Homotopy-coherent interchange and equivariant little disk operads

Natalie Stewart

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INI Equivariant Homotopy Theory in Context

Queen's University Belfast, April 7 2025







In trace methods for Real algebraic K-theory, THH has a Real analogue:¹

$$\begin{array}{cccc} \operatorname{Alg}_{\mathbb{E}_1}(\operatorname{Sp}) & \operatorname{Alg}_{\mathbb{E}_\sigma}(\operatorname{Sp}_{C_2}) \\ & & & & & & & \downarrow \operatorname{ThR} \\ \operatorname{Sp} & & & & & & & \downarrow \operatorname{ThR} \end{array}$$

In this, σ is the sign representation and \mathbb{E}_{σ} -algebras are a genuine-equivariant version of rings with anti-involution.

Question (c.f. Dotto-Moi-Patchkoria-Reeh² '17)

What algebraic structure does THR of highly structured C₂-ring spectra have?

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Observation

THH can be given a symmetric monoidal structure, so we may lift

$$\operatorname{Alg}_{\mathcal{O} \otimes \mathbb{E}_{1}}(\operatorname{Sp}) \simeq \operatorname{Alg}_{\mathcal{O}} \operatorname{Alg}_{\mathbb{E}_{1}}^{\otimes}(\operatorname{Sp}) \xrightarrow{----} \operatorname{Alg}_{\mathcal{O}}(\operatorname{Sp})$$

$$\downarrow \qquad \qquad \downarrow \qquad$$

Theorem (Dunn³ '88, Lurie⁴ '09)

 $\mathbb{E}_n\simeq\mathbb{E}_{n-1}\otimes\mathbb{E}_1$; hence THH takes \mathbb{E}_n -rings to \mathbb{E}_{n-1} -rings

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Conjecture

 $\mathbb{E}_V \otimes \mathbb{E}_W \simeq \mathbb{E}_{V \oplus W}$

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Statement of the additivity theorem

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$$\mathrm{Alg}_{\mathbb{E}_V} \underline{\mathrm{Alg}}^\otimes_{\mathbb{E}_W} \left(\operatorname{Sp}_G
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Given V, W orthogonal G-representations, we have

$$\mathbb{E}_{V} \otimes \mathbb{E}_{W} \simeq \mathbb{E}_{V \oplus W};$$

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$$\mathrm{Alg}_{\mathbb{E}_{V}} \underline{\mathrm{Alg}}_{\mathbb{E}_{W}}^{\otimes} (\mathrm{Sp}_{G}) \simeq \mathrm{Alg}_{\mathbb{E}_{V \oplus W}} (\mathrm{Sp}_{G}) .$$

Corollary

THR of $\mathbb{E}_{V \oplus \sigma}$ -rings has a natural \mathbb{E}_{V} -ring structure

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¹² Ben Szczesny, Equivariant Framed Little Disk Operads are Additive, 2024, arXiv: 2410, 20235 [math. AT].

- May '72: 6 $C_n \otimes C_m$ and C_{n+m} agree on **connected spaces**.
- **1** Dunn '88: $^7C_1^{\otimes n} \simeq C_n$ w.r.t. a **point-set** tensor product.
- **2** Brinkmeier '00:⁸ $C_n \otimes C_m \simeq C_{n+m}$ w.r.t. a **point-set** tensor product.
- **3** Rourke-Sanderson'00: 9 $D_V \otimes D_W$ and $D_{V \oplus W}$ agree on G-connected G-spaces.
- 4 Lurie '09: ${}^{10}\mathbb{E}_n^{\otimes} \overset{\mathbb{N}}{\otimes} \mathbb{E}_m^{\otimes} \simeq \mathbb{E}_{n+m}^{\otimes}$ with respect to a **homotopical** tensor product.
- **5** Fiedorowicz-Vogt '15:¹¹ Dunn & Brinkmeier's result extends to **cofibrant** \mathbb{E}_n -**operads**.
- **6** Szczesny '24: 12 $D_V \otimes D_W \simeq D_{V \oplus W}$ w.r.t. a **point-set** tensor product.
- 7 S. '25: $\mathbb{E}_V^{\otimes} \overset{\text{\tiny BV}}{\otimes} \mathbb{E}_W^{\otimes} \simeq \mathbb{E}_{V \oplus W}^{\otimes}$ with respect to a **homotopical** tensor product.

