Appendices

A General Age-Specific Mortality Model with An Example Indexed by Child or Both Child and Adult Mortality

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Appendix A SVD Algebra

Below I rearrange the basic SVD relationship to derive useful additional relationships. $\mathbf{X}$ is an $A \times L$ matrix of rank ρ.

$$\mathbf{X} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} \tag{A.1}$$

$$\begin{aligned}
\begin{bmatrix} | & & | \\ \mathbf{x}_1 & \dots & \mathbf{x}_L \\ | & & | \end{bmatrix} &= \begin{bmatrix} | & & | \\ \mathbf{u}_1 & \dots & \mathbf{u}_\rho \\ | & & | \end{bmatrix} \begin{bmatrix} s_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & s_\rho \end{bmatrix} \begin{bmatrix} — & \mathbf{v}_1 & — \\ & \vdots & \\ — & \mathbf{v}_\rho & — \end{bmatrix} \\
&= \begin{bmatrix} | & & | \\ \mathbf{u}_1 & \dots & \mathbf{u}_\rho \\ | & & | \end{bmatrix} \begin{bmatrix} — & s_1\mathbf{v}_1 & — \\ & \vdots & \\ — & s_\rho\mathbf{v}_\rho & — \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^{\rho} u_{1i}s_i v_{1i} & \dots & \sum_{i=1}^{\rho} u_{1i}s_i v_{Li} \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{\rho} u_{Ai}s_i v_{1i} & \dots & \sum_{i=1}^{\rho} u_{Ai}s_i v_{Li} \end{bmatrix} \\
&= \begin{bmatrix} | & & | \\ \sum_{i=1}^{\rho} s_i v_{1i}\mathbf{u}_i & \dots & \sum_{i=1}^{\rho} s_i v_{Li}\mathbf{u}_i \\ | & & | \end{bmatrix} && \text{(A.2)} \\
&= \sum_{i=1}^{\rho} \begin{bmatrix} | & & | \\ s_i v_{1i}\mathbf{u}_i & \dots & s_i v_{Li}\mathbf{u}_i \\ | & & | \end{bmatrix} \\
&= \sum_{i=1}^{\rho} \begin{bmatrix} s_i v_{1i}u_{1i} & \dots & s_i v_{Li}u_{1i} \\ \vdots & \ddots & \vdots \\ s_i v_{1i}u_{Ai} & \dots & s_i v_{Li}u_{Ai} \end{bmatrix} \\
&= \sum_{i=1}^{\rho} s_i \begin{bmatrix} u_{1i} \\ \vdots \\ u_{Ai} \end{bmatrix} \begin{bmatrix} v_{1i} \dots v_{Li} \end{bmatrix} && \text{(A.3)}
\end{aligned}$$

$$\mathbf{X} = \sum_{i=1}^{\rho} s_i \mathbf{u}_i \mathbf{v}_i^{\mathrm{T}} \tag{A.4}$$

From Equation A.2 we have

$$\mathbf{x}_\ell = \sum_{i=1}^{\rho} s_i v_{\ell i} \mathbf{u}_i \ . \tag{A.5}$$

Appendix B SVD Component Values

Table B.1: SVD Component ($s_{zi}\mathbf{u}_{zi}$) Values.

Age	Female c_1	Female c_2	Female c_3	Female c_4	Male c_1	Male c_2	Male c_3	Male c_4
0	-936.95	-33.16	11.72	-13.73	-921.04	-46.96	-6.01	5.06
1	-1080.59	-69.60	8.18	-16.96	-1070.20	-80.39	-4.71	-0.69
2	-1123.82	-68.05	3.59	-14.79	-1109.59	-74.46	-6.31	-0.68
3	-1148.09	-64.22	-1.27	-21.16	-1133.10	-73.11	-13.70	4.48
4	-1163.32	-58.34	5.40	-16.53	-1148.08	-66.00	-12.17	1.72
5	-1175.85	-56.72	-0.23	-15.00	-1158.79	-62.01	-11.91	2.86
6	-1185.33	-54.45	-8.74	-15.44	-1168.01	-61.65	-14.05	5.66
7	-1196.32	-53.28	-13.97	-13.08	-1174.82	-57.85	-14.39	6.15
8	-1204.07	-49.23	-21.60	-14.27	-1181.83	-53.43	-14.09	8.67
9	-1209.47	-46.70	-19.17	-10.49	-1188.26	-51.33	-17.18	4.91
10	-1212.17	-46.07	-23.97	-5.57	-1190.60	-48.94	-13.46	2.28
11	-1212.90	-46.11	-20.14	1.05	-1191.27	-44.78	-14.10	1.63
12	-1209.27	-42.39	-15.25	1.02	-1187.99	-40.81	-12.22	1.05
13	-1202.60	-40.48	-14.08	3.13	-1181.99	-38.30	-10.80	1.61
14	-1192.50	-35.74	-8.83	8.28	-1169.13	-29.97	-4.67	-3.34
15	-1181.22	-30.53	-8.30	15.14	-1155.50	-25.90	-3.69	-4.27
16	-1169.84	-26.00	-1.01	13.21	-1137.42	-17.28	0.07	-6.94
17	-1163.40	-24.97	-0.38	10.70	-1122.31	-12.59	3.15	-6.64
18	-1156.47	-21.56	0.62	14.40	-1107.24	-6.06	6.17	-11.11
19	-1153.54	-21.99	2.42	14.31	-1101.20	-7.97	8.25	-11.60
20	-1151.34	-23.45	-0.74	13.01	-1097.20	-10.12	8.21	-12.62
21	-1150.63	-25.01	-1.99	14.31	-1094.93	-11.03	10.34	-14.69
22	-1148.73	-26.37	0.50	13.59	-1093.42	-12.02	11.18	-12.00
23	-1147.91	-27.89	0.22	14.69	-1092.89	-12.25	12.36	-12.51
24	-1146.03	-27.70	1.25	13.92	-1093.31	-12.62	12.25	-11.73
25	-1143.37	-25.83	0.73	12.46	-1093.03	-11.73	13.46	-10.89
26	-1140.86	-24.49	3.41	11.03	-1093.36	-11.37	13.55	-11.68
27	-1138.56	-23.73	3.60	9.06	-1092.15	-8.96	14.43	-10.34
28	-1136.38	-24.69	3.33	12.61	-1091.06	-8.39	15.14	-8.62
29	-1131.46	-18.97	6.45	5.78	-1089.78	-7.34	14.46	-8.24
30	-1127.61	-18.24	5.08	5.57	-1088.36	-9.69	14.00	-7.34
31	-1126.46	-17.80	3.34	7.78	-1086.40	-5.35	14.87	-7.00
32	-1121.50	-16.87	4.56	8.25	-1083.30	-6.38	14.15	-4.70
33	-1118.14	-15.77	2.80	10.07	-1080.61	-4.93	14.49	-3.96
34	-1113.15	-12.56	7.35	7.97	-1077.47	-3.92	14.91	-2.74
35	-1107.92	-9.14	7.30	5.22	-1074.16	-4.61	14.13	-1.13
36	-1104.37	-8.56	6.18	5.63	-1070.69	-2.66	13.36	-1.02
37	-1100.74	-7.26	5.27	5.58	-1067.70	-3.06	12.87	-1.81

