



Applications of optimal reinforcement layouts for concrete slabs via digital fabrication methods

Leanna BRADBURY^a*, Andrew LIEW^b and Matthew GILBERT^a

^a Department of Civil and Structural Engineering, University of Sheffield, UK

* leannabradbury97@gmail.com

^b Unipart Construction Technologies, Advanced Manufacturing Park, Rotherham, UK

Abstract

For time and cost reasons, reinforced concrete floor systems often employ simple reinforcement layouts. However, this can mean that the amount of reinforcement required is greatly in excess of that needed. Although optimal reinforcement layouts were the subject of research in the 1960s, with researchers identifying alternative layouts that required up to 37 % less reinforcement, practical methods of constructing these layouts were not available at that time. Given the increasing availability of digital manufacturing techniques, coupled with the need for material efficiency in the light of the climate emergency and the high embodied carbon associated with steel and concrete, it is of interest to now revisit this area. In this paper, grillage layout optimization is used in conjunction with the proven laser-cutting digital manufacturing technique; grillage layout optimization allows the most efficient reinforcement layout to be identified for a slab of given loading, geometry and support conditions. The identified reinforcement can then be detailed in line with code of practice rules, prior to being transformed into a ‘reinforcing plate’ that can be laser-cut from a steel sheet. In the paper, experimental results derived from small-scale tests are presented. These indicate that the load carrying capacities of optimally reinforced slabs are 33 % higher than those of traditionally reinforced slabs, when all slabs contain the same mass of reinforcement. Issues such as the importance of using ‘nibs’ to achieve adequate concrete-to-steel bond when using laser-cut steel plate as reinforcement are also briefly discussed.

Keywords: Optimization, grillage, laser-cut, reinforcement, digital fabrication, concrete, flat slab

1. Introduction

Reinforced concrete remains the most used building material in modern construction [1], with one of its main applications being concrete floor structures, in particular the concrete flat slab. RC floor systems contribute around 50-60 % of the total building mass for concrete construction, and with gross floor area requirements worldwide set to double by 2060 [2], innovations into making materially efficient designs are a key topic of interest in the research community as well as within industry practices.

Considering concrete slab construction, reinforcement is used to provide strength to resist tensile stresses, whilst the concrete provides strength to resist compressive stresses. Traditional reinforcement design practice for concrete flat slabs involve the use of solid circular steel bars, which are welded or tied together with wire in an orthogonal grid format to form a reinforcing mesh. Multiple meshes can be used in conjunction; however these must be manually tied together to prevent movement during concrete pouring, a task that was identified in Manolache et al (2010) [3] as being the most physically demanding task for a rebar installer. Another issue that arises with standardized reinforcing meshes are the finite set of available sizes / lengths, which often leads to engineers having to round up to the next available size

of mesh. Additionally, as the provided reinforcement is usually constant over the span of a slab, sizing of the reinforcement is based on the largest design moment, which leads to more steel being used than necessary. The reinforcement must also be cut to fit the size of the slab, leading to material wastage amounts in the region of 3-5 % of the total steel volume [4], and potentially more in the case of larger diameter bars. Nevertheless, a uniform reinforcing mesh allows for a simple fabrication process and allows a certain degree of design safety. As reinforcement accounts for around 25 % of the total cost of concrete flat slab construction [5], judicious structural design of slab reinforcement is needed to prevent unnecessary material costs as well as wastage.

Structural optimization techniques can offer a solution to materially-inefficient reinforcement design, by allowing new design possibilities to be explored for concrete flat slab reinforcement, which utilize more efficient load paths. Optimization techniques for the design of flat slab reinforcement were first explored in the 1960s and early 1970s, with methods of designing reinforced concrete slabs with minimum steel volume proposed; these used equilibrium methods that, in the context of slab design, could be reduced to simple elementary beam analysis [6]. This led to experimental work involving a range of alternative reinforcement layouts for a uniformly loaded square slab, with up to 37.5 % material savings possible; all tested samples achieved a higher than predicted failure load. However, despite the potential steel savings, it proved time-intensive to fabricate the associated optimal reinforcement layouts from standard rebar and thus these were considered to be of ‘purely academic interest’ [7].

In the present study, optimal reinforcement layouts are revisited, now utilizing the numerical grillage optimization method developed in [8]. This optimization technique applies a ground structure approach, whereby a planar network of potential beams are used to identify the most efficient load path for a slab of given loading, geometry and support conditions. In doing so, bending moments are determined for each beam in the optimal configuration and thus can be used to determine the reinforcement required, in line with code of practice rules. A simple union function is then used to combine all reinforcing strips to form a singular reinforcing plate.

One common issue highlighted within almost all previous work on optimal reinforcement design is the constructability of the layouts that are produced. To address this issue, the digital manufacturing technique of laser-cutting was used, which allows for an automated and accurate process of fabricating optimal reinforcing layouts from steel plate. Similar implementations of steel plate reinforcement for concrete construction have been previously explored for beam [9] and column [10] applications; however to date there do not appear to have been similar research studies involving use of steel plate reinforcement in slab elements. As steel plate typically has a smooth surface finish, as opposed to traditional ribbed rebar, the innovative ‘nib’ detailing developed by [11] has been used; the inclusion of ‘nibs’ (defined undulations) on the side of each steel strip has been shown to increase the bond strength of the steel plate by six-fold.

