**Table 1. Runtime (seconds) for calculating <Z>**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| N | Qiskit(CPU) | Pennylane(CPU) | Pennylane(GPU) | TensorCircuit(CPU) | TensorCircuit(GPU) | Qulacs(ConeSim,CPU) | Cirq | Project Q |
| 4 | 0.04 | 0.14 | 0.30 | 0.99 | 23.48 | 0.001 | 0.10 | 0.495874 |
| 5 | 0.07 | 0.14 | 0.01 | 1.25 | 23.48 | 0.001 | 0.16 | 0.598717 |
| 6 | 0.10 | 0.14 | 0.01 | 0.81 | 23.48 | 0.001 | 0.16 | 0.907146 |
| 7 | 0.16 | 0.15 | 0.01 | 0.88 | 23.48 | 0.001 | 0.17 | 1.324505 |
| 8 | 0.29 | 0.15 | 0.02 | 4.02 | 23.55 | 0.001 | 0.18 | 2.436873 |
| 9 | 0.66 | 0.15 | 0.07 | 4.05 | 23.76 | 0.001 | 0.20 | 5.485286 |
| 10 | 1.97 | 0.17 | 0.02 | 10.34 | 4.99 | 0.002 | 0.21 | 15.78357 |
| 11 | 6.20 | 0.17 | 0.02 | 4.17 | 4.57 | 0.003 | 0.22 | 52.9837 |
| 12 | 21.76 | 0.19 | 0.02 | 4.24 | 2.62 | 0.01 | 0.24 | 201.6772 |
| 13 | 113.73 | 0.24 | 0.03 | 4.37 | 3.09 | 0.01 | 0.27 | 818.0197 |
| 14 | 390.56 | 0.30 | 0.14 | 4.43 | 7.94 | 0.24 | 0.30 | 3377.31 |
| 15 | -err | 0.40 | 0.05 | 4.57 | 4.58 | 0.40 | 0.35 | 14975.93 |
| 16 | -err | 0.61 | 0.07 | 4.72 | 5.31 | 0.42 | 0.43 | -time |
| 17 | -err | 1.04 | 0.12 | 4.94 | 4.93 | 0.49 | 0.64 | -time |
| 18 | -err | 2.04 | 0.24 | 6.02 | 6.85 | 0.36 | 1.09 | -time |
| 19 | -err | 4.17 | 0.44 | 10.24 | 22.25 | 0.75 | 2.09 | -time |
| 20 | -err | 9.36 | 0.82 | 18.06 | 22.52 | 1.69 | 3.66 | -time |
| 21 | -err | 16.81 | 1.61 | 34.75 | 22.59 | 7.67 | 6.57 | -time |
| 22 | -err | 40.98 | 3.21 | 67.99 | 22.68 | 10.96 | 12.12 | -time |
| 23 | -err | 76.55 | 6.54 | 133.32 | 6.42 | 26.16 | 28.75 | -time |
| 24 | -err | 153.76 | 12.73 | 278.71 | 8.35 | 73.18 | 57.22 | -time |
| 25 | -err | 302.07 | 25.72 | 558.05 | 12.38 | 143.95 | 107.61 | -time |
| 26 | -err | 606.58 | 50.54 | 1359.98 | 22.49 | 275.42 | 279.55 | -time |
| 27 | -err | 1214.95 | 98.77 | 2935.85 | -tensor\_limit | 575.93 | 564.45 | -time |
| 28 | -err | 2477.76 | 195.96 | 5947.25 | -tensor\_limit | 954.99 | 1055.78 | -time |
| 29 | -err | 4912.13 | 403.38 | -memory | -tensor\_limit | 3229.91 | -err | -time |
| 30 | -err | -time | 783.16 | -memory | -tensor\_limit | 5947.75 | -time | -time |
| 31 | -err | -memory | -memory | -memory | -tensor\_limit | -time | -time | -time |

