In the case that $e > n + r + 1$, we have confirmed that there exists an iso $\varphi^r_{2n+r} : W \to V'$ for appropriate subrepresentation V' and quotient W of M. Define the following coefficients:

$$\begin{aligned}
\Phi_3(q^{1/2}) &:= q^{3/2} + 1 \\
[3]_{q^{1/2}} &:= q^{3/2} + q + q^{1/2} + 1 \\
q^{1/2}\,[2]_{q^{1/2}} &:= q + q^{1/2}
\end{aligned}$$

Then, we may empirically compute the following:

$$\begin{aligned}
\varphi^0_6 = \varphi^1_5 &= \begin{bmatrix} 0 & 0 & -\Phi_3(q^{1/2}) & 0 & 0 \\ 0 & -\Phi_3(q^{1/2}) & [3]_{q^{1/2}} & 0 & 0 \\ 0 & 0 & [3]_{q^{1/2}} & 0 & -\Phi_3(q^{1/2}) \\ -[3]_{q^{1/2}} & [3]_{q^{1/2}} & q^{1/2}\,[2]_{q^{1/2}} & 0 & [3]_{q^{1/2}} \\ [3]_{q^{1/2}} & 0 & [3]_{q^{1/2}} & -[3]_{q^{1/2}} & 0 \end{bmatrix} \\
\varphi^3_5 &= \begin{bmatrix} -q^{1/2}\,[2]_{q^{1/2}} & [3]_{q^{1/2}} & q^{1/2} \\ q^{1/2} & q^{3/2} & 0 \\ 0 & -q-1 & \end{bmatrix} \\
\varphi^0_4 &= \begin{bmatrix} 0 & -[3]_{q^{1/2}} \\ -1 & 1 \end{bmatrix} \\
\varphi^2_4 &= \begin{bmatrix} 0 & [3]_{q^{1/2}} & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
\varphi^1_3 &= \begin{bmatrix} 0 & -[3]_{q^{1/2}} \\ q^{1/2}\,[2]_{q^{1/2}} & -q^{1/2}\,[2]_{q^{1/2}} \end{bmatrix}
\end{aligned}$$

$$\iota_{3,2,1} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\iota_{3,4,0} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\iota_{3,4,1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\iota_{3,2,4} = \begin{bmatrix} -1 & -1 & 0 & 1 & 1 \end{bmatrix}$$

$$\iota_{3,6,0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\iota_{3,4,3} = \begin{bmatrix} 1 & 0 & -1 & -1 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\iota_{3,6,1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