The structure of the paper is as follows: Section 2. introduces the methodology used for designing steel plate reinforcement, including details about the grillage optimization formulation, engineering design for transforming beam bending moments into areas of reinforcement, as well as modelling and fabrication considerations for creating a steel reinforcing plate. Section 3. then outlines a typical design problem involving a simply supported, uniformly loaded square slab, with results shown for a typical reinforcement layout as well as for the new proposed optimal layout. Section 4. validates the use of optimal reinforcement, with results presented for a range of small-scale prototype tests. Finally, Section 5. summarizes the findings of this work.

2. Methodology

In this section, a proposed new reinforcement design methodology is briefly introduced, where the goal is to identify the minimum volume steel reinforcement layout for a slab with given geometry, loading and support conditions. For demonstration purposes, Figure 1 shows each key step in the methodology, for the example of a simply-supported square slab, subjected to a uniformly distributed load.

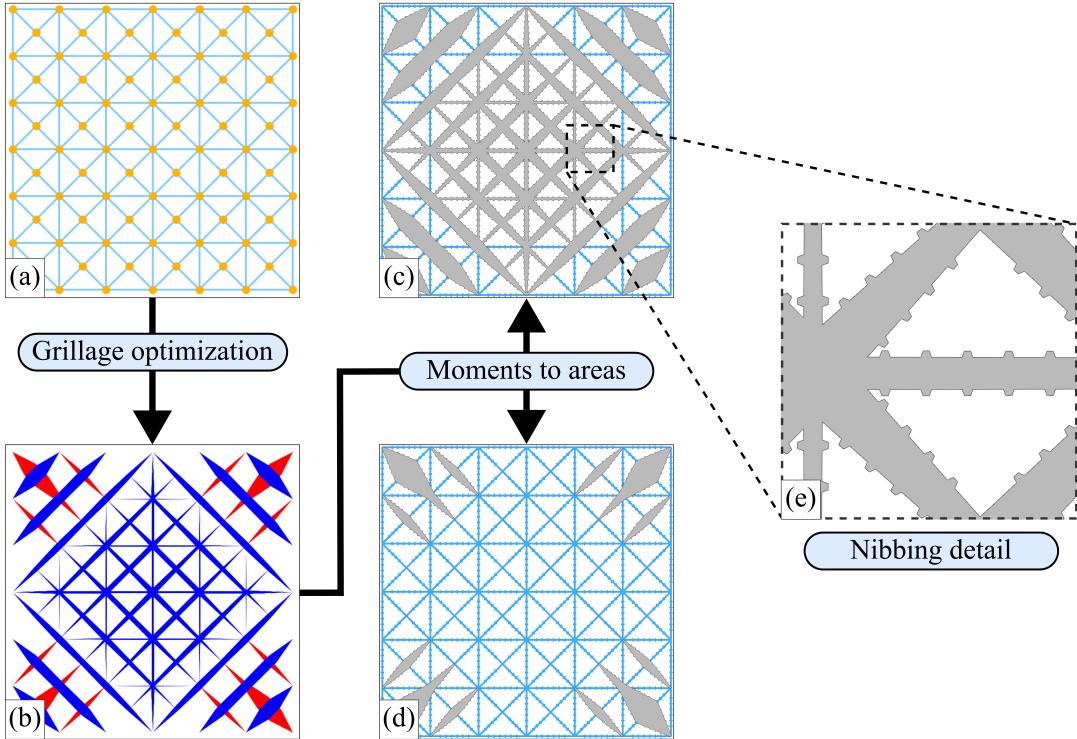


Figure 1: Key steps in the proposed design methodology, considering a uniformly loaded simply supported square slab: (a) adjacent connectivity ground structure, comprising nodes (orange dots) and beams (light blue lines); (b) resulting beam layout with line thickness corresponding to the bending moment magnitudes (sagging and hogging moments shown in blue and red respectively); (c and d) sagging and hogging reinforcement layouts with extra material added for minimum detailing shown in light blue; (e) close up of nibbing detail for increased bond strength.

Firstly, the plastic grillage design optimization method detailed in [8] is employed, which builds upon previously developed layout optimization techniques for truss applications [12]. A ground structure approach is adopted, which utilizes a design domain consisting of a two-dimensional array of beams and nodes, as seen in Figure 1a. The problem can be posed as the following linear programming (LP) formulation, which can be solved by modern well-developed solvers:

$$\min_{\mathbf{a}, \mathbf{q}} \quad V = \mathbf{l}^T \mathbf{a} \quad (1a)$$

$$\text{s.t.} \quad \mathbf{B}\mathbf{q} = \mathbf{f} \quad (1b)$$

$$-m_p^- \mathbf{a} \leq \mathbf{q} \leq m_p^+ \mathbf{a} \quad (1c)$$

$$\mathbf{a} \geq \mathbf{0}, \quad (1d)$$

In this formulation for each beam within the ground structure, it is assumed that the cross-sectional area

varies linearly along the length of the beam; the total volume of material can then be found by summing the volumes of all beams (Eq. [1a]). An equilibrium constraint (Eq. [1b]) is also included, such that moment equilibrium in the x and y directions (i.e., in-plane) and force equilibrium in the z direction (i.e., out-of-plane) is satisfied at each node. A yield condition is also implemented by specifying a limiting moment per unit area for both sagging and hogging (Eq. [1c]).

