Monad Transformers Step by Step

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Abstract

In this tutorial, we describe how to use monad transformers in order to incrementally add functionality to Haskell programs. It is not a paper about *implementing* transformers, but about *using* them to write elegant, clean and powerful programs in Haskell. Starting from an evaluation function for simple expressions, we convert it to monadic style and incrementally add error handling, environment passing, state, logging and input/output by composing monad transformers.

1 Introduction

This paper gives a step-by-step introduction to monad transformers in Haskell.

Monads are a remarkably elegant way for structuring programs in a flexible and extensible way. They are especially interesting in a lazy functional language like Haskell, because they allow the integration of side-effects into otherwise purely functional programs. Furthermore, by structuring a program with monads, it is possible to hide much of the necessary book-keeping and plumbing necessary for many algorithms in a handful of definitions specific for the monad in use, removing the clutter from the main algorithm.

Monad transformers offer an additional benefit to monadic programming: by providing a library of different monads and types and functions for combining these monads, it is possible to create custom monads simply by composing the necessary monad transformers. For example, if you need a monad with state and error handling, just take the *StateT* and *ErrorT* monad transformers and combine them. The goal of this paper is to give a gentle introduction to the use of monad transformers by starting with a simple function and extending it step by step with various monadic operations in order to extend its functionality. This paper is *not* about the theory underlying the monad transformer concept, and not about their implementation (except for what is necessary for successfully using them).

The reader is expected to be familiar with functional and basic monadic programming, such as the use of the *Monad* class and **do** notation in Haskell. Everything else will be explained on the fly.

The Haskell programs in this paper use language features which are not in the current Haskell'98 standard, since they use the non-standard library modules *Control.Monad.Error* etc. Both the hierarchical module names of these modules and some of their implementation details are beyond Haskell'98. Nevertheless, these extensions are well supported in current versions of the Glasgow Haskell compiler (GHC) [1] and the Hugs Haskell interpreter [2]. The programs have been tested using version 6.5 of GHC and version 20050308 of Hugs¹.

The monad transformer modules are inspired by a paper by Mark P. Jones [3], which gives a very readable introduction to monadic programming, but is less practical than this paper.

This document has been converted from a literate Haskell script using Andres Löh's 1hs2TeX² preprocessor. The script is executable by both GHC and Hugs. The literate Haskell source file

¹You need to pass the -98 option to Hugs in order to allow the required Haskell extensions.

²http://www.iai.uni-bonn.de/~loeh/

Transformers.lhs is available from my homepage³.

1.1 Example Program

As a running example, an interpreter for a simple programming language will be used throughout the paper. All the code will be located in a module called *Transformers*, which has the following header:

```
module Transformers where
import Control.Monad.Identity
import Control.Monad.Error
import Control.Monad.Reader
import Control.Monad.State
import Control.Monad.Writer
import Data.Maybe
import qualified Data.Map as Map
```

Several of the imported modules beginning with *Control.Monad* are only needed when you use the monad transformers defined there. The *Data.Maybe* module defines useful functions for dealing with optional values of type *Maybe* a, and the module *Data.Map* defines finite maps. These will be used to define environments (variable-value mappings) in our little interpreter.

The following data types for modelling programs in that language will be used:

The Name type is simply a shorthand for the standard String type. It is used to make clear when we are talking about variable names and not about general strings. The Exp data type has variants for literal integers (constants), variables, addition, λ expressions (abstractions) and function application. The programs which are to be evaluated will be made up of the Exp data type, whereas the results are from the Value type. Values are either integers or functions (closures). The Env component of a FunVal is the environment in which the corresponding λ -abstraction was evaluated.