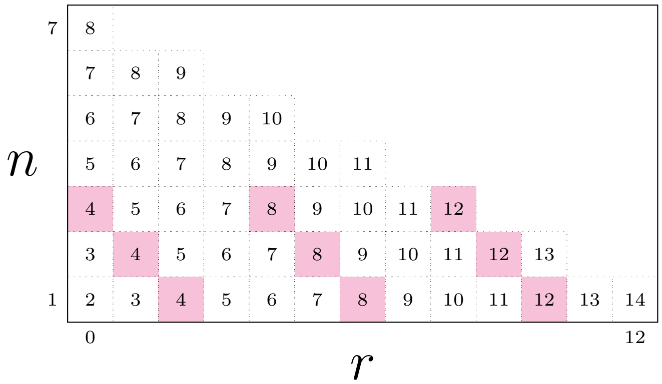
$$\iota_{4,2,2} = \begin{bmatrix} 1 & \alpha & 1 \end{bmatrix}$$
$$\iota_{4,2,1} = \begin{bmatrix} 1 & \frac{1}{2}\alpha & \frac{1}{2}\alpha & 1 & \frac{1}{2}\alpha \end{bmatrix}$$
$$\iota_{4,4,2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \alpha & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & -\alpha & 0 \\ 0 & 0 & 1 & 0 & 0 & \alpha & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & -\alpha & -1 \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{2}\alpha & \frac{1}{2}\alpha & 1 & \frac{1}{2}\alpha \end{bmatrix}$$
$$\iota_{4,6,0} = \begin{bmatrix} \alpha & 1 & 1 & \alpha & 1 \end{bmatrix}$$
$$\iota_{4,6,1} = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{2}\alpha & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 1 & 1/2 & \alpha & \frac{1}{2} \\ 0 & 1 & 0 & 0 & -1 & 0 & \frac{1}{2}\alpha & \frac{1}{2}\alpha & 1 & 0 & \alpha & \frac{1}{2}\alpha & -1 & \frac{1}{2}\alpha \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & \alpha & 1 & 0 & -\alpha & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & \alpha & 0 & 0 & -1 & -\alpha & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2}\alpha & \frac{1}{2}\alpha & 0 & 0 & 0 & \frac{1}{2}\alpha & -1 & \frac{1}{2}\alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{2}\alpha & \frac{1}{2}\alpha & 1 & \frac{1}{2}\alpha \end{bmatrix}$$
$$\iota_{5,2,3} = \begin{bmatrix} 1 & \beta & \beta & 1 \end{bmatrix}$$
$$\iota_{5,4,2} = \begin{bmatrix} \beta & \beta & 1 & 1 & \beta+1 & \beta & \beta & \beta & 1 \end{bmatrix}$$
$$\iota_{5,4,3} = \begin{bmatrix} 1 & \beta & \beta & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \beta & \beta & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta & \beta & 1 & 0 & 0 & 0 \\ \beta & \beta & 1 & 0 & 1 & \beta+1 & \beta & 0 & \beta & \beta & 0 & 1 & 0 & 0 \\ -\beta-1 & -\beta & -\beta & 0 & -\beta & -\beta-1 & -\beta-1 & 0 & -\beta-1 & -\beta-1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \beta & 0 & 0 & \beta & 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\iota_{5,6,1} = \begin{bmatrix} 1 & \alpha & \alpha & \alpha & -\beta & \alpha & -\beta & \alpha & -\beta & 1 & \alpha & \alpha & \alpha & -\beta \end{bmatrix}$$
$$\iota_{6,2,4} = \begin{bmatrix} 1 & \gamma & 2 & \gamma & 1 \end{bmatrix}$$
$$\iota_{6,4,3} = \begin{bmatrix} \gamma & 2 & \gamma & 1 & 1 & 2\gamma & 3 & \gamma & 2 & 2\gamma & 2 & 2 & \gamma & 1 \end{bmatrix}$$
$$\iota_{6,4,3} = \begin{bmatrix} 1 & \delta & \varepsilon & \varepsilon & \delta & 1 \end{bmatrix}$$

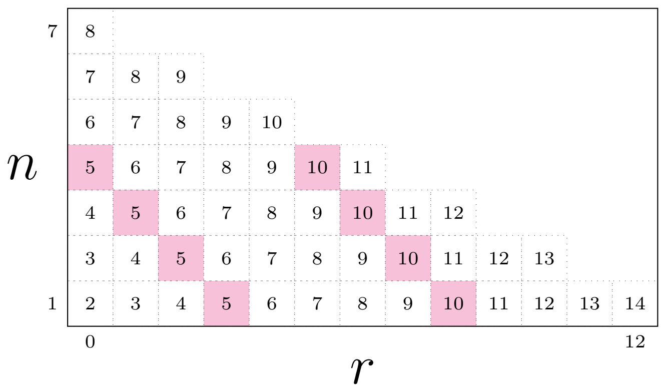
[1 -v$^3+2*v2-v^3+2*v1]e = 6n = 4r = 3[-v^3+2*v2-v^3+2*v11-2*v^3+4*v3-v^3+2*v2-2*v^3+4*v22-v^3+2*v1]e = 7n = 2r = 5[1-v^5+v^4-v^3+v^2+1-v^5+v^2+1-v^5+v^2+1-v^5+v^4-v^3+v^2+11]$