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Table B.1 – continued from previous page

Age	Female c_1	Female c_2	Female c_3	Female c_4	Male c_1	Male c_2	Male c_3	Male c_4
38	-1094.12	-2.89	8.72	1.83	-1062.20	-0.46	13.20	0.72
39	-1090.82	-2.94	7.54	3.83	-1057.99	0.93	12.19	1.11
40	-1085.24	-0.76	7.23	-1.63	-1052.80	0.47	12.42	3.52
41	-1082.33	1.69	5.56	4.84	-1049.25	3.81	11.62	2.59
42	-1075.23	4.66	7.83	0.86	-1042.86	4.15	10.92	4.09
43	-1071.29	7.40	7.22	0.72	-1038.29	6.43	10.81	4.16
44	-1066.68	9.32	6.81	2.81	-1033.42	7.17	9.96	4.64
45	-1060.98	11.20	8.65	0.74	-1027.21	8.53	9.43	5.37
46	-1057.00	12.13	7.43	2.33	-1022.75	10.18	8.90	5.78
47	-1051.54	15.63	7.58	-0.23	-1017.33	11.64	8.46	6.39
48	-1045.73	16.36	8.00	-0.95	-1011.78	11.87	7.88	6.94
49	-1040.47	18.54	7.63	-0.71	-1006.15	13.43	7.24	7.04
50	-1033.96	17.76	9.10	-0.34	-999.72	13.49	6.96	7.74
51	-1031.05	19.28	6.44	0.45	-996.01	15.33	6.06	7.10
52	-1023.98	18.89	8.22	-1.61	-988.80	15.82	5.84	7.91
53	-1019.78	19.71	7.23	-0.54	-983.93	16.36	4.88	7.82
54	-1014.12	20.30	7.80	-1.40	-977.96	17.61	4.54	8.12
55	-1009.29	21.14	7.38	-1.83	-972.75	18.76	4.24	7.88
56	-1003.48	21.72	8.29	-2.04	-967.22	19.36	3.27	8.30
57	-998.59	22.51	7.49	-2.29	-962.01	20.42	2.70	8.15
58	-992.29	21.89	7.48	-2.51	-955.80	20.34	2.26	8.97
59	-986.94	21.81	6.71	-2.17	-950.19	20.73	2.06	8.40
60	-979.16	20.10	8.03	-3.48	-942.99	20.12	2.60	9.34
61	-975.10	22.18	6.87	-3.10	-939.37	20.98	1.49	7.97
62	-967.30	20.92	7.97	-3.98	-931.97	20.56	1.40	8.82
63	-961.69	20.57	5.70	-3.83	-926.33	20.60	0.70	8.60
64	-955.12	20.60	6.64	-4.46	-920.32	20.73	0.61	8.61
65	-948.15	20.55	7.34	-4.69	-914.10	20.68	0.18	8.85
66	-943.03	21.03	5.33	-4.73	-909.64	21.20	-0.42	7.36
67	-936.14	21.54	5.33	-4.83	-903.47	21.23	-0.82	7.68
68	-928.93	20.87	5.68	-5.41	-897.20	21.16	-1.12	7.57
69	-922.59	21.38	4.17	-4.98	-891.54	21.25	-1.85	6.68
70	-913.48	19.74	6.15	-6.22	-884.19	19.87	-1.63	7.30
71	-908.95	21.20	2.97	-5.41	-880.11	21.13	-3.31	5.43
72	-899.67	20.19	4.22	-6.38	-871.97	20.06	-2.64	6.54
73	-892.80	20.73	3.31	-6.32	-865.88	20.16	-2.89	6.12
74	-885.02	20.99	4.04	-6.51	-859.29	19.87	-3.06	6.20
75	-877.49	20.87	3.32	-6.72	-852.88	19.79	-3.05	6.09
76	-870.47	21.70	2.69	-6.79	-846.79	20.07	-3.93	5.58
77	-864.06	22.39	-0.07	-6.23	-841.34	20.30	-5.17	3.90
78	-855.36	22.31	2.09	-6.60	-833.79	19.95	-4.30	4.93

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Table B.1 – continued from previous page

Age	Female c_1	Female c_2	Female c_3	Female c_4	Male c_1	Male c_2	Male c_3	Male c_4
79	-848.64	23.25	0.51	-5.77	-827.80	20.27	-4.64	3.72
80	-840.20	23.70	1.81	-6.15	-820.48	20.26	-3.97	4.05
81	-835.03	25.82	0.08	-4.94	-815.67	21.48	-4.45	2.57
82	-825.99	25.30	0.02	-5.28	-807.64	20.72	-4.99	2.46
83	-818.71	26.03	-0.96	-4.85	-801.23	20.93	-5.51	1.66
84	-810.77	26.13	-1.23	-4.80	-793.98	20.61	-6.07	1.45
85	-804.07	27.08	-2.13	-4.28	-788.26	21.52	-6.47	0.48
86	-796.85	28.16	-2.96	-4.05	-781.64	22.01	-7.25	-0.07
87	-790.39	29.15	-3.93	-3.33	-775.61	22.68	-7.75	-0.94
88	-783.66	29.79	-4.56	-2.94	-769.53	23.01	-8.12	-1.60
89	-777.25	31.15	-5.70	-2.32	-763.73	23.88	-8.67	-2.60
90	-769.53	30.66	-5.39	-2.38	-757.04	23.78	-8.76	-2.77
91	-764.89	32.91	-7.18	-1.17	-752.48	25.56	-9.64	-4.31
92	-757.65	32.96	-7.38	-1.17	-745.93	25.75	-10.08	-4.41
93	-751.63	33.85	-8.09	-0.76	-740.47	26.50	-10.58	-5.04
94	-745.44	34.35	-8.50	-0.49	-734.99	26.89	-11.03	-5.41
95	-738.62	34.54	-9.08	-0.27	-729.09	26.18	-11.11	-6.11
96	-732.69	35.20	-9.65	0.09	-723.83	26.60	-11.45	-6.63
97	-726.91	35.85	-10.20	0.44	-718.73	27.02	-11.77	-7.11
98	-721.30	36.47	-10.71	0.78	-713.78	27.43	-12.06	-7.56
99	-715.87	37.07	-11.19	1.10	-709.00	27.83	-12.33	-7.98
100	-710.62	37.64	-11.64	1.42	-704.39	28.22	-12.57	-8.37
101	-705.57	38.18	-12.06	1.71	-699.96	28.60	-12.78	-8.72
102	-700.72	38.69	-12.44	1.99	-695.71	28.96	-12.97	-9.04
103	-696.07	39.16	-12.79	2.26	-691.64	29.30	-13.13	-9.32
104	-691.64	39.60	-13.10	2.50	-687.75	29.62	-13.27	-9.57
105	-687.42	39.99	-13.37	2.73	-684.05	29.93	-13.38	-9.78
106	-683.42	40.34	-13.60	2.93	-680.54	30.21	-13.47	-9.96
107	-679.64	40.65	-13.80	3.11	-677.22	30.46	-13.53	-10.11
108	-676.08	40.91	-13.96	3.28	-674.08	30.70	-13.57	-10.22
109	-672.74	41.13	-14.09	3.42	-671.12	30.91	-13.59	-10.31

