In the case that $e > n+r+1$, we have confirmed that there exists an iso $\varphi^r_{2n+r} : W \to V'$ for appropriate subrepresentation V' and quotient W of M. Define the following coefficients:

$$\begin{aligned}\Phi_3(q^{1/2}) &:= q^{3/2}+1\\ [3]_{q^{1/2}} &:= q^{3/2}+q+q^{1/2}+1\\ q^{1/2}\,[2]_{q^{1/2}} &:= q+q^{1/2}\end{aligned}$$

Then, we may empirically compute the following:

$$\varphi^0_6 = \varphi^1_5 = \begin{bmatrix} 0 & 0 & -\Phi_3(q^{1/2}) & 0 & 0\\ 0 & -\Phi_3(q^{1/2}) & [3]_{q^{1/2}} & 0 & 0\\ 0 & 0 & [3]_{q^{1/2}} & 0 & -\Phi_3(q^{1/2})\\ -[3]_{q^{1/2}} & [3]_{q^{1/2}} & q^{1/2}\,[2]_{q^{1/2}} & 0 & [3]_{q^{1/2}}\\ [3]_{q^{1/2}} & 0 & [3]_{q^{1/2}} & -[3]_{q^{1/2}} & 0 \end{bmatrix}$$

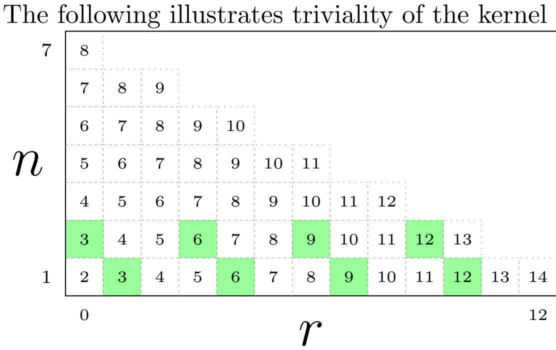
$$\varphi^3_5 = \begin{bmatrix} -q^{1/2}\,[2]_{q^{1/2}} & [3]_{q^{1/2}} & q^{1/2}\\ q^{1/2} & q^{3/2} & 0\\ 0 & -q-1 & \end{bmatrix}$$

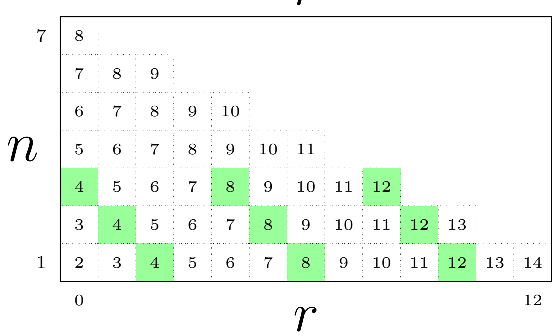
$$\varphi^0_4 = \begin{bmatrix} 0 & -[3]_{q^{1/2}}\\ -1 & 1 \end{bmatrix}$$

