The Logic of Indexicals

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1 The Topic

- we begin by talking only about a few indexicals: "I", "you", "yesterday", "today", and "tomorrow"

- Kaplan's semantics: indexicals get their value from the context of utterance, which is formally represented as a sequence of objects (agent, time, place, world, etc). Once we calculate the referents, the truth value is calculated relative to the world and time of the context of utterance.

- Frege quote:

    If a time-indication is conveyed by the present tense one must know when the sentence was uttered in order to grasp the thought correctly. Therefore the time of utterance is part of the expression of the thought. If someone wants to say today what he expressed yesterday using the word 'today', he will replace this word with 'yesterday'. Although the thought is the same its verbal expression must be different in order that the change of sense which would otherwise be effected by the differing times of utterance may be canceled out. (Frege ([1918] 1997, p. 332))

- not sure about "same thought". But these arguments seem valid:

    (1) Josh, the weatherman, says "It's raining today". The next day, Josh says "It rained yesterday".

    (2) Jerry says to Elaine: "If you want to go to the movies, we'll go to the movies." Elaine replies: "I do want to go to the movies."

    (3) James Madison says to Napoleon: "I'm taller than you are". Napoleon replies (correctly): "You're not taller than I am".[1]

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[1] Napoleon was 5′6.5″ (1.69m); Madison was shorter: 5′4″ (1.63m). I don't know if they ever met, but Madison was the Secretary of State of the US during the Louisiana Purchase, and Napoleon was the selling party.

- can we have a logic which counts these as valid?

- Soames says no: if the sentences in (1) were said on the same day, and it had not rained the previous day, the argument would not be valid. In general, no argument with indexicals in it comes out valid, since those expressions always risk to change referent mid-argument

2   Yes, We Can

- look at (1): it's a good argument because the day of the premise is exactly one day before the day of the conclusion. It is valid because in all sequences of contexts which have the days in that relation, if the premise is true in its context, the conclusion is true in its context

- note: arguments are made up of context-sentence pairs, not sentences alone

- more generally, validity doesn't require that we look at all sequences of contexts; just at the relevant ones. So we're interested in sequences of contexts which have certain abstract properties: (non)identities between participants, relations between days.

- think of an *argument* as a sequence of the form $\langle [c_1, \phi_1], [c_2, \phi_2], \ldots [c_n, \phi_n] \rangle$, where $n \geqslant 1$, and each pair $[c_i, \phi_i]$ is a pair of a context and a sentence.

- think of a *context* as (S, A, D, W), where S is the speaker, A is the addressee, D is the day (represented by a positive integer), and W is the world.

Definition 1. Given an argument $\langle [c_1, \phi_1], [c_2, \phi_2], \ldots [c_n, \phi_n] \rangle$, the sequence of its sentences $\langle \phi_1, \phi_2, \ldots \phi_n \rangle$ is its *conversational thread*.

Definition 2. Given an argument $\langle [c_1, \phi_1], [c_2, \phi_2], \ldots [c_n, \phi_n] \rangle$, the sequence of its contexts $\langle c_1, c_2, \ldots c_n \rangle$ is its *conversational situation*.

Definition 3. A conversational situation
$$\langle (S_1, A_1, D_1, W_1), (S_2, A_2, D_2, W_2), \ldots (S_n, A_n, D_n, W_n) \rangle$$
is *proper* iff $(\forall i, j \in [1, n]) W_i = W_j \wedge (S_i$ and A_i exist at W_i on $D_i)$.

Definition 4. Two conversational situations $\langle c_1, c_2, \ldots c_m \rangle$ and $\langle c_1', c_2', \ldots c_n' \rangle$ are *similar* iff

    (a) $m = n$

    (b) $(\forall i, j \in [1, n]) A_i = A_j \leftrightarrow A_i' = A_j'$

    (c) $(\forall i, j \in [1, n]) S_i = S_j \leftrightarrow S_i' = S_j'$

    (d) $(\forall i, j \in [1, n]) A_i = S_j \leftrightarrow A_i' = S_j'$ and

    (e) $(\forall k \in \mathbb{N})(\forall i, j \in [1, n]) D_i - D_j = k \leftrightarrow D_i' - D_j' = k$

Definition 5. An argument $\langle [c_1, \phi_1], [c_2, \phi_2], \ldots [c_n, \phi_n] \rangle$, where we think of $[c_n, \phi_n]$ as the conclusion, is *valid* iff for any conversational situation $\langle c_1', c_2', \ldots c_n' \rangle$ similar to its conversational situation, if $(\forall i \in [1, n - 1]) \phi_i$ is true in c_i', then ϕ_n is true in c_n'.

Definition 6. $[c_1, \phi]$ is a *logical truth* iff for any $\langle c_2 \rangle$ which is similar to $\langle c_1 \rangle$, $[c_2, \phi]$ is true.

3   The Art of Symbolization

- a step in an argument is a context-sentence pair. So symbolization depends on empirical, non-semantic knowledge, and validity depends on worldly facts

  - against Quine: "the business of formal logic is describable as that of finding statement forms which are logical, in the sense of containing no constants beyond the logical vocabulary, and (extensionally) valid, in the sense that all statements exemplifying the form in question are true" (Quine (1953, p. 456))

- so it's OK that this turns out valid:

  (4) ⟨[(Margaret Thatcher, Margaret Thatcher, D, W), "I am identical to you"]⟩

4   Why Not a Logic on Contents?

- ⟨[(Paul N, Paul D, October 31, 2011, @), T$\phi \leftrightarrow \phi$]⟩, where ϕ is "I am taller than you", is valid in LI, but the proposition is not valid in a logic on contents

  - note that it would remain valid even if we counted as proper contexts without an addressee. Unlike in [Demons], we only look at *some* contexts.

- "Hesperus is Phosphorus" expresses a proposition of the form $\alpha = \alpha$ (so, valid in a logic on contents). But it's not valid in any logic which looks at sentences.

  - why is that? Is it because "H = P" depends on worldly facts? I hope not
  - the difference is that semantic facts do not determine the coreference of "H" and "P"; contrast with "I" and "you" in (4)

5   What about Demonstratives?

- what is the difference between demonstratives ("this", "that") and indexicals ("I", "today")?

- look at "you"

  - a bit like demonstratives: the speaker must *intend* to address X; it can fail to refer (e.g. if addressing a hallucination)
  - a bit like indexicals: it has a constant character ("I" refers to the speaker; "you" to the addressee); the speaker may intend to refer to X, but if she's not addressing X, it won't work

- it depends on your theoretical goals. For logical purposes, "you" is an indexical, because it depends on a robust feature of contexts

  - a feature of a context is robust if it is a feature that any context may have; it's not there only to provide a semantic value to some expression

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- – any utterance of "H = P" has a speaker, and takes place on a particular day; it may also have an addressee
- – a demonstratum is only needed to give a demonstrative its referent

- for "you", the operative intention is to address X (2-place). For "this", it is to refer to X by this use of "this" (3-place)

- all this is independent of the possibility of multiple occurrences

- so what about a logic for demonstratives?

  - – we need more from contexts: (possibly gappy sequences of) demonstrata
  - – for most theories of the mechanism of reference of demonstratives, we only get demonstrata in contexts where there is an occurrence of a demonstrative
  - – so unlike for indexicals, we cannot evaluate a sentence at a context where nothing is uttered. Logic of speech acts?
  - – otherwise, just like before: we care about identities and non-identities between speakers, addressees, and demonstrata, and relations between days

References

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Quine, W. V. O. (1953). Mr. Strawson on Logical Theory. *Mind, 62*(248), 433–451.