Appendix C SVD Sum of Squares and SVD-Comp Calibration using HMD Data

The *total sum of squares* explained by each component of the SVD should not be interpreted like the variances identified by the eigenvalues of a PCA. Standard PCA operates on an appropriately rescaled and *centered* data cloud so that each new component has the same standardized scale and the ‘eigenvalue’ variances are real variances. In this application the SVD operates on the raw data cloud of logit-transformed mortality rates, all of which are negative. Consequently the data cloud is not centered and the dimensions of the cloud do not have the same scale. The first SVD component effectively locates the cloud with respect the origin, and because of this geometric reality, it must explain the vast majority of the total sum of squares associated with the data cloud. Another consequence is that the first component relates to the overall level of mortality and reflects the characteristic, underlying age pattern of mortality, while the remaining components describe age-specific deviations around the underlying age pattern. To better describe how much of this age-specific variability is associated with each additional component *after the first*, we calculate the fraction of the remaining total sum of squares associated with each of them. The ‘remaining total sum of squares’ is the total sum of squares remaining after subtracting the sum of squares associated with the first component. For female components 2-4, those values are 0.558890, 0.042194, and 0.034791, totaling 0.635875; and for males 0.587236, 0.073129, and 0.037035, totaling 0.697401. This indicates that the next three components after the first capture the bulk of the age-specific variation in the HMD mortality schedules. The next few components after these add little explanatory power so that it does not seem prudent to include more than four total components.

With respect to the SVs, the sum of the squares of the SVs is the total sum of squares in the original dataset (or cloud), so as either the number of points in the data cloud or the number of dimensions of the cloud increases, so will the total sum of squares and the values of the SVs, especially the first few. Consequently, the scale of the SVs is dependent on the ‘size’ of the dataset over which the SVD is calculated, and hence the scale of the components $s_i\mathbf{u}_i$ is also dependent on the size of the dataset. In contrast the magnitude of the LSVs is constrained to be unity, but this means that the elements of the LSVs will be smaller as the number of elements increases, or as the number of points in the original dataset increases. All this is to explain that the scale of the components is not fixed and depends on the size of the dataset over which the SVD is calculated. Critically, this affects only the magnitude of the components, not their age patterns, and in practice none of this matters at all because the weights in Equation 11 can incorporate a factor that accounts for scale.

Appendix D Estimated Regression Coefficients

Table D.1: Female RSV Models: $v_{\ell i} = f_i({}_5q_{0\,\ell}, {}_{45}q_{15\,\ell})$

	Right Singular Vector Elements			
	$\mathbf{v}_1$	$\mathbf{v}_2$	$\mathbf{v}_3$	$\mathbf{v}_4$
${}_5q_0$	0.016***	0.504***	0.812***	–1.930***
	(0.001)	(0.042)	(0.094)	(0.093)
logit(${}_5q_0$)	–0.005***	–0.158***	–0.207***	0.536***
	(0.0004)	(0.012)	(0.028)	(0.028)
logit$({}_5q_0)^2$	–0.001***	–0.030***	–0.024***	0.106***
	(0.0001)	(0.003)	(0.006)	(0.006)
logit$({}_5q_0)^3$	–0.0001***	–0.002***	–0.002***	0.007***
	(0.00001)	(0.0002)	(0.0004)	(0.0004)
${}_{45}q_{15}$	–0.003***	–0.002	–0.079***	0.049***
	(0.0001)	(0.005)	(0.010)	(0.010)
logit$({}_{45}q_{15})^2$	0.0004***	0.013***	0.024***	–0.014***
	(0.00002)	(0.001)	(0.002)	(0.002)
logit$({}_{45}q_{15})^3$	–0.00002***	0.002***	–0.003***	–0.002***
	(0.00000)	(0.0002)	(0.0004)	(0.0004)
${}_5q_0 \times {}_{45}q_{15}$	–0.0004***	–0.006***	–0.043***	0.004**
	(0.00002)	(0.001)	(0.002)	(0.002)
Constant	0.006***	–0.287***	–0.354***	0.931***
	(0.001)	(0.021)	(0.048)	(0.047)
Observations	4,626	4,626	4,626	4,626
R^2	0.966	0.861	0.318	0.324
Adjusted R^2	0.966	0.861	0.316	0.323
F Statistic (df = 8; 4617)	16,498.880***	3,570.534***	268.630***	277.217***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table D.2: Male RSV Models: $v_{\ell i} = f_i({}_5q_{0\,\ell}, {}_{45}q_{15\,\ell})$