The outputs of interest are contained in the vector \mathbf{q} , which contains the bending moments at the ends of each beam within the grillage, as visualized in Figure [1b]. These bending moment values then form the basis of reinforcement design, by generating the total area of reinforcement required, by considering each beam in the grillage model as a rectangular reinforced concrete strip. In doing so, the following equation can be used:

$$A_s = \frac{m_i}{z \cdot f_{yd}} \quad (2)$$

where m_i represents the moment at either end of a given beam, along with a given steel design strength, $f_{yd} = f_y/\gamma_M$, which corresponds to the yield strength of the steel divided by a given partial factor of safety. Also z represents the internal lever arm between compression and tension forces.

Typical CAD / modelling software can be used to model the plan area required for each beam reinforcing strip, by dividing the calculated A_s by a chosen steel plate thickness, t . Once all in-plane strips have been identified, a minimum detailing check is conducted to ensure every strip has a width adequate for laser-cutting and safe handling. All reinforcing strips can then be unioned together, and the desired nibbing detail added along strip edges (see Figure [1e]), to form a single planar outline, as shown in Figure [1c/d], with the added material for minimum detailing highlighted in blue. Once a final singular region of reinforcement is modelled, the reinforcement can then be exported digitally and laser-cut from a steel plate of the desired thickness and design strength.

3. Example design problem: simply supported square slab

To quantify the potential material savings (or load increase possible for layouts of equivalent volume of steel), a traditional orthogonal reinforcing layout is compared with an optimal reinforcing layout, for the case of a simply supported square slab ($L \times L$), subject to a uniformly distributed load, w . To keep the solutions general, irrespective of a chosen steel strength and thickness, only bending moment capacities, m_x or m_y depending on the orientation of the beam, were considered (which are proportional to an area of reinforcement required). In the optimal design, an orthogonal grid structure, rotated by 45° , has been used, as it was observed in Figure [1] that these diagonal members form a vast majority of the reinforcement required. Furthermore, by restricting the reinforcement to two perpendicular principle directions, labelled as x_i and y_i respectively, the Von Mises criterion is valid, and the overall complexity of the reinforcing plate is reduced. A moment volume, M_v , for each design was calculated, which is equivalent to $\iint (|m_x| + m_y|) dx dy$. For this section, only flexural moment capacities have been considered and additional material required for fabrication purposes has been neglected, as this is dependent on the steel selected and does not contribute to the overall capacity of the design.

For the design of traditional slab reinforcement, a uniform orthogonal layout is often selected and designed using the Hillerborg strip method, using equal reinforcement in the x and y directions. As this reinforcement is sized based on the maximum bending moment along the strip, the moment capacity selected remains constant along the strip's length and is calculated as:

$$m_x = m_y = dx \cdot \frac{wL^2}{16} \quad (3)$$

with dx equivalent to the spacing between strips. Therefore, for a slab of size $(L \times L)$, the total moment volume was calculated as $M_{v,\text{orth}} = 0.125 w_{\text{orth}} L^4$, with the reinforcement layout shown in Figure 2. For the optimal design case, the proposed design methodology outlined in Section 2 was used, with an equivalent spacing of beams in the ground structure selected (dx). The resultant bending moments were then used to calculate the moment volume, by summing the average bending moment of each beam multiplied by its length and effective width (both equal to dx). Subsequently, for the design example shown in Figure 2, the total moment volume for a slab of size $(L \times L)$ was calculated to be $M_{v,\text{opt}} = 0.0633 w_{\text{opt}} L^4$.