**Table 2. Memory Usage(MiB) for calculating <Z>**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| N | Qiskit(CPU) | Pennylane(CPU) | Pennylane(GPU) | TensorCircuit(CPU) | TensorCircuit(GPU) | Qulacs(ConeSim,CPU) | Cirq | Project Q |
| 4 | 4.53 | 5.39 | 5.94 | 5.79 | 4.38 | 0.01 | 59.45 | 7.44 |
| 5 | 4.55 | 5.39 | 5.80 | 5.83 | 4.42 | 0.01 | 59.47 | 7.44 |
| 6 | 4.61 | 5.39 | 5.81 | 5.89 | 4.44 | 0.01 | 59.47 | 7.46 |
| 7 | 4.81 | 5.4 | 3.82 | 5.92 | 4.48 | 0.01 | 59.48 | 7.46 |
| 8 | 5.57 | 5.41 | 5.83 | 6.00 | 4.50 | 0.01 | 59.50 | 7.48 |
| 9 | 12.25 | 5.42 | 5.84 | 6.05 | 4.55 | 0.01 | 59.51 | 7.5 |
| 10 | 48.27 | 5.45 | 5.87 | 6.13 | 4.59 | 0.01 | 59.54 | 13.14 |
| 11 | 192.28 | 5.5 | 5.92 | 6.16 | 4.65 | 0.01 | 59.59 | 38.78 |
| 12 | 768.30 | 5.61 | 6.00 | 6.26 | 4.72 | 0.01 | 59.67 | 138.01 |
| 13 | 3072.32 | 5.83 | 6.16 | 6.38 | 4.84 | 0.01 | 59.84 | 528.48 |
| 14 | 12288.33 | 6.27 | 6.48 | 6.61 | 5.07 | 0.01 | 60.32 | 2077.57 |
| 15 | -err | 8.56 | 7.11 | 12.05 | 5.46 | 0.01 | 64.24 | 8247.58 |
| 16 | -err | 15.57 | 11.53 | 23.30 | 6.25 | 0.01 | 72.04 | -time |
| 17 | -err | 29.57 | 21.04 | 45.59 | 7.77 | 0.01 | 87.59 | -time |
| 18 | -err | 57.57 | 40.06 | 90.67 | 10.81 | 0.01 | 118.64 | -time |
| 19 | -err | 113.59 | 78.07 | 180.72 | 20.14 | 0.01 | 180.62 | -time |
| 20 | -err | 225.59 | 154.08 | 360.74 | 40.41 | 0.01 | 304.66 | -time |
| 21 | -err | 449.57 | 306.09 | 720.83 | 81.19 | 0.01 | 552.71 | -time |
| 22 | -err | 897.58 | 610.10 | 1440.75 | 163.21 | 0.01 | 1048.75 | -time |
| 23 | -err | 1793.58 | 1218.12 | 2880.76 | 328.18 | 0.01 | 2040.79 | -time |
| 24 | -err | 3585.59 | 2434.13 | 5760.68 | 660.0 | 0.02 | 4024.83 | -time |
| 25 | -err | 7169.6 | 4866.14 | 11520.78 | 1327.44 | 0.02 | 7992.87 | -time |
| 26 | -err | 14337.62 | 9730.12 | 23040.78 | 2670.07 | 0.02 | 15928.91 | -time |
| 27 | -err | 28673.63 | 19458.14 | 46080.80 | -tensor\_limit | 0.02 | 31800.95 | -time |
| 28 | -err | 57345.64 | 38914.15 | 92160.84 | -tensor\_limit | 0.02 | 63544.99 | -time |
| 29 | -err | 114689.7 | 77824.17 | -memory | -tensor\_limit | 0.02 | -err | -time |
| 30 | -err | -time | 155650.2 | -memory | -tensor\_limit | 0.02 | -time | -time |
| 31 | -err | -memory | -memory | -memory | -tensor\_limit | -time | -time | -time |

Revised Table Guide:

* "err" indicates errors occurring during execution due to framework issues.
* "memory" signifies a need for increased unified memory allocation.
* "time" denotes the necessity for an extended time limit when running on the NCSA Delta high-performance computer.
* "tensor\_limit" points to limitations in the framework's tensor data structure, preventing it from handling the required size.