Since the example for using monad transformers will be an interpreter for the little language defined above, we start by defining an evaluation function. This function is not monadic and will serve as a kind of "reference implementation". The definition of the interpreter function, called *eval0*, is straightforward.

```
eval0 :: Env \rightarrow Exp \rightarrow Value

eval0 \ env \ (Lit \ i) = IntVal \ i

eval0 \ env \ (Var \ n) = fromJust \ (Map.lookup \ n \ env)

eval0 \ env \ (Plus \ e1 \ e2) = \mathbf{let} \ IntVal \ i1 = eval0 \ env \ e1
```

³http://uebb.cs.tu-berlin.de/~magr/pub/Transformers.lhs

```
IntVal \ i2 = eval0 \ env \ e2
\mathbf{in} \ IntVal \ (i1 + i2)
eval0 \ env \ (Abs \ n \ e) = FunVal \ env \ n \ e
eval0 \ env \ (App \ e1 \ e2) = \mathbf{let} \ val1 = eval0 \ env \ e1
val2 = eval0 \ env \ e2
\mathbf{in} \ \mathbf{case} \ val1 \ \mathbf{of}
FunVal \ env' \ n \ body \rightarrow eval0 \ (Map.insert \ n \ val2 \ env') \ body
```

Integer literals simply evaluate to themselves (packaged up in the Value data type), variables evaluate to the values to which they are bound in the environment. The use of the $fromJust^4$ function is necessary because the Map.lookup function returns a Maybe value. Note that the use of this function introduces an error condition: when a variable is used which is not bound anywhere using a λ expression, the program will halt with an error message. Addition is implemented by simply evaluating both operands and returning their sum. Whenever one of the addition operands evaluates to a non-number, the pattern matching in the let expression will fail, also terminating the program with an error message. Abstractions simply evaluate to functional values, which capture the environment in which they are evaluated. Function application proceeds similar to addition, by first evaluating the function and the argument. The first expression must evaluate to a functional value, whose body is then evaluated in the captured environment, extended with the binding of the function parameter to the argument value. The case expression used here to deconstruct the functional value introduces another error possibility. In later sections of this text, we will handle these error cases using an error monad, which gives us more control over their handling.

The definition of *eval0* could be shortened a little bit, for example, the **let** expression in the *Plus* case seems superfluous. Nevertheless, the definition given here will make it easier to relate it to the monadic versions defined below.

The following example expression,

$$12 + ((\lambda x \to x)(4+2))$$

can be used to test this interpreter and all of the others we will define shortly.

For example, entering

eval0 Map.empty exampleExp

in ghci will give the answer

IntVal 18

2 Monad Transformers

The goal of using monad transformers is to have control over aspects of computations, such as state, errors, environments etc. It is a bit tedious to reformulate an already written program in monadic style, but once that is done, it is relatively easy to add, remove or change the monads involved.

In this section, we will first rewrite the example program from Section 1.1 in monadic style and then extend the data types and function definitions with various monad transformer types and functions step by step.

⁴ from Just has the type Maybe $\alpha \to \alpha$

2.1 Converting to Monadic Style

In order to use monad transformers, it is necessary to express functions in monadic style. That means that the programmer needs to impose sequencing on all monadic operations using **do** notation, and to use the *return* function in order to specify the result of a function.

First, we define a monad in which the evaluator will be defined. The following type synonym defines Eval1 α as a synonym for the type Identity α . Identity is a monad imported from Control.Monad.Identity, which is perhaps the simplest monad imaginable: it defines the standard return and \gg operations for constructing operations in the monad, and additionally a function runIdentity to execute such operations. Other than that, the identity monad has no effect. In some sense, we will use this monad as a "base case", around which other monad transformers can be wrapped. For readability, we also define a function runEval1, which simply calls runIdentity.

```
type Eval1 \alpha = Identity \ \alpha

runEval1 :: Eval1 \alpha \rightarrow \alpha

runEval1 ev = runIdentity ev
```