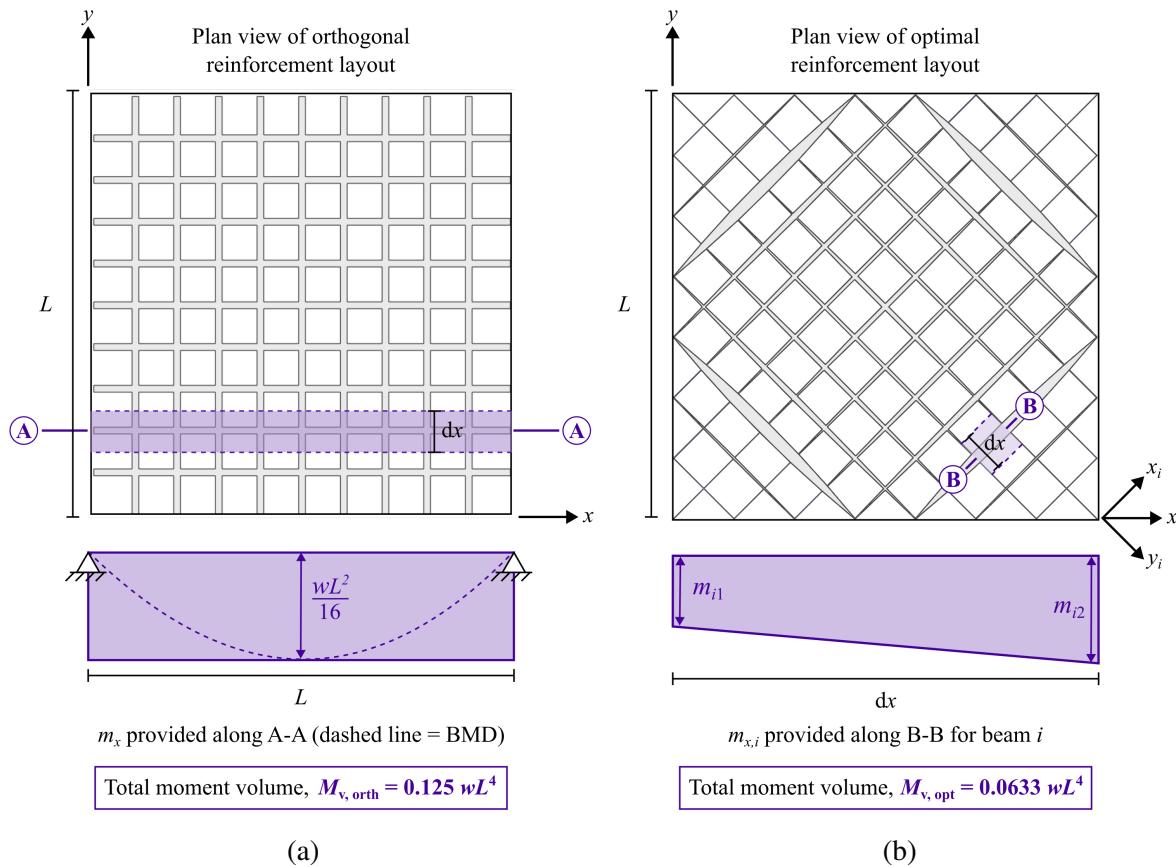


Figure 2: Reinforcement designs: (a) traditional uniform orthogonal layout; (b) optimal layout calculated using the proposed design methodology. For each case, a representative strip is highlighted, with the volume of this contributing to the total moment volume, M_v of each design.

Considering the optimal reinforcing layout in comparison to the traditional uniform orthogonal layout, this indicates that a saving of 49.4 % in moment volume (and thus also the volume of reinforcement required) can be achieved in this case. However, one key difference between the two designs is the assumption of a constant bending moment capacity along the strip in the case of the orthogonal design. This additional moment resistance added may subsequently alter the actual true capacity of the slab, whereas in the case of the optimal design, the moment resistance is only present where it has been identified to be needed. To amend this, the well-established upper bound method of yield-line analysis was conducted, using the LimitState:SLAB software [13], to calculate an upper bound ultimate load carrying capacity for the orthogonal design.

It was found that for the given moment carrying capacity stated in Equation 3, and with the design hogging moment capacity of the slab assumed to be zero, a load factor (LF) of 1.35 is achievable. Note

that this load factor value may differ in reality, as the tensile strength of the concrete selected may contribute to the hogging moment capacity of the slab, therefore increasing this load factor further (to a limit of 1.5). For the determined load factor of 1.35, the moment volume for the orthogonal design can be more accurately written as:

$$M_{v,\text{orth}} = 0.125 \cdot \frac{w}{\text{LF}} L^4$$

$$M_{v,\text{orth}} = 0.09259 \cdot w L^4 \quad (4)$$

Alternatively, for designs of equivalent volume, an expected increase on the load carrying capacity can be found by equating $M_{v,\text{orth}}$ and $M_{v,\text{opt}}$ giving the following relationship:

$$w_{\text{opt}} = 1.4628 \cdot w_{\text{orth}} \quad (5)$$

The result is a 46.3 % expected increase in load carrying capacity for the optimal reinforcement layout design, when compared to a traditional uniform orthogonal layout of equivalent volume. It is worth noting that additional material added for fabrication purposes may further reduce this expected load increase, as a percentage of the total volume for the optimal design will be included for non-flexural reasons. However, it is expected that the overall effect will be minimal, as this issue is only prevalent in the corner sections, and a significant improvement on the efficiency of the reinforcement layout can still be achieved.

4. Experimental validation

To verify the use and benefits of laser-cut optimal reinforcement, in comparison to traditional methods of steel reinforcement design, a series of small-scale experimental tests were carried out on $600 \times 600 \times 30$ mm square slab samples. To limit the numbers of variables involved, designs with equivalent volumes of reinforcement were used, such that the ultimate load capacities of each could be observed and compared. Two steel plate thicknesses of 1 mm and 0.8 mm were used for two optimal design cases, to test the influence of the steel plate area utilization vs thickness ratio. The traditional orthogonal design used the 1 mm steel plate thickness. The influence of adding extra reinforcement in areas of high shear, as well as the inclusion of a nibbing detail to provide enhanced bond strength, were also tested when using the 1 mm plate thickness. The final six designs can be seen in Figure [3].