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^{6).} P. May. The geometry of iterated loop spaces. Vol. Vol. 271. Lecture Notes in Mathematics. Springer-Verlag, Berlin-New York, 1972, pp. viii+175.

 $^{^{7}} Gerald \ Dunn.\ "Tensor product\ of\ operads\ and\ iterated\ loop\ spaces".\ In:\ \textit{J. Pure\ Appl.\ Algebra\ 50.3\ (1988)},\ pp.\ 237-258.$

⁸ Michael Brinkmeier. On Operads. Thesis (Ph.D.)–Universitat Osnabrück. 2000.

⁹Colin Rourke and Brian Sanderson. "Equivariant configuration spaces". In: J. London Math. Soc. (2) 62.2 (2000), pp. 544–552.

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- Nardin-Shah '22:¹⁴ G-operads are a type of fibration over a G-categorica version $\mathbb{F}_{G,*}$ of Segal's category Γ^{op} .
- Barkan-Haugseng-Steinebrunner '22: ¹⁵ G-operads are also **fibrations over the effective Burnside** 2-category Span(\mathbb{F}_G).
- S. '25:¹⁶ homotopy-coherent interchange is corepresented by BV tensor products and G-operads are monadic over G-symmetric sequences.
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¹⁰ Natalie Stewart. Equivariant operads, symmetric sequences, and Boardman-Vogt tensor products. 2025. arXiv: 2501.02129 [math.CT].

¹⁷ Natalie Stewart, On tensor products with equivariant commutative operads, 2025, arXiv, 2504, 02143, [math, AT]

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Sidebar: S-ary spaces in Mike's G-operads

When considering the *homotopy theory* of Mike's G-operads, we usually define *graph subgroups*: given $(H) \subset G$ and $S \in \mathbb{F}_H$ define

$$\Gamma_{S} = \{(h, \rho_{S}(h)) \mid h \in H, \ \rho_{S} \colon H \to \Sigma_{|S|}\} \subset G \times \Sigma_{S}.$$

(these are transitive $G \times \Sigma_n$ -sets with free underlying Σ_n -sets)

Definition

The S-ary structure space of \mathcal{O} is the fixed points

$$\mathcal{O}(S) = \mathcal{O}(|S|)^{\Gamma_S}$$

A map of Mike's G-operads $\varphi \colon \mathcal{O} \to \mathcal{P}$ is a weak equivalence if $\mathcal{O}(S) \to \mathcal{P}(S)$ is an equivalence for all $(H) \subset G$ and $S \in \mathbb{F}_H$.

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Sidebar: operads on the other side of Elmendorf's theorem

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Given \mathcal{O} a Mike's G-operad, we have operadic composition maps¹⁸

$$\gamma \colon \mathcal{O}(S) \otimes \bigotimes_{[H/K_i] \in \operatorname{Orb}(S)} \mathcal{O}(T_i) \to \mathcal{O}\left(\coprod_{[H/K_i] \in \operatorname{Orb}(S)} \operatorname{Ind}_{K_i}^H T_i\right),$$

operadic restriction maps

Res:
$$\mathcal{O}(S) \to \mathcal{O}\left(Res_K^H S\right)$$
,

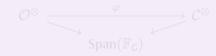
and equivariant symmetric group action

$$\rho \colon \operatorname{Aut}_{H}(S) \times \mathcal{O}(S) \to \mathcal{O}(S).$$

¹⁸ Peter Bonventre and Luís A. Pereira. "Genuine equivariant operads". In: Adv. Math. 381 (2021), Paper No. 107502, 133.