Based on the Eval1 monad, we now rewrite the eval0 function as eval1:

```
eval1
                               :: Env \rightarrow Exp \rightarrow Eval1 \ Value
eval1 env (Lit i)
                               = return \$ Int Val i
eval1 \ env \ (Var \ n)
                               = Map.lookup \ n \ env
eval1 \ env \ (Plus \ e1 \ e2) = \mathbf{do} \ IntVal \ i1 \leftarrow eval1 \ env \ e1
                                       IntVal\ i2 \leftarrow eval1\ env\ e2
                                       return \$ Int Val (i1 + i2)
eval1 \ env \ (Abs \ n \ e)
                               = return \$ FunVal \ env \ n \ e
eval1 \ env \ (App \ e1 \ e2) = \mathbf{do} \ val1 \leftarrow eval1 \ env \ e1
                                       val2 \leftarrow eval1 \ env \ e2
                                       case val1 of
                                           FunVal\ env'\ n\ body \rightarrow
                                              eval1 (Map.insert n val2 env') body
```

The first thing to note is that the cases for *Lit* and *Abs* use the *return* function for specifying their result.⁵ The next is that the *Var* case does not need a *fromJust* call anymore: The reason is that *Map.lookup* is defined to work within any monad by simply calling the monad's *fail* function – this fits nicely with our monadic formulation here. (The *fail* function of the *Maybe* monad returns *Nothing*, whereas the *fail* function in the *Identity* monad throws an exception, which will lead to different error messages.)

The Plus and App cases now evaluate their subexpressions using **do**-notation, binding their results ti variables. In the Plus case, the result is returned using return, whereas in th App case, the function value is further discriminated like in the eval0 function above.

In order to test this interpreter, we have to evaluate the monadic action obtained by applying eval1 to our example expression example Exp. This is done by calling runEval1 defined earlier:

```
\label{eq:continuous_continuous_continuous} run \textit{Eval1 (eval1 Map.empty exampleExp)} \\ \\ \text{gives}
```

IntVal 18

To recapitulate: conversion to monadic form consists mainly of returning function results using the *return* function, and sequencing of monadic actions using **do** notation or the \gg or \gg (monadic bind) functions.

⁵The (\$) operator is function application with low precedence and mainly used to avoid parentheses.

Note: The type of *eval1* could be generalized to

```
eval1 :: Monad \ m \Rightarrow Env \rightarrow Exp \rightarrow m \ Value,
```

because we do not use any monadic operations other than return and \gg (hidden in the **do** notation). This allows the use of eval1 in any monadic context, so that instead of

```
runEval1 (eval1 Map.empty exampleExp)
```

we could write

```
eval1 Map.empty exampleExp
```

at the ghci prompt. In some contexts, this is a nice feature, but in general you will be using some operations specific to a particular monad, and this forces your operation to stay within that special monad.

2.2 Adding Error Handling

We have already seen that our evaluation function is partial, that means it will terminate with an error message for some inputs, for example for expressions with unbound variables or type errors.

Using monad transformers, we simply go to our local monad transformer library and take the ErrorT monad transformer, using it to extend the basic Eval1 monad to Eval2.

```
type Eval2 \alpha = ErrorT String Identity \alpha
```

The String type argument to ErrorT is the type of exceptions, that is the values which are used to indicate error conditions. We use String here to keep things simple, in a real implementation we might want to include source code locations (in a compiler) or time stamps (in some kind of web application).

The function for running a computation in the Eval2 monad changes in two ways. First, the result of evaluation is now of type $Either\ String\ \alpha$, where the result $Left\ s$ indicates that an error has occurred with error message s, or $Right\ r$, which stands for successful evaluation with result r. Second, we need to call the function runErrorT on the given computation to yield an Identity computation, which can in turn be evaluated using runIdentity.

```
runEval2 :: Eval2 \alpha \rightarrow Either\ String\ \alpha

runEval2 ev = runIdentity\ (runErrorT\ ev)
```