All slab samples were simply-supported on all edges against a reaction frame and were uniformly loaded via gradual inflation of an airbag, as demonstrated in Figure [4]. Displacements were measured at slab midpoint using a 25 mm linear variable differential transformer displacement transducer, whilst pressures were measured via a transducer placed close to the airbag inlet. All slab specimens were load tested until the point of failure, or when displacements became very large (> 20 mm).

Two samples of each design were tested, giving twelve samples in total. The average pressure vs displacement curves for each design are shown in Figure [5], with the pressure achieved at first yield indicated.

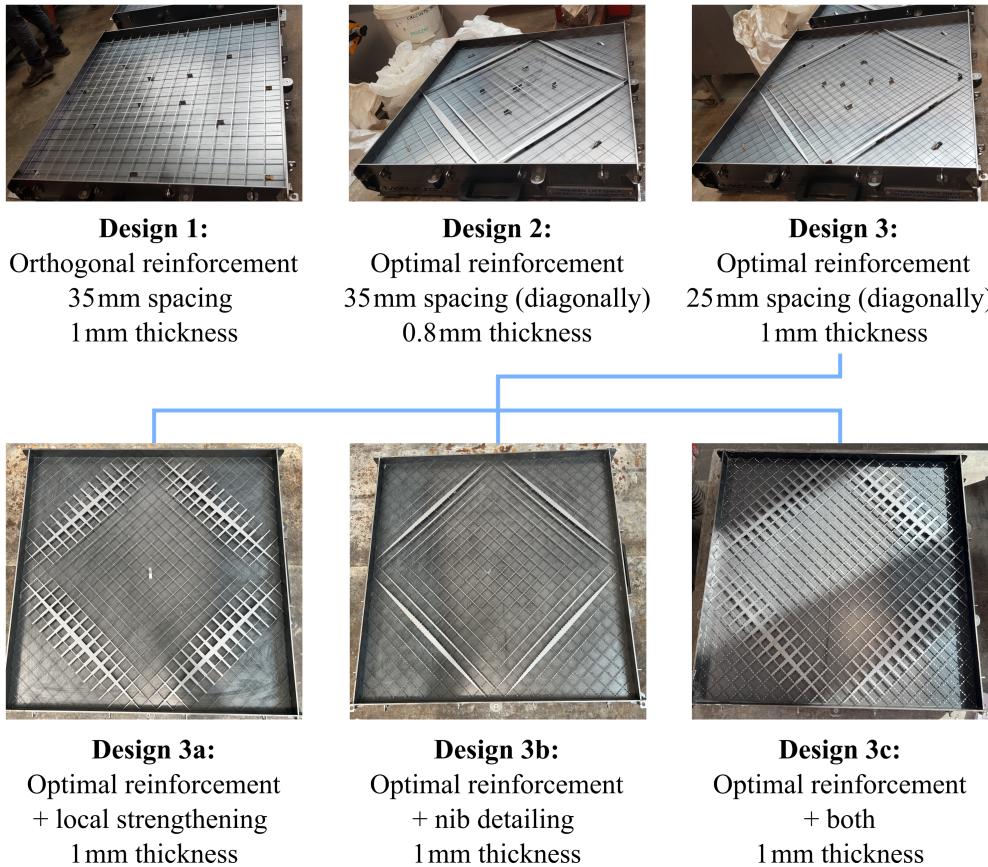


Figure 3: Six reinforcement designs prepared for the experimental tests. Designs 1-3 have identical steel volumes and were used to determine the increase in load-carrying capacity achievable when using the proposed design methodology. Designs 3a-3c are a variation of design 3, whereby additional material was added in the form of local strengthening and/or nibs, and were tested to verify whether these additions improved the shear behaviour and/or bond strength.

As demonstrated in Figure 5(a), Designs 2 and 3 achieved a 11.0 % and 33.9 % increase respectively in load carrying capacity when compared to Design 1. As all designs have an equivalent volume of reinforcement, this increase in load carrying capacity results in improved efficiency for the same mass of steel. Regarding failure mechanisms, Design 1 displayed a ductile response with cracks appearing with little warning and then continuously opening until the end of the test. Design 2 and Design 3 had an initially ductile behaviour followed by a brittle shear failure. This corresponded to hairline cracks forming at corners, gradually widening until brittle shear failure of the middle section of the slab, forming a diamond shape segment spanning from the midpoint of each support, as shown in Figure 6a.

