CONNECTIVITY OF EQUIVARIANT CONFIGURATION SPACES AND $\mathbb{E}_V$-ALGEBRAIC WIRTHMÜLLER MAPS

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Abstract. We provide conditions on a locally smooth G-manifold and finite G-set S under which its spaces of equivariant configurations $\mathrm{Conf}_S^G(X)$ are n-connected; they are sharp when X is a real G-representation. This specializes to give simple sharp conditions under which the fibers of the (finite-index) $\mathbb{E}_V$-algebra Wirthmüller maps are n-connected.

Throughout this paper, we fix G a Lie group.

Definition 1. If $H \subset G$ is a closed subgroup, $S \in \mathbb{F}_H$ an H-set with finitely many orbits, and X a topological space with G-action, we denote the subspace of H-equivariant embeddings $S \hookrightarrow M$ by

$$\mathrm{Conf}_S^H(X) \subset \mathrm{Map}^H(S, X). \qquad \triangleleft$$

Nonequivariantly, the homotopy type of configurations spaces in X is a rich source of homeomorphism-invariants of X [LS04]. In this paper, we study some rudiments of an equivariant lift of this in the smooth setting. Namely, in Section 1, we supply sufficient conditions for a smooth G-manifold M such that its nonempty configurations spaces $\mathrm{Conf}_S^G(M)$ are all d-connected.

We have a particular application in mind: the structure spaces of the *little V-disks operad* are configuration spaces in smooth G-manifolds, and connectivity statements of G-operads translate to structural statements about their algebras. In particular, we prove the following, where $\mathbb{E}_V$ is the *finite-index proper equivariant little V-disks G-operad* of [Ste25c].[1]

Theorem A. *Suppose $H \subset G$ is a compact closed subgroup, V an orthogonal G-representation, and $S \in \mathbb{F}_H$ a finite H-set admitting an embedding into $\mathrm{Res}_H^G V$. The following conditions of (H, S, V, n) are equivalent.*

(i) Both of the following are satisfied:
   - *(a) If $\left|S^H\right| \geq 2$, then $\dim V^H \geq n+2$, and*
   - *(b) For all orbits $[H/K] \subset S$ and intermediate subgroups $K \subset J \subset H$, we have*

$$\dim V^K \geq \dim V^J + n + 2;$$

(ii) the space $\mathrm{Conf}_S^H(V)$ is n-connected; and

(iii) the S-indexed $\mathbb{E}_V$-H-space Wirthmüller map

$$W_{S,(X_K)} \colon \coprod_K^S X_K \to \prod_K^S X_K$$

is n-connected for all $(X_K) \in \prod_{[H/K]\subset S} \mathrm{Alg}_{\mathbb{E}_{\mathrm{Res}_K^G V}}(\underline{\mathcal{S}}_K)$.

Here, the $[H/K]$-*indexed Wirthmüller map* in a G-category is the comparison map $\mathrm{Ind}_K^H X \to \mathrm{CoInd}_K^H X$, which is adjunct to the map $X \to \mathrm{Res}_K^H \mathrm{CoInd}_K^H X \simeq \prod_{g \in [K\backslash H/K]} \mathrm{CoInd}_{gKg^{-1}}^H \mathrm{Res}_{gKg^{-1}}^H X$ which projects to the identity when $g = e$ and 0 otherwise. For instance, the *Wirthmüller isomorphism theorem* (for finite-index inclusions) states that this is an equivalence in the equivariant Spanier-Whitehead category [Wir75].

The S-indexed Wirthmüller map is a combination of this with the ordinary semiadditive norm map, and is centered in [CLL24; Nar16]; in a pointed G-category, writing $\coprod_{K_i}^H X_{K_i} := \coprod_{S=\coprod_i [H/K_i]} \mathrm{Ind}_{K_i}^H X_{K_i}$ and similar for

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[1] In the case that G is finite, this recovers all other notions of $\mathbb{E}_V$.

$\prod_{K_i}^H X_{KM_i}$, the S-Wirthmüller map is classified by the diagonal matrix

$$W_{X,S} = 1\begin{bmatrix} W_{X_{K_1},[H/K_1]} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & W_{X_{K_\ell},[H/K_\ell]} \end{bmatrix}$$

We briefly mention an easy corollary to this in equivariant stable homotopy theory; define the full subcategory G-spectra with genuine equivariant homotopy concentrated in a given interval

$$\mathrm{Sp}_{G,[d,d+n]} := \left\{X \in \mathrm{Sp}_G \;\middle|\; \forall \ell \notin [d,d+n], H \subset G, \quad \text{we have } \pi_\ell^H(X) \simeq 0\right\} \subset \mathrm{Sp}_G.$$

Corollary B. *Fix $n \in \mathbb{N} \cup \{\infty\}$, $d \in \mathbb{Z}$, and V an orthogonal G-representation such that*

$$\dim V^G,\ \dim\left(V^K/V^H\right) \in \{0\} \cup [n,\infty] \qquad \forall\, K \subset H \subset G \tag{1}$$

Then, the forgetful functor yields an equivalence

$$U\colon \mathrm{Alg}_{\mathbb{E}_{\infty V}}\left(\underline{\mathrm{Sp}}_{G,[d,d+n-2]}\right) \xrightarrow{\sim} \mathrm{Alg}_{\mathbb{E}_V}\left(\underline{\mathrm{Sp}}_{G,[d,d+n-2]}\right)$$

Remark 2. The *complete* case of Eq. (1) would simply state that $\dim V^G, \dim\left(V^k/V^H\right) \geq n$ for all $K \subset H \subset G$; this is simply the condition appearing in Theorem A simultaneously for the fixed (V,n) and for all (H,S). It's worthwhile to note that the case $n = 2$ of this has appeared in [Lew92], where such V would be called *sufficiently large*, and it would be concluded that $[\Sigma_+^V G/H, \Sigma_+^V G/K] \simeq \pi_0 \mathrm{Map}_{\mathrm{Sp}_G}\left(\Sigma_+^{\rho\infty} G/H, \Sigma_+^{\rho\infty} G/K\right)$ by a correction of a result of Namboodiri [Nam83]; this constructs a natural Mackey structure on $\underline{\pi}_V(-)$. We may view Theorem A and Corollary B simultaneously as *algebraic analogues* of this stability result, as *incomplete analogues*, and as analogues which allow for n-types for all $n \in [0,\infty]$ rather than simply 0-types. ◁

1. Configuration spaces in smooth G-manifolds

Definition 3 ([Bre72, § IV])**.** If M is a smooth manifold with a continuous G-action, we say that the action is *locally smooth* if, for each point $x \in M$, there exists a real orthogonal $\mathrm{stab}_G(x)$-representation V_x and a trivializing open neighborhood

$$x \in \coprod_{G/\mathrm{stab}_G(x)} V_x \hookrightarrow M,$$

where for a topological H-space X, we write $\coprod_{G/\mathrm{stab}_G(x)} X := G \times_H X$ as a topological G-space. In this case, we say that M with its action is a *locally smooth G-manifold.* ◁

Smooth actions on manifolds admit well-behaved tubular neighborhoods; for example, [Bre72, Cor VI.2.4] proves that smooth actions are locally smooth. On the other hand, if M is a locally smooth G-manifold, then the inclusion $M_{(H)} \hookrightarrow M$ of points with orbit isomorphic to G/H is a locally closed topological submanifold [Bre72, Thm IV.3.3], which is smooth if M is smooth [Bre72, Cor VI.2.5].

We begin this section in Section 1.1 by proving the following.

Theorem 4 (Equivariant Fadell-Neuwirth fibration)**.** *Fix M a locally smooth G-manifold, $S,T \in \mathbb{F}_G$ a pair of orbit-finite G-sets, and $\iota\colon S \hookrightarrow M$ a G-equivariant configuration. The following is a homotopy-Cartesian square:*

$$\begin{tikzcd} \mathrm{Conf}_T^G(M - \iota(S)) \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & \mathrm{Conf}_{S \sqcup T}^G(M) \arrow[d, "U"] \\ \{\iota\} \arrow[r, hook] & \mathrm{Conf}_S^G(M) \end{tikzcd}$$

Thus the long exact sequence in homotopy for $T = G/H$ yields means for computing homotopy groups of $\mathrm{Conf}_S^G(M)$ inductively on the cardinality of the orbit set $|S_G|$, with inductive step hinging on homotopy of

$$\mathrm{Conf}_{G/H}^G(M - \iota(S)) \simeq (M - \iota(S))_{(H)}.$$

Remark 5. In the case that T and S satisfy the condition that, for all $x \in T$ and $y \in S$, $\mathrm{stab}_G(x)$ and $\mathrm{stab}_G(y)$ are non-conjugate, U is actually a trivial fiber bundle [BQV23]. Furthermore, defining the *unordered* configuration space $C_S^G(M) := \mathrm{Conf}_S^G(M)/\mathrm{Aut}_G(S)$, we acquire a splitting of principal bundles

$$\begin{tikzcd} \mathrm{Conf}_S^G(M) \arrow[d] & \simeq & \prod_{(H)\subset G} \mathrm{Conf}_{S_{(H)}}^G(M) \arrow[d] & \simeq & \prod_{(H)\subset G} \mathrm{Conf}_{S_{(H)}}^G\left(M_{(H)}\right) \arrow[d] \\ C_S^G(M) & \simeq & \prod_{(H)\subset G} C_{S_{(H)}}^G(M) & \simeq & \prod_{(H)\subset G} C_{S_{(H)}/G}\left(M_{(H)}/G\right) \end{tikzcd}$$

Unfortunately, this does not obviate Theorem 4, as it appears to often be more convenient to determine invariants of $\mathrm{Conf}_{n\cdot[G/H]}^G(M)$ inductively using Theorem 4 than to determine those invariants on $C_{S_{(H)}/G}\left(M_{(H)}/G\right)$ and pull back along the defining principal $\mathrm{Aut}_G(n\cdot[G/H]) \simeq W_G H \wr \Sigma_n$-bundle. ◃

We denote by $[\mathcal{O}_G]^{\mathrm{op}}$ the subconjugacy lattice of closed subgroups of G, and we let

$$\mathrm{Istrp}(M) = \{\mathrm{stab}_x(G) \mid x \in M\} \subset [\mathcal{O}_G]^{\mathrm{op}}$$

be the full subposet spanned by conjugacy classes (H) for which $M_{(H)}$ is nonempty. We are inspired to make the following definition.