We can now simply change the type of our eval1 function, giving the following version, called eval2a.

```
:: Env \rightarrow Exp \rightarrow Eval 2 \ Value
eval2a
eval2a env (Lit i)
                                = return \$ Int Val i
eval2a \ env \ (Var \ n)
                                = Map.lookup \ n \ env
eval2a \ env \ (Plus \ e1 \ e2) = \mathbf{do} \ IntVal \ i1 \leftarrow eval2a \ env \ e1
                                        IntVal~i2 \leftarrow eval2a~env~e2
                                        return \$ Int Val (i1 + i2)
eval2a \ env \ (Abs \ n \ e)
                                = return \$ FunVal \ env \ n \ e
eval2a \ env \ (App \ e1 \ e2) = \mathbf{do} \ val1 \leftarrow eval2a \ env \ e1
                                        val2 \leftarrow eval2a \ env \ e2
                                        case val1 of
                                            FunVal\ env'\ n\ body \rightarrow
                                               eval2a (Map.insert n val2 env') body
```

This version can be run using the runEval2 function defined above. When we apply this function to our example expression, the result only varies in that it is wrapped in a Right constructor:

```
runEval2 \ (eval2a \ Map.empty \ exampleExp) \Rightarrow Right \ (IntVal \ 18)
```

But unfortunately, when given an invalid expression, the error reporting of the ErrorT transformer is not used. We have to modify our definition in order to give useful error messages:

```
:: Env \rightarrow Exp \rightarrow Eval2 \ Value
eval2b
eval2b env (Lit i)
                                = return \$ Int Val i
                                = Map.lookup \ n \ env
eval2b \ env \ (Var \ n)
eval2b \ env \ (Plus \ e1 \ e2) = \mathbf{do} \ e1' \leftarrow eval2b \ env \ e1
                                        e2' \leftarrow eval2b \ env \ e2
                                        case (e1', e2') of
                                           (IntVal\ i1, IntVal\ i2) \rightarrow
                                              return \$ Int Val (i1 + i2)
                                           \_ \rightarrow throwError "type error"
eval2b \ env \ (Abs \ n \ e)
                                = return \$ FunVal \ env \ n \ e
eval2b \ env \ (App \ e1 \ e2) = \mathbf{do} \ val1 \leftarrow eval2b \ env \ e1
                                        val2 \leftarrow eval2b \ env \ e2
                                        case val1 of
                                           FunVal\ env'\ n\ body \rightarrow
                                              eval2b (Map.insert n val2 env') body
                                           \_ \rightarrow throwError "type error"
```

Now, when we try to evaluate an invalid expression, we get an error message, packaged in the *Left* constructor. So by matching against the result of an evaluation, we can distinguish between normal and error results.

```
runEval2\ (eval2a\ Map.empty\ (Plus\ (Lit\ 1)\ (Abs\ "x"\ (Var\ "x")))) \Rightarrow Left\ "type\ error"
```

But wait a minute! What is about $Map.lookup \ n \ env$? Shouldn't we check whether it returns Nothing and generate an appropriate error message? As mentioned above, Map.lookup returns it result in an arbitrary monad, and the Control.Monad.Error module gives the necessary definitions so that it works just out of the box:

```
runEval2 \ (eval2b \ Map.empty \ (Var "x")) \Rightarrow Left "Data.Map.lookup: Key not found"
```

A little bit of closer inspection of function eval2b reveals that we can do even shorter (better?) we exploiting the fact that monadic binding in a **do** expression uses the fail function whenever a pattern match fails. And, as we have seen, the fail function does what we want.

```
\begin{array}{lll} eval2c & :: Env \rightarrow Exp \rightarrow Eval2 \ Value \\ eval2c \ env \ (Lit \ i) & = return \ \$ \ Int Val \ i \\ eval2c \ env \ (Var \ n) & = Map.lookup \ n \ env \\ eval2c \ env \ (Plus \ e1 \ e2) = \mathbf{do} \ Int Val \ i1 \leftarrow eval2c \ env \ e1 \\ & Int Val \ i2 \leftarrow eval2c \ env \ e2 \\ & return \ \$ \ Int Val \ (i1 + i2) \\ eval2c \ env \ (Abs \ n \ e) & = return \ \$ \ Fun Val \ env \ n \ e \\ eval2c \ env \ (App \ e1 \ e2) & = \mathbf{do} \ Fun Val \ env' \ n \ body \leftarrow eval2c \ env \ e1 \\ & val2 & \leftarrow eval2c \ env \ e2 \\ & eval2c \ (Map.insert \ n \ val2 \ env') \ body \end{array}
```