Definition

- A *G*-preoperad is a functor $\mathcal{O}^{\otimes} \to \operatorname{Span}(\mathbb{F}_G)$ with cocartesian lifts over backwards maps.
- A G-operad is required to satisfy "Segal conditions."
- An \mathcal{O} -algebra in \mathcal{C}^{\otimes} is a functor preserving cocartesian arrows





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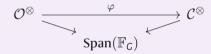
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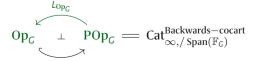
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Construction

$$\mathcal{O}(S) \xrightarrow{\hspace{1cm}} \operatorname{Mor}(\mathcal{O}^{\otimes})$$

$$\downarrow \qquad \qquad \downarrow$$

$$\operatorname{nd}_{H}^{G}S = \operatorname{Ind}_{H}^{G}S \rightarrow [G/H] \} \longrightarrow \operatorname{Mor}(\operatorname{Span}(\mathbb{F}_{G}))$$

The underlying G-symmetric sequence

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Construction

Given $\mathcal{O}^{\otimes} \in \operatorname{Op}_{C}$, $H \subset G$, and $S \in \mathbb{F}_{G}$, define the **structure space**

$$\begin{array}{ccc} \mathcal{O}(S) & \longrightarrow & \operatorname{Mor}(\mathcal{O}^{\otimes}) \\ & & \downarrow & & \downarrow \\ \left\{\operatorname{Ind}_{H}^{G}S = \operatorname{Ind}_{H}^{G}S \to [G/H]\right\} & \longrightarrow & \operatorname{Mor}\left(\operatorname{Span}(\mathbb{F}_{G})\right) \end{array}$$

The underlying G-symmetric sequence

Interchange and little V-disks

Natalie Stewart

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Proposition (S.¹⁹)

If \mathcal{O}^{\otimes} has "one color" then is it conservatively identified by $(\mathcal{O}(S))_{\substack{H \subset G \\ S \in \mathbb{F}_H}}$

¹⁹ Natalie Stewart. Equivariant operads, symmetric sequences, and Boardman-Vogt tensor products. 2025. arXiv: 2501.02129 [math.CT].

Example

$$\mathbb{E}_{V}(S) := \operatorname{Emb}^{H,\operatorname{Affine}}(S \cdot D(V), D(V)) \simeq \operatorname{Conf}_{S}^{H}(V)$$

$$\operatorname{End}_X(S) \simeq \operatorname{Map}_{\mathcal{C}(G/H)}\left(\left(\operatorname{Res}_H^GX\right)^{\otimes S},\operatorname{Res}_H^GX\right)$$

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Example

The little V-disks G-operad has S-ary structure space given by H-equivariant configurations of S in V:

$$\mathbb{E}_{V}(S) := \text{Emb}^{H,\text{Affine}} \left(S \cdot D(V), D(V) \right) \simeq \text{Conf}_{S}^{H}(V).$$

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Example

Given a G-symmetric monoidal category C^{\otimes} : Span(\mathbb{F}_{G}) \to Cat $_{\infty}$, its unstraightening is a G-operad. Given $X \in \mathcal{C}(G/G)$, there is an **endomorphism** G-operad with

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$$End_{X}(S) \simeq Map_{\mathcal{C}(G/H)}\left(\left(Res_{H}^{G}X\right)^{\otimes S}, Res_{H}^{G}X\right)$$

In particular, an \mathbb{E}_{V} -algebra in \mathcal{C} consists of an object $X \in \mathcal{C}_{G}$ and homotopy-coherently compatible maps

$$\operatorname{\mathsf{Conf}}^{\mathsf{H}}_{\mathsf{S}}(V) o \operatorname{\mathsf{Map}}_{\mathcal{C}(\mathsf{G}/\mathsf{H})}\left(\left(\operatorname{\mathsf{Res}}^{\mathsf{G}}_{\mathsf{H}}X\right)^{\otimes \mathsf{S}},\operatorname{\mathsf{Res}}^{\mathsf{G}}_{\mathsf{H}}X\right).$$

Example (Horev-Klang-Zou²⁰ '20)

Let $\underline{\mathcal{S}}_G^{G-\times}$ be the *cartesian structure* on *G*-spaces. Then, for all $X \in \mathcal{S}_G$, we have $\Omega^V X \in \mathrm{Alg}_{\mathbb{R}_V}(\mathcal{S}_G)$.