Definition 6. Fix a G-set S with finitely many orbits. A locally smooth G-manifold M is

- $\geq d$*-dimensional at the isotropy of* S if $M_{(H)}$ is $\geq d$-dimensional for each $[G/H] \subset S$;
- $(d-2)$*-connected at the isotropy of* S if $M_{(H)}$ is $(d-2)$-connected for each $[G/H] \subset S$. ◃

In Section 1.2, we observe that Theorem 4 implies the following.

Corollary 7. *Fix G a compact Lie group, M a locally smooth G-manifold, and S a G-set with finitely many orbits. If M is $\geq d$-dimensional and $(d-2)$-connected at the isotropy of S, then $\mathrm{Conf}_S^G(M)$ is $(d-2)$-connected.*

In order to identify applications of this theorem, we give sufficient conditions for M to be $(d-2)$-connected at each orbit type. Note by repeatedly applying [Bre72, Thm IV.3.1] that the subspace $M_{\supseteq(H)} \subset M$ consisting of points of orbit type $[G/K]$ a quotient of $[G/H]$ is a disjoint union of closed submanifolds. In Section 1.3, we use this to prove the following.

Proposition 8. *Suppose that M is a smooth G-manifold satisfying the following conditions:*

(a) M is $\geq d$-dimensional at the isotropy of S.
(b) $M_{\supseteq(H)}$ is $(d-2)$-connected whenever $[G/H] \subset S$.
(c) $\mathrm{codim}(M_{\supseteq(K)} \hookrightarrow M_{\supseteq(H)}) \geq d$ for each $[G/H] \subset S$ and $H \subset K$.
(d) $\mathrm{Istrp}(M)$ is finite (e.g. G compact and M finite type, c.f. [Bre72, Thm IV.10.5]).

Then M is $(d-2)$-connected at the isotropy of S.

1.1. A Fadell-Neuwirth fibration for equivariant configurations.

Our strategy for Theorem 4 mirrors that of Knudsen in the notes [Knu18]. In particular, we would like to use Quillen's theorem B [Qui73], which requires us to construct $\mathrm{Conf}_S^H(M)$ as a classifying space. In fact, there is a general scheme to do this:

Lemma 9 ([DI04, Thm 2.1], via [Knu18, Thm 4.0.2])**.** *If $\mathcal{B}$ is a topological basis for X such that all elements of $\mathcal{B}$ are weakly contractible, then the canonical map*

$$|\mathcal{B}| = \mathrm{hocolim}_{\mathcal{B}} * \to X$$

is a weak equivalence, where on the left $\mathcal{B}$ is considered as a poset under inclusion.

To use this, define an elementwise-contractible basis for $\mathrm{Conf}_S^G(M)$ by

$$\widetilde{\mathcal{B}}_S^G(M) := \left\{ (X,\sigma) \;\middle|\; \exists (V_x) \in \prod_{[x]\in \mathrm{Orb}_S} \mathbf{Rep}_{\mathbb{R}}^{\mathrm{orth}}(\mathrm{stab}_G([x])), \text{ s.t. } \coprod_{[x]}^{S} V_x \simeq X \subset M,\; \sigma : S \xrightarrow{\sim} \pi_0(U) \right\},$$

where for all tuples $(Y_x) \in \prod_{[x] \in \mathrm{Orb}_S} \mathrm{Top}_{\mathrm{stab}_G([x])}$, we write

$$\coprod_{[x]}^{S} Y_x := \coprod_{[x] \in \mathrm{Orb}(S)} \left(G \times_{\mathrm{stab}_G([x])} Y_x \right) \in \mathrm{Top}_G$$

for the *indexed disjoint union of* Y_x. We fix $\mathcal{B}_S^G(M) \subset \widetilde{\mathcal{B}}_S^G(M)$ the smaller basis consisting of open sets (X, σ) possessing neighborhoods $(X, \sigma) \subset (X', \sigma)$ such that the associated embeddings factor as

$$\begin{tikzcd} \coprod_U^S D(V_U)^\circ \arrow[r, phantom, "\simeq"] \arrow[d, hook, "\exists"'] & \coprod_U^S V_U \arrow[d, hook, "X"] \\ V'_U \arrow[r, hook, "X'"] & M \end{tikzcd} \tag{2}$$

where $D(V_U)^\circ$ denotes the open unit V_U-disk; that is, open sets in $\mathcal{B}_S^G(M)$ consist of collections of configurations possessing a fixed common neighborhood resembling disjoint unions of real orthogonal representations, subject to the condition that there is "space on all sides" of the neighborhood. This is functorial in two ways:

- given a summand inclusion $S \hookrightarrow T \sqcup S$, the forgetful map $\mathrm{Conf}_{T \sqcup S}^G(M) \to \mathrm{Conf}_S^G(M)$ preserves basis elements, inducing a map $\mathcal{B}_{T \sqcup S}^G(M) \to \mathcal{B}_S^G(M)$.
- any open embedding $\iota : M \hookrightarrow N$ induces a map $\mathrm{Conf}_T^G(M) \hookrightarrow \mathrm{Conf}_T^G(N)$ preserving basis elements, inducing a map $\mathcal{B}_S^H(M) \to \mathcal{B}_S^H(N)$.

To summarize, we've observed the proof of following lemma.

Lemma 10. *Given $H \subset G$ and $S, T \in \mathbb{F}_H$, there is an equivalence of arrows*

$$\begin{tikzcd} \left|\mathcal{B}_{T \sqcup S}^G(M)\right| \arrow[r, phantom, "\simeq"] \arrow[d] & \mathrm{Conf}_{T \sqcup S}^G(M) \arrow[d] \\ \left|\mathcal{B}_S^G(M)\right| \arrow[r, phantom, "\simeq"] & \mathrm{Conf}_S^G(M) \end{tikzcd}$$

Thus we can characterize the homotopy fiber of U using Quillen's theorem B and the following.

Proposition 11. *For $(X_S, \sigma_S) \le (X'_S, \sigma'_S) \in \mathcal{B}_S^G(M)$, and an S-configuration $\mathbf{x} \in X_S$, we have a diagram*

$$\begin{tikzcd} \mathcal{B}_T^G(M - \mathbf{x}) & \mathcal{B}_T^G(M - \overline{X}_S) \arrow[l, "\varphi"] \arrow[d, phantom, "\wr"] & \mathcal{B}_T^G(M - \overline{X'}_S) \arrow[l] \arrow[ll, bend right, "\varphi"'] \arrow[d, phantom, "\wr"] \\ & ((X_S, \sigma_S) \downarrow U) & \left((X'_S, \sigma'_S) \downarrow U\right) \arrow[l] \end{tikzcd}$$

such that the maps φ induce weak equivalences on classifying spaces.

We will power this with the following observation:

Observation 12. Recall that an embedding of topological G-spaces $f : Y \hookrightarrow Z$ is a *G-isotopy equivalence* if there exists another G-equivariant embedding $g : Z \hookrightarrow Y$ and a pair of G-equivariant isotopies $gf \sim \mathrm{id}_Z$, $fg \sim \mathrm{id}_Y$. If $f : Y \to Z$ is a G-isotopy equivalence, then postcomposition with f induces a $G \times \Sigma_n$-isotopy equivalence $\mathrm{Conf}_n(Y) \hookrightarrow \mathrm{Conf}_n(Z)$; indeed, postcomposition with f and g induce G-equivariant embeddings, and postcomposition with the isotopies $gf \sim \mathrm{id}_Z$, $fg \sim \mathrm{id}_Y$ yields equivariant isotopies $\mathrm{Conf}_n(g) \circ \mathrm{Conf}_n(f) \sim \mathrm{Conf}_n(gf) \sim \mathrm{Conf}_n(\mathrm{id}_Z) \sim \mathrm{id}_{\mathrm{Conf}_n(Z)}$ and similar for fg.

