Problem Set 1

Problems

1. Identify the normalization condition that describes a wavefunction $\Phi(x)$ expanded in terms of a complete orthonormal set of basis functions $\{\phi_i(x)\}$ with coefficients $\{c_i\}$.

Since $\{\phi_i\}$ are a complete orthonormal set of basis, $\Phi(x)$ can be expanded as $\sum_i c_i\phi_i(x)$ for some unique set of coefficients $\{c_i\}$. Then, the normalization condition of $\Phi(x)$, $|\Phi(x)|^2 = 1$, can be written as $\int |\sum_i c_i\phi_i(x)|^2\,dx = \int \sum_{i,j} c_i^* c_j \phi_i^* \phi_j\,dx$.

$$\begin{aligned}
1 &= \int |\Phi(x)|^2\,dx \\
&= \int \left|\sum_i c_i\phi_i(x)\right|^2\,dx \\
&= \int \left(\sum_i c_i\phi_i(x)\right)\left(\sum_j c_j^*\phi_j^*(x)\right)\,dx \\
&= \int \left[\sum_i \sum_j c_i c_j^* \phi_i(x)\phi_j^*(x)\right]\,dx \\
&= \sum_i \sum_j c_i c_j^* \int \phi_i(x)\phi_j^*(x)\,dx \\
&= \sum_i \sum_j c_i c_j^* \langle \phi_i|\phi_j\rangle \\
&= \sum_i \sum_j c_i c_j^* \delta_{ij} \\
&= \sum_i c_i c_i^* \\
&= \sum_i |c_i|^2
\end{aligned}$$

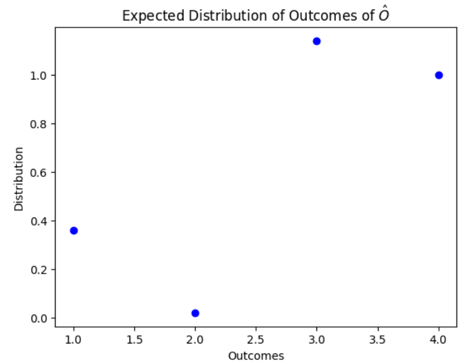

Figure 1: Plot of the probability of getting the eigenvalue j_i in a measurement of the property O

2. Consider the wavefunction

$$\Psi(x) = 0.6i\phi_1(x) + 0.1\phi_2(x) + c_3\phi_3(x) - 0.5\phi_4(x),$$

where $\Psi(x)$ is normalized, $\hat{O}\phi_j(x) = j\phi_j(x)$, and $\{\phi_j(x)\}$ form an orthonormal set.

(a) Calculate the magnitude of c_3.

$$\begin{aligned} 1 &= 0.6i \cdot (-0.6i) + 0.1^2 + |c_3|^2 + 0.5^2 \\ &= 0.36 + 0.01 + |c_3|^2 + 0.25 \\ &= 0.62 + |c_3|^2 \\ |c_3|^2 &= 0.38 \\ c_3 &= \sqrt{0.38} \end{aligned}$$

(b) Plot the expected distribution of the outcomes of repeated measurements of the operator $\hat{O}$.

(c) Compute $\langle \hat{O} \rangle$.

$$\int \Psi^*(x)\, \hat{O}\, \Psi(x)\, dx$$
$$= \int \Psi^*(x)\, \hat{O}\, (0.6i\phi_1(x) + 0.1\phi_2(x) + c_3\phi_3(x) - 0.5\phi_4(x))\ dx$$
$$= \int \Psi^*(x) \Big(0.6i\hat{O}\phi_1(x) + 0.2\hat{O}\phi_2(x) + 3c_3\hat{O}\phi_3(x) - 2\hat{O}\phi_4(x)\Big)\ dx$$
$$= \int (0.6i\phi_1(x) + 0.1\phi_2(x) + c_3\phi_3(x) - 0.5\phi_4(x))^*$$
$$\Big(0.6i\hat{O}\phi_1(x) + 0.2\hat{O}\phi_2(x) + 3c_3\hat{O}\phi_3(x) - 2\hat{O}\phi_4(x)\Big)\ dx.$$
$$= \int (0.6i\phi_1(x) + 0.1\phi_2(x) + c_3\phi_3(x) - 0.5\phi_4(x))^*$$
$$(0.6i \cdot 1\phi_1(x) + 0.2 \cdot \phi_2(x) + 3c_3 \cdot \phi_3(x) - 2 \cdot 4\phi_4(x))\ dx.$$
$$= 0.36 + 0.02 + 3|c_3|^2 + 1 \quad \text{(used normalization condition of basis)}$$
$$= 0.36 + 0.02 + 1.14 + 1$$
$$= 2.52$$

3. Prove that Hermitian operators only have real eigenvalues.

$$\hat{H}^\dagger = \hat{H}$$
$$\hat{H}\Psi = \alpha\Psi$$
$$\int \Psi^*\, \hat{H}\, \Psi\, dx = \alpha \int |\Psi|^2\, dx$$

Complex conjugate of the above:

$$\int \Psi\, \hat{H}^\dagger\, \Psi^*\, dx = \alpha^* \int |\Psi|^2\, dx$$
$$\int \Psi^*\, \hat{H}\, \Psi\, dx = \alpha^* \int |\Psi|^2\, dx$$
$$\alpha = \alpha^* \quad \text{(since } \Psi \neq 0\text{)}$$

4. Complete the Python exercise in `problemset1.ipynb`.

(a) Download and install Python and Jupyter Notebook.

(b) Run the Jupyter notebook `problemset1.ipynb` using `shift-enter` to run each cell.

i. Method 1: Run `Anaconda Navigator`, open `JupyterLab`, and import the notebook with the `Upload file` icon (icon with an up arrow).

ii. Method 2: Navigate to the folder on your computer that contains `problemset1.ipynb`, open the command line (terminal), and type `jupyter notebook problemset1.ipynb`.

(c) To gain a greater understanding of the type of Python code used in class, test how the output of the program varies as you change variables, function names, etc.

(d) Complete and submit the answer to the problem marked "Problem Set Exercise" at the end of `problemset1.ipynb` alongside this problem set. Be careful to always label axes.