	Right Singular Vector Elements			
	$\mathbf{v}_1$	$\mathbf{v}_2$	$\mathbf{v}_3$	$\mathbf{v}_4$
${}_5q_0$	0.010***	0.328***	–0.398***	1.840***
	(0.001)	(0.039)	(0.075)	(0.094)
logit(${}_5q_0$)	–0.003***	–0.112***	0.108***	–0.515***
	(0.0003)	(0.012)	(0.022)	(0.028)
logit(${}_5q_0$)^2	–0.001***	–0.021***	0.024***	–0.098***
	(0.0001)	(0.002)	(0.005)	(0.006)
logit(${}_5q_0$)^3	–0.00005***	–0.002***	0.002***	–0.006***
	(0.00000)	(0.0002)	(0.0003)	(0.0004)
${}_{45}q_{15}$	–0.002***	–0.008***	0.110***	–0.056***
	(0.0001)	(0.003)	(0.005)	(0.007)
logit(${}_{45}q_{15}$)^2	0.0001***	0.002***	–0.002***	–0.004***
	(0.00001)	(0.0004)	(0.001)	(0.001)
logit(${}_{45}q_{15}$)^3	–0.00001***	0.001***	–0.001***	–0.0001
	(0.00000)	(0.0001)	(0.0002)	(0.0003)
${}_5q_0 \times {}_{45}q_{15}$	–0.00003***	–0.001*	–0.004***	–0.002***
	(0.00001)	(0.0004)	(0.001)	(0.001)
Constant	0.009***	–0.198***	0.147***	–0.891***
	(0.0004)	(0.020)	(0.038)	(0.048)
Observations	4,626	4,626	4,626	4,626
R^2	0.974	0.877	0.553	0.296
Adjusted R^2	0.974	0.877	0.552	0.295
F Statistic (df = 8; 4617)	21,628.820***	4,127.574***	714.055***	243.055***
Note:				*p<0.1; **p<0.05; ***p<0.01

Table D.3: Adult Mortality Models: $\text{logit}({}_{45}q_{15})_{z\ell} = f({}_5q_{0\ z\ell})$

	logit(${}_{45}q_{15}$)	
	Female	Male
${}_5q_0$	–10.659*** (1.721)	3.337 (2.453)
logit(${}_5q_0$)	3.965*** (0.510)	0.219 (0.734)
logit(${}_5q_0$)^2	0.687*** (0.103)	0.075 (0.149)
logit(${}_5q_0$)^3	0.045*** (0.007)	0.014 (0.010)
Constant	5.851*** (0.874)	–0.938 (1.251)
Observations	4,626	4,626
R^2	0.934	0.794
Adjusted R^2	0.934	0.794
F Statistic (df = 4; 4621)	16,337.530***	4,465.503***
Note:		*p<0.1; **p<0.05; ***p<0.01

Table D.4: Infant Mortality Models: $\text{logit}({}_1q_0)_{z\ell} = f({}_5q_{0\ z\ell})$

	logit(${}_1q_0$)	
	Female	Male
logit(${}_5q_0$)	0.660*** (0.004)	0.690*** (0.004)
logit(${}_5q_0$)^2	–0.038*** (0.001)	–0.037*** (0.001)
Constant	–0.950*** (0.006)	–0.827*** (0.005)
Observations	4,626	4,626
R^2	0.996	0.996
Adjusted R^2	0.996	0.996
F Statistic (df = 2; 4623)	513,798.600***	574,994.400***
Note:		*p<0.1; **p<0.05; ***p<0.01

Appendix E Additional Figures

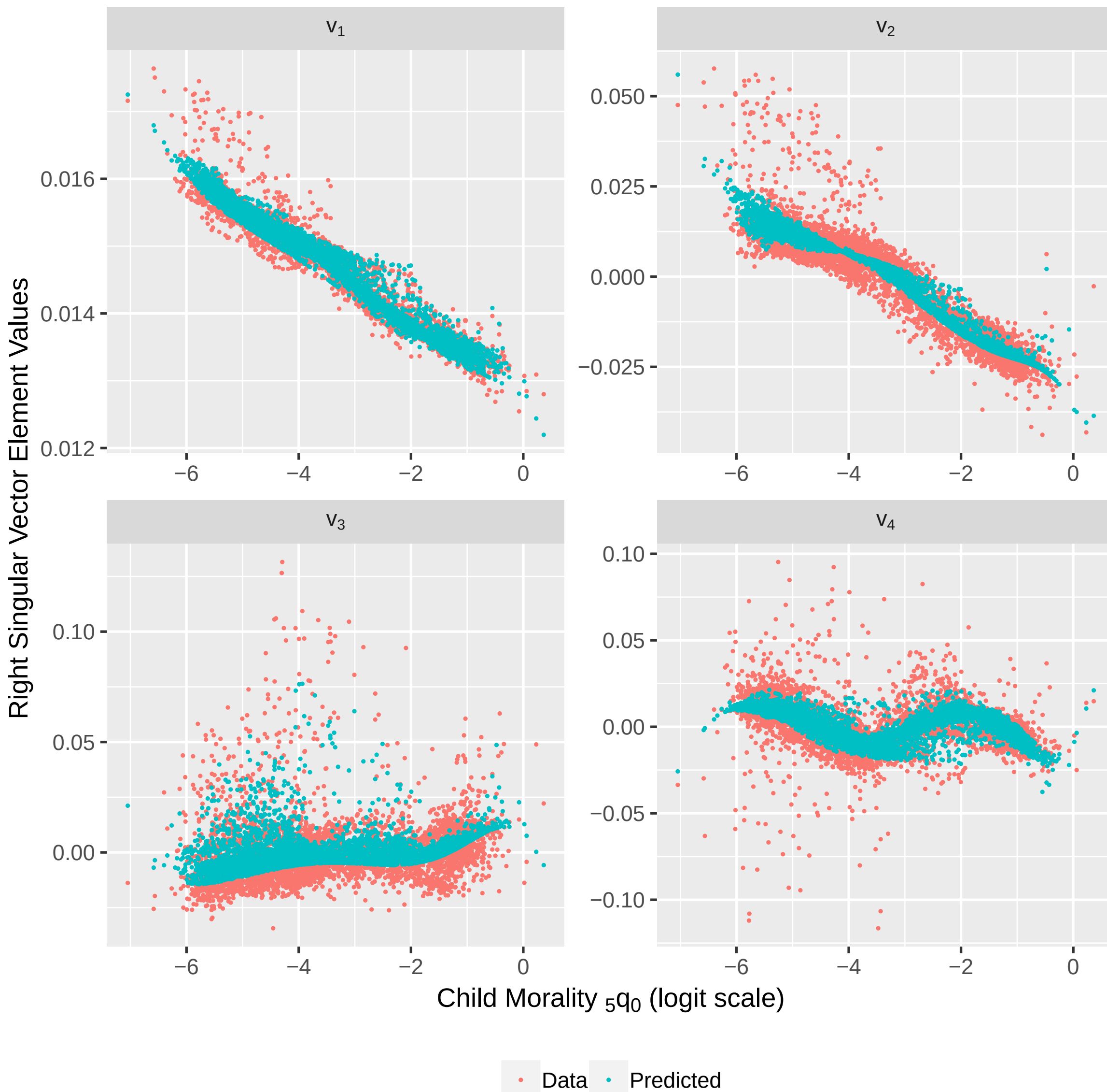

Figure E.1: Right Singular Vector Element Values for Females. Values and predictions from model in Equation 12 on the logit scale by logit($_5q_0$). The predicted values are based on both $_5q_0$ and $_{45}q_{15}$ which explains why they appear as a cloud rather than a curve.