In particular, the vertical arrows in the following diagram are isotopy equivalences

$$\begin{tikzcd} \mathrm{Conf}_S^H(X) \arrow[r, phantom, "\simeq"] \arrow[d] & \mathrm{Conf}_{|S|}(X)^{\Gamma_S} \arrow[r, phantom, "\simeq"] \arrow[d] & \mathrm{Map}^G\left(G \times \Sigma_{|S|}/\Gamma_S, \mathrm{Conf}_{|S|}(X)\right) \arrow[d] \\ \mathrm{Conf}_S^H(Y) \arrow[r, phantom, "\simeq"] & \mathrm{Conf}_{|S|}(Y)^{\Gamma_S} \arrow[r, phantom, "\simeq"] & \mathrm{Map}^G\left(G \times \Sigma_{|S|}/\Gamma_S, \mathrm{Conf}_{|S|}(Y)\right) \end{tikzcd}$$

where $\Gamma_S = \{(h, \rho_S(h)) \mid h \in H\} \subset G \times \Sigma_{|S|}$ is the graph subgroup corresponding with an H-set S with action map $\rho_S : H \to \Sigma_{|S|}$. Hence f induces a homotopy equivalence $\mathrm{Conf}_S^H(X) \xrightarrow{\sim} \mathrm{Conf}_S^H(Y)$. ◃

Proof of Proposition 11. The maps φ are each induced by the open inclusions $M - \overline{X}_S \hookrightarrow M - \mathbf{x}$, so the top horizontal arrows commute. The equivalences $\mathcal{B}_T^G(M - \overline{X_S}) \simeq ((X_S, \sigma_S) \downarrow U)$ simply follow by unwinding definitions. Thus we're left with proving that φ induces an equivalence on classifying spaces

$$\begin{tikzcd} \mathrm{Conf}_T^G(M - \mathbf{x}) \arrow[d, phantom, "\wr"] & \mathrm{Conf}_T^G(M - \overline{X}_s) \arrow[l] \arrow[d, phantom, "\wr"] \\ \left|\mathcal{B}_T^G(M - \mathbf{x})\right| & \left|\mathcal{B}_T^G(M - \overline{X}_s)\right| \arrow[l] \end{tikzcd}$$

By Observation 12, it suffices to show that $M - \overline{X_S} \hookrightarrow M - \mathbf{x}$ is a G-isotopy equivalence. In fact, by Eq. (2), it suffices to prove that the inclusion $f : V - D(V) \hookrightarrow V - \{0\}$ is a G-isotopy equivalence. But this is easy; scaling is equivariant, so we may define the G-equivariant embedding $g : V - \{0\} \to V - D(V)$ by $g(x) = \frac{1+|x|}{|x|} \cdot x$. Then, each of the equivariant isotopies $gf \sim \mathrm{id}, fg \sim \mathrm{id}$ can be taken as restrictions of $h(t,x) = \frac{1-t+|x|}{|x|} \cdot x$. □

We are ready to conclude our equivariant homotopical lift of [FN62, Thm 1].

Proof of Theorem 4. By the above analysis, we may replace our diagram with a homotopy equivalent diagram given by the geometric realiztion of the following diagram of posets, and prove that it is homotopy Cartesian

$$\begin{tikzcd} \mathcal{B}_T^G(M - \iota(S)) \arrow[r] \arrow[d] & \mathcal{B}_{T \sqcup S}^G(M) \arrow[d] \\ \{\iota\} \arrow[r, hook] & \mathcal{B}_S^G(M) \end{tikzcd}$$

By Quillen's theorem B [Qui73, Thm B], it suffices to prove two statements:

- for all basis elements (X_S, σ_S), The canonical map $((X_s, \sigma_s) \downarrow U) \to \mathcal{B}_T^G(M - \iota(S))$ induces a weak equivalence on classifying spaces, and
- for all inclusions of basis elements $(X_S, \sigma_S) \subset (X'_S, \sigma'_S)$, the canonical map $\left((X'_S, \sigma'_S) \downarrow U\right) \to ((X_S, \sigma_S) \downarrow U)$ induces a weak equivalence on classifying spaces.

In fact, both statements follow immediately from Proposition 11, with the second using two-out-of-three. □

1.2. **Connectivity at various orbits.** To prove Corollary 7, we begin with a lemma.

Lemma 13. *If M is a locally smooth G-manifold which is at least d-dimensional and $(d-2)$-connected at the isotropy of S and $\iota\colon [G/H] \hookrightarrow M$ a configuration, then $M - \iota(G/H)$ is at least d-dimensional and $(d-2)$-connected at the isotropy of S.*

Proof. We have

$$(M - \iota(G/H))_{(K)} = \begin{cases} M_{(K)} & G/K \neq G/H \\ M_{(H)} - \iota(G/H) & G/K = G/h, \end{cases}$$

so the only nontrivial case is $H = K$, in which case we're tasked with verifying that the complement of a discrete set of points in a d-dimensional $(d-2)$-connected manifold is $(d-2)$-connected. This is a well known fact in algebraic topology which follows quickly from the Blakers-Massey theorem. □

Proof of Corollary 7. If $d - 2 < 0$, there is nothing to prove, so assume that $d - 2 \geq 0$. We induct on $|S_G|$ with base case 1, i.e. with $S = G/H$. In this case, $\mathrm{Conf}_{G/H}^G(M) = M_{(H)}$ is $(d-2)$-connected by assumption.

For induction, fix some $S \sqcup G/H \in \mathbb{F}_G$ and inductively assume the theorem when $|T_G| \leq |S_G|$. Then, note that $\mathrm{Conf}_S^G(M)$ is $(d-2)$-connected by assumption and $\mathrm{Conf}_{G/H}^G(M - \iota(S))$ $(d-2)$-connected by the inductive hypothesis combined with Lemma 13. Thus Theorem 4 expresses $\mathrm{Conf}_{S \sqcup G/H}^G(M)$ as the total space of a homotopy fiber sequence with connected base and fiber, so it is connected. Furthermore, examining the long exact sequence associated with Theorem 4, we find that

$$\begin{tikzcd}
0 \arrow[r] \arrow[d, phantom, "\wr"] & \pi_k \operatorname{Conf}^G_{S \sqcup G/H}(M) \arrow[r] \arrow[dr] & 0 \arrow[d, phantom, "\wr"] \\
\pi_k \operatorname{Conf}^G_{G/H}(M - \iota(S)) \arrow[ur] & & \pi_k \operatorname{Conf}^G_S(M)
\end{tikzcd}$$

is exact for $0 < k \leq d-2$; hence $\operatorname{Conf}^G_{S \sqcup G/H}(M)$ is $(d-2)$-connected, completing the induction. $\square$

1.3. **Some sufficient conditions for connectivity at various orbits.** We begin with the following observations:

Observation 14. If M satisfies the conditions of Proposition 8, then $M_{\supseteq(H)}$ does as well. ◃

Observation 15. If M satisfies the conditions of Proposition 8 for $d-2 \geq 0$, then in particular, the orbit space M_G are a union of path-connected topological spaces $\left(M_{\supseteq(H)}\right)_G$ along path-connected intersections, so M_G is path-connected. In particular, by [Bre67, Thm IV.3.1], M has a *principle orbit type*, i.e. a minimal element of $(H_{\min}) \in \mathrm{Istrp}(M)$. Hence we have underlying space

$$M^e = M_{(H_{\min})},$$

so the underlying space M^e is $\geq d$-dimensional and $(d-2)$-connected. ◃

We will strengthen Proposition 8. Pick an order on $\mathrm{Istrp}(M) = (G/H_1, \ldots, G/H_n, G/G)$, and write

$$\begin{aligned}
M_k &= M - \bigcup_{i<k} M_{\supseteq(H_i)} \\
\widetilde{M}_k &= M_{\supseteq(H_k)} - \bigcup_{i<k} M_{\supseteq(H_k)} \cap M_{\supseteq(H_i)} \\
&= M_{\supseteq(H_k)} - \bigcup_{\substack{(K)\supseteq(H_k H_i) \\ i<k}} M_{\supseteq(K)}
\end{aligned}$$

Lemma 16. *For all k, the space M_k is $(d-2)$-connected.*

Proof. We induct in two ways:

- First, we inductively assume we have proved the lemma at full strength when G is replaced with any proper subgroup $H \subsetneq G$ such that $[G/H] \in \mathrm{Istrp}(M)$; since $\mathrm{Istrp}(M)$ is finite, this begins with the base case in which case there are no such proper subgroups.
- Second, we inductively assume that we have proved the lemma for all $k' < k$; this begins with the base case that $k = 1$, in which case we have $M_1 = M$, which is $(d-2)$-connected by Observation 15.

Under these assumptions, note that $\widetilde{M}_{k-1} \subset M_{k-1}$ is a $(d-2)$-connected closed submanifold of codimension $\geq d$ in a $(d-2)$-connected smooth manifold with complement is M_k. Thus it possesses a tubular neighborhood $\widetilde{M}_{k-1} \subset \tau(\widetilde{M}_{k-1}) \subset M_{k-1}$, and "hemmed gluing" presents a homotopy pushout square

$$\begin{tikzcd}
\partial\tau\widetilde{M}_{k-1} \arrow[r] \arrow[d, "\tilde{\iota}"] & M_k \arrow[d, "\iota"] \\
\widetilde{M}_{k-1} \arrow[r] \arrow[ur, phantom, "\ulcorner", very near end] & M_{k-1}
\end{tikzcd}$$

The boundary $\partial\tau\left(\tilde{M}_{k-1}\right)$ is the total space of a c-sphere bundle over a $(d-2)$-connected space, where

$$c = \operatorname{codim}(M_{\leq(H_k)} \hookrightarrow M) - 1 > d-2.$$

The long exact sequence in homotopy reads

$$\begin{tikzcd}[column sep=small]
\pi_1(S^c) \arrow[r] & \pi_1\left(\partial\tau\left(\widetilde{M}_{k-1}\right)\right) \arrow[r] & \pi_1\left(\widetilde{M}_{k-1}\right) \arrow[r] \arrow[dr] & 0 \arrow[r] \arrow[d, phantom, "\wr"] & \pi_0\left(\partial\tau\left(\widetilde{M}_{k-1}\right)\right) \arrow[r] & \pi_0\left(\widetilde{M}_{k-1}\right) \arrow[r] & 0 \\
& & & \pi_0(S^c V \arrow[ur] & & &
\end{tikzcd}$$

so $\partial\tau\widetilde{M}_{k-1}$ is connected, and when $d-2 \geq 1$, $\partial\tau\widetilde{M}_{k-1}$ is simply connected. Furthermore, at degree $0 < \ell \leq (d-2)$ the Gysin sequence in integral homology reads

$$\begin{tikzcd}
0 \arrow[r] \arrow[d, phantom, "\wr"] & H^\ell(\partial\tau\widetilde{M}_{k-1}) \arrow[r] \arrow[dr] & 0 \arrow[d, phantom, "\wr"] \\
H^\ell(\widetilde{M}_{k-1}) \arrow[ur] & & H^{\ell-c}(\widetilde{M}_{k-1})
\end{tikzcd}$$

so $\partial\tau\tilde{M}_{k-1}$ has vanishing cohomology in degrees $0 < \ell \leq d-2$. Hurewicz' theorem then implies that $\partial\widetilde{M}_{k-1}$ is $(d-2)$-connected.