Example (loc. cit.)

Let $\underline{\mathrm{Sp}}_{\mathsf{G}}^{\otimes}$ be the HHR G-symmetric monoidal structure. If $f \colon \Omega^{\mathsf{V}} \mathsf{X} \to \underline{\mathrm{Pic}}(\underline{\mathrm{Sp}}_{\mathsf{G}})$ is a V-loop map, then $\mathrm{Th}(f) \in \mathrm{Alg}_{\mathbb{E}_V}(\mathrm{Sp}_{\mathsf{G}})$.

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²⁰⁾ Jeremy Hahn, Asaf Horev, Inbar Klang, Dylan Wilson, and Foling Zou. Equivariant nonabelian Poincaré duality and equivariant factorization homology of Thom spectra. 2024. arXiv. 2006. 13348. [math. 87]

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Model categorist: (co) fibrantly replace, then apply a construction to monoids in *G*-symmetric sequences.

 ∞ -categorist: apply a G-preoperadic construction, then L_{Op_G} -localize.

Shared goal: model corepresenting object for **pairings** (aka interchanging algebras, bifunctors, etc.) akin to May.²¹

$$\mathcal{O}^{\otimes} imes \mathcal{P}^{\otimes} \stackrel{ ext{"pairing"}}{\longrightarrow} \mathcal{Q}^{\otimes} \ \downarrow^{\pi} \ \mathcal{S}pan(\mathbb{F}_G) imes \mathsf{Span}(\mathbb{F}_G) \stackrel{\wedge}{\longrightarrow} \mathsf{Span}(\mathbb{F}_G)$$

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 $\mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$ lifts to a "pointwise" *G*-symmetric monoidal category $\mathrm{Alg}_{\mathcal{O}}^{\otimes}(\mathcal{C})$.

Proposition (S.²²

$$(-)\overset{\scriptscriptstyle{\mathbb{N}}}{\otimes}\mathcal{O}^{\otimes}$$
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$$Alg_{\mathcal{O}}\underline{Alg}_{\mathcal{P}}^{\otimes}(\mathcal{C}) \simeq Alg_{\mathcal{O}\otimes\mathcal{P}}(\mathcal{C}).$$

Also, have "wreath" operator \ and natural L_{Op}, -equivalences

$$\mathcal{O}^{\otimes} \overset{\text{\tiny inv}}{\otimes} \mathcal{P}^{\otimes} \leftarrow \mathcal{O}^{\otimes} \times \mathcal{P}^{\otimes} \rightarrow \mathcal{O}^{\otimes} \wr \mathcal{P}^{\otimes}.$$

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²² Natalie Stewart. Equivariant operads, symmetric sequences, and Boardman-Vogt tensor products. 2025. arXiv: 2501.02129 [math.CT].

Definition

$$\mathcal{O}^{\otimes} \overset{\text{\tiny inv}}{\otimes} \mathcal{P}^{\otimes} := L_{\operatorname{Op}_G} \left(\mathcal{O}^{\otimes} \times \mathcal{P}^{\otimes} \longrightarrow \operatorname{Span}(\mathbb{F}_G)^2 \overset{\wedge}{\longrightarrow} \operatorname{Span}(\mathbb{F}_G) \right)$$

 $\mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$ lifts to a "pointwise" G-symmetric monoidal category $\mathrm{Alg}_{\mathcal{O}}^{\otimes}(\mathcal{C})$.

