In the case that $e > n+r+1$, we have confirmed that there exists an iso $\varphi^r_{2n+r} : W \to V'$ for appropriate subrepresentation V' and quotient W of M. Define the following coefficients:

$$\begin{aligned}
\Phi_3(q^{1/2}) &:= q^{3/2}+1\\
[3]_{q^{1/2}} &:= q^{3/2}+q+q^{1/2}+1\\
q^{1/2}\,[2]_{q^{1/2}} &:= q+q^{1/2}
\end{aligned}$$

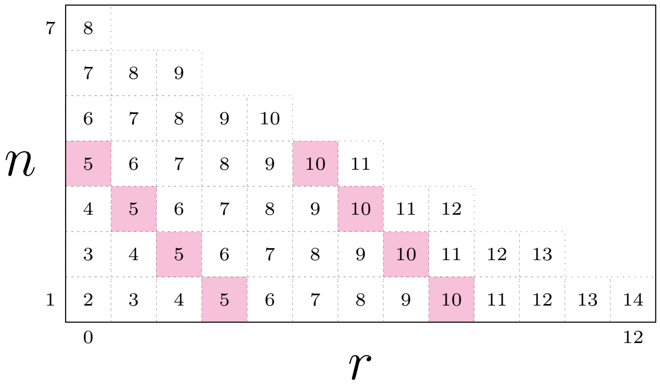
Then, we may empirically compute the following:

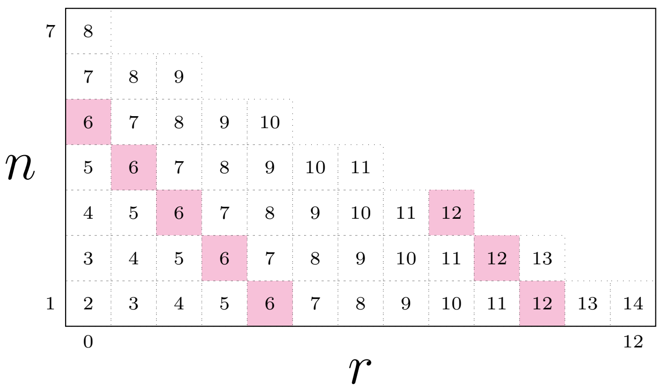
$$\begin{aligned}
\varphi^0_6 = \varphi^1_5 &= \begin{bmatrix} 0 & 0 & -\Phi_3(q^{1/2}) & 0 & 0 \\ 0 & -\Phi_3(q^{1/2}) & [3]_{q^{1/2}} & 0 & 0 \\ 0 & 0 & [3]_{q^{1/2}} & 0 & -\Phi_3(q^{1/2}) \\ -[3]_{q^{1/2}} & [3]_{q^{1/2}} & q^{1/2}\,[2]_{q^{1/2}} & 0 & [3]_{q^{1/2}} \\ [3]_{q^{1/2}} & 0 & [3]_{q^{1/2}} & -[3]_{q^{1/2}} & 0 \end{bmatrix}\\
\varphi^3_5 &= \begin{bmatrix} -q^{1/2}\,[2]_{q^{1/2}} & [3]_{q^{1/2}} & q^{1/2} \\ q^{1/2} & q^{3/2} & 0 \\ 0 & -q-1 & \end{bmatrix}\\
\varphi^0_4 &= \begin{bmatrix} 0 & -[3]_{q^{1/2}} \\ -1 & 1 \end{bmatrix}\\
\varphi^2_4 &= \begin{bmatrix} 0 & [3]_{q^{1/2}} & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\\
\varphi^1_3 &= \begin{bmatrix} 0 & -[3]_{q^{1/2}} \\ q^{1/2}\,[2]_{q^{1/2}} & -q^{1/2}\,[2]_{q^{1/2}} \end{bmatrix}
\end{aligned}$$

The following illustrates triviality of the kernel at various e:

n													
7	8												
	7	8	9										
	6	7	8	9	10								
	5	6	7	8	9	10	11						
	4	5	6	7	8	9	10	11	12				
	3	4	5	**6**	7	8	**9**	10	11	**12**	13		
1	2	**3**	4	5	**6**	7	8	**9**	10	11	**12**	13	14
r	0												12

n													
7	8												
	7	8	9										
	6	7	8	9	10								
	5	6	7	8	9	10	11						
	4	5	6	7	**8**	9	10	11	**12**				
	3	**4**	5	6	7	**8**	9	10	11	**12**	13		
1	2	3	**4**	5	6	7	**8**	9	10	11	**12**	13	14
r	0												12