In particular, this together with $(d-3)$-connectivity of the homotopy fiber S^c implies that $\widetilde{\iota}$ is a $(d-2)$-connected map, so its homotopy pushout ι is $(d-2)$-connected. Since M_{k-1} is a $(d-2)$-connected space by assumption, this implies that M_k is $(d-2)$-connected, completing the induction. □

We conclude the following equivalent statement of Proposition 8.

Proposition 17. *Suppose that $M_{(\supseteq H)}$ is $\geq d$-dimensional,* $\mathrm{Istrp}(M)$ *is finite, and for all inclusions $(H) \subset (K)$ in* $\mathrm{Istrp}(M)$*, the following conditions are satisfied:*

(b) $M_{\supseteq(K)}$ *is $(d-2)$-connected, and*

(c) $\mathrm{codim}\left(M_{\supseteq(K)} \hookrightarrow M_{\supseteq(H)}\right) \geq d$.

Then, $M_{(H)}$ is $(d-2)$-connected.

Proof. By Observation 14 we may assume $H = G$. This is precisely what is shown in Lemma 16 when $k = n+1$. □

Warning 18. Neither the conditions of Proposition 8 or of Corollary 7 are stable under restrictions for general G-manifolds; for instance, let $G = C_2 \times C_2$ be the Klein 4 group, and $H, H' \subset G$ a pair of distinct order-2 subgroups. Write σ, σ' for the inflated orthogonal G-representations from the sign representation of H and H'. Then for any $n \in \mathbb{N}$, define the $(2n-2)$-dimensional smooth G-manifold

$$M := (n-2)\cdot\sigma \oplus n \sqcup S\left((2n-1)\cdot\sigma'\right),$$

Note that M satisfies Proposition 8 for n, but its restriction to H is not $(n-2)$-connected at each orbit type when $n-2 \geq 0$, as $\left(\mathrm{Res}_H^G M\right)_{(e)} \simeq S^{2n-1} \sqcup D^n$ is not connected. ◁

1.4. **A variation: Fadell-Neuwirth fibrations of G-spaces.** Recall the *classifying G-space/G-groupoid $B_G\Sigma_n$ for genuine G-equivariant principal Σ_n-bundle*; one may observe that, unstraightening the coefficient system $\mathcal{O}_G \to \mathrm{Top}$ classifying $B_G\Sigma_n$ under Elmendorf's theorem, the structure fibration $B_G\Sigma_n \to \mathcal{O}_G$ is equivalent to the forgetful functor $\mathcal{O}_{G\times\Sigma_n,\Gamma_n} \to \mathcal{O}_G$ from the orbit category for the family of graph subgroups. In particular, we acquire a forgetful functor

$$\mathrm{Top}_{G\times\Sigma_n} \to \mathrm{Top}_{G\times\Sigma_n,\Gamma_n} \simeq \mathrm{Fun}(B_G\Sigma_n, \mathrm{Top}).$$

We let $\underline{\mathrm{Conf}}_n(M)$ denote the $B_G\Sigma_n$-space underlying $\mathrm{Conf}_n(M)$ with its $G\times\Sigma_n$ action.

Under the identification $\mathrm{Fun}(\mathcal{O}_G, B_G\Sigma_n) \simeq (B_G\Sigma_n)^G \simeq \{S \in \mathrm{Set}_G \mid |S| = n\}$, we acquire a functor $\chi_S \colon \mathcal{O}_G \to B_G\Sigma_S$, and we define the G-space

$$\underline{\mathrm{Conf}}_S(M) := \chi_S^* \underline{\mathrm{Conf}}_{|S|}(M).$$

Of course, this is determined under Elmendorf's theorem by the fixed points

$$\underline{\mathrm{Conf}}_S(M)^H \simeq \begin{cases} \mathrm{Conf}_S^H(M) & |G\colon H| < \infty \\ \varnothing & \text{otherwise,} \end{cases}$$

with the usual restrictions under Elmendorf's theorem. Moreover, this is *functorial*, in that a G-set summand inclusion $S \subset T$ yields a commutative diagram

$$\begin{tikzcd}
\mathcal{O}_G \arrow[r, "\chi_T"] \arrow[dr, "\chi_S"'] & B_G\Sigma_{|T|} \arrow[d, "r"] & \ni & \left[G\times\Sigma_{|T|}/(h, \rho_R(H) \mid h \in H)\right] \arrow[d, mapsto] \\
& B_G\Sigma_{|S|} & \ni & \left[G\times\Sigma_{|S|}/\left(h, \rho_{R\cap S^e}(H) \mid h \in \rho_R^{-1}(S^e)\right)\right]
\end{tikzcd}$$

yielding a mate natural transformation $\chi_T^* \implies \chi_S^* r_!$. Note that $r_! \underline{\mathrm{Conf}}_{|T|}(M) \simeq \underline{\mathrm{Conf}}_{|S|}(M)$, so we get a map

$$U_S^T : \underline{\mathrm{Conf}}_T(M) \to \underline{\mathrm{Conf}}_S(M),$$

natural in embeddings in M and satisfying $U_R^S U_S^T \sim U_R^T$. Similarly, given a configuration ι of a set of cardinality n in M, we have an evident $G \times \Sigma_n$ map

$$\mathrm{Conf}_m(M - \iota(n)) \to \mathrm{Conf}_{n+m}(M).$$

Now, this yields a commutative square

$$\begin{tikzcd} \underline{\mathrm{Conf}}_T(M - \iota(S)) \arrow[r] \arrow[d] & \underline{\mathrm{Conf}}_{T \sqcup S}(M) \arrow[d] \\ E_{\text{f.i.sub}} \arrow[r, "\{\iota\}"] & \underline{\mathrm{Conf}}_S(M) \end{tikzcd} \tag{3}$$

where the G-space $E_{G,\text{f.i.}}$ is determined up to homotopy by its fixed points

$$E_{G,\text{f.i.sub}}^H \simeq \begin{cases} * & |G : H| < \infty \\ \varnothing & \text{otherwise.} \end{cases}$$

Our variation of the equivariant Fadell-Neuwirth fibration is the following.

Theorem 19 (G-space Fadell-Neuwirth fibration)**.** *Eq. (3) is equivalent to a homotopy pullback square.*

Proof. Observe that the H-genuine fixed points of Eq. (3) is the equivariant Fadell-Neuwirth square of Theorem 4 for the restricted H-set, and apply the aforementioned theorem and Elmendorf's theorem. □

2. Representations, homotopy-coherent algebra, and configuration spaces

2.1. **Connectivity of $\mathrm{Conf}_S^H(V)$.** We begin by verifying that Conditions (i) and (ii) are equivalent.

Proposition 20. *If a real orthogonal G-representation V satisfies the conditions of Theorem A, then the smooth G-manifold $V - \{0\}$ is at least d-dimensional and $(d-2)$-connected at the isotropy of S.*

Proof. We may write V as a filtered (homotopy) colimit $V = \bigcup_i V_i$ with V_i a finite dimensional real orthogonal G-representation with $\min(i,d)$-codimensional fixed points; then, if V_i is $(i-2)$-connected for each i, taking a colimit, this implies that V is d-connected. Hence it suffices to prove this in the case we that V is finite dimensional.

In this case, G acts smoothly on V, and we make the following observations:

(a) $(V - \{0\})_{(H)} = \bigcup_{H' \in (H)} \left(V^{H'} - \bigcup_{H' \subset K} V^K \right) - \{0\}$ is either empty or $\max_{H' \in (H)} \left| V^{H'} \right| \geq d$-dimensional.
(b) $V_{\leq (H)} = \bigcup_{H' \in (H)} V^H$ is a union of contractible spaces along contractible intersections, so it is contractible and $\geq d$-dimensional; by the same argument as Lemma 13, $(V - \{0\})_{(H)} = V_{(H)} - \{0\}$ is $(d-2)$-connected.
(c) when $(K), (H) \in \mathrm{Istrp}(M)$,

$$\begin{aligned} \mathrm{codim}((V - \{0\})_{\leq (K)} \hookrightarrow (V - \{0\})_{\leq (H)}) &= \mathrm{codim}(V_{\leq (K)} \hookrightarrow V_{\leq (H)}) \\ &= \min_{\substack{H' \subset K \\ V^{H'} \neq V^K}} \left(\left| V^{H'} \right| - \left| V^K \right| \right) \\ &= \min_{\substack{H' \subset K \\ V^{H'} \neq V^K}} \left| V^H / V^K \right| \\ &\geq d \end{aligned}$$

by assumption, and it is nonzero since $V_{(H)}$ is nonempty.
(d) $\mathrm{Istrp}(V)$ is finite since V is finite dimensional.