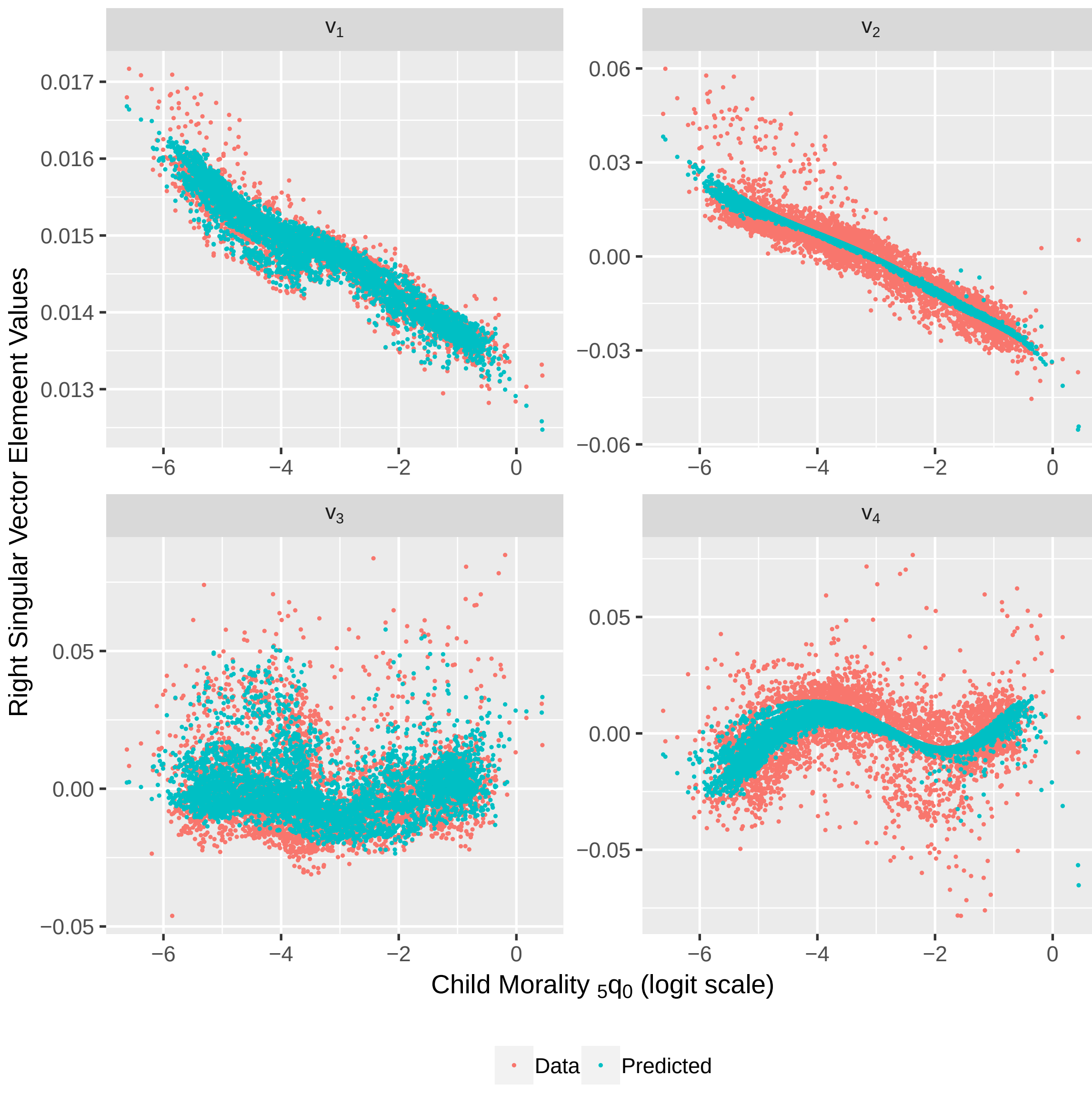

Figure E.2: Right Singular Vector Element Values for Males. Values and predictions from model in Equation 12 on the logit scale by logit($_5q_0$). The predicted values are based on both $_5q_0$ and $_{45}q_{15}$ which explains why they appear as a cloud rather than a curve.

