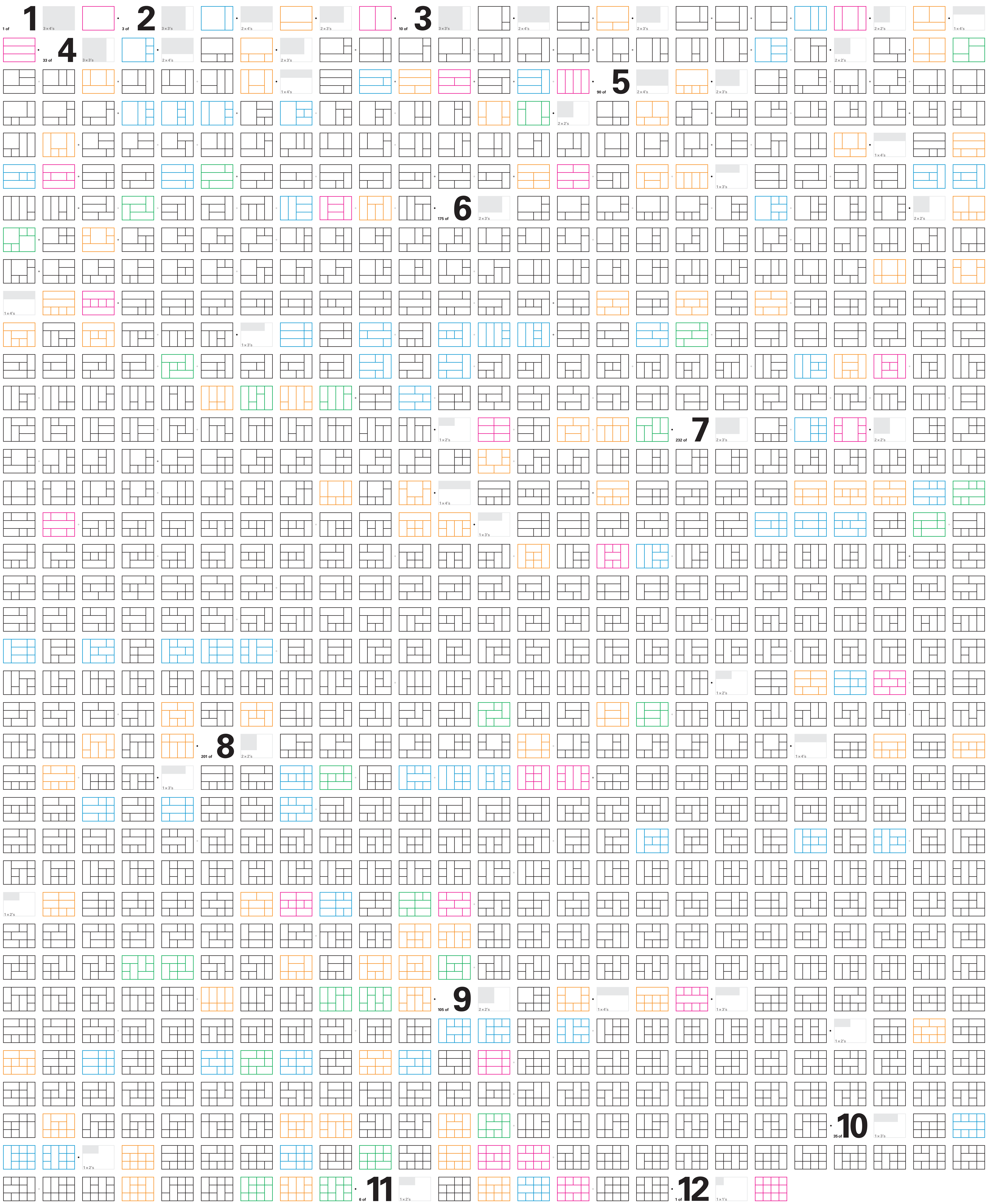

The 892 unique ways to partition a 3 × 4 grid

This poster illustrates a change in design practice. Computation-based design—that is, the use of algorithms to compute options—is becoming more practical and more common. Design tools are becoming more computation-based; designers are working more closely with programmers; and designers are taking up programming.