Thus Proposition 8 applies, proving the proposition. □

Proposition 21. *If a finite-index inclusion of subgroups $K \subset H$ has $V^H \hookrightarrow V^K$ a proper inclusion of codimension $< d$, then $\mathrm{Conf}_{[G/K]}^G(V) \simeq V_{(K)}$ is not $(d-2)$-connected.*

Proof. This never occurs when V is 0-dimensional. If V^G is $0 < c < d$-dimensional, then we may directly see $\mathrm{Conf}^G_{2\cdot *_G}(V) = \mathrm{Conf}_2(V^G) = S^{c-1}$ is not $(d-2)$-connected, as it has nontrivial π_{c-1}. Thus we assume that V^G is $\geq d$-dimensional, so that V^H is $\geq d$-dim for all H.

Fix $c \coloneqq \min_{K \subsetneq H' \in \mathrm{Istrp}(V)} \mathrm{codim}\left(V^{H'} \hookrightarrow V^K\right)$. We may replace V with the real orthogonal G-representation $V^K = V_{(\geq K)}$. We're left with proving that $V_{(K)} = V - \bigcup_{K \subsetneq H' \in \mathrm{Istrp}(V)} V^{H'}$ is not $(d-2)$-connected. Pick an order $(H_i)_{1 \leq i \leq n}$ on $\mathrm{Istrp}(V) - \{(K)\}$ so that $c = \mathrm{codim}\left(V^{H'} \hookrightarrow V^K\right)$, and set the notation

$$V_\ell \coloneqq V - \bigcup_{i=1}^{\ell-1} V^{H_i}$$
$$\widetilde{V}_\ell \coloneqq V^{H_\ell} - \bigcup_{i=1}^{\ell-1} V^{H_i} \cap V^{H_\ell}$$

so that $V_1 = V \simeq *$ and $V_{n+1} = V_{(K)}$. Furthermore, note that $V_2 = V - V^{H_1} \simeq V^{H_1} \times S\left(V/V^{H_1}\right) \simeq S^{c-1}$; in particular, its reduced homology is

$$\widetilde{H}_m(V_2) = \begin{cases} \mathbb{Z} & n = c-1; \\ 0 & \text{otherwise.} \end{cases}$$

We argue via induction on ℓ that V_ℓ $\widetilde{H}_m(V_{\ell-1}) = 0$ when $m < c-1$ and that $\widetilde{H}_{c-1}(V_\ell)$ is nontrivial. The end of this induction implies the proposition; indeed, if $c-1 = 0$ then this directly implies that $V_{n+1} = V_{(H)}$ has at least two path components, and if $c-1 > 0$, then Hurewicz' theorem will imply that

$$\pi_{c-1}(V_{(K)})_{\mathbf{Ab}} = \pi_{c-1}(V_{n+1})_{\mathbf{Ab}} \simeq \widetilde{H}_{c-1}(V_{n+1}) \neq 0.$$

The base case $\ell = 2$ is satisfied by the above computation of $\widetilde{H}_m(V_2)$, so we inductively assume the statement is true for $\ell - 1$. Write $c_\ell \coloneqq \mathrm{codim}\left(V^{H_\ell} \hookrightarrow V\right)$. Note that the normal bundle of $V^{H_\ell} \subset V$ is a trivial D^{c_ℓ}-bundle; this restricts to the (trivial) normal bundle of $\widetilde{V}_{\ell-1} \subset V_{\ell-1}$, so the bounding $S^{c_\ell - 1}$ sphere bundle $\partial\tau\widetilde{V}_{\ell-1} \to V_\ell$ is trivial. Thus "hemmed gluing" presents a homotopy pushout square

$$\begin{tikzcd} S^{c_{\ell-1}} \times \widetilde{V}_{\ell-1} \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & V_\ell \arrow[d] \\ \widetilde{V}_{\ell-1} \arrow[r] & V_{\ell-1} \end{tikzcd}$$

If $c_\ell > c$, the left vertical arrow (hence the right vertical arrow) is a homology isomorphism in degrees $\leq c-1$, proving the inductive step. Furthermore, if $c_\ell = c$, then the vertical arrows are homology isomorphisms in degrees $\leq c-2$ and the associated map $\widetilde{H}_c(S^{c-1} \times \widetilde{V}_{\ell-1}) \to \widetilde{H}_c(\widetilde{V}_{\ell-1})$ is an isomorphism. This implies that $H_m(\widetilde{V}_\ell) = 0$ when $m < c-1$ and the Mayer-Vietoris sequence restricts to a short exact sequence

$$\begin{tikzcd} 0 \arrow[r] \arrow[dr] & \mathbb{Z} \oplus \widetilde{H}_{c-1}\left(\widetilde{V}_{\ell-1}\right) \arrow[r] \arrow[d, phantom, "\wr"] & \widetilde{H}_{c-1}\left(\widetilde{V}_{\ell-1}\right) \oplus \widetilde{H}_{c-1}(V_\ell) \arrow[r] & \widetilde{H}_{c-1}(V_{\ell-1}) \arrow[r] & 0, \\ & \widetilde{H}_{c-1}\left(S^{c-1} \times \widetilde{V}_{\ell-1}\right) \arrow[ur] & & & \end{tikzcd}$$

so that $\widetilde{H}_{c-1}(V_\ell) \neq 0$, as desired. □

Corollary 22. *Conditions (i) and (ii) are equivalent.*

Proof. We begin by noting

$$\mathrm{Conf}^H_S(V) = \begin{cases} \mathrm{Conf}^H_{S - *_H}(\mathrm{Res}^G_H(V - \{0\})) & S^H \neq \varnothing, \\ \mathrm{Conf}^H_S(\mathrm{Res}^G_H(V - \{0\})) & \text{otherwise.} \end{cases}$$

so it suffices to show $\mathrm{Conf}^H_S(\mathrm{Res}^G_H(V - \{0\}))$ to be $(d-2)$-connected or empty. Noting that the condition of having d-codmimensional fixed points is restriction-stable, this follows by Corollary 7 and Proposition 20. □

2.2. **Connectivity of Wirthmüller maps.** The following result of [Ste25a] is central.

Proposition 23 ([Ste25a])**.** *Let I be a unital weak indexing system. Then, the following conditions are equivalent:*

(a) for all $S \in \underline{\mathbb{F}}_I$, the space $\mathcal{O}(S)$ is n-connected.

(b) for all $S \in \underline{\mathbb{F}}_I$ and S-tuples $(X_K) \in \underline{\mathrm{Alg}}_{\mathbb{E}_V}(\underline{\mathcal{S}}_G)_S$, the Wirthmüller map

$$W_{S,(X_K)} \colon \coprod_K^S X_K \to \prod_K^S X_K$$

is n-connected.

(c) for all $S \in \underline{\mathbb{F}}_I$ and S-tuples $(X_K) \in \underline{\mathrm{Alg}}_{\mathbb{E}_V}(\underline{\mathcal{S}}_{G,\leq n})_S$, the Wirthmüller map

$$W_{S,(X_K)} \colon \coprod_K^S X_K \to \prod_K^S X_K$$

is an equivalence.

Corollary 24. *Let $\mathcal{O}^{\otimes}$ be a unital G-operad. Then, the collection of arities*

$$\underline{\mathbb{F}}_{\mathcal{O}}^{\mathrm{n-conn}} := \left\{ S \;\middle|\; \forall (X_K) \in \underline{\mathrm{Alg}}_{\mathbb{E}_V}(\underline{\mathcal{S}}_G)_S,\ W_{S,(X_K)} \text{ is } n\text{-connected} \right\}$$

is a unital weak indexing system.

Proof. This follows by combining Proposition 23 with [Ste25b]. □

The condition that $\mathbb{E}_V(S)$ is n-connected cuts out a unital weak indexing system as well.

Proposition 25. *Let $\underline{\mathbb{F}}^{\mathrm{n-conn}(V)} \subset \underline{\mathbb{F}}_G$ be the collection containing each S such that $\mathrm{Conn}_S^H(V)$ is n-connected. Then, $\underline{\mathbb{F}}^{\mathrm{n-conn}(V)}$ is a unital weak indexing system.*

Before showing this, we will see how it proves Theorem A.

Proof of Theorem A. After Corollary 22, it suffices to show that Condition (i) and Proposition 17 are equivalent, i.e. $\underline{\mathbb{F}}^{\mathrm{n-conn}(V)} = \underline{\mathbb{F}}_{\mathbb{E}_V}^{\mathrm{n-conn}}$; indeed, Proposition 25 verifies that $S \in \underline{\mathbb{F}}^{\mathrm{n-conn}(V)}$ if and only if $\underline{\mathbb{F}}_{I_S} \subset \underline{\mathbb{F}}^{\mathrm{n-conn}(V)}$, which together with Proposition 23 implies that

$$\underline{\mathbb{F}}^{\mathrm{n-conn}(V)} = \left\{ S \;\middle|\; \forall\, T \in \underline{\mathbb{F}}_{I_S},\ \mathbb{E}_V(T) \text{ is } n\text{-connected} \right\} = \underline{\mathbb{F}}_{\mathbb{E}_V}^{\mathrm{n-conn}}. \qquad \square$$

For the rest of this section, we verify Proposition 25 via a series of small claims.

Lemma 26. *Let V be an orthogonal G-representation.*

(1) For all H, $\varnothing_H \in \mathbb{F}_H^{\mathrm{n-conn}(V)}$.

