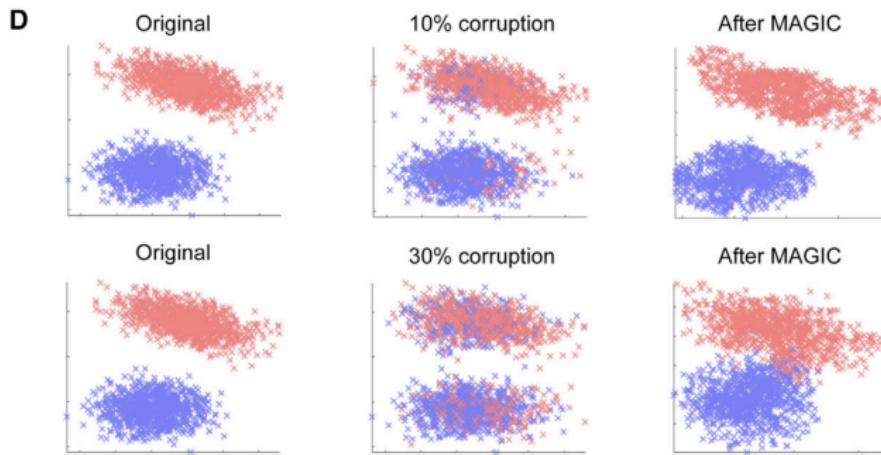


Figure: from van Dijk *et al.* MAGIC can restore noisy data, when features are artificially corrupted in a synthetic dataset.

Our Good Friend Flow Cytometry

Using MAGIC makes it easier to identify well-characterized cell-types.

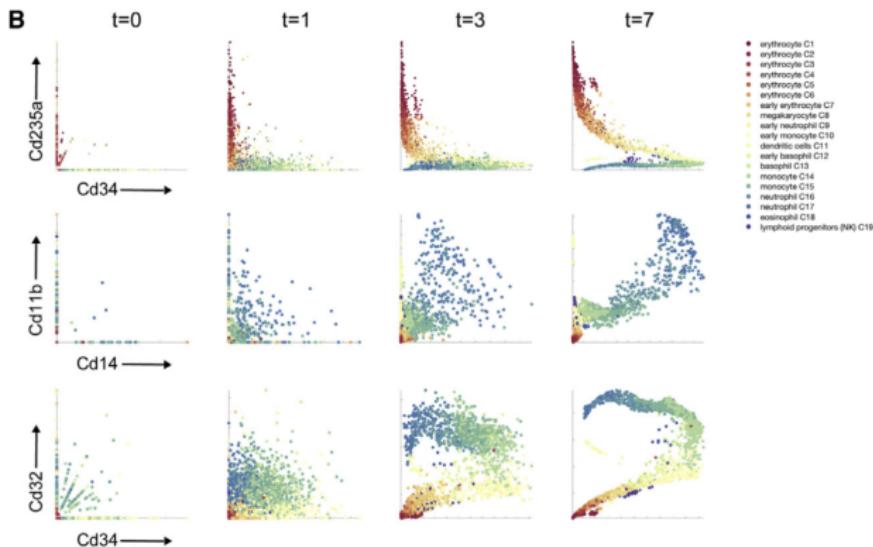


Figure: from van Dijk *et al.* Cell Systems. 2018.

Visualizing Single-Cell Data with PHATE

A Dimensionality Reduction Method for Single Cells

PHATE aims to preserve between-cell population similarities according to the way that cells differentiate.