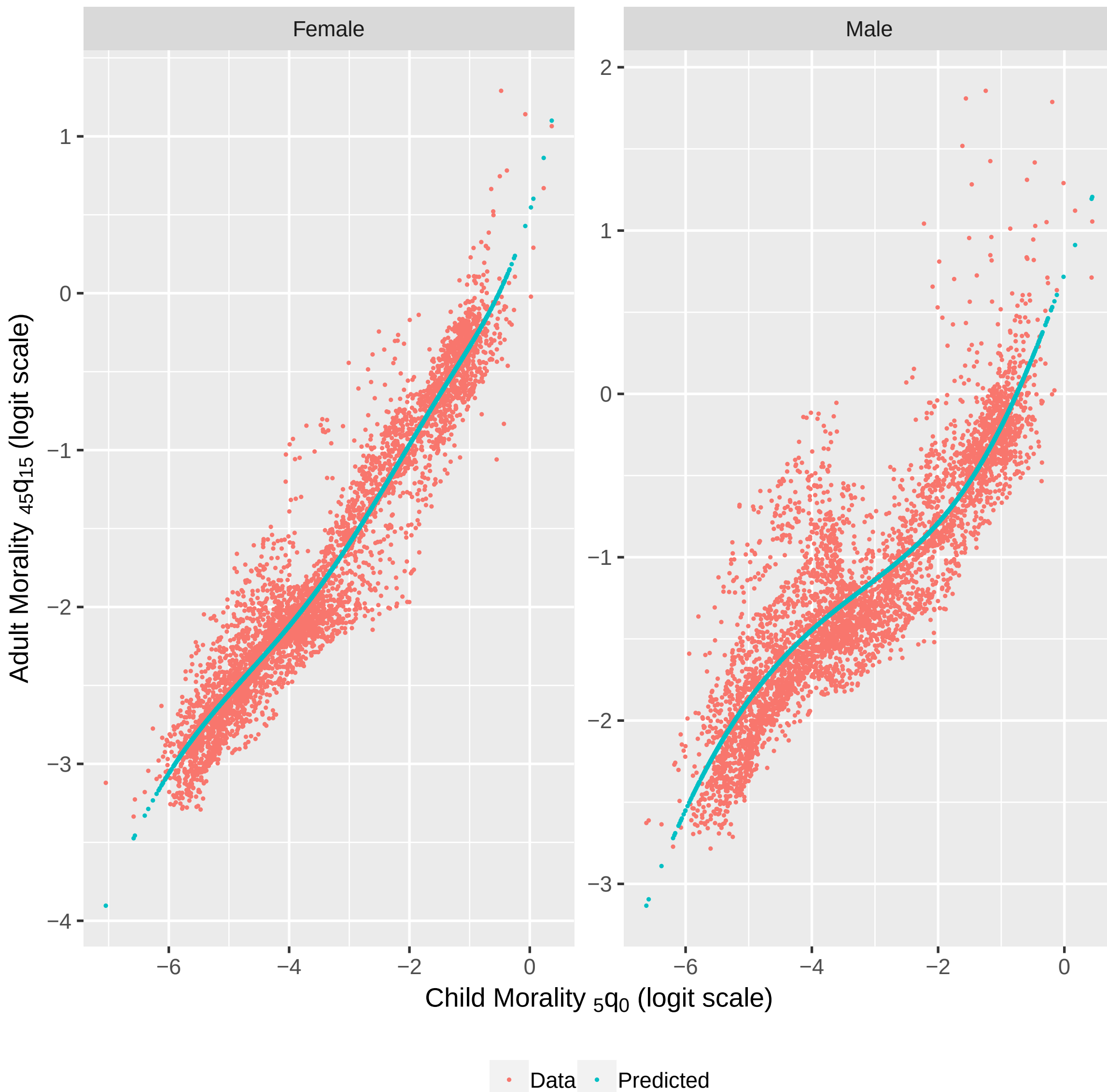

Figure E.3: Adult vs. Child Mortality. Values and predictions from model in Equation 13 on the logit scale by logit($_5q_0$).

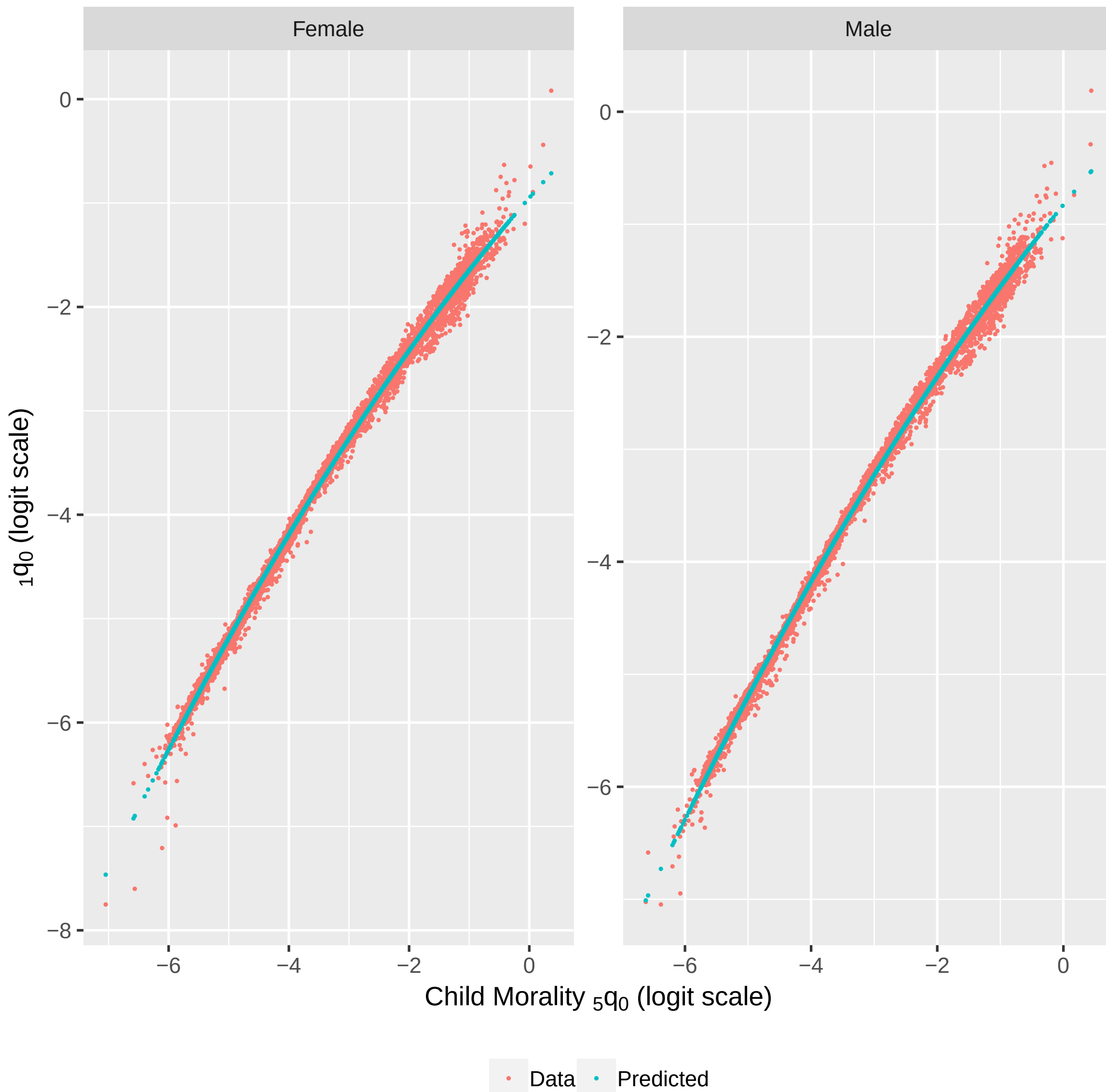

Figure E.4: Age 0 Probability of Dying $_1q_0$ vs. Child Mortality. Values and predictions from model in Equation 14 on the logit scale by logit($_5q_0$).