Proposition (S.²²)

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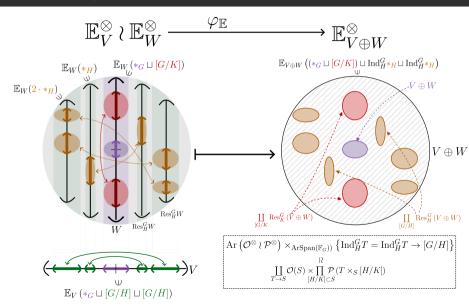
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Definition

A *G-category* is a coefficient system of ∞ -categories $\mathcal{C}\colon \mathcal{O}_{\mathcal{C}}^{\operatorname{op}} \to \operatorname{\mathsf{Cat}}$

Example

The terminal *G*-category has $*_G(G/H) = *$

Definition

A G-functor $\mathcal{C} \to \mathcal{D}$ is a natural transformation $F \colon \mathcal{C} \implies \mathcal{D}$. A G-object is a G-functor $X \colon *_G \to \mathcal{C}$.

The corresponding theory of *G*-(co)limits, *G*-adjunctions formalizes "*H*-set indexed (co)products." Moral for approximations: model (reflective) localizations of the underlying *G*-category.

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Given $\mathcal{P}^{\otimes} \in \operatorname{POp}_{\mathsf{C}}$, we let $\mathcal{P}^{\operatorname{act}}_{/P}$ be the ∞ -category of arrows $P' \to P$ projecting to a forward map $T = T \to S$.

Definition

A map of G-preoperads $\alpha\colon \mathcal{P}^\otimes\to\mathcal{O}^\otimes$ with \mathcal{O}^\otimes a "one color" G-operad is a **weak** approximation if

- 11 The G-category of colors UP has a terminal G-object, and
- 2 For all $P \in \mathcal{P}^{\otimes}$ and $T \to \pi(P)$, the map of spaces

$$B\left(\mathcal{P}_{/P}^{act}\times_{\mathbb{F}_{G,/\pi(P)}}\left\{T\to\pi(P)\right\}\right)\to\prod_{[H/K]\subset\pi(P)}\mathcal{O}\left(T\times_{\pi(P)}[H/K]\right)$$

is a weak equivalence.

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Proposition (Harpaz²³ '19 + reinterpretation)

If $\alpha\colon \mathcal{P}^\otimes \to \mathcal{O}^\otimes$ is a weak approximation, pullback is fully faithful

$$\mathrm{Alg}_{\mathcal{O}}(\mathcal{S}_G) \subset \mathrm{Alg}_{\mathcal{P}}(\mathcal{S}_G)$$

with image the \mathcal{P} -monoids whose "color" G-functors $U\mathcal{P} \to \underline{\mathcal{S}}_G$ are constan

Weak approximations can be made to have many colors; a weak approximation α is a **strong approximation** if $UP \to UO$ is an equivalence.

Proposition (S.²⁴)

 $\mathrm{Alg}_{(-)}(\mathcal{S}_G)$ detects L_{Op_G} -equivalences when $U\mathcal{P} \to U\mathcal{O}$ is an equivalence; in particular, strong approximations are L_{Op_G} -equivalences.

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²³ Yonatan Harpaz. Little cubes algebras and factorization homology (course notes).

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Proof sketch.

Examine the free O-G-space monad:²⁵

$$\left(T_{\mathcal{O}}X\right)^{H}\simeq\coprod_{S\in\mathbb{F}_{H}}\left(\mathcal{O}(S)\times\left(X^{S}\right)^{H}\right)_{h\operatorname{Aut}_{H}S}$$

The inclusion $\operatorname{Aut}_H(S) \subset \operatorname{End}_H(S) = \left(S^S\right)^H$ yields natural splitting

$$(T_{\mathcal{O}}S)^H \simeq \mathcal{O}(S) \sqcup Junk$$

Use monadicity of $\mathrm{Alg}_{\mathcal{O}}(\mathcal{S}_G) o \mathcal{S}_G$ and conservativity of $(\mathcal{O}(S))_{\substack{H \subset G \\ S \in \mathbb{F}_f}}$

²⁵ Natalie Stewart, Equivariant operads, symmetric sequences, and Boardman-Voat tensor products, 2025, arXiv: 2501,02129 [math.CT].