Above, you see the 892 unique ways to partition a 3 × 4 grid into unit rectangles. For many years, designers have used grids to unify diverse sets of content in books, magazines, screens, and other environments. The 3 × 4 grid is a common example. Yet even in this simple case, generating all the options has—until now—been almost impossible.

Patch Kessler designed algorithms to generate all the possible variations, identify unique ones, and sort them—not only for 3 × 4 grids but also for any n × m grid. He instantiated the algorithms in a MATLAB program, which output PDFs, which Thomas Gaskin imported into Adobe Illustrator to design the poster.

Rules for generating variations
The rule system that generated the variations in the poster was suggested by Bill Drenttel and Jessica Helfand who noted its relationship to the tatami mat system used in Japanese buildings for 1300 years or more. In 2006, Drenttel and Helfand obtained U.S. Patent 7124360 on this grid system—"Method and system for computer screen layout based on recombinant geometric modular structure".

The tatami system uses 1 × 2 rectangles. Within a 3 × 4 grid, 1 × 2 rectangles can be arranged in 5 ways. They appear at the end of section 6.

Unit rectangles (1 × 1, 1 × 2, 1 × 3, 1 × 4; 2 × 2, 2 × 3, 2 × 4; 3 × 3, 3 × 4) can be arranged in a 3 × 4 grid in 3,164 ways. Many are almost the same—mirrored or rotated versions of the same configuration. The poster includes only unique variations—one version from each mirror or rotation group. Colors indicate the type and number of related non-unique variations. The variations shown in black have 3 related versions; blue, green, and orange have 1 related version; and magenta variations are unique, because mirroring and rotating yields the original, thus no other versions. (See the table to the right for examples.)

Rules for sorting
The poster groups variations according to the number of non-overlapping rectangles. The large figures indicate the beginning of each group. The sequence begins in the upper left and proceeds from left to right and top to bottom. Each group is further divided into sub-groups sharing the same set of elements. The sub-groups are arranged according to the size of their largest element from largest to smallest. Squares precede rectangles of the same area; horizontals precede verticals of the same dimensions. Within sub-groups, variations are arranged according to the position of the largest element, preceding from left to right and top to bottom. Variations themselves are oriented so that the largest rectangle is in the top left. Black dots separate groups by size. Gray dots separate groups by orientation.

Where to learn more
Grids have been described in design literature for at least 50 years. French architect Le Corbusier describes grid systems in his 1946 book, *Le Modulor*. Swiss graphic designer Karl Gerstner describes a number of grid systems or "programmes" in his 1964 book, *Designing Programmes*. The classic work on grids for graphic designers is Josef Muller-Brockman's 1981 book, *Grid Systems*.

Patch Kessler explores the mathematical underpinnings of grid generation in his paper "Arranging Rectangles". www.mechanicaldust.com/Documents/Partitions_05.pdf

Thomas Gaskin has created an interactive tool for viewing variations and generating HTML. www.3x4grid.com

Design: Thomas Gaskin
Creative Direction: Hugh Dubberly
Algorithms: Patrick Kessler
Patent: William Drenttel + Jessica Helfand

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703 × Black
Asymmetric
Changed by horizontal reflection, vertical reflection, and 180° rotation.

61 × Blue
Top-bottom symmetry
Changed by horizontal reflection and 180° rotation.

26 × Green
Rotational symmetry
Changed by horizontal and vertical reflection.

76 × Orange
Left-right symmetry
Changed by vertical reflection and 180° rotation.

26 × Magenta
All three symmetries combined
Unchanged by horizontal reflection, vertical reflection, or 180° rotation.