n
7 8
7 8 9
6 7 8 9 10
5 6 7 8 9 10 11
4 5 6 7 8 9 10 11 12
3 4 5 6 7 8 9 10 11 12 13
1 2 3 4 5 6 7 8 9 10 11 12 13 14
0
12
r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6 7 8 9 10
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4 5 6 7 8 9 10 11 12
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1 2 3 4 5 6 7 8 9 10 11 12 13 14
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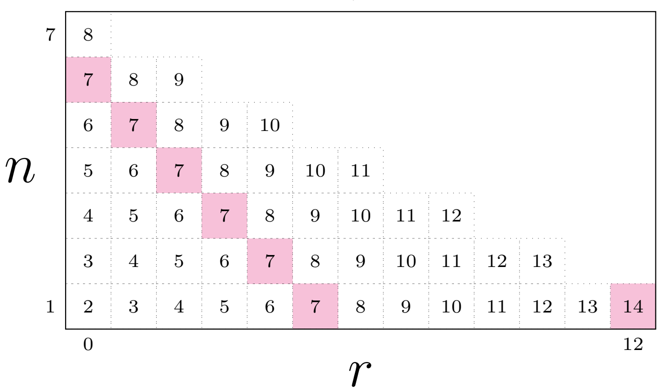
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4 5 6 7 8 9 10 11 12
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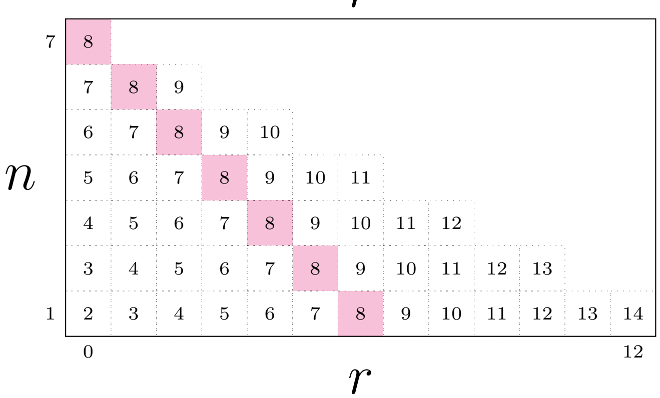
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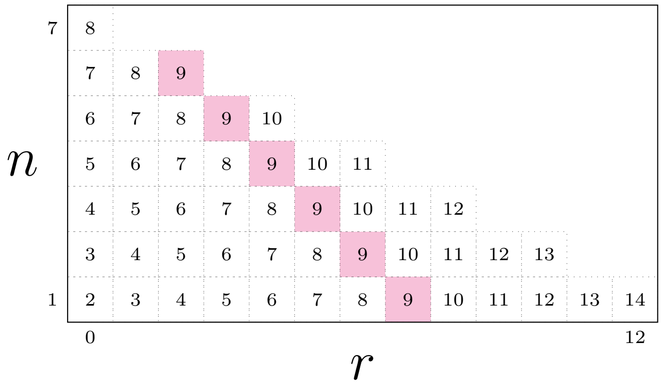
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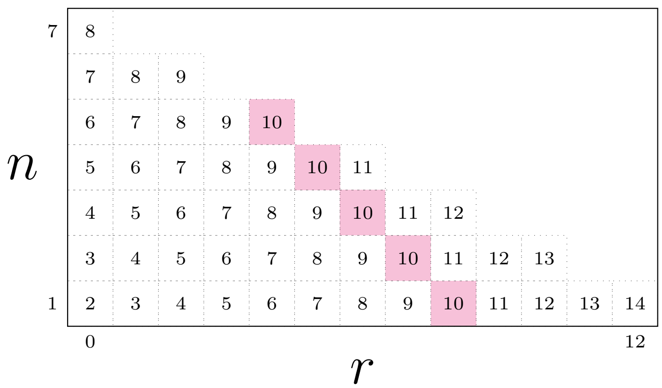
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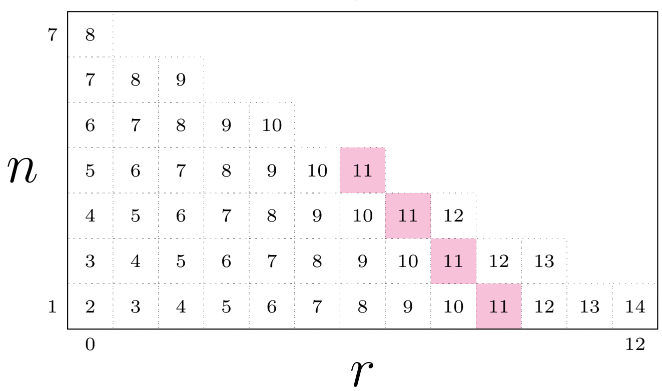
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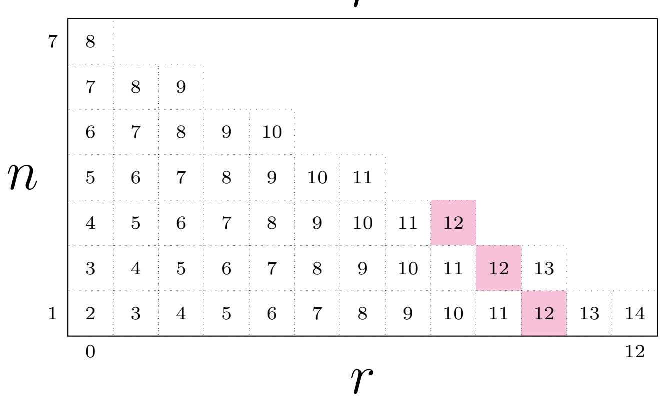
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5 6 7 8 9 10 11
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2 3 4 5 6 7 8 9 10 11 12 13 14
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n

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	7	8	9										
	6	7	8	9	10								
	5	6	7	8	9	10	11						
	4	5	6	7	8	9	10	11	12				
	3	4	5	6	7	8	9	10	11	12	13		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
	0												12

r