- This lab will cover Binary Search Trees & Binary Trees.
- It is assumed that you have reviewed chapter 8 & 11 of the textbook. You may want to refer to the text and your lecture notes during the lab as you solve the problems.
- When approaching the problems, think before you code. Doing so is good practice and can help you lay out possible solutions.
- Think of any possible test cases that can potentially cause your solution to fail!
- You must stay for the duration of the lab. If you finish early, you may help other students. If you don't finish by the end of the lab, we recommend you complete it on your own time. Ideally you should not spend more time than suggested for each problem.
- Your TAs are available to answer questions in lab, during office hours, and on Ed.

Vitamins (30 minutes)

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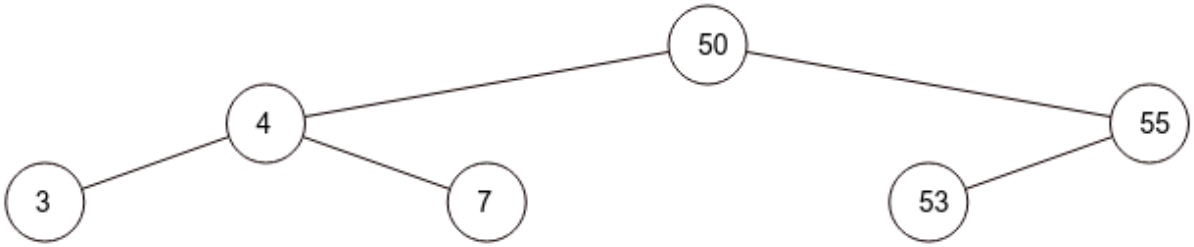
1. Given the following Binary Search Tree, perform the following operations cumulatively:



Perform the following operations (10 minutes):

1. Insert 2
2. Delete 7
3. Insert 6
4. Insert 8
5. Delete 55
6. Insert 56
7. Pre Order Traversal
8. Post Order Traversal
9. In Order Traversal
10. Level Order Traversal

2. (10 minutes)

a. Given the following traversal of a **binary tree**. Restore the tree:

   Preorder:
   ```
   7 4 3 8 9 1 6 5 2
   ```

   Inorder:
   ```
   3 8 4 7 1 6 9 5 2
   ```

b. If you are given just the preorder, can this **binary tree** be unique? If not, give a counterexample (two different trees with this same preorder).

   Preorder:
   ```
   5 2 1 3 4 9 7 6 8
   ```

c. Now, if you are given just the preorder of a **binary search tree**, will this tree be unique?

   Preorder:
   ```
   5 2 1 3 4 9 7 6 8
   ```

3. A student suggested the following implementation for checking whether a LinkedBinaryTree object is also a Binary Search Tree. (10 minutes)

```
def is_BST(root):

    if (root.left is None and root.right is None):
        return True

    elif root.left and root.right:
        check_left = root.left.data < root.data
        check_right = root.right.data > root.data
        return check_left and check_right and is_BST(root.left)
and is_BST(root.right)

    elif root.left:
        check_left = root.left.data < root.data
        return check_left and is_BST(root.left)

    elif root.right:
        check_right = root.right.data > root.data
        return check_right and is_BST(root.right)
```

Is there a problem with this function? If so, draw a simple binary tree that will cause this function to return an incorrect result.

ex) either draw **a BST** that makes is_BST return **False**

or a **non BST** that makes is_BST return **True**.

Coding

In this section, it is strongly recommended that you solve the problem on paper before writing code.

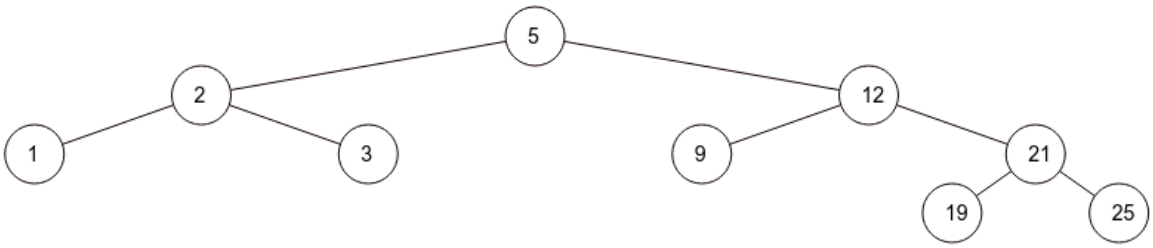
Download the **BinarySearchTreeMap.py** file on BrightSpace.

1. Given a non-empty `BinarySearchTreeMap`, implement a function that will return a tuple containing the minimum and maximum key in the tree. The function should be **iterative**. (10 minutes)

```
def min_max_BST(bst):
''' Returns a tuple containing the min and max keys in the
binary search tree'''
```

What is the worst-case run time of your function?

