Problem Set 4

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Exercise 1

$$
\begin{aligned}
\frac{1}{e^{z_t}K_t^\alpha - Ae^{z_t}K_t^\alpha} &= \beta E_t \frac{\alpha e^{z_{t+1}}(Ae^{z_t}K_t^\alpha)^{\alpha-1}}{e^{z_{t+1}}(Ae^{z_t}K_t^\alpha)^\alpha - Ae^{z_{t+1}}(Ae^{z_t}K_t^\alpha)^\alpha} \\
&= \beta E_t \frac{\alpha(Ae^{z_t}K_t^\alpha)^{-1}}{1-A} \\
\frac{1}{e^{z_t}K_t^\alpha(1-A)} &= \beta E_t \frac{\alpha(Ae^{z_t}K_t^\alpha)^{-1}}{1-A} \\
A &= \beta\alpha
\end{aligned}
$$

Therefore, $K_{t+1} = \beta\alpha e^{z_t}K_t^\alpha$

Exercise 2

1. $c_t = (1-\tau)[w_t l_t(r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $\frac{1}{c_t} = \beta E_t\{\frac{1}{c_{t+1}}[(r_{t+1}-\delta)(1-\tau)+1]\}$
3. $\frac{a}{1-l_t} = \frac{1}{c_t}w_t(1-\tau)$
4. $r_t = \alpha e^{z_t}K_t^{\alpha-1}L_t^{1-\alpha}$
5. $w_t = (1-\alpha)e^{z_t}K_t^\alpha L_t^{-\alpha}$
6. $\tau[w_t l_t + (r_t-\delta)k_t] = T_t$
7. $z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z;\ \epsilon_t^z \sim i.i.d.(0,\sigma_z^2)$

Exercise 3

1. $c_t = (1-\tau)[w_t l_t(r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\}$
3. $\frac{a}{1-l_t} = c_t^{-\gamma}w_t(1-\tau)$
4. $r_t = \alpha e^{z_t}K_t^{\alpha-1}L_t^{1-\alpha}$
5. $w_t = (1-\alpha)e^{z_t}K_t^\alpha L_t^{-\alpha}$

6. $\tau[w_t l_t + (r_t - \delta)k_t] = T_t$
7. $z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z;\ \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Exercise 4

1. $c_t = (1-\tau)[w_t l_t (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\}$
3. $a(1-l_t)^{-\xi} = c_t^{-\gamma} w_t (1-\tau)$
4. $r_t = \alpha e^{z_t} K_t^{\eta-1}[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1-n}{n}}$
5. $w_t = (1-\alpha)e^{z_t} L_t^{\eta-1}[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1-n}{n}}$
6. $\tau[w_t l_t + (r_t - \delta)k_t] = T_t$
7. $z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z;\ \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Exercise 5

1. $c_t = (1-\tau)[w_t (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\}$
3. $r_t = \alpha K_t^{\alpha-1}(e^{z_t})^{(1-\alpha)}$
4. $w_t = (1-\alpha)K_t^{\alpha}(e^{z_t})^{-\alpha}$
5. $\tau[w_t + (r_t - \delta)k_t] = T_t$
6. $z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z;\ \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Steady-state versions of these equations are given by

1. $\bar{c} = (1-\tau)[\bar{w}(\bar{r} - \delta)\bar{k}] + \bar{T}$
2. $\bar{c}^{-\gamma} = \beta E_t\{\bar{c}^{-\gamma}[(\bar{r} - \delta)(1-\tau) + 1]\}$
3. $\bar{r} = \alpha \bar{k}^{\alpha-1}(e^{\bar{z}})^{(1-\alpha)}$
4. $\bar{w} = (1-\alpha)\bar{k}^{\alpha}(e^{\bar{z}})^{-\alpha}$
5. $\tau[\bar{w} + (\bar{r} - \delta)\bar{k}] = \bar{T}$
6. $0 = \epsilon_t^{\bar{z}};\ \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

The algebraic solution of steady state capital as a function of $\bar{z}$ is

$$\bar{k} = \Big[\frac{1}{(e^{\bar{z}})^{1-\alpha}}\Big(\frac{1-\beta}{\alpha\beta(1-\tau)} + \frac{\delta}{\alpha}\Big)\Big]^{\frac{1}{\alpha-1}}$$

Exercise 6

1. $c_t = (1-\tau)[w_t(r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t\{c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\}$
3. $a(1-l_t)^{-\xi} = c^{-\gamma} w_t(1-\tau)$
4. $r_t = \alpha K_t^{\alpha-1}(L_t e^{z_t})^{(1-\alpha)}$
5. $w_t = (1-\alpha)K_t^{\alpha}(L_t e^{z_t})^{-\alpha}$
6. $\tau[w_t l_t + (r_t - \delta)k_t] = T_t$
7. $z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z;\ \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Steady-state versions of these equations are given by

1. $\bar{c} = (1-\tau)[\bar{w}(\bar{r} - \delta)\bar{k}] + \bar{T}$
2. $\bar{c}^{-\gamma} = \beta E_t\{\bar{c}^{-\gamma}[(\bar{r} - \delta)(1-\tau) + 1]\}$
3. $a(1-\bar{l})^{-\xi} = c^{-\gamma}\bar{w}(1-\tau)$
4. $\bar{r} = \alpha \bar{k}^{\alpha-1}(\bar{l}e^{\bar{z}})^{(1-\alpha)}$
5. $\bar{w} = (1-\alpha)\bar{k}^{\alpha}(\bar{l}e^{\bar{z}})^{-\alpha}$
6. $\tau[\bar{w}\bar{l} + (\bar{r} - \delta)\bar{k}] = \bar{T}$
7. $0 = \epsilon_t^{\bar{z}};\ \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

The algebraic solution of steady state capital as a function of $\bar{z}$ is

$$\bar{k} = \Big[\frac{1}{(e^{\bar{z}})^{1-\alpha}}\Big(\frac{1-\beta}{\alpha\beta(1-\tau)} + \frac{\delta}{\alpha}\Big)\Big]^{\frac{1}{\alpha-1}}$$