The drawback of this function is that the error messages only talks about "pattern match failure", with no specific information about why the pattern match fails. Thus, in order to get good error messages, it is better to provide our own calls to *throwError*. This is what we'll do for the final version of the error handling evaluation.

```
:: Env \rightarrow Exp \rightarrow Eval2 \ Value
eval2
eval2 env (Lit i)
                              = return \$ Int Val i
eval2 \ env \ (Var \ n)
                              = case Map.lookup n env of
                                    Nothing \rightarrow throwError ("unbound variable: " ++ n)
                                    Just\ val \rightarrow return\ val
eval2 \ env \ (Plus \ e1 \ e2) = \mathbf{do} \ e1' \leftarrow eval2 \ env \ e1
                                      e2' \leftarrow eval2 \ env \ e2
                                      case (e1', e2') of
                                         (Int Val \ i1, Int Val \ i2) \rightarrow
                                            return \$ Int Val (i1 + i2)
                                         \_ \rightarrow throwError "type error in addition"
eval2 \ env \ (Abs \ n \ e)
                              = return \$ FunVal \ env \ n \ e
eval2 \ env \ (App \ e1 \ e2) = \mathbf{do} \ val1 \leftarrow eval2 \ env \ e1
                                      val2 \leftarrow eval2 \ env \ e2
                                      case val1 of
                                         FunVal\ env'\ n\ body \rightarrow
                                            eval2 (Map.insert \ n \ val2 \ env') \ body
                                         \_ \rightarrow throwError "type error in application"
```

Note: The Control.Monad.Error module provides another function for catching errors raised using throwError, called catchError:: $m\ a \to (e \to m\ a) \to m\ a$ for arbitrary error monads. It can be used for either handling errors locally or passing them on to the surrounding calling context.

2.3 Hiding the Environment

One way to make the definition of the evaluation function even more pleasing is to hide the environment from all function definitions and calls. Since there is only one place where the environment is extended (for function application) and two places where it is actually used (for variables and λ expressions), we can reduce the amount of code by hiding it in all other places. This will be done by adding a ReaderT monad transformer in order to implement a reader monad. A reader monad threads a value through all computations which can be read and modified for nested computations. In contrast to state monads (which will be introduced in Section 2.4), an encapsulated computation cannot change the value used by surrounding computations.

We start by simply wrapping a ReaderT constructor around our previous monad.

```
type Eval3 \alpha = ReaderT \ Env \ (ErrorT \ String \ Identity) \ \alpha
```

The run function runEval3 must be slightly modified, because we need to pass in the initial environment. The reason is that we will remove the environment parameter from the evaluation function.

```
runEval3 :: Env \rightarrow Eval3 \ \alpha \rightarrow Either \ String \ \alpha
runEval3 \ env \ ev = runIdentity \ (runErrorT \ (runReaderT \ ev \ env))
eval3 \ :: Exp \rightarrow Eval3 \ Value
eval3 \ (Lit \ i) = return \ IntVal \ i
```

```
eval3 (Var n)
                     = \mathbf{do} \ env \leftarrow ask
                              case Map.lookup n env of
                                   Nothing \rightarrow throwError ("unbound variable: " ++ n)
                                   Just\ val \rightarrow return\ val
eval3 (Plus \ e1 \ e2) = \mathbf{do} \ e1' \leftarrow eval3 \ e1
                                e2' \leftarrow eval3 \ e2
                                case (e1', e2') of
                                   (IntVal\ i1, IntVal\ i2) \rightarrow
                                     return \$ IntVal (i1 + i2)
                                   \_ \rightarrow throwError "type error in addition"
eval3 (Abs \ n \ e)
                        = \mathbf{do} \ env \leftarrow ask
                                return $ FunVal env n e
eval3 (App \ e1 \ e2) = \mathbf{do} \ val1 \leftarrow eval3 \ e1
                                val2 \leftarrow eval3 \ e2
                                case val1 of
                                   FunVal\ env'\ n\ body \rightarrow
                                      local (const (Map.insert n val2 env'))
                                        (eval3\ body)
                                   \_ \rightarrow throwError "type error in application"
```