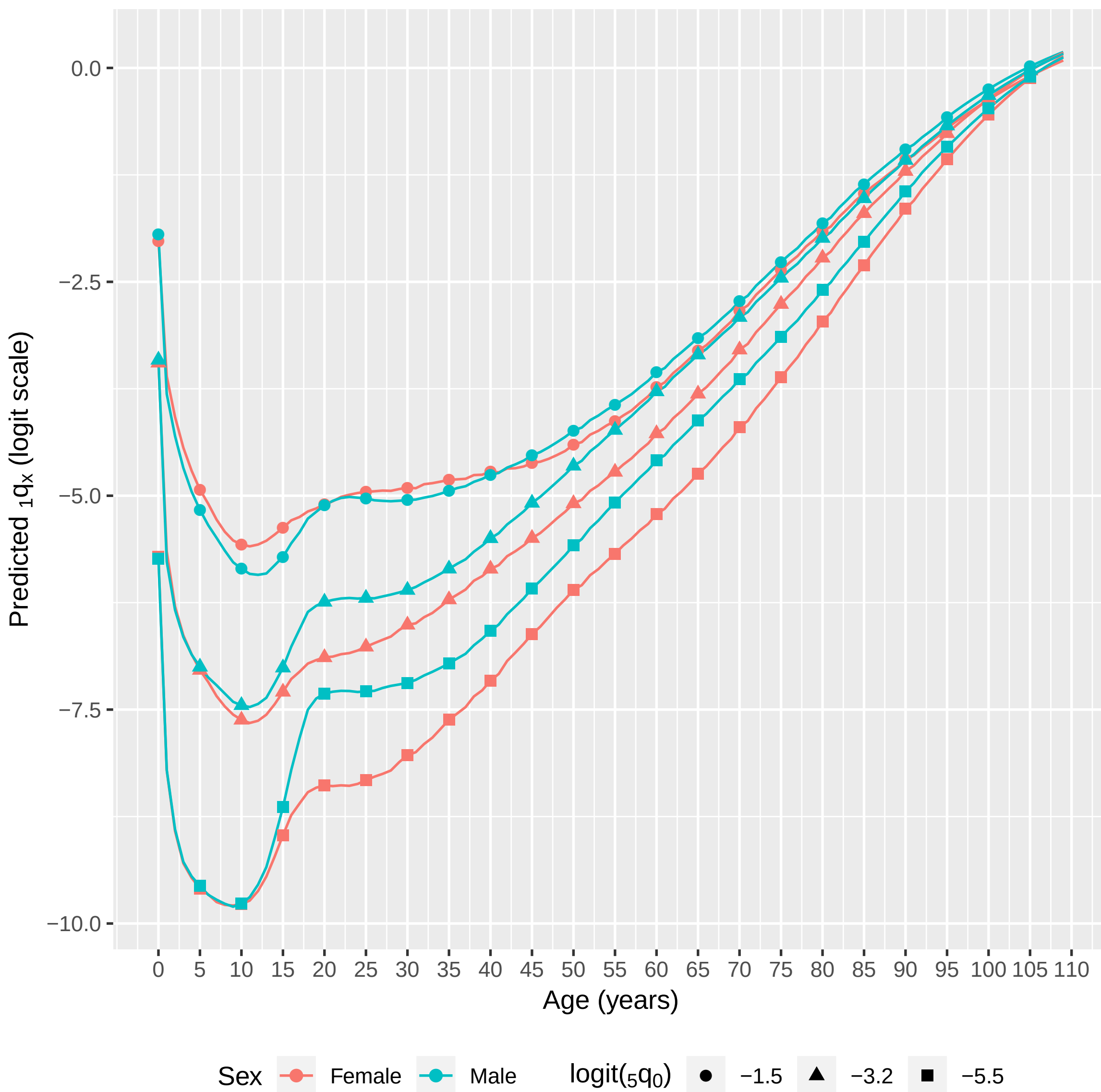

Figure E.5: Predicted $_1q_x$ at Three Levels of $_5q_0$. As $_5q_0$ increases the relationship between female and male mortality changes, and female mortality generally exceeds male mortality between ages roughly 10 and 40 for high levels of $_5q_0$. It has been verified that this reflects the real change in this relationship embodied in the HMD life tables.