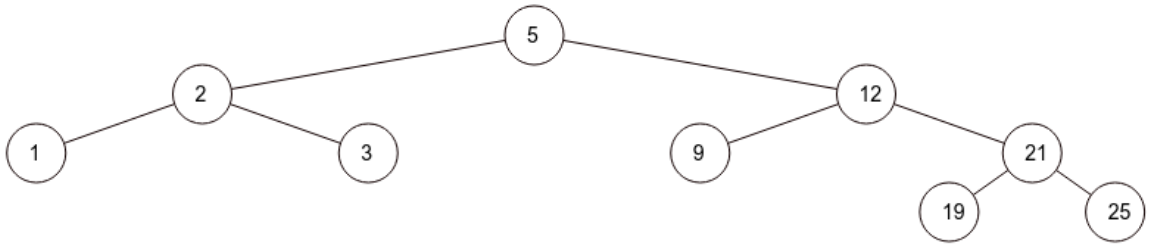
ex) min_and_max will return (1, 25) for this Binary Search Tree



2. Given a binary search tree and a number, n, write an **iterative** function that will find the greatest number that is one of the values of the given tree and is less than or equal to n. If there is no such value, then return -1. Your implementation should run in O(h) worst case, where h is the height of the tree. There are no constraints on space. (15 minutes)

```
def glt_n(bst, n): #glt = greatest less than
''' Returns the greatest number in the binary search tree
less than or equal to n'''
```

ex)



Input : n = 21 20

Output : 21

Input: n = 4

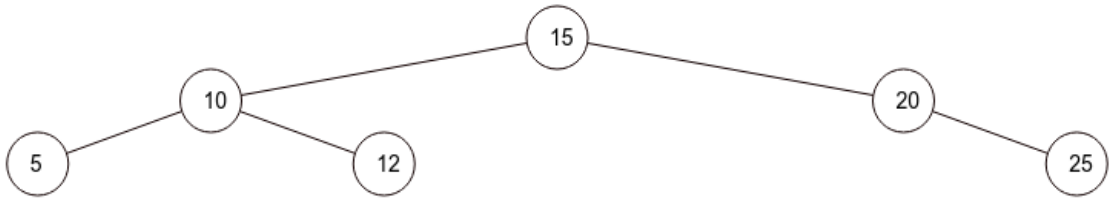
Output: 3

3. Given two Binary Search Trees consisting of unique positive elements, write a program to check whether the two BSTs contain the same set of elements or not. Although there are no time or space constraints, what is the worst-case run time and extra space complexity of your function? (30 minutes)

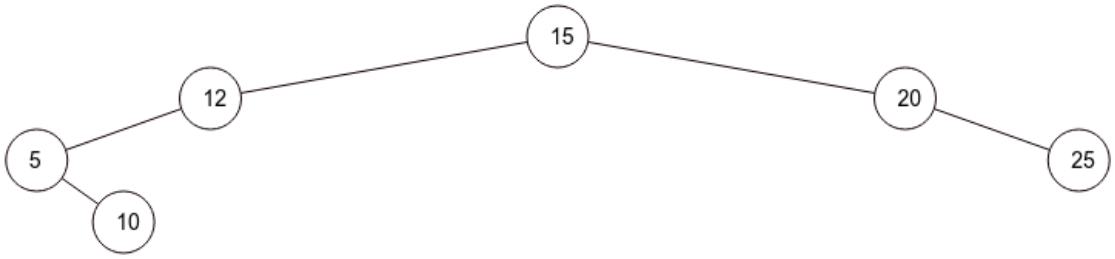
```
def compare_BST(bst1, bst2):
''' Returns true if the two binary search trees contain the
same set of elements and false if not'''
```

Given the two trees, the program should return True.

BST 1



BST 2



4. Let's fix the is_BST function that was incorrectly defined in the vitamins section of this lab. Given the root node of a LinkedBinaryTree, implement a function that will return True if the tree is a Binary Search Tree and False if not. Try to aim for linear implementation.(40 minutes)

   The helper function should return a tuple triplet containing the min and max of the current subtree, and a bool value. (min, max, bool)

```
def is_BST(root):
     return is_BST_helper(root)[2]


def is_BST_helper(root):
''' Returns a tuple (min, max, bool)''' 5484
```

Optional - Coding

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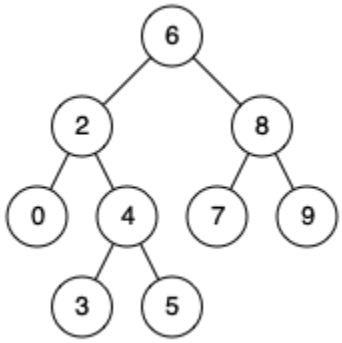
5. Given two nodes in a tree, implement a function to find their lowest common ancestor. The lowest common ancestor is the deepest node from which one can get to the two input nodes. Your implementation should run in **O(h) worst case**.



   A. Implement such function for two nodes in a **BST** (Binary Search Tree)

```
def lca_BST(bst, node1, node2):
```

   B. Implement such function for two nodes in a **BT** (Binary Tree)

```
def lca_BT(bst, node1, node2):
```

Note: Don't worry about the run-time and extra space complexity. Simply come up with solutions, which you may improve upon if you're able to.

Hint: Optimal implementations for parts A and B **should not** follow the same approach. For part A, you should utilize the property of a Binary Search Tree.