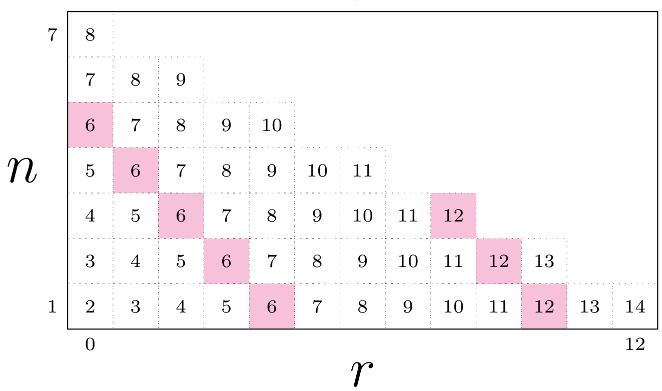
The following illustrates triviality of the kernel at various e:

7	8												
	7	8	9										
	6	7	8	9	10								
n	5	6	7	8	9	10	11						
	4	5	6	7	8	9	10	11	12				
	3	4	5	6	7	8	9	10	11	12	13		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
	0											12	

r


7 8
7 8 9
6 7 8 9 10
n
5 6 7 8 9 10 11
4 5 6 7 8 9 10 11 12
3 4 5 6 7 8 9 10 11 12 13
1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 12
r