For our running example, we now have to evaluate

```
runEval3 Map.empty (eval3 exampleExp)
```

In all places where the current environment is needed, it is extracted from the hidden state of the reader monad using the ask function. In the case of function application, the local function is used for modifying the environment for the recursive call. Local has the type $r \to r) \to m$ $a \to m$ a, that is we need to pass in a function which maps the current environment to the one to be used in the nested computation, which is the second argument. In our case, the nested environment does not depend on the current environment, so we simply pass in a constant function using const.

Note: In addition to *ask*, a function *asks* is predefined, which expects a function mapping the environment to a value. This can be used to extract individual components of the environment by applying *asks* to record selector functions.

2.4 Adding State

Another important application of monads is to provide mutable state to otherwise purely functional code. This can be done using a *State* monad, which provides operations for specifying an initial state, querying the current state and changing it.

As an example, suppose that we want to add profiling capabilities to our little interpreter. We define the new monad by wrapping a StateT constructor around the innermost monad, Identity. (In the case of State and Error monads, the order of these constructor matters, as we will see below.) The state maintained in our example is a simple integer value, but it can be any data type we wish. Normally, it will be a record holding the complete state necessary for the task at hand.

```
type Eval4 \alpha = ReaderT \ Env \ (ErrorT \ String \ (StateT \ Integer \ Identity)) \ \alpha
```

The return type of the function runEval4 changes, because the final state is returned together with the evaluation result (error or value). Additionally, we give the initial state as an additional parameter so that we gain some flexibility (this can be used, for example, to start a computation in the final state of another one).

```
runEval4 :: Env \rightarrow Integer \rightarrow Eval4 \alpha \rightarrow (Either\ String\ \alpha, Integer)

runEval4 env st ev = runIdentity\ (runStateT\ (runErrorT\ (runReaderT\ ev\ env))\ st)
```

For our simple example, we only want to count the number of evaluation steps, that is the number of calls to the *eval4* function. All modification happens in a little helper function *tick*, which gets the hidden state from the computation, increases the counter and stores it back.

```
tick :: Eval4 ()

tick = \mathbf{do} \ st \leftarrow get

put \ (st + 1)
```

By adding a call to the *tick* function in each case, we can count the number of applications.

```
:: Exp \rightarrow Eval \checkmark Value
eval4 (Lit i)
                       = do tick
                               return \$ Int Val i
eval4 (Var n) = \mathbf{do} \ tick
                               env \leftarrow ask
                               case Map.lookup n env of
                                  Nothing \rightarrow throwError ("unbound variable: " ++ n)
                                  Just\ val \rightarrow return\ val
eval4 (Plus e1 e2) = do tick
                               e1' \leftarrow eval 4 e1
                               e2' \leftarrow eval 4 e2
                               case (e1', e2') of
                                  (IntVal\ i1, IntVal\ i2) \rightarrow
                                    return \$ Int Val (i1 + i2)
                                  \_ \rightarrow throwError "type error in addition"
eval4 (Abs \ n \ e) = \mathbf{do} \ tick
                               env \leftarrow ask
                               return $ FunVal env n e
eval4 (App e1 e2) = \mathbf{do} \ tick
                               val1 \leftarrow eval4 \ e1
                               val2 \leftarrow eval4 \ e2
                               case val1 of
                                  FunVal\ env'\ n\ body \rightarrow
                                    local (const (Map.insert n val2 env'))
                                       (eval 4 bod y)
                                  \_ \rightarrow throwError "type error in application"
```

Evaluating our example expression yields:

```
(Right (IntVal 18),8)
```

meaning that the evaluation was successful, returned the integer 18 and took 8 reduction steps.