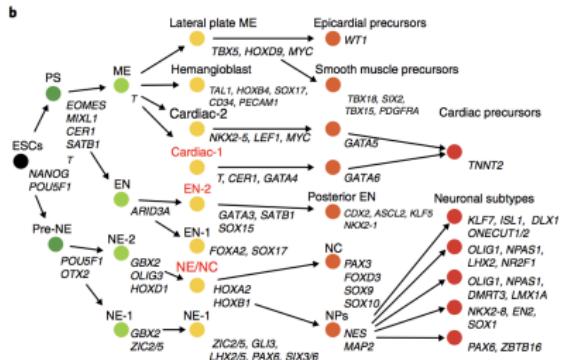


Figure: from Moon et al. Nature Biotechnology. 2019

Latent Structure

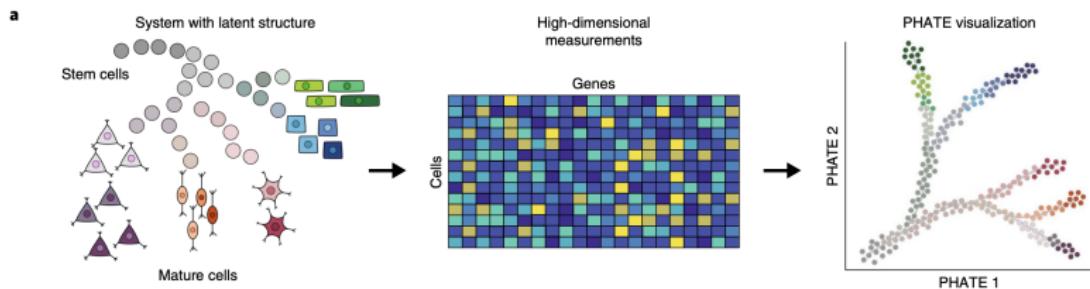


Figure: from Moon *et al.* Nature Biotechnology. 2019

PHATE Overview

Table 1 | General steps in the PHATE algorithm

Input: Data matrix, algorithm parameters (Methods)

Output: The PHATE visualization

- (1) Compute the pairwise distances from the data matrix.
- (2) Transform the distances to affinities to encode local information.
- (3) Learn global relationships via the diffusion process.
- (4) Encode the learned relationships using the potential distance.
- (5) Embed the potential distance information into low dimensions for visualization.

Figure: from Moon *et al.* Nature Biotechnology. 2019

The First Few Steps are Identical to MAGIC

- **Step 1:** Cell \times Cell distance matrix (based on Euclidean distance between cells)
- **Step 2:** Convert to Cell \times Cell row-stochastic affinity matrix \rightarrow, \mathbf{P} , such that $\sum_j \mathbf{P}_{ij} = 1$.
- **Step 3:** Power $\mathbf{P} \rightarrow \mathbf{P}^t$.

A Complementary Method to Determine Optimal t

- The choice of t will determine how much noise you want to filter. Small eigenvalues of \mathbf{P} correspond to noise, and will ultimately decrease towards 0.
- Let $[\eta(t)]_i = \lambda_i^t / \sum_{j=0}^{N-1} \lambda_j^t$ be the probability distribution defined by the non-negative eigenvalues of \mathbf{P}^t .

The von Neumann entropy $H(t)$ is then computed based on $[\eta(t)]_i$ and decreases as $t \rightarrow \infty$.

$$H(t) = - \sum_{i=1}^N [\eta(t)]_i \log([\eta(t)]_i) \quad (6)$$

Relationship Between t and $H(t)$

Higher t means less noise, but the implications of t are super powerful!

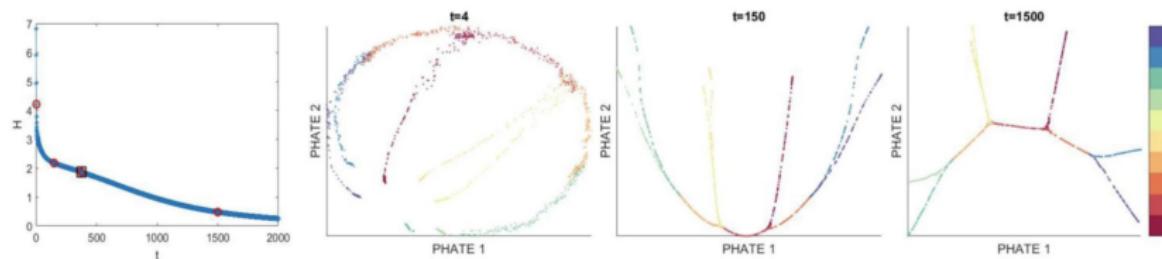


Figure: from Moon et al. Nature Biotechnology. 2019

Computing Potential Distances

- Potential distance is computed as a divergence between the associated diffusion probability distributions of the two cells to all other cells