(2) $S \in \mathbb{F}_H^{\mathrm{n-conn}(V)}$ if and only if $2 \cdot *_H \in \mathbb{F}_H^{\mathrm{n-conn}(V)}$ whenever $\left|S^H\right| \geq 2$ $[H/K] \in \mathbb{F}_H^{\mathrm{n-conn}(V)}$ whenever $[H/K] \subset S$.

(3) If $S \in \mathbb{F}_K^{\mathrm{n-conn}(V)}$ and $[H/K] \in \mathbb{F}_H^{\mathrm{n-conn}(V)}$, then $\mathrm{Ind}_K^H S \in \mathbb{F}_K^{\mathrm{n-conn}(V)}$.

(4) If $V_{(H)}$ and $V_{(K)}$ are $(n-2)$-connected, then $V_{(H\cap K)}$ is $(n-2)$-connected.

(5) $\underline{\mathbb{F}}^{\mathrm{n-conn}(V)}$ is closed under restriction.

Proof. (1) follows by contractibility of $\mathrm{Conf}_{\varnothing_H}^H(V)$; (2) follows by unwinding Corollary 22.

For (3), by (2), it suffices to prove the statement in the case that $S = [K/J]$. In this case, fix some $x \in H - J$, and note that it suffices to prove that $\mathrm{codim}\left(V^{\langle J,x\rangle} \hookrightarrow V^J\right) \geq n$. In fact, if $x \in K$, then this follows by Corollary 22 for $[K/J]$, so assume that $x \notin K$. Then, we get an intersection diagram

$$\begin{tikzcd} V^H \arrow[r, hook] & V^{\langle K,x\rangle} \arrow[r, hook, "b"] \arrow[d, hook] \arrow[dr, phantom, "\lrcorner"] & V^K \arrow[d, hook] \\ & V^{\langle J,x\rangle} \arrow[r, hook, "a"] & V^J \end{tikzcd} \tag{4}$$

where arrows are decorated with their codimension. We're tasked with proving that $a \geq n$, and by linear algebra, $a \geq b$, so it suffices to prove that $b \geq n$. By assumption, V embeds $[H/K]$, so $b > 0$; since $[H/K] \in \mathbb{F}_H^{\text{n-conn}(V)}$, we have $b \geq n$ by Corollary 22.

The proof of (4) is similar. Let $x \in G$ be an element not contained in $H \cap K$ satisfying the property that x does not stabilize $V^{H \cap K}$. Writing $J \coloneqq H \cap K$, we're tasked with proving that

$$\mathrm{codim}(V^{\langle J,x\rangle} \hookrightarrow V^J) \geq n.$$

We may assume without loss of generality that $x \notin K$, so we once again form Eq. (4) and find that $a \geq n$. To prove (5), note that the orbit types $[H/J]$ appearing in $\mathrm{Res}_H^G S$ are precisely those isomorphic with $J = H \cap gKg^{-1}$ with $[G/K] \subset S$, so this follows from (4) and Corollary 22. □

Proof of Proposition 25. First note that $*_H, \varnothing_H \in \mathbb{F}_H^{\text{n-conn}(V)}$ by Lemma 26, and $\mathbb{F}_H^{\text{n-conn}(V)}$ is closed under restriction by Lemma 26, so it suffices to prove that $\underline{\mathbb{F}}^{\text{n-conn}(V)}$ is closed under self-indexed coproducts. Fix some $S \in \mathbb{F}_H^{\text{n-conn}(V)}$ and $(T_K) \in \mathbb{F}_S^{\text{n-conn}(V)}$, and write $T \coloneqq \coprod_K^S T_K$.

By Lemma 26, we're faced with the case $T = 2 \cdot *_H$ or $T = [H/J]$ for $K \neq H$. In the former case, it follows that either $S = 2 \cdot *_H = T$ or $S = *_H$ and $T_H = 2 \cdot *_H = T$, so $T \in \underline{\mathbb{F}}^{\text{n-conn}(V)}$. In the latter case, $S = [H/K]$ and $T = [K/J]$, and the result follows from Lemma 26. □

2.3. A corollary about $\mathbb{E}_V$-ring spectra.

Definition 27. Let $\mathrm{Cat}^{\mathrm{st},trex}$ denote the ∞-category whose objects are stable ∞-categories with t-structure and whose morphisms are right t-exact functors. The *∞-category of fiberwise-stable I-symmetric monoidal ∞-categories with t-structure* is the full subcategory

$$\mathrm{Cat}_I^{\mathrm{st},trex,\otimes} \subset \mathrm{CMon}_I\left(\mathrm{Cat}^{\mathrm{st},trex}\right).$$

whose restriction functors are additionally left t-exact. ◁

In this case, $\mathcal{C}_{\geq a} \hookrightarrow \mathcal{C}$ is a well-defined G-symmetric monoidal subcategory and $\mathcal{C}_{\geq a} \to \mathcal{C}_{[a,b]}$ a well-defined G-symmetric monoidal localization.

Proposition 28. *Suppose $\mathcal{C}$ is a fiberwise-stable I-symmetric monoidal ∞-category with t-structure and $\mathcal{O}^{\otimes}$ a k-connected G-operad. Then, the forgetful functor*

$$\mathrm{CAlg}_{A\mathcal{O}}\left(\mathcal{C}_{[d,d+k]}\right) \to \mathrm{Alg}_{\mathcal{O}}\left(\mathcal{C}_{[d,d+k]}\right)$$

is an equivalence.

Proof. It suffices to note that $\mathcal{C}_{H,[d,d+k]}$ is a k-category and apply [Ste25b]. □

Example 29. For all $H \subset G$, we give $\mathrm{Sp}_H = \left(\underline{\mathrm{Sp}}_G\right)_H$ the homotopy t-structure, whose 0-connectives agree with slice 0-connectives, (see [Wil17] in the case $n = 0$). Its restriction functors are evidently t-exact and its binary tensors and norms are right t-exact [HHR16, Prop 4.26, Prop 4.33]. Corollary B follows immediately by combining Theorem A and Proposition 28. ◁

2.4. **A remark about homology.** One motivation for Theorem 4 is to contribute to the burgeoning area of *homology operations and cellularity* for $\mathbb{E}_V$-algebras. One expects a central role to be played by the *homology G-symmetric sequence*

$$\underline{H}_*^G(\mathbb{E}_V; R) \colon \coprod_n B_G\Sigma_n \longrightarrow \underline{\mathcal{S}}_G \xrightarrow{\underline{H}_*^G} \underline{\mathrm{Mack}}_G(\mathrm{Mod}_R)^{\mathbb{Z}},$$

where $\underline{\mathrm{Mack}}_G(\mathrm{Mod}_R)$ is the G-*category* of R-module Mackey functors, and the homology of *free $\mathbb{E}_V$-algebras*

$$\mathrm{Fr}_{\mathbb{E}_V} X \simeq \coprod_{n\in\mathbb{N}} (\mathrm{Conf}_n(V) \times X^n)_{h_G\Sigma_n} \simeq \underline{\mathrm{colim}}_{S \in \underline{\Sigma}_G} \underline{\mathrm{Conf}}_S^H(V)_{h\,\mathrm{Aut}_S(H)}$$

(this notation $\underline{\mathrm{colim}}$ notation refers to a G-colimit, see [Ste25b]). There are many ways to compute the latter; at any rate, in the case $X = *_G$, the Serre spectral sequence applied to the equivariant Borel fibration

$$\mathrm{Conf}_n(V) \to C_n(V) \simeq \mathrm{Conf}_n(V)_{h_G\Sigma_n} \to B_G\Sigma_n$$

should yield an *equivariant homotopy orbit spectral sequence* of R-module Mackey functors

$$E^2_{p,q}\colon \underline{H}^G_p(B_G\Sigma_n; \underline{H}^G_q(\mathrm{Conf}_n(V);R)) \implies \underline{H}^G_{p+q}(C_n(V);R).$$

There is a subtly here; $H_q(\mathrm{Conf}_n(V);R)$ must be remembered *as an equivariant local coefficient system on* $B_G\Sigma_n$, i.e. as a G-functor $B_G\Sigma_n \to \underline{\mathrm{Mack}}_G(\mathrm{Mod}_R)$. Thus in both cases, under the equivalence

$$\mathrm{Fun}_G\left(B_G\Sigma_n, \underline{\mathrm{Mack}}_G(\mathrm{Mod}_R)^{\mathbb{Z}}\right) \simeq \underline{\mathrm{Mack}}_{G\times\Sigma_n,\Gamma_n}(\mathrm{Mod}_R)^{\mathbb{Z}}$$

between G-equivariant Mackey functors with Σ_n-action and graph subgroup-genuine $G\times\Sigma_n$-Mackey functors, one is inspired to compute the $G\times\Sigma_n$-equivariant Bredon homology of $\underline{\mathbb{E}}_V(n) \simeq \mathrm{Conf}_n(V)$ with constant coefficients. Indeed, this Bredon homology Mackey functor has Γ_S-value

$$\underline{H}^G_*(\mathbb{E}_V;R)(\Gamma_S) \simeq H_*\left(\mathrm{Conf}_n(V)^{\Gamma_S};R\right) \simeq H_*\left(\mathrm{Conf}^H_S(V);R\right).$$

Thus a potentially helpful tool the R-module Serre spectral sequence for the Fadell-Neuwirth fibration

$$E^2_{p,q} = H_p\left(\mathrm{Conf}^G_{S\sqcup[G/H]}(M); H_q\left((M-S)_{(H)};R\right)\right) \implies H_{p+q}\left(\mathrm{Conf}^G_{S\sqcup[G/H]}(M);R\right).$$

The reader should note that $M=V$ and V admits an embedding of G/H, we may describe $\mathrm{Conf}^G_{[G/H]}(V-n\cdot[G/H]) \simeq (V-n\cdot[G/H])_{(H)}$ as a complement of a finite set in a central subspace complement

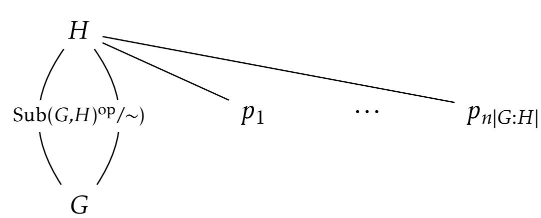
$$(V-n\cdot[G/H])_{(H)} = V^H - \bigcup_{H\subsetneq J} V^J - n\cdot[G/H],$$

and in particular the representation theory of V can be used to determine the homology via Goresky-MacPherson's variant of the Orlik-Solomon algebra [GM88, p. 238]. Indeed, let

$$P_V = \left\{V^{H_i}\cap\cdots\cap V^{H_n} \subset V \mid H_i,\dots,H_n \subset G\right\}$$

be the *poset of flats* for the subspace arrangement of fixed point loci of subroups.