Note: When the type of the Eval4 monad is changed to the following (StateT and ErrorT are swapped), the interpretation of the monad changes.

```
type Eval4' \alpha = ReaderT \ Env \ (StateT \ Integer \ (ErrorT \ String \ Identity)) \ \alpha
```

Instead of returning a result (error or normal) and a state, either an error or a result together with the final state is returned, as can be seen in the type of the corresponding run function:

```
runEval4' :: Env \rightarrow Integer \rightarrow Eval4' \alpha \rightarrow (Either\ String\ (\alpha, Integer))

runEval4'\ env\ st\ ev = runIdentity\ (runErrorT\ (runStateT\ (runReaderT\ ev\ env)\ st))
```

The position of the reader monad transformer does not matter, since it does not contribute to the final result.

Note: The *State* monad also provides an additional function, *gets* which applies a projection function to the state before returning it. There is also a function *modify* which can be used to change the internal state by applying a function over it.

2.5 Adding Logging

The last monad transformer in the toolbox which will be described here is WriterT. It is in some sense dual to ReaderT, because the functions it provides let you add values to the result of the computation instead of using some values passed in.

```
type Eval5 \ \alpha = ReaderT \ Env \ (ErrorT \ String \ (WriterT \ [String] \ (StateT \ Integer \ Identity))) \ \alpha
```

Similar to StateT, WriterT interacts with ErrorT because it produces output. So depending on the order of ErrorT and WriterT, the result will include the values written out or not when an error occurs. The values to be written out will be lists of strings. When you read the documentation for the WriterT monad, you will notice that the type of the output values is restricted to be a member of the type class Monoid. This is necessary because the methods of this class are used internally to construct the initial value and to combine several values written out.

The running function is extended in the same way as earlier.

```
 \begin{array}{ll} \mathit{runEval5} & :: \mathit{Env} \rightarrow \mathit{Integer} \rightarrow \mathit{Eval5} \ \alpha \rightarrow ((\mathit{Either} \ \mathit{String} \ ), \mathit{Integer}) \\ \mathit{runEval5} \ \mathit{env} \ \mathit{st} \ \mathit{ev} = \\ \mathit{runIdentity} \ (\mathit{runStateT} \ (\mathit{runWriterT} \ (\mathit{runErrorT} \ (\mathit{runReaderT} \ \mathit{ev} \ \mathit{env}))) \ \mathit{st}) \\ \end{array}
```

In the evaluation function, we illustrate the use of the writer monad by writing out the name of each variable encountered during evaluation.

```
:: Exp \rightarrow Eval5 \ Value
eval5
                       = do tick'
eval5 (Lit i)
                               return \$ Int Val i
eval5 (Var n)
                       = do tick'
                               tell[n]
                               env \leftarrow ask
                               case Map.lookup n env of
                                  Nothing \rightarrow throwError ("unbound variable: " ++ n)
                                  Just\ val \rightarrow return\ val
eval5 (Plus \ e1 \ e2) = \mathbf{do} \ tick'
                               e1' \leftarrow eval5 \ e1
                               e2' \leftarrow eval5 \ e2
                               case (e1', e2') of
                                  (Int Val \ i1, Int Val \ i2) \rightarrow
                                     return \$ Int Val (i1 + i2)
                                  \_ \rightarrow throwError "type error in addition"
eval5 (Abs \ n \ e)
                       = do tick'
                               env \leftarrow ask
```

```
return \$ Fun Val \ env \ n \ e
eval5 \ (App \ e1 \ e2) = \mathbf{do} \ tick'
val1 \leftarrow eval5 \ e1
val2 \leftarrow eval5 \ e2
\mathbf{case} \ val1 \ \mathbf{of}
Fun Val \ env' \ n \ body \rightarrow
local \ (const \ (Map.insert \ n \ val2 \ env'))
(eval5 \ body)
- \rightarrow throwError \ "type \ error \ in \ application"
tick' :: Eval5 \ ()
tick' = \mathbf{do} \ st \leftarrow get
put \ (st + 1)
```