As indicated in Figure 5(b), the additional material added for local strengthening in the case of Design 3a resulted in a 4 % increase in load carrying capacity, but more importantly, successfully suppressed the brittle shear failure previously observed, with a typical X-shaped failure mechanism observed, and with cracks gradually widening until the end of the test. The load-carrying capacity of Design 3b, that included the nibbing detail, increased by 9.6 %; however the brittle shear failure previously seen in the case of Design 3 was observed. Design 3c, which included both additions, displayed a load-carrying capacity increase of 10.9 %, with the brittle shear failure successfully suppressed; the associated failure mechanism is shown in Figure 6b.

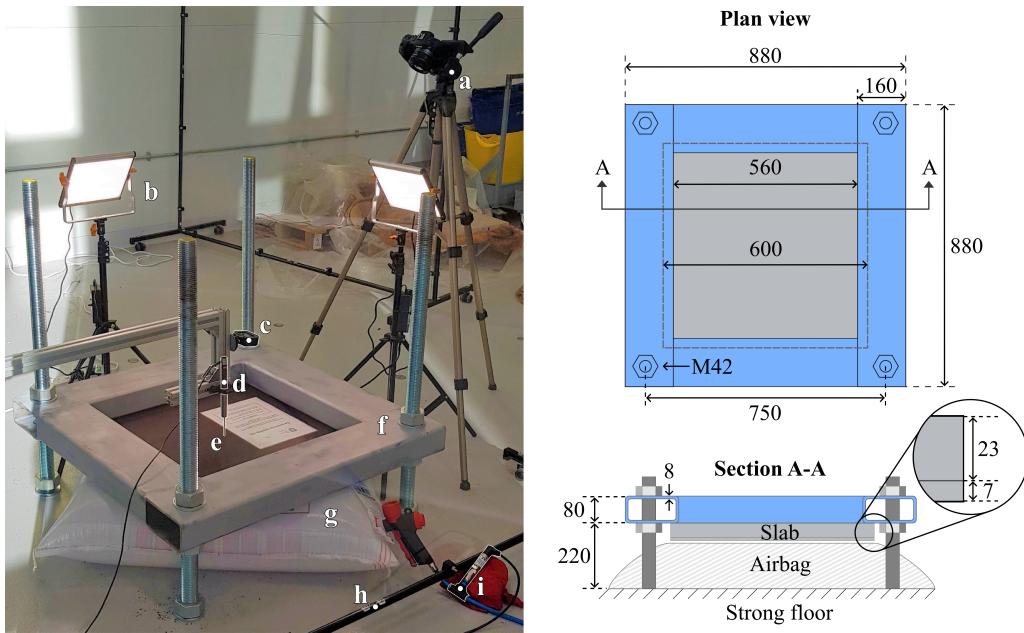


Figure 4: Experimental set-up with dimensions (in mm): (a) DSLR camera and tripod; (b) lighting; (c) Go Pro camera; (d) LVDT displacement transducer; (e) slab sample; (f) reaction frame; (g) Exporta 'PPW Premium Dunnage' air bag; (h) safety screen; (i) pressure transducer.