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²⁵ Natalie Stewart Faujuariant onergds, symmetric sequences, and Boardman-Voat tensor products, 2025, arXiv: 2501,02129 [math.CT].

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Proposition (Dugger-Isaksen²⁶ '01)

If X is a topological space and $\mathfrak{O} \subset \mathscr{P}(X)$ a basis of contractible open subsets,

$$B\mathfrak{O} \xrightarrow{\sim} X$$

$$\mathrm{B}\mathfrak{O}_{\mathsf{S}}^{\mathsf{H}}(\mathsf{V}) \stackrel{\sim}{\longrightarrow} \mathrm{Conf}_{\mathsf{S}}^{\mathsf{H}}(\mathsf{V}) \simeq \mathbb{E}_{\mathsf{V}}(\mathsf{S}).$$

²⁶ Daniel Dugger and Daniel C. Isaksen, "Topological hypercovers and A¹-realizations", In: Math. Z. 246.4 (2004), pp. 667–689.

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Proposition (Dugger-Isaksen²⁶ '01)

If *X* is a topological space and $\mathfrak{O}\subset \mathscr{P}(X)$ a basis of contractible open subsets, then we get a weak equivalence

$$B\mathfrak{O} \xrightarrow{\sim} X$$

Corollary

Let $\mathfrak{O}_{S}^{H}(V) \subset \operatorname{Conf}_{S}^{H}(V)$ be the basis of configurations in affinely $\coprod_{S} D(V)$ -shaped invariant subspaces of D(V). We get a weak equivalence

$$B\mathfrak{O}_{S}^{H}(V) \stackrel{\sim}{\longrightarrow} Conf_{S}^{H}(V) \simeq \mathbb{E}_{V}(S).$$

²⁶ Daniel Dugger and Daniel C. Isaksen. "Topological hypercovers and A¹-realizations". In: Math. Z. 246.4 (2004), pp. 667–689.

Proposition (Dugger-Isaksen²⁶, '01)

If X is a topological space and $\mathfrak{O} \subset \mathscr{P}(X)$ a basis of contractible open subsets, then we get a weak equivalence

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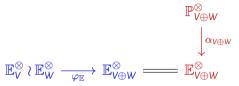
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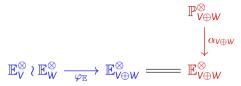
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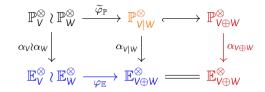
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Here, $\mathbb{P}_{V|W}^{\infty}$ is the "G-preoperadic image, i.e. "decomposed little disks."

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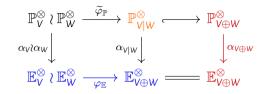
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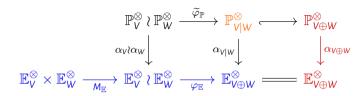
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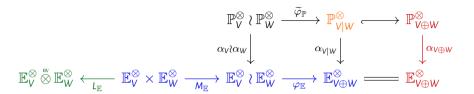
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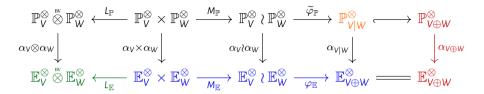
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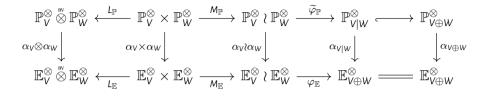
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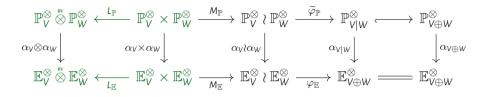
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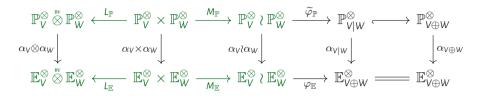
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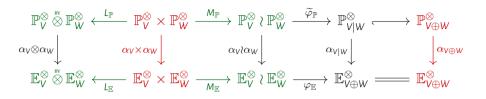
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²⁷ Yonatan Harpaz, Little cubes algebras and factorization homology (course notes).