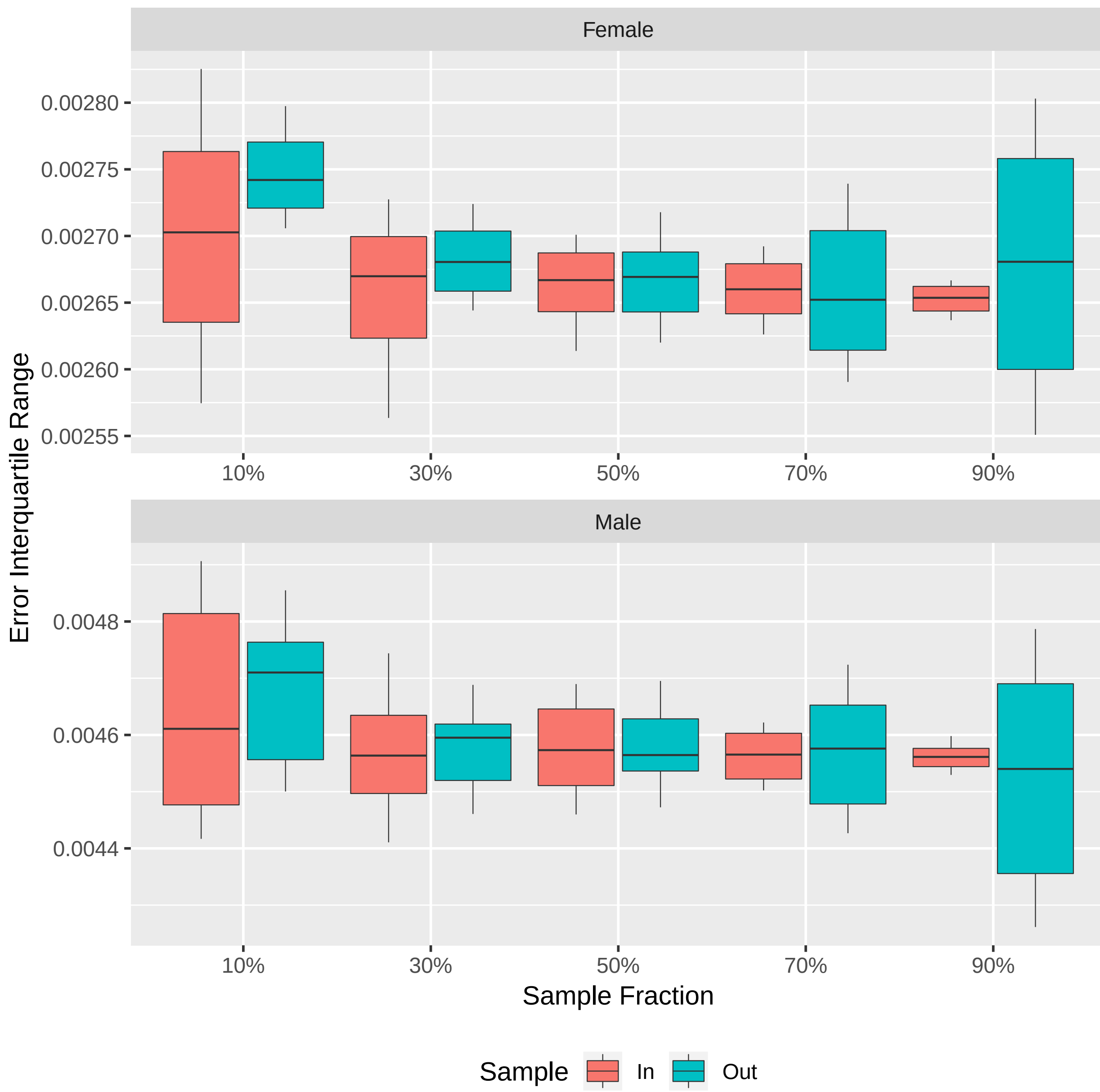

Figure E.6: Interquartile Range of Prediction Error by Sample Fraction. 50 samples for each sample fraction. For each sample, the interquartile range is calculated across all ages and all mortality schedules in each sample category (in/out). Whiskers extend to 10% and 90% quantiles.

Appendix F Additional Error Summary Tables

Table F.1: Weighted age-specific absolute errors in ${}_5\widehat{\mathbf{q}}_x$. Errors summed across all 4,610 HMD life tables 'SC' is SVD-Comp and 'LQ' is Log-Quad'.

	Female			Male		
x (years)	SC	LQ	SC-LQ	SC	LQ	SC-LQ
0	1.2853	1.3090	-0.0237	1.5220	1.5410	-0.0189
1-4	1.4072	1.2973	0.1099	2.0149	1.5462	0.4687
5-9	0.7749	0.7394	0.0355	0.8823	0.8655	0.0167
10-14	0.5104	0.4880	0.0224	0.5191	0.4865	0.0326
15-19	0.6319	0.7043	-0.0724	0.8852	0.8436	0.0416
20-24	0.7768	0.8569	-0.0801	1.6894	1.6212	0.0682
25-29	0.7677	0.8477	-0.0800	1.5660	1.5058	0.0602
30-34	0.7599	0.8016	-0.0417	1.5014	1.4589	0.0426
35-39	0.8349	0.8364	-0.0014	1.6744	1.6227	0.0517
40-44	0.9492	0.9192	0.0300	1.9525	1.8984	0.0541
45-49	1.1050	1.0901	0.0149	2.3541	2.3200	0.0341
50-54	1.4472	1.4440	0.0032	2.8997	2.9286	-0.0289
55-59	1.8920	1.9108	-0.0188	3.4558	3.6015	-0.1457
60-64	2.4297	2.5125	-0.0828	4.1335	4.4517	-0.3182
65-69	2.9800	3.0998	-0.1198	4.5528	4.9328	-0.3800
70-74	3.8689	4.0145	-0.1456	4.7049	5.1552	-0.4503
75-79	4.4725	4.5281	-0.0556	4.2939	4.5880	-0.2941
80-84	4.2773	4.3839	-0.1066	3.0989	3.3037	-0.2049
85-89	3.0512	3.0506	0.0006	1.7148	1.7796	-0.0648
90-94	1.4773	1.5106	-0.0333	0.6722	0.7137	-0.0415
95-99	0.3769	0.4027	-0.0258	0.1301	0.1460	-0.0160
100-104	0.0464	0.0523	-0.0059	0.0135	0.0158	-0.0023
105-109	0.0029	0.0033	-0.0005	0.0008	0.0010	-0.0002
0-109	36.1254	36.8028	-0.6774	46.2320	47.3273	-1.0954

Table F.2: Weighted age-specific absolute errors in $\widehat{\mathbf{e}}_x$. Errors summed across all 4,610 HMD life tables. 'SC' is SVD-Comp and 'LQ' is Log-Quad'.

	Female			Male		
x (years)	SC	LQ	SC-LQ	SC	LQ	SC-LQ
0	411.52	415.65	-4.13	620.60	632.39	-11.79
1-4	450.97	460.34	-9.37	659.43	662.35	-2.92
5-9	417.10	426.68	-9.58	610.07	635.42	-25.35
10-14	396.12	405.35	-9.23	592.54	616.71	-24.18
15-19	382.15	389.85	-7.71	582.31	606.31	-24.00
20-24	362.52	367.74	-5.23	560.51	583.80	-23.29
25-29	340.79	344.28	-3.49	521.48	546.22	-24.74
30-34	321.55	324.58	-3.02	486.38	513.70	-27.32
35-39	304.96	309.33	-4.37	452.17	481.53	-29.36
40-44	288.67	294.55	-5.88	415.85	445.72	-29.87
45-49	270.77	277.11	-6.34	375.47	403.51	-28.04
50-54	249.49	255.68	-6.19	329.12	354.70	-25.57
55-59	224.76	230.18	-5.42	277.21	299.19	-21.99
60-64	196.33	201.32	-4.98	221.43	239.46	-18.03
65-69	165.55	168.51	-2.96	164.63	177.36	-12.73
70-74	131.49	133.31	-1.82	112.60	120.07	-7.47
75-79	94.70	95.74	-1.04	68.37	71.43	-3.06
80-84	59.91	60.46	-0.55	36.18	36.82	-0.64
85-89	31.20	31.11	0.10	16.02	15.92	0.09
90-94	12.49	12.45	0.05	5.57	5.51	0.06
95-99	3.24	3.23	0.01	1.17	1.16	0.00
100-104	0.46	0.46	0.00	0.14	0.14	-0.00
105-109	0.03	0.03	0.00	0.01	0.01	0.00
110+	0.01	0.00	0.00	0.00	0.00	0.00
0+	5,116.80	5,207.94	-91.14	7,109.26	7,449.46	-340.20

Table F.3: Total Absolute Errors in $\mathbf{e}_0$.

Value	Female	Male
Log-Quad	6,211	8,789
SVD-Comp, C=1	11,421	12,641
SVD-Comp, C=2	6,372	8,762
SVD-Comp, C=3	6,310	8,727
SVD-Comp, C=4	6,149	8,625
SVD-Comp, C=1 - Log-Quad	5,209	3,852
SVD-Comp, C=2 - Log-Quad	161	-27
SVD-Comp, C=3 - Log-Quad	99	-62
SVD-Comp, C=4 - Log-Quad	-62	-164