9/13/2023 [1]

$$\frac{d\delta_i}{dt} = \omega_s \cdot (\omega_i - 1) \quad ①$$

δ_i not in per unit.

ω_i is in per unit, that is, its range is around 1.

$$\frac{d\omega_i}{dt} = \left[P_{mi} - P_{ei} - D_i\left(\omega_i \cdot \omega_s - \omega_s\right)\right] / (2H_i) \quad ②$$

$\rightarrow D_i \omega_s(\omega_i - 1)$

from ① $\omega_i = \frac{1}{\omega_s} \cdot \frac{d\delta_i}{dt} + 1$ ③

③ $\Rightarrow \frac{d\omega_i}{dt} = \frac{1}{\omega_s} \cdot \frac{d^2\delta_i}{dt^2}$ ④

plug ③ & ④ into ②:

$$\Rightarrow \frac{1}{\omega_s} \cdot \frac{d^2\delta_i}{dt^2} = \frac{1}{2H_i}\left[P_{mi} - P_{ei} - D_i \omega_s \cdot \frac{1}{\omega_s} \cdot \frac{d\delta_i}{dt}\right]$$

$$\Rightarrow \frac{1}{\omega_s} \cdot \frac{d^2\delta_i}{dt^2} = \frac{1}{2H_i}\left(P_{mi} - P_{ei} - D_i \cdot \frac{d\delta_i}{dt}\right) \quad ⑤$$

☆ use quantum circuit to represent δ_i, calculate $\frac{d\delta_i}{dt}$ & $\frac{d^2\delta_i}{dt^2}$

☆ then used ③ & ④ to obtain ω_i & $\frac{d\omega_i}{dt}$ → not necessary.

☆ need to satisfy ⑤.

in [WSCC 2nd order_09132023.ipynb]

du[7] = 0 = $-P\_e1 + f_1(\delta_1, \delta_2, \delta_3)$ ⑥

du[8] = 0 = $-P\_e2 + f_2(\delta_1, \delta_2, \delta_3)$ ⑦

du[9] = 0 = $-P\_e3 + f_3(\delta_1, \delta_2, \delta_3)$ ⑧

f_1, f_2, f_3 are long expression in the ipynb file.

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Plug 6, 7, 8 into 5, we obtain:

$$\frac{1}{\omega_s} \cdot \frac{d^2\delta_i}{dt^2} = \frac{1}{2H_i}\left(P_{mi} - f_i(\delta_1, \delta_2, \delta_3) - D_i \cdot \frac{d\delta_i}{dt}\right) \quad (9)$$

for $i = 1, 2, 3$, we have

$$\begin{cases} \dfrac{d^2\delta_1}{dt^2} = \dfrac{\omega_s}{2H_1}\left(P_{m1} - f_1(\delta_1, \delta_2, \delta_3) - D_1 \dfrac{d\delta_1}{dt}\right) & (9.1) \\ \dfrac{d^2\delta_2}{dt^2} = \dfrac{\omega_s}{2H_2}\left(P_{m2} - f_2(\delta_1, \delta_2, \delta_3) - D_2 \dfrac{d\delta_2}{dt}\right) & (9.2) \\ \dfrac{d^2\delta_3}{dt^2} = \dfrac{\omega_s}{2H_3}\left(P_{m3} - f_3(\delta_1, \delta_2, \delta_3) - D_3 \dfrac{d\delta_3}{dt}\right) & (9.3) \end{cases}$$

steps: 1. use 3 quantum circuits to represent $\delta_1, \delta_2, \delta_3$

they may use the same structure, but different θ.

each circuit can use 2 qubit, like the single machine system.

2. calculate $\frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}, \frac{d^2\delta_1}{dt^2}, \frac{d^2\delta_2}{dt^2}, \frac{d^2\delta_3}{dt^3}$

via finite derivative or DQC

3. objective function or loss function is set

according to (9.1), (9.2), (9.3) & boundary/initial conditions

of $\delta_1, \delta_2, \delta_3, \omega_1, \omega_2, \omega_3$ ← used

4. use 3 to calculate $\omega_1, \omega_2, \omega_3$

5. plot $\delta_1, \delta_2, \delta_3, \omega_1, \omega_2, \omega_3$ & compare with julia data

in the same figure.