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- Simple ∞ -category theory shows that $\alpha_V \times \alpha_W$ is a weak approximation.

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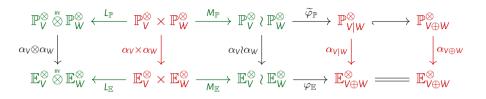
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- 4 A variant of Dunn's strategy²⁸ shows that $\alpha_{V|W}$ is a weak approximation.

27 Yonatan Harpaz, Little cubes algebras and factorization homology (course notes).

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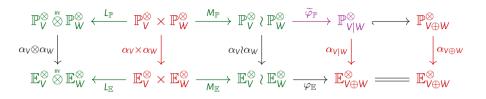
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²⁸ Gerald Dunn. "Tensor product of operads and iterated loop spaces", In: 1, Pure Appl, Algebra 50.3 (1988), pp. 237-258,



- **1** $L_{\mathbb{P}}$ and $L_{\mathbb{E}}$ are $L_{Op_{\mathbb{C}}}$ -equivalences by fiat.
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- **3** Simple ∞-category theory shows that $\alpha_V \times \alpha_W$ is a weak approximation.
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- **5** Explicit 1-category theory shows that $\widetilde{\varphi}_{\mathbb{P}}$ is a strong approximation.

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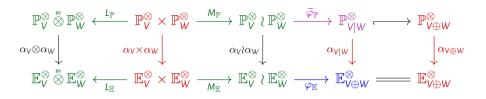
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²⁷ Yonatan Harpaz. Little cubes algebras and factorization homology (course notes).

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- **1** $L_{\mathbb{P}}$ and $L_{\mathbb{E}}$ are L_{Op_c} -equivalences by fiat.
- 2 A variant of Harpaz's strategy²⁹ shows that $M_{\mathbb{P}}$ and $M_{\mathbb{E}}$ are $L_{\mathrm{Op}_{\mathbb{C}}}$ -equivalences.
- Simple ∞ -category theory shows that $\alpha_V \times \alpha_W$ is a weak approximation.
- 4 A variant of Dunn's strategy³⁰ shows that $\alpha_{V|W}$ is a weak approximation.
- **5** Explicit 1-category theory shows that $\widetilde{\varphi}_{\mathbb{P}}$ is a strong approximation.
- Routine bookkeeping then shows that $Alg_{\mathbb{E}_{V \times \mathbb{E}_W}}(\mathcal{S}_G) \xrightarrow{\sim} Alg_{\mathbb{E}_V \times \mathbb{E}_W}(\mathcal{S}_G)$.

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²⁹ Yonatan Harpaz. Little cubes algebras and factorization homology (course notes).

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Define $\mathbb{E}_{B_{-}^{\text{lin}}\text{Top}(n)}^{\otimes}$ to have colors the linearizable *G*-actions on \mathbb{R}^n and operations the topological embeddings.

$$\mathbb{E}_{X}^{\otimes} := L_{\mathrm{Op}_{G}} \left(\mathbb{E}_{\beta_{G}^{\mathrm{lin}} \mathrm{Top}(n)}^{\otimes} \times_{\beta_{G}^{\mathrm{lin}} \mathrm{Top}(n)^{G-\sqcup}} X^{G-\sqcup} \right).$$

$$\underline{\text{colim}}_{x \in X} \mathbb{E}_{T_x}^{\otimes} \overset{\sim}{-\!\!\!\!-\!\!\!\!-\!\!\!\!-}} \mathbb{E}_X^{\otimes}$$

Interchange and little V-disks

Natalie Stewart

Define $\mathbb{E}_{B_{-}^{\text{lin}}\text{Top}(n)}^{\otimes}$ to have colors the linearizable *G*-actions on \mathbb{R}^n and operations the topological embeddings.