7 8
7 8 9
6 7 8 9 10
n
5 6 7 8 9 10 11
4 5 6 7 8 9 10 11 12
3 4 5 6 7 8 9 10 11 12 13
1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 12
r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7 8 9
6 7 8 9 10
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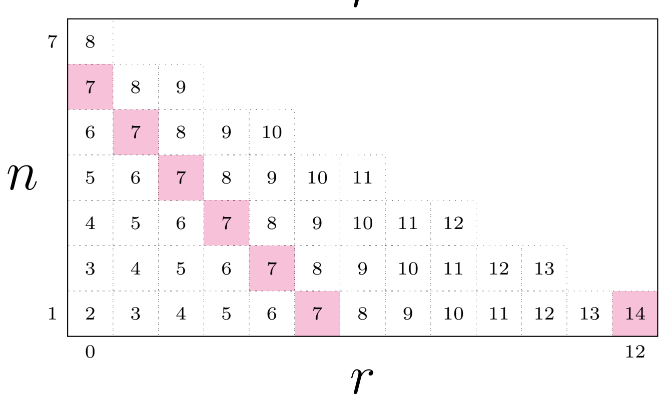
0 12
r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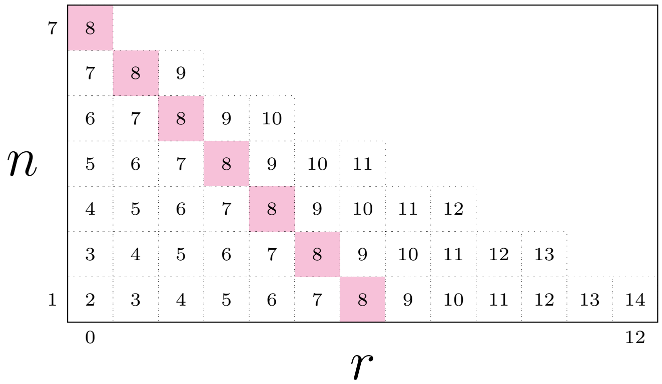
r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6 7 8 9 10
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3 4 5 6 7 8 9 10 11 12 13
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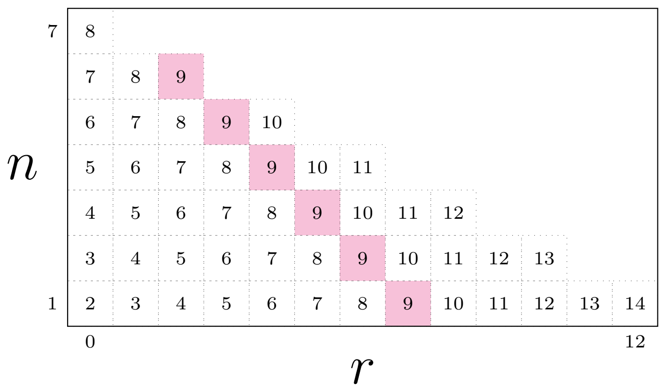
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3 4 5 6 7 8 9 10 11 12 13
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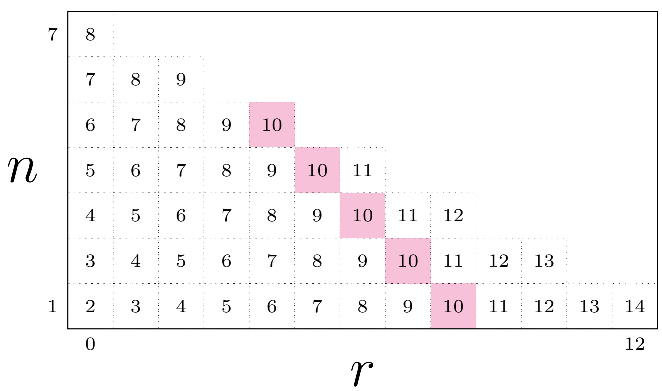
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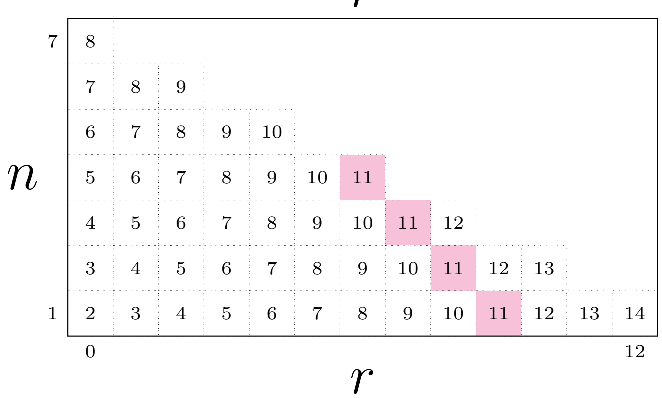
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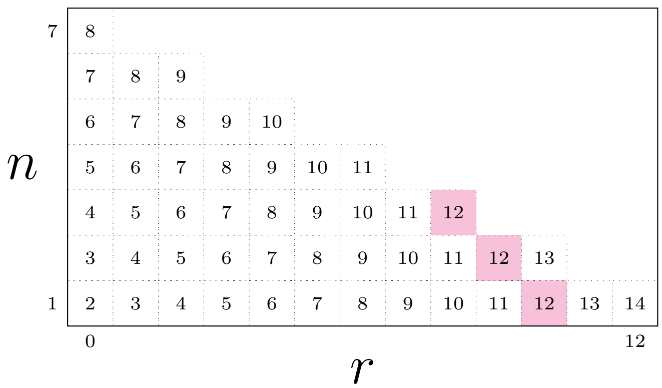
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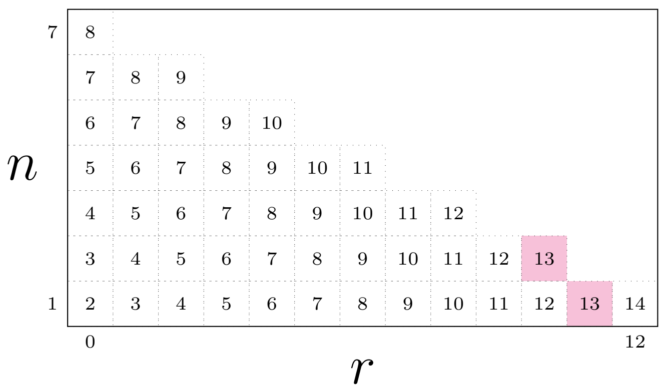
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6 7 8 9 10
n
5 6 7 8 9 10 11
4 5 6 7 8 9 10 11 12
3 4 5 6 7 8 9 10 11 12 13
1
2 3 4 5 6 7 8 9 10 11 12 13 14
0
12
r