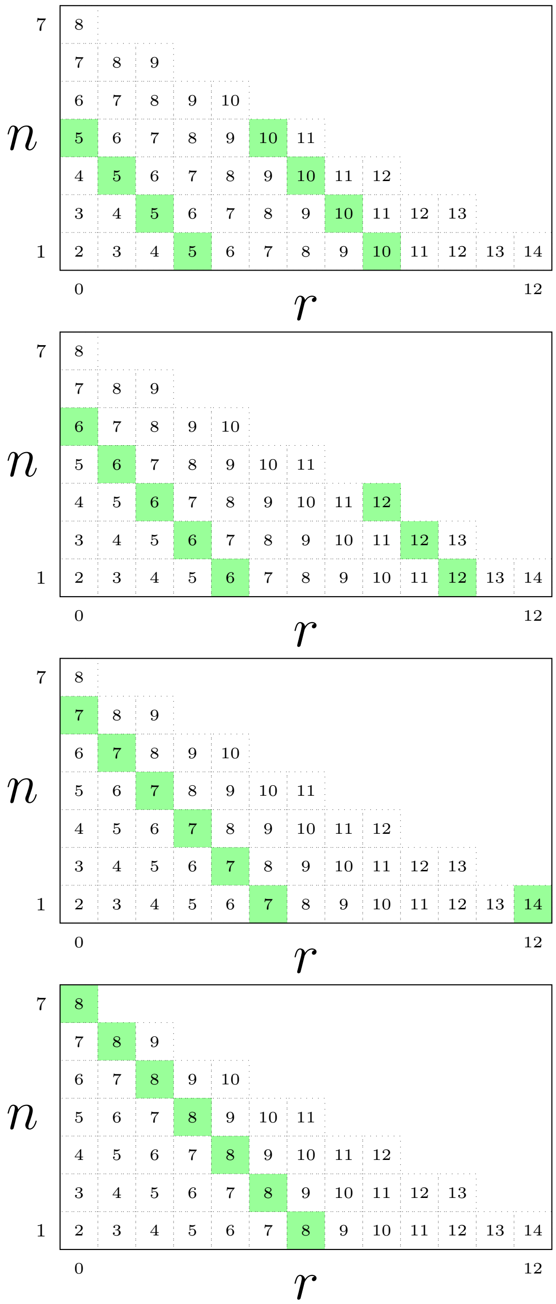
$$\varphi^2_4 = \begin{bmatrix} 0 & [3]_{q^{1/2}} & 0\\ 1 & -1 & 0\\ -1 & 0 & 1 \end{bmatrix}$$

$$\varphi^1_3 = \begin{bmatrix} 0 & -[3]_{q^{1/2}}\\ q^{1/2}\,[2]_{q^{1/2}} & -q^{1/2}\,[2]_{q^{1/2}} \end{bmatrix}$$

The following illustrates triviality of the kernel at various e:



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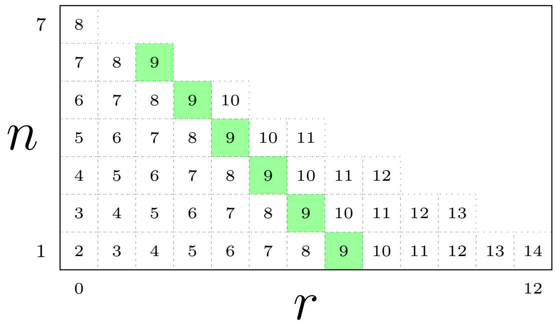
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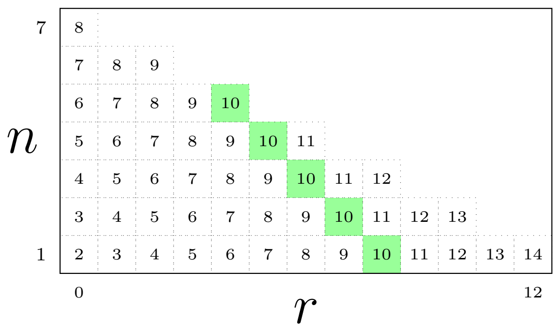
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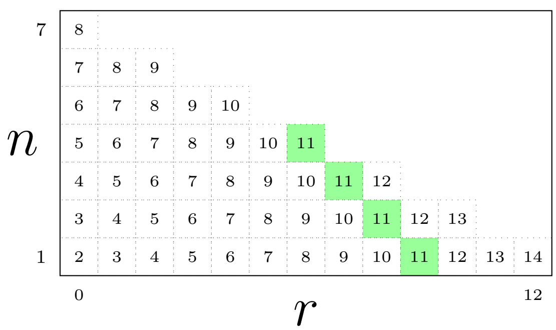
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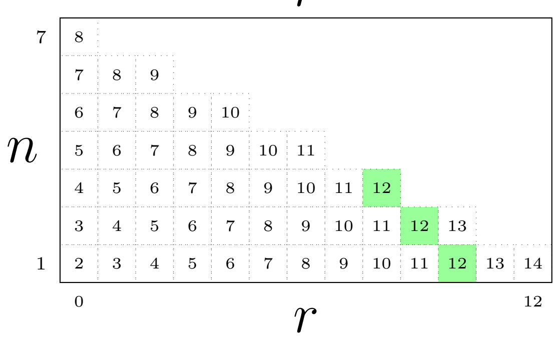
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