Given a G-space X with stalkwise-linearizable equivariant \mathbb{R}^n -bundle $T_{\bullet}: X \to B_C^{\text{lin}} \text{Top}(n)$, we define the assembly

$$\mathbb{E}_{X}^{\otimes} := L_{\operatorname{Op}_{G}} \left(\mathbb{E}_{\mathcal{B}_{G}^{\operatorname{lin}} \operatorname{Top}(n)}^{\otimes} \times_{\mathcal{B}_{G}^{\operatorname{lin}} \operatorname{Top}(n)^{G-\sqcup}} X^{G-\sqcup} \right).$$

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Interchange and little V-disks

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Theorem (S.³¹)

There is a natural equivariant colimit expression of operads

$$\underline{\text{colim}}_{x \in X} \mathbb{E}_{T_x}^{\otimes} \overset{\sim}{-\!\!\!\!-\!\!\!\!-\!\!\!\!-}} \mathbb{E}_X^{\otimes}$$

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³¹ Natalie Stewart. On homotopical additivity of equivariant little disks operads (draft on website). 2025.

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³¹ Natalie Stewart. On homotopical additivity of equivariant little disks operads (draft on website), 2025.

Corollary (S.)

Given stalkwise-linearizable equivariant \mathbb{R}^k -bundles $X \to B_G^{\text{lin}} \text{Top}(n)$ and $Y \to B_G^{\text{lin}} \text{Top}(m)$, fixing the direct sum structure

$$X \times Y \longrightarrow B_G^{\text{lin}} \text{Top}(n) \times B_G^{\text{lin}} \text{Top}(m) \stackrel{\oplus}{\longrightarrow} B_G^{\text{lin}} \text{Top}(n+m),$$

there is an equivalence.

$$\mathbb{E}_X^{\otimes} \overset{\scriptscriptstyle{\mathrm{BV}}}{\otimes} \mathbb{E}_Y^{\otimes} \overset{\sim}{-\!\!\!\!-\!\!\!\!-\!\!\!\!-}} \mathbb{E}_{X\times Y}^{\otimes}.$$

Other cases that are covered by the original argument:

- (Equivariant) swiss cheese
- Stratified little disks

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Let $\underline{\mathrm{Disk}}_G^{X-\mathrm{fr},\sqcup} \subset \underline{\mathrm{Mfld}}_G^{X-\mathrm{fr},\sqcup}$ be the G-symmetric monoidal full category whose objects are disjoint unions of X-framed G-disks and whose mapping spaces are

Corollary (S., c.f. Dwyer-Hess-Knudsen³² '19)

There is a G-symmetric monoidal equivalence

X-framed equivariant disk embeddings.

$$\underline{\mathrm{Disk}}_{G}^{X-\mathrm{fr},\sqcup} \Box \underline{\mathrm{Disk}}_{G}^{Y-\mathrm{fr},\sqcup} \simeq \underline{\mathrm{Disk}}_{G}^{X\times Y-\mathrm{fr},\sqcup}$$

Here, \square is the box product of semi-Mackey functors valued in Cat $_{\infty}$.

³² William Dwyer, Kathryn Hess, and Ben Knudsen. Configuration spaces of products. 2018. arXiv: 1710.05093 [math.AT].

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Corollary (S., c.f. Dwyer-Hess-Knudsen³² '19)

There is a G-symmetric monoidal equivalence

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This presentation was made in Beamer, with figures via tikz-cd and Inkscape, presented via Impressive. The tex is on my website. The title slide is $H_{C_2}^{\star}$ (* C_2 ; $\mathbb Z$) [6].