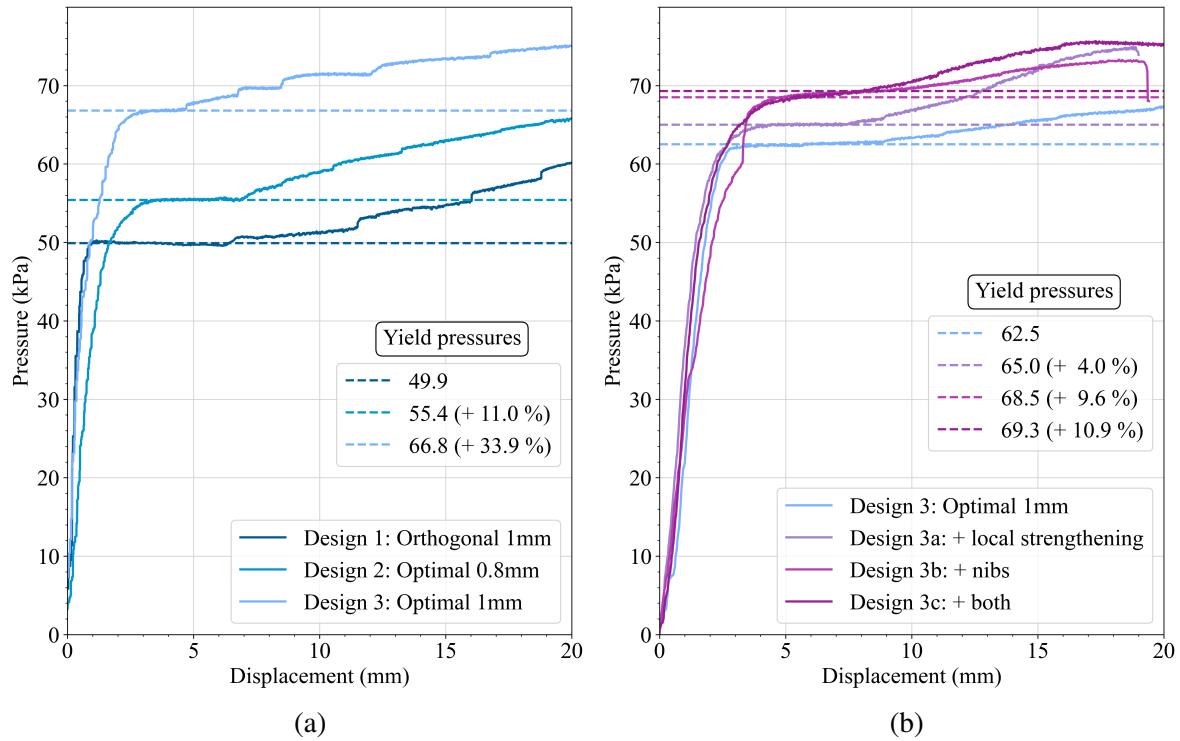


Figure 5: Average pressure vs displacement results: (a) test series 1 with comparisons drawn between traditional orthogonal design (Design 1) and optimal designs (Design 2-3); (b) test series 2 with comparisons drawn between 1 mm optimal design and same design with additions made for local strengthening and/or nibbing detail (Design 3a-3c).

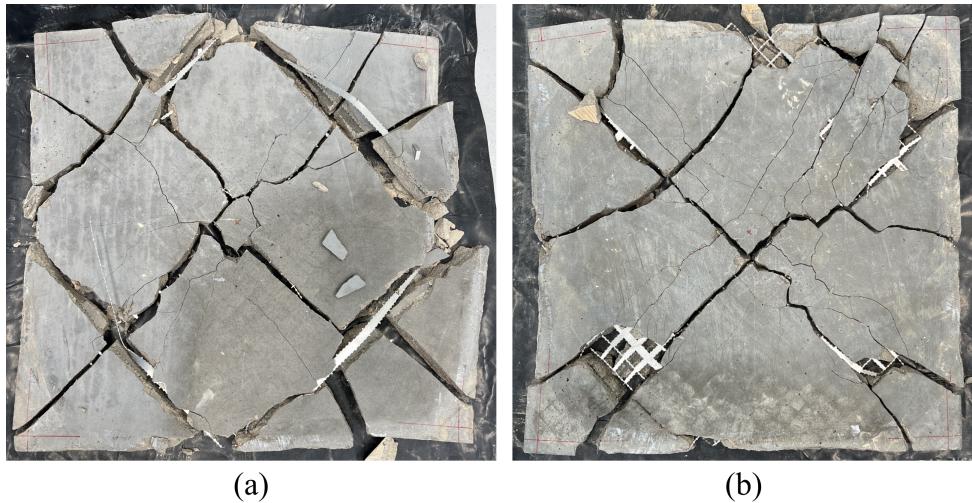


Figure 6: Observed failure mechanisms for: (a) optimal 1 mm reinforcement (Design 3) case; (b) equivalent optimal design with local strengthening and nibbing detail added (Design 3c).

Table I presents the average yield pressure attained by each design, along with the observed percentage increase in load-carrying capacity.

Table 1: Summary of experimental results, showing yield pressure and the observed % increase in load carrying capacity.

Design	Description	Average yield pressure (kPa)	% increase
Comparison between orthogonal design and optimal designs			
1	Orthogonal 1 mm	49.9	-
2	Optimal 0.8 mm	55.4	11.0
3	Optimal 1 mm	66.8	33.9
Comparison between optimal 1 mm design with/without additions			
3	Optimal 1 mm	62.5	-
3a	+ local strengthening	65.0	4.0
3b	+ nibs	68.5	9.6
3c	+ both	69.3	10.9

5. Conclusions

This paper has presented a new methodology for the optimal design of reinforcement for concrete slabs. Building on previous research in the area, the presented methodology takes advantage of a modern numerical design method and digital fabrication techniques. Specifically, numerical layout optimization is used to identify the most efficient layout of grillage members; this also indicates the resultant bending moments that can subsequently be transformed into reinforcement strips of the appropriate area, in line with code of practice rules. Simple geometric modelling techniques are then used to combine these strips into a reinforcing plate, which can be fabricated using the modern laser cutting digital manufacturing technique. A design example involving a uniformly loaded simply supported square slab is presented in the paper, suggesting that a load increase in the range of 46.3 % can be achieved in the case of the optimally reinforced design, compared to a traditional orthogonal layout of the same volume. Results from small-scale prototype tests have been presented to verify the approach, with an increase in load-carrying capacity of up to 33.9 % observed, relative to the traditional orthogonally reinforced layout.

The addition of material to provide local strengthening and a nibbing detail showed improvements in the shear behaviour and bond strength, with a brittle shear failure mode successfully suppressed. Work is currently underway to further verify the proposed design methodology, considering various aspects not covered here (e.g., different geometries, load cases, support types, etc). It is envisaged that implementation of optimal reinforcing layouts at an industry scale could lead to significant material savings, contributing to reducing the environmental impact of the construction industry on the planet.