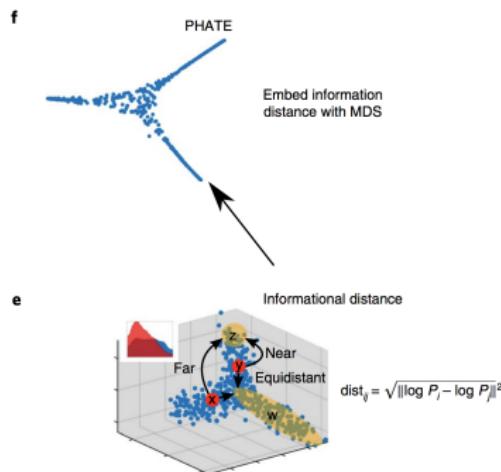


Figure: from Moon et al. Nature Biotechnology. 2019

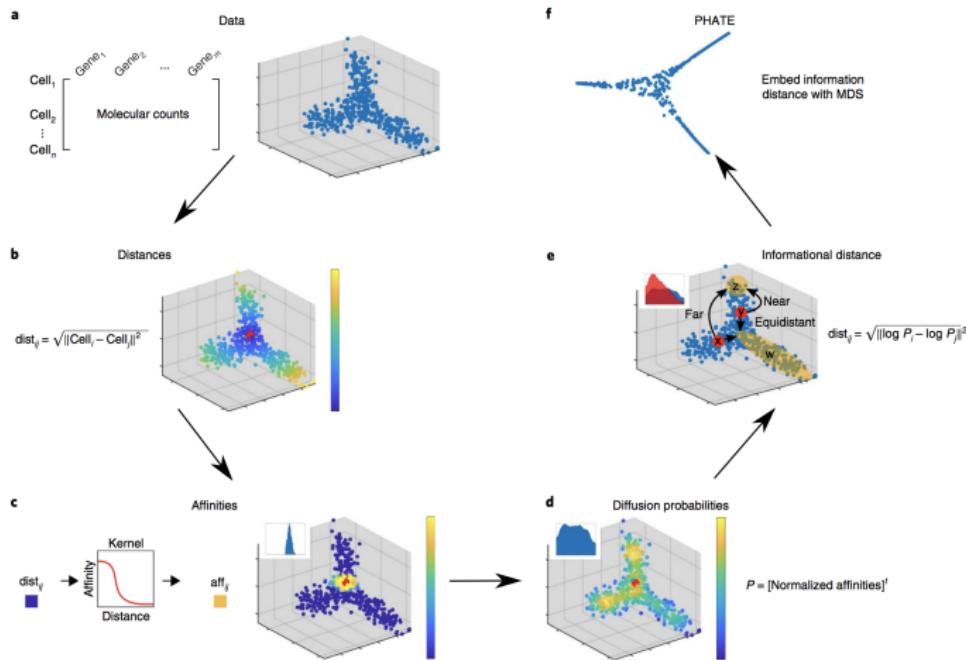
Potential Distances, Written Formally

- $U_{\mathbf{x}}^t = -\log(P_{\mathbf{x}}^t)$
- Then $\text{PD}(\mathbf{x}, \mathbf{y}) = \|U_{\mathbf{x}}^t - U_{\mathbf{y}}^t\|_2$

Then use a classic algorithm called multi-dimensional scaling to project all points in two dimensions. Apply to,

$$\mathfrak{V}^t(\mathbf{x}, \mathbf{y}) = \|U_{\mathbf{x}}^t - U_{\mathbf{y}}^t\|_2, \mathbf{x}, \mathbf{y} \in \mathcal{X}$$

Recap



PHATE preserves branch structure

As expected tSNE is preserving local similarities, but not necessarily distances between groups of points.