Observation 30. The map $\mathrm{Sub}(G,H)\to P_V^{\mathrm{op}}$ is a poset quotient by the equivalence relation setting $H\sim H'$ whenever $V^H = V^{H'}$; in particular, the poset of flats of the subspace arrangement for $(V-n\cdot[G/H])_{(H)}$ is



i.e. it is $\mathrm{Sub}(G,H)^{\mathrm{op}}\sim$ with $n\cdot|G\colon H|$ additional points, whose only relations are that they are beneath the maximal element of $\mathrm{Sub}(G,H)^{\mathrm{op}}/\sim$. Writing $\mathfrak{S} := \mathrm{Sub}(G,H)^{\mathrm{op}}/\sim$, Goresky-MacPherson's theorem computes

$$H_*\left((V-n\cdot[G/H])_{(H)};\mathbb{Z}\right) \simeq \mathbb{Z}[\dim V^H - 1]^{\oplus n|G\colon H|} \oplus \bigoplus_{J\in\mathfrak{S}} H^{\dim V^H - \dim V^J - * - 1}\left(B\mathfrak{S}_{(J,H]}, B\mathfrak{S}_{(J,H)};\mathbb{Z}\right)$$

where we write $H^{-1}(\varnothing,\varnothing)=\mathbb{Z}$. We set $A_{V,(H)} := \bigoplus_{J\in\mathfrak{S}} H^{\dim V^H - \dim V^J - * - 1}\left(B\mathfrak{S}_{(J,H]}, B\mathfrak{S}_{(J,H)};\mathbb{Z}\right)$. ◁

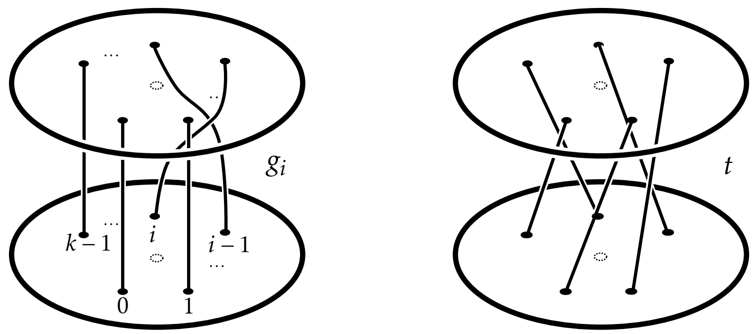
In particular, the equivariant Fadell-Neuwirth Serre spectral sequences look similar to each other:

$$E^2_{p,q} = H_p\left(\mathrm{Conf}^G_{n\cdot[G/H]}(V); \mathbb{Z}\left[\dim V^H - 1\right]_q^{\oplus n|G\colon H|} \oplus A_{V,(H),q}\right) \implies H_{p+q}\left(\mathrm{Conf}^G_{(n+1)\cdot[G/H]}(V);\mathbb{Z}\right).$$

3. Low-dimensional computations

Fix an orthogonal G-representation V and a chain of subgroups $K\subset H\subset G$. Define

$$X := \left(\mathrm{Res}^G_H V\right)_{(K)}/H.$$



$\mathbb{R}^2 - \{0\} \simeq \lambda_{\omega,(C_n)}/C_n$ $\qquad$ $\mathbb{R}^2 - \{0\} \simeq \lambda_{\omega,(C_n)}/C_n$

Figure 1. Movies illustrating the Coxeter generators for $\pi_1 C_k\left(R^2 - \{0\}\right) \simeq \mathrm{CBr}_k$.

Identifying $\mathrm{Aut}_H(n \cdot [H/K]) \simeq N_H K \wr \Sigma_n$, the long exact sequence in homotopy associated with the quotient map $\mathrm{Conf}^H_{n\cdot[H/K]}(V) \to C_n(X)$ reads

$$\begin{tikzcd}
 & 0 \arrow[r] & \pi_\ell \mathrm{Conf}^H_{n\cdot[H/K]}(V) \arrow[r] & \pi_\ell C_n(X) \arrow[r] & 0 \\
 & 0 \arrow[r] & \pi_1 \mathrm{Conf}^H_{n\cdot[H/K]}(V) \arrow[r] & \pi_1 C_n(X) \arrow[dll] & \\
N_H K \wr \Sigma_n \arrow[r] & \pi_0 \mathrm{Conf}^H_{n\cdot[H/K]}(V) \arrow[r] & \pi_0 C_n(X) \arrow[r] & 0 &
\end{tikzcd} \tag{5}$$

In particular low-dimensional examples, we can compute $\pi_\ell C_n(X)$ explicitly.

Example 31. Suppose V is trivial. Then,

$$\mathrm{Conf}^G_S(V) \simeq \begin{cases} \mathrm{Conf}_{S^G}(V^G) & S = n \cdot *_G; \\ \varnothing & \text{otherwise.} \end{cases}$$

More generally, if $N \subset G$ is a normal subgroup, then

$$\mathrm{Conf}^G_S\left(\mathrm{Infl}^G_{G/N} V\right) \simeq \begin{cases} \mathrm{Conf}_{S^N}(V) & S = S^N; \\ \varnothing & \text{otherwise.} \end{cases}$$

We will usually use this to reduce to the case that V is faithful. $\triangleleft$

Using this, we immediately get an answer in the 1-dimensional case.

Proposition 32. *Suppose V is 1-dimensional. Then,* $\mathrm{Conf}^G_S(V)$ *is* discrete*, and identified with the* $\mathrm{Aut}_G(S)$*-set*

$$\mathrm{Conf}^G_S(V) \simeq \begin{cases} \mathrm{Aut}_G(S)/e & S = \epsilon *_G \sqcup n \cdot [G/\ker(V)] \textit{ for } \epsilon \in \{1,2\} \textit{ and } n \in \mathbb{N}, \\ \varnothing & \text{otherwise.} \end{cases}$$

Proof. Since $O(1) \simeq C_2$, we have $|G \colon \ker(V)| \le 2$. The trivial example is Example 31, and the nontrivial example is reduced by Example 31 to the nontrivial 1-dimensional orthogonal C_2-representation σ. In this case, the empty spaces are easy to determine; moreover, note that $C_n(\sigma - \{0\}/C_2) \simeq C_n(\mathbb{R}^1)$ is contractible. In particular, applying Eq. (5) yields that $\mathrm{Conf}^G_S(V)$ is an $\mathrm{Aut}_G(S) = C_2 \wr \Sigma_n$-torsor, as desired. $\square$

For our penultimate example, note that orbits yield a continuous map $\mathrm{Conf}^H_S(V) \longrightarrow \mathrm{Conf}_{S_H}(V_H)$.

Example 33. Suppose H is finite and we know a-priori that $\left(\mathrm{Res}^G_H V\right)_{(e)} \simeq \coprod_{[H/e]}\left(\left(\mathrm{Res}^G_H V\right)_{(e)}/H\right)$, i.e. the H-submanifold of $\mathrm{Res}^G_H V$ of points with stabilizer e is topologically induced from a nonequivariant disk. Then, the map $\mathrm{Conf}_{k\cdot[H/e]} H(V) \to \mathrm{Conf}_k(V_H)$ can be identified with the projection

$$\mathrm{Conf}_k(V_H) \times_{\Sigma_k} H \wr \Sigma_k \longrightarrow \mathrm{Conf}_k(V_H).$$

In particular, whenever $\dim V = \dim V^G + 1$ and $x \in V - V^G$, the orbit-stabilizer theorem implies that $\ker(V) \subset G$ is an index-2 subgroup, and it's follows by nontriviality of V that $V \simeq \mathrm{Infl}^G_{G/\ker(V)} V^G \oplus \sigma$.