2.6 What about I/O?

Until now, we have not considered one importand aspect: input and output. How do we integrate I/O into the monadic definitions we have developed so far? It is not possible to define an I/O monad transformer, because the execution of I/O operations in Haskell cannot be arbitrarily nested into other functions or monads, they are only allowed in the monad IO. Fortunately, the monad transformer library provides us with the infrastructure to easily integrate I/O operations into our framework: we simply substitute IO where we have used Identity! This is possible because Identity is the innermost monad, and as we have seen, the function runIdentity for evaluating actions in this monad is always applied last.

```
type Eval6 \ \alpha = ReaderT \ Env \ (ErrorT \ String \ (WriterT \ [String] \ (StateT \ Integer \ IO))) \ \alpha
```

The return type of runEval6 is wrapped in an IO constructor, which means that the running an Eval6 computation does not directly yield a result, but an I/O computation which must be run in order to get at the result. Accordingly, the runIdentity invocation disappears.

```
runEval6 :: Env \rightarrow Integer \rightarrow Eval6 \ \alpha \rightarrow IO \ ((Either \ String \ \alpha, [String]), Integer)

runEval6 \ env \ st \ ev =

runStateT \ (runWriterT \ (runErrorT \ (runReaderT \ ev \ env))) \ st
```

In the eval6 function we can now use I/O operations, with one minor notational inconvenience: we have to invoke the operations using the function liftIO, which lifts the I/O computation into the currently running monad. As an example, we chose to print out each integer constant as soon as it is evaluated. (We don't think this is good style, but it illustrates the point and sometimes makes a good debugging technique.)

```
\begin{array}{lll} eval6 & :: Exp \rightarrow Eval6 \ Value \\ eval6 \ (Lit \ i) & = \mathbf{do} \ tick'' \\ & \quad liftIO \$ \ print \ i \\ & \quad return \$ \ Int Val \ i \\ eval6 \ (Var \ n) & = \mathbf{do} \ tick'' \\ & \quad tell \ [n] \\ & \quad env \leftarrow ask \\ & \quad \mathbf{case} \ Map.lookup \ n \ env \ \mathbf{of} \\ & \quad Nothing \rightarrow throw Error \ ("unbound \ variable: " ++ n) \\ & \quad Just \ val \rightarrow return \ val \\ eval6 \ (Plus \ e1 \ e2) = \mathbf{do} \ tick'' \\ & \quad e1' \leftarrow eval6 \ e1 \end{array}
```

```
e2' \leftarrow eval6 \ e2
                                case (e1', e2') of
                                   (Int Val \ i1, Int Val \ i2) \rightarrow
                                      return \$ IntVal (i1 + i2)

ightarrow throw Error "type error in addition"
                        = do tick''
eval6 (Abs \ n \ e)
                                env \leftarrow ask
                                return $ FunVal env n e
eval6 (App \ e1 \ e2) = \mathbf{do} \ tick''
                                val1 \leftarrow eval6 \ e1
                                val2 \leftarrow eval6 \ e2
                                case val1 of
                                   FunVal\ env'\ n\ body \rightarrow
                                      local (const (Map.insert n val2 env'))
                                         (eval6 body)
                                   \_ 
ightarrow throw Error "type error in application"
tick'' :: Eval6 ()
tick'' = \mathbf{do} \ st \leftarrow get
              put (st + 1)
```

3 Conclusion

Monad transformers are a powerful tool in the toolbox of a functional programmer. This paper introduces several of the monad transformers available in current Haskell implementations, and shows how to use and combine them in the context of a simple functional interpreter.

We have not covered all monad transformers presently implemented in Haskell (e.g., continuation and list monad transformers) and recommend to read the library documentation available from the Haskell web site for additional information.

The use of monad transformers makes it very easy to define specialized monads for many applications, reducing the temptation to put everything possibly needed into the one and only monad hand-made for the current application.

Happy hacking in Haskell!

References

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