**Project of Discrete Event Systems - A.A. 2021/22**

Group #07

Prepare a document where you describe how you accomplished the project (theoretical tools used, simulation techniques applied, etc.) and the results you obtained. Please use figures and/or tables to explain the results. Provide also the Matlab code you produced.

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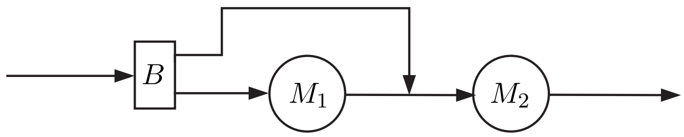
Consider the production system shown in the figure, composed of a one-place buffer $B$ followed by the series interconnection of two machines $M_1$ and $M_2$.




A fraction $q = 2/5$ of the arriving parts needs to be processed in $M_2$ only, while the other parts need to be processed in both machines. Parts arriving when $B$ is full, are rejected. If $M_1$ terminates a job, and $M_2$ is working, $M_1$ keeps the part (and therefore it remains unavailable for another job) until $M_2$ terminates the ongoing job. Parts arriving from $M_1$ have prority to $M_2$ over those waiting in $B$.

1. Model the logic of the system using a stochastic state automaton.

Assume that raw parts arrive at the system as generated by a Poisson process, and processing times in $M_1$ and $M_2$ have exponential distributions. In the following, let $t_\delta$ denote the minimum time such that all the state probabilities settle around their limit values $\pm\delta$, with $\delta = 0.001$. We say that the system is practically at steady state for $t \geq t_\delta$.

2. For a fixed $\varepsilon > 0$, determine the parameters of the event lifetime distributions so that all the limit state probabilities (computed analytically) belong to the interval $[(1-\varepsilon)\frac{1}{n}, (1+\varepsilon)\frac{1}{n}]$, where $n$ is the number of states, and $t_\delta \in [10, 15]$ min. By trial and error, make $\varepsilon$ as small as you can, and describe the difficulties encountered in making it smaller.

3. Estimate the limit state probabilities using simulations over a time horizon such that the steady state is reached. Show the trend of the variance of the estimates versus the number of samples using tables and/or figures, discussing them in the light of what you know from the theory.

4. Estimate the probability that at least five arriving parts are rejected over the interval $[t_\delta, 2.5t_\delta]$.

5. Estimate $\lambda_{eff}$ and $\mu_{eff}$ at steady state using simulations, verifying the condition $\lambda_{eff} = \mu_{eff}$ with an error not exceeding 0.001.

6. Estimate $E[S_\Sigma]$, $E[X_\Sigma]$ and $\lambda_\Sigma$ at steady state using simulations for the subsystem $\Sigma$ formed by $B$ and $M_1$, verifying the Little's law with an error not exceeding 0.01.

To compute the estimates in points 5) and 6), rely on definitions of the quantities of interest, rather than on expressions based on specific assumptions, such as the use of exponential distributions. However, it is advisable to use those expressions to compare your estimates, and check the correct implementation of the estimation procedures.

Finally, consider the file `dati_gruppo_07.mat`, containing measurements of the lifetimes of the events.

7. Verify whether the system admits steady state, generating the lifetimes of the events according to the empirical distributions estimated with measured data.

8. In case of a positive answer to the previous question, repeat points 5) and 6).