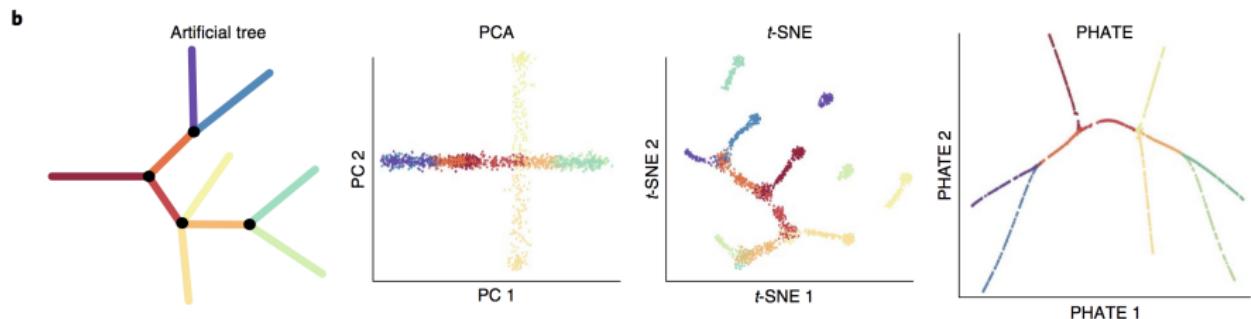


Figure: from Moon et al. Nature Biotechnology. 2019

And Preserves Cellular Differentiation Patterns!

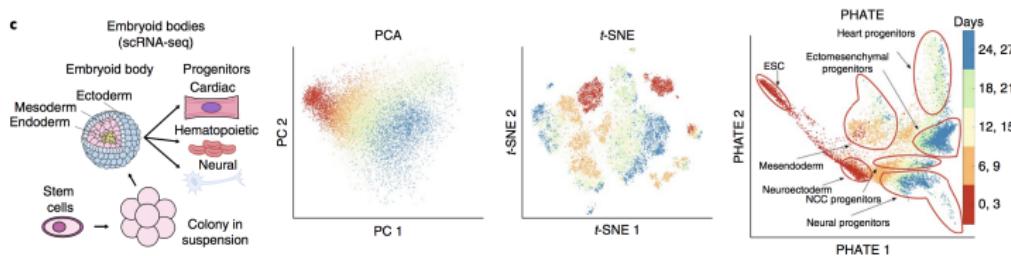


Figure: from Moon *et al.* Nature Biotechnology. 2019

Compare Phate to Other Dimension Reduction Techniques

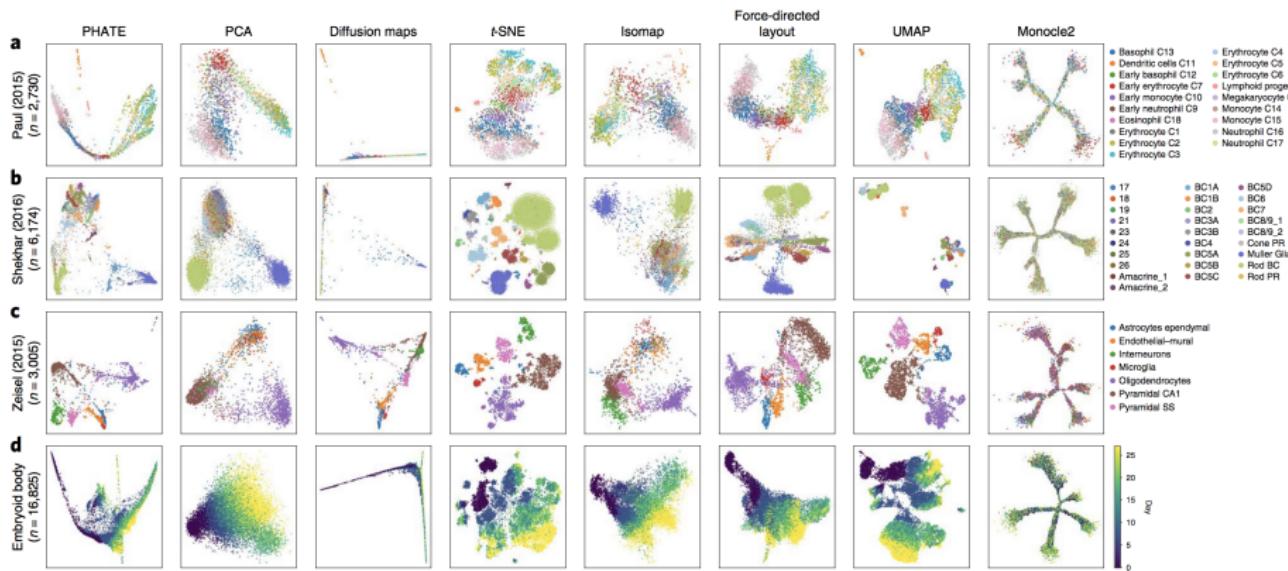


Figure: from Moon *et al.* Nature Biotechnology. 2019