When $G = C_2$, $([n] \oplus \sigma)_{(C_2)} \simeq \coprod_{[C_2/e]} D^{n+1}$ is free, so the above characterizes its configuration spaces. In general, if V is $(d+1)$-dimensional, this gives the formula

$$\mathrm{Conf}^G_S(V) \simeq \begin{cases} \mathrm{Conf}_a\left(\mathbb{R}^d\right) \times \mathrm{Conf}_b\left(\mathbb{R}^{d+1}\right) \times_{\Sigma_b} C_2 \wr \Sigma_b & S = a *_G + b[G/\ker(V)] \text{ and either } a \leq 1 \text{ or } d \geq 1; \\ \varnothing & \text{otherwise.} \end{cases}$$

For instance, this characterizes $\mathrm{Conf}^G_S(\rho_{C_2}) \simeq \mathbb{E}_{\rho_{C_2}}(S)$. ◁

Example 34. Suppose $G = C_n$ for $n \neq 0$, ω is a complex nth root of unity, and λ_ω is the 2-dimensional orthogonal G-representation on which a distinguished generator $x \in C_n$ acts by ω. Then, $\lambda_k \simeq \mathrm{Infl}^{C_n}_{C_n/C_n^{\mathrm{order}(\omega)}} \lambda_\omega$, so by Example 31 we may assume without loss of generality that ω is primitive.

Moreover, given a divisor $m|n$, we have $\mathrm{Res}^{C_n}_{C_m} \lambda_\omega \simeq \lambda_{\omega^{n/m}}$, so it suffices to consider C_n-equivariant configurations. In this case, we have $\lambda_{\omega,(C_n)} = \lambda - \{0\}$, on which C_n acts freely with orbit space homeomorphic to the annulus $\mathbb{R}^2 - \{0\}$. It is known that the fundamental group of $C_k(\mathbb{R}^2 - \{0\})$ is the *cylindrical braid group* on k letters [DH98], i.e. the braid group associated with the Coxeter graph B_k, as in

$$t \overset{4}{\text{——}} g_1 \text{———} \cdots \text{———} g_{k-1};$$

that is,

$$\begin{aligned} \pi_1 C_k(\lambda_{\omega,(C_n)}/C_n) \simeq & \mathrm{CBr}_k \\ & := \left\langle t, g_1, \dots, g_{n-1} \;\middle|\; t g_1 t g_1 = g_1 t g_1 t,\; g_i g_{i+1} g_i = g_{i+1} g_i g_{i+1},\; g_i g_j = g_j g_i \text{ if } |i-j| \geq 2 \right\rangle. \end{aligned}$$

Writing elements of $C_n \wr \Sigma_k$ as $(c_1, \dots, c_n; \sigma)$, the Deck map $\pi_1 C_k(\lambda_{\omega,(C_n)}) = \mathrm{CBr}_k \longrightarrow C_n \wr \Sigma_k$ then takes

$$\begin{aligned} g_i &\mapsto (0, \dots, 0, 0; \tau_i), \\ t &\mapsto (0, \dots, 0, 1; \tau_1 \tau_2 \cdots \tau_{n-1}), \end{aligned}$$

where τ_i is the ith simple transposition (see Fig. 1). The projected homomorphism $\mathrm{CBr}_k \to \Sigma_k$ takes the underlying permutation; the ith projected pointed set map $\mathrm{CBr}_k \to C_n$ unravels the path to a loop of the standard configuration of $\mathbb{Z}$ in the universal cover $\mathbb{R} \times \mathbb{R}$ and evaluates $\lfloor h_1(i) \rfloor$ modulo n. Combining everything we've found, we have

$$\mathrm{Conf}^G_S(\lambda_\omega) \simeq \begin{cases} B\ker\left(\mathrm{CBr}_{|S - S^{C_n}|/n} \to C_n \wr \Sigma_{|S - S^{C_n}|/n}\right) & \left|V^{C_n}\right| \leq 1 \\ \varnothing & \text{otherwise.} \end{cases}$$

In essence, the nonempty ordered C_n-equivariant configuration spaces in λ_ω are classifying spaces for the group of cylindrical braids which are pure and have 0 winding modulo n, and $\mathrm{Aut}_{C_n}(\epsilon *_{C_n} \sqcup k[C_n/e]) \simeq C_n \wr \Sigma_k$ acts by permuting the underlying C_n-set. Together with Examples 31 and 33, we have completely characterized computes all equivariant configuration spaces in ≤ 2-dimensional C_n-representations ◁

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References

[BQV23] Eva Belmont, J. D. Quigley, and Chase Vogeli. *Bredon homological stability for configuration spaces of G-manifolds*. 2023. arXiv: `2311.02459 [math.AT]`. URL: `https://arxiv.org/abs/2311.02459` (cit. on p. 3).

[Bre67] Glen E. Bredon. *Equivariant cohomology theories*. Lecture Notes in Mathematics, No. 34. Springer-Verlag, Berlin-New York, 1967, vi+64 pp. (not consecutively paged) (cit. on p. 6).

[Bre72] Glen E. Bredon. *Introduction to compact transformation groups*. Vol. 46. Pure and Applied Mathematics. Academic Press, New York-London, 1972, pp. xiii+459. URL: https://jfdmath.sitehost.iu.edu/seminar/Bredon,Introduction_to_Compact_Transformation_Groups.pdf (cit. on pp. 2, 3).

[CLL24] Bastiaan Cnossen, Tobias Lenz, and Sil Linskens. *Parametrized higher semiadditivity and the universality of spans*. 2024. arXiv: 2403.07676 [math.AT] (cit. on p. 1).

[DH98] Tammo tom Dieck and Reinhard Häring-Oldenburg. "Quantum groups and cylinder braiding". In: *Forum Math.* 10.5 (1998), pp. 619–639. ISSN: 0933-7741,1435-5337. DOI: 10.1515/form.10.5.619. URL: https://doi.org/10.1515/form.10.5.619 (cit. on p. 14).

[DI04] Daniel Dugger and Daniel C. Isaksen. "Topological hypercovers and A^1-realizations". In: *Math. Z.* 246.4 (2004), pp. 667–689. ISSN: 0025-5874,1432-1823. DOI: 10.1007/s00209-003-0607-y. URL: https://people.math.rochester.edu/faculty/doug/otherpapers/dugger-hypercover.pdf (cit. on p. 3).

[FN62] Edward Fadell and Lee Neuwirth. "Configuration spaces". In: *Math. Scand.* 10 (1962), pp. 111–118. ISSN: 0025-5521,1903-1807. DOI: 10.7146/math.scand.a-10517. URL: https://www.mscand.dk/article/download/10517/8538 (cit. on p. 5).

[GM88] Mark Goresky and Robert MacPherson. *Stratified Morse theory*. Vol. 14. Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)]. Springer-Verlag, Berlin, 1988, pp. xiv+272. ISBN: 3-540-17300-5. DOI: 10.1007/978-3-642-71714-7. URL: https://doi.org/10.1007/978-3-642-71714-7 (cit. on p. 12).

[HHR16] M. A. Hill, M. J. Hopkins, and D. C. Ravenel. "On the nonexistence of elements of Kervaire invariant one". In: *Ann. of Math. (2)* 184.1 (2016), pp. 1–262. ISSN: 0003-486X. DOI: 10.4007/annals.2016.184.1.1. URL: https://people.math.rochester.edu/faculty/doug/mypapers/Hill_Hopkins_Ravenel.pdf (cit. on p. 11).

[Knu18] Ben Knudsen. *Configuration spaces in algebraic topology*. 2018. arXiv: 1803.11165 [math.AT]. URL: https://arxiv.org/abs/1803.11165 (cit. on p. 3).

[Lew92] L. Gaunce Lewis Jr. "The equivariant Hurewicz map". In: *Trans. Amer. Math. Soc.* 329.2 (1992), pp. 433–472. ISSN: 0002-9947,1088-6850. DOI: 10.2307/2153946. URL: https://doi.org/10.2307/2153946 (cit. on p. 2).

[LS04] Riccardo Longoni and Paolo Salvatore. *Configuration spaces are not homotopy invariant*. 2004. arXiv: math/0401075 [math.AT]. URL: https://arxiv.org/abs/math/0401075 (cit. on p. 1).

[Nam83] Unni Namboodiri. "Equivariant vector fields on spheres". In: *Trans. Amer. Math. Soc.* 278.2 (1983), pp. 431–460. ISSN: 0002-9947,1088-6850. DOI: 10.2307/1999164. URL: https://doi.org/10.2307/1999164 (cit. on p. 2).

[Nar16] Denis Nardin. *Parametrized higher category theory and higher algebra: Exposé IV – Stability with respect to an orbital ∞-category*. 2016. arXiv: 1608.07704 [math.AT] (cit. on p. 1).

[Qui73] Daniel Quillen. "Higher algebraic K-theory. I". In: *Algebraic K-theory, I: Higher K-theories (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972)*. Vol. Vol. 341. Lecture Notes in Math. Springer, Berlin-New York, 1973, pp. 85–147. URL: https://sma.epfl.ch/~hessbell/topo_alg/Quillen.pdf (cit. on pp. 3, 5).

[Ste25a] Natalie Stewart. *A homotopically equivariant Eckmann-Hilton argument (forthcoming)*. 2025 (cit. on p. 10).

[Ste25b] Natalie Stewart. *Equivariant operads, symmetric sequences, and Boardman-Vogt tensor products*. 2025. arXiv: 2501.02129 [math.CT] (cit. on pp. 10, 11).

[Ste25c] Natalie Stewart. *On homotopical additivity of equivariant little disks operads (forthcoming)*. 2025 (cit. on p. 1).

[Wil17] Dylan Wilson. *On categories of slices*. 2017. arXiv: 1711.03472 [math.AT] (cit. on p. 11).

[Wir75] Klaus Wirthmüller. "Equivariant S-duality". In: *Arch. Math. (Basel)* 26.4 (1975), pp. 427–431. ISSN: 0003-889X,1420-8938. DOI: 10.1007/BF01229762. URL: https://doi.org/10.1007/BF01229762 (cit. on p. 1).