The Castaway: An Analysis of Escape and Capure from a Shark Patrolled Island

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***Abstract*—The following paper analyzes the problem of whether or not a castaway can escape from a circular island that is closely guarded by a hungry shark. The castaway desires to move from the interior of the island to its exterior while avoiding capture. The shark, on the other hand, desires to capture and eat the castaway. The castaway can move in any direction within the island at speed s, while the shark is restricted to motion along the island's circumference at speed $4s$. If the shark and the castaway arrive at the boundary simultaneously then the shark succeeds in capture, but if the person arrives before the shark then the castaway succeeds in escape. The shark and the castaway are modeled as points on a 2D plane. The best strategies for each entity are outlined. The strategies are implemented with ROS and C++. With mathematical and imperial results, it is shown that the castaway can escape the island.**

I. Introduction

The following problem requires careful analysis of the strategies of the castaway and the shark, defined as entities of the island. Each strategy must be unbiased and modeled separately. After each strategy is defined, an analysis must be made to define which entity will be able to accomplish their goal. Section II defines the problem statement and the physical restrictions of the island world. Section III proposes te strategies for the shark and the castaway. Section IV describes the steps taken to implement each strategy. Section V analyzes the results and finally Section VI gives a brief conclusion.

II. Problem Statement

The following restrictions are given by the problem:

1) In a circular island of radius, $R = 1$, there is a castaway who made a boat to escape.
2) The castaway can move in any direction within the island with speed s.
3) Outside the island, there is a shark that can move in the boundary of the island with speed $4s$.
4) The starting location of the castaway is arbitrary.
5) If the castaway can reach the boundary and the shark is not there, then the castaway can use the boat and escape. If he reaches the boundary and the shark is there, then the shark eats him.

The following questions need to be answered:

1) Can the castaway escape?
2) What is the best strategy for the shark?
3) What is the best strategy for the castaway?

III. Defining the strategies

The shark projects the position of the castaway onto the closest point on the circumference of the circle. The shark evaluates the direction of motion based on the shortest distance to the projected point. Note that if the castaway is at point $P(x_c, y_c)$ then the projected point,$P(x_p, y_p)$, can be calculated with equations 1,2 and 3. For the purposes of simplicity, the center of the circle (x_{center}, y_{center}) is assumed to be $(0, 0)$ for every calculation hereon after.

$$\theta = arctan(y_c, x_c) \tag{1}$$

$$x_p = Rcos(\theta) \tag{2}$$

$$x_p = Rsin(\theta) \tag{3}$$

Given the circle with radius R, the circumference of the island C is defined by equation 4. The distance, d, that an entity will travel at a speed s for a time t is given by equation 5. If the castaway travels from the center of the circle directly to the island shore then the time, t_c, it takes for the castaway to reach the shore is defined by equation 6. If the shark starts to move along the circumference of the circle to meet the human at the opposite end then the shark will need to travel half the circumference of the circle and the time, t_s, it takes the shark to get to the other side is defined by equation 7. The conclusion, equation 8, states that the shark will get to the opposte side faster than the castaway hence the shark will eat the castaway if the castaway's strategy is to head directly for the shore. The Straight For Shore (SFS) strategy is insufficient for escape.

$$C = 2\pi R \tag{4}$$

$$d = st \tag{5}$$

$$t_c = \frac{R}{s} \tag{6}$$

$$t_s = \frac{\pi R}{4s} \tag{7}$$

$$t_s < t_c \tag{8}$$

Another option for the castaway is to try to move out towards the circumference of the circle while staying antipodal to the shark. The antipodal point of a point on the surface of a sphere is the point which is diametrically opposite to another point. [1] The antipodal point of the shark on the circle will be the point that is $180°$ rotated around the center. Is is important to calculate the terminal distance from the center of the circle at which the castaway can no longer both offset tangentially to maintain being antipodal to the shark and advance toward the shore. If the terminal distance is greater than the radius of the island then the castaway can escape solely with this Antipodal strategy. Substituting equation 9 into equation 1, we obtain equation 10 and 11 which define how far a shark is travelling at any given time along the circumference of the island and how far the castaway is travelling at any given distance from the center respectively. Knowing that the castaway is trying to offset the same angle in the same amount of time as the shark, we can arrive at equation 12, which tells us that the castaway can only offset the shark's distance up until a quarter of the island's distance.

$$d = R\theta \tag{9}$$

$$R\theta = 4st \tag{10}$$

$$R_c\theta = st \tag{11}$$

$$R_c = R/4 \tag{12}$$

Pure motion strategies don't work well for the castaway so the castaway must be clever and put the two strategies together. The castaway will use the Antipodal strategy to get as close to the shore as possible, which is at $R/4$ and then the castaway will use SFS strategy to go the rest of the way. After the terminal distance of the Antipodal strategy the castaway only has $0.75R$ to go and from equations 6 and 7 it can be deduced that the castaway can travel a distance of $0.785R$ in the same time that the shark travels half of the circumference of the circle. Hence the castaway can arrive to the shore.

IV. Implementation with ROS

Two ROS nodes are launched, one for each entity. The node of the castaway behaves according to the algorithm below.

Algorithm 1: Castaway strategy

```
while R_castaway < R_island do
    if R_castaway < R_island/4 then
        Move antipodal;
    else
        Move SFS;
    end
end
```

The node of the shark behaves according to the algorithm below.

Algorithm 2: Shark strategy

```
while R_castaway != R_shark and θ_castaway != θ_shark do
    θ_diff = θ_shark − θ_castaway;
    θ_diff = (θ_diff + 180)%360 − 180 ;
    Increment θ_shark in direction of θ_diff;
end
```

V. Results

The algorithms described above have been implemented in ROS and C++ and the source code is publicly available [2]. There are two termination conditions; either the shark gets close enough to the person or the person gets close enough to the shore. It can be seen that the second condition is met first every time.

VI. Conclusion

It has been shown that the castaway can escape a shark that travels four times faster along the circumference of a circle with radius R. Future work includes research into the udnerstanding of shark and person behavior at various speeds with various circle sizes.

References

[1] John C. Polking *Incidence Relations on a Sphere*, April 15, 1999, https://math.rice.edu/ pcmi/sphere/gos2.html

[2] https://github.com/xTrials/island_ws