References

- [1] G. Cement and C. Association, *Concrete future*, 2021. [Online]. Available: <https://gccassociation.org/concretefuture/>.
- [2] U. N. E. Programme, *Global Status Report for Buildings and Construction: Towards a zero-emission, Efficient and Resilient Buildings and Construction Sector*. 2021, p. 29. [Online]. Available: <https://www.unep.org/resources/report/2021-global-status-report-buildings-and-construction>.
- [3] C. T. Manolache, S. Nadeau, and J. Arteau, “Social sustainability through occupational health and safety prevention in the construction industry,” 2010.
- [4] K. Kwon, D. Kim, and S. Kim, “Cutting waste minimization of rebar for sustainable structural work: A systematic literature review,” *Sustainability*, vol. 13, no. 11, p. 5929, 2021.
- [5] M. Aldwaik and H. Adeli, “Cost optimization of reinforced concrete flat slabs of arbitrary configuration in irregular highrise building structures,” *Structural and Multidisciplinary Optimization*, vol. 54, pp. 151–164, 2016.
- [6] G. I. N. Rozvany, “Rational approach to plate design,” *American Concrete Institute Journal Proceedings*, vol. 63, 1966. DOI: [10.14359/7663](https://doi.org/10.14359/7663).
- [7] M. A. Muspratt, “Destructive tests on rationally designed slabs,” *Magazine of Concrete Research*, vol. 22, pp. 25–36, 1970. DOI: [10.1680/macr.1970.22.70.25](https://doi.org/10.1680/macr.1970.22.70.25).
- [8] K. Bolbotowski, L. He, and M. Gilbert, “Design of optimum grillages using layout optimization,” *Structural and Multidisciplinary Optimization*, vol. 58, pp. 851–868, 2018. DOI: [10.1007/s00158-018-1930-6](https://doi.org/10.1007/s00158-018-1930-6).
- [9] N. K. Subedi and P. S. Baglin, “Plate reinforced concrete beams: Experimental work,” *Engineering Structures*, vol. 21, pp. 232–254, 1999. DOI: [10.1016/S0141-0296\(97\)00171-5](https://doi.org/10.1016/S0141-0296(97)00171-5).
- [10] H. Sezen and M. Shamsai, “High-strength concrete columns reinforced with prefabricated cage system,” *Journal of Structural Engineering*, vol. 134, pp. 750–757, 2008. DOI: [10.1061/\(ASCE\)0733-9445\(2008\)134:5\(750\)](https://doi.org/10.1061/(ASCE)0733-9445(2008)134:5(750)).
- [11] M. Takalloozadeh, M. Gilbert, D. Allen, and G. Torelli, “Experimental and numerical assessment of the bond behaviour of laser-cut reinforcement,” Manuscript in preparation.
- [12] M. Gilbert and A. Tyas, “Layout optimization of large-scale pin-jointed frames,” *Engineering Computations*, vol. 20, pp. 1044–1064, 2003. DOI: [10.1108/02644400310503017](https://doi.org/10.1108/02644400310503017).
- [13] LimitState Ltd, *LimitState:SLAB*, version 2.3. [Online]. Available: <https://www.limitstate.com/slab>.



*Proceedings of the IASS 2024 Symposium
Redefining the Art of Structural Design*
August 26-30, 2024, Zurich Switzerland
Philippe Block, Giulia Boller, Catherine De Wolf,
Jacqueline Pauli, Walter Kaufmann (eds.)

Copyright Declaration

Before publication of your paper in the Proceedings of the IASS Annual Symposium 2024, the Editors and the IASS Secretariat must receive a signed Copyright Declaration. The completed and signed declaration may be uploaded to the EasyChair submission platform or sent as an e-mail attachment to the symposium secretariat (papers@iass2024.org). A scan into a .pdf file of the signed declaration is acceptable in lieu of the signed original. In the case of a contribution by multiple authors, either the corresponding author or an author who has the authority to represent all the other authors should provide his or her address, phone and E-mail and sign the declaration.

Paper Title: Applications of optimal reinforcement layouts for concrete slabs via digital fabrication methods

Author(s): Leanna Bradbury*, Andrew Liew**, and Matthew Gilbert*

Affiliation(s): * Department of Civil and Structural Engineering, University of Sheffield, UK.

** Unipart Construction Technologies, Advanced Manufacturing Park, Rotherham, UK.

Address: * Department of Civil and Structural Engineering, University of Sheffield, UK

Phone: 07809572956

E-mail: leannabradbury97@gmail.com

I hereby license the International Association for Shell and Spatial Structures to publish this work and to use it for all current and future print and electronic issues of the Proceedings of the IASS Annual Symposia. I understand this licence does not restrict any of the authors' future use or reproduction of the contents of this work. I also understand that the first-page footer of the manuscript is to bear the appropriately completed notation:

Copyright © 2024 by <name(s) of all of the author(s)>

Published by the International Association for Shell and Spatial Structures (IASS) with permission

If the contribution contains materials bearing a copyright by others, I further affirm that (1) the authors have secured and retained formal permission to reproduce such materials, and (2) any and all such materials are properly acknowledged by reference citations and/or with credits in the captions of photos/figures/tables.

Printed name: Leanna Bradbury

Signature: 

Location: University of Sheffield, UK

Date: 27/06/24