Integrated Control and Planning for Mobile Robots

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1 Motivation

Mobile Robots are a class of **underactuated** systems with inherent **non-holonomic constraints** that restrict their maneuverability. In practice, they are often deployed in space-constrained operating workspaces. Therefore, motion planning for such systems becomes a complex task, and it is significantly challenging when the exact obstacle configuration is not known beforehand. The mathematical models (unicycle, bicycle,etc) for mobile robots are typically driftless , and exhibit the following structure,

$$\dot{X} = g(X, U) \quad ; \quad X \in Q \subseteq \mathbb{R}^n, U \in [U_m, U_M] \subset \mathbb{R}^m \quad ; \quad m < n \tag{1}$$

Here, $g(X) \in \mathbb{R}^{n \times m}$ is typically a lipschitz function. The configuration space is given by $\mathcal{Q}$ and the set of obstacles is denoted by $\mathcal{O}_i$. From these definitions, the free configuration space $\mathcal{Q}_{\text{free}}$ is obtained as follows,

$$\mathcal{Q}_{\text{free}} = \{X \in \mathcal{Q} \mid X \cap (Q \cap (\cup_i \mathcal{O}_i)) = \emptyset\} \tag{2}$$

Suppose the initial conditions are given by X_0 at an initial time instant t_0 and the target goal is given by X_f at a time instant $t = t_f$, a candidate motion plan $X_{\text{ref}}(t) \in Q$ is deemed admissible for navigation under the following conditions,

- $X_{\text{ref}}(t)$ is a solution to (1), with $X_{\text{ref}}(t_0) = X_0$ and $X_{\text{ref}}(t_f) = X_f$.
- $X_{\text{ref}}(t) \in \mathcal{Q}_{\text{free}}, \; \forall t \in [t_0, t_f]$

Under such conditions, the resulting motion plan $X_{\mathbf{ref}}(t)$ is compliant for tracking through a careful design of a low-level tracking controller. (The final time argument condition could be optionally relaxed if required). *During operation in unknown environments, such amiable motion plans must be generated during run-time to accomplish the mission objectives based on the sensed information.*

1.1 Integrated Planning and Control (IPC)

IPC algorithms **seek motion plans as a sequence of feedback controllers over domains** in the navigable regions of the environment, rather than an explicit path/trajectory in the configuration space of the system. The IPC motion plans exhibit the following properties,

- Each domain $\mathcal{D}_k$ is positively invariant under **feedback controller** $\mathcal{F}_k$, and is associated with a goal set $\mathcal{W}_k \subset \mathcal{D}_k$. Essentially, any initial state $X(t) \in \mathcal{D}_k$ evolves such that it converges onto $\mathcal{W}_k$ under the influence of $\mathcal{F}_k$ while remaining within $\mathcal{D}_k$ during the entire duration. Given a system as in (1), the following holds true,
  - If $X(t = t_0) \in \mathcal{D}_k$, then $X(t = t_f) \in \mathcal{W}_k$ and $X(t) \in \mathcal{D}_k, \; \forall t \in [t_0, t_f]$
- The domains $\mathcal{D}_k$ are sequenced in such a way that their goal sets $\mathcal{W}_k$ lies on **at least one other domain** $\mathcal{D}_i$, and each $\mathcal{D}_i \subset Q_{\text{free}}$. (This ensures overall connectivity between the domains $\mathcal{D}_k$)

$$\mathcal{W}_k \in \mathcal{D}_i, \quad i \neq k$$

  - The initial configuration $\mathcal{S}$ in the navigation query is such that, $\mathcal{S} \in \bigcup \mathcal{D}_k$.
  - The final configuration $\mathcal{T}$ in the navigation query is such that $\mathcal{T} \in \mathcal{W}_k$ for some 'k'.

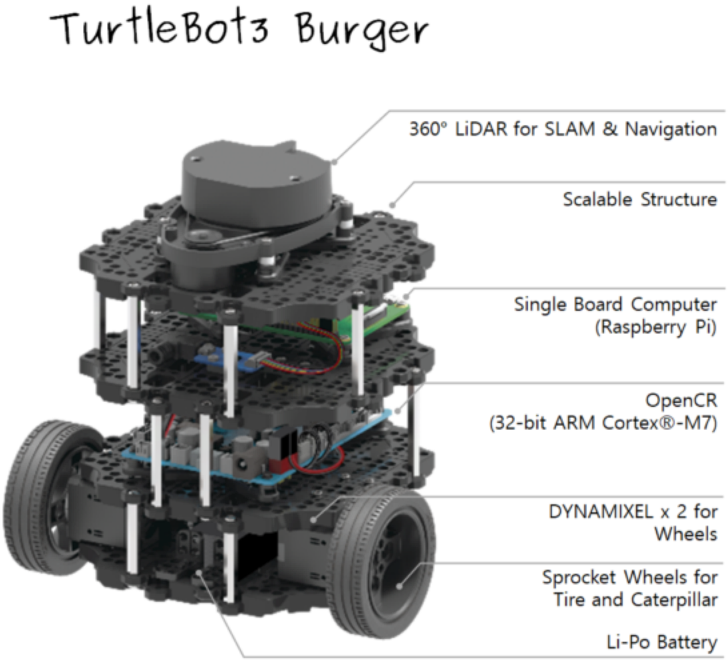

Figure 1: Caption

Depending on the current state of the system, the $\mathcal{F}_k$ associated with the corresponding $\mathcal{D}_k$ is applied to the system. Suppose the domains $\mathcal{D}_k$ are sequenced in such a way that each $\mathcal{W}_k$ lies on a unique $\mathcal{D}_i$, $i \neq k$, the system's trajectories are guaranteed to safely converge onto $\mathcal{T}$ when the above-mentioned structures are established.

Mathematically, the geometric structure of $\mathcal{D}_k$ is a consequence of the underlying system dynamics for which $\mathcal{F}_k$ is designed for stability. **Essentially, $\mathcal{D}_k$ can be thought of as a geometric manifestation of the feedback controller $\mathcal{F}_i$. Therefore, given the existence of a feedback controller, the problem of mobile robot navigation effectively becomes a problem of computational geometry (involving the sequencing of $\mathcal{D}_k$) which can often be solved much faster than optimization/optimal control problems.** This can potentially suit real-time implementations where online decision-making in the face of uncertainties is vital during operation in unknown/dynamic environments.