

Problem Set 4: DSGE

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Exercise 1

$$\begin{aligned}\frac{1}{e^{z_t} K_t^\alpha - A e^{z_t} K_t^\alpha} &= \beta E_t \frac{\alpha e^{z_{t+1}} (A e^{z_t} K_t^\alpha)^{\alpha-1}}{e^{z_{t+1}} (A e^{z_t} K_t^\alpha)^\alpha - A e^{z_{t+1}} (A e^{z_t} K_t^\alpha)^\alpha} \\ &= \beta E_t \frac{\alpha (A e^{z_t} K_t^\alpha)^{-1}}{1 - A} \\ \frac{1}{e^{z_t} K_t^\alpha (1 - A)} &= \beta E_t \frac{\alpha (A e^{z_t} K_t^\alpha)^{-1}}{1 - A} \\ A &= \beta \alpha\end{aligned}$$

Therefore, $K_{t+1} = \beta \alpha e^{z_t} K_t^\alpha$

Exercise 2

1. $c_t = (1 - \tau)[w_t l_t (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $\frac{1}{c_t} = \beta E_t \left\{ \frac{1}{c_{t+1}} [(r_{t+1} - \delta)(1 - \tau) + 1] \right\}$
3. $\frac{a}{1-l_t} = \frac{1}{c_t} w_t (1 - \tau)$
4. $r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha}$
5. $w_t = (1 - \alpha) e^{z_t} K_t^\alpha L_t^{-\alpha}$
6. $\tau[w_t l_t + (r_t - \delta)k_t] = T_t$
7. $z_t = (1 - \rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z; \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Exercise 3

1. $c_t = (1 - \tau)[w_t l_t (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t \left\{ c_{t+1}^{-\gamma} [(r_{t+1} - \delta)(1 - \tau) + 1] \right\}$
3. $\frac{a}{1-l_t} = c_t^{-\gamma} w_t (1 - \tau)$
4. $r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha}$
5. $w_t = (1 - \alpha) e^{z_t} K_t^\alpha L_t^{-\alpha}$

6. $\tau[w_t l_t + (r_t - \delta)k_t] = T_t$
7. $z_t = (1 - \rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z; \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Exercise 4

1. $c_t = (1 - \tau)[w_t l_t (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t \{c_{t+1}^{-\gamma} [(r_{t+1} - \delta)(1 - \tau) + 1]\}$
3. $a(1 - l_t)^{-\xi} = c_t^{-\gamma} w_t (1 - \tau)$
4. $r_t = \alpha e^{z_t} K_t^{\eta-1} [\alpha K_t^\eta + (1 - \alpha) L_t^\eta]^{\frac{1-n}{n}}$
5. $w_t = (1 - \alpha) e^{z_t} L_t^{\eta-1} [\alpha K_t^\eta + (1 - \alpha) L_t^\eta]^{\frac{1-n}{n}}$
6. $\tau[w_t l_t + (r_t - \delta)k_t] = T_t$
7. $z_t = (1 - \rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z; \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Exercise 5

1. $c_t = (1 - \tau)[w_t (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t \{c_{t+1}^{-\gamma} [(r_{t+1} - \delta)(1 - \tau) + 1]\}$
3. $r_t = \alpha K_t^{\alpha-1} (e^{z_t})^{(1-\alpha)}$
4. $w_t = (1 - \alpha) K_t^\alpha (e^{z_t})^{-\alpha}$
5. $\tau[w_t + (r_t - \delta)k_t] = T_t$
6. $z_t = (1 - \rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z; \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Steady-state versions of these equations are given by

1. $\bar{c} = (1 - \tau)[\bar{w}(\bar{r} - \delta)\bar{k}] + \bar{T}$
2. $\bar{c}^{-\gamma} = \beta E_t \{\bar{c}^{-\gamma} [(\bar{r} - \delta)(1 - \tau) + 1]\}$
3. $\bar{r} = \alpha \bar{k}^{\alpha-1} (e^{\bar{z}})^{(1-\alpha)}$
4. $\bar{w} = (1 - \alpha) \bar{k}^\alpha (e^{\bar{z}})^{-\alpha}$
5. $\tau[\bar{w} + (\bar{r} - \delta)\bar{k}] = \bar{T}$
6. $0 = \epsilon_t^z; \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

The algebraic solution of steady state capital as a function of \bar{z} is

$$\bar{k} = \left[\frac{1}{(e^{\bar{z}})^{1-\alpha}} \left(\frac{1-\beta}{\alpha\beta(1-\tau)} + \frac{\delta}{\alpha} \right) \right]^{\frac{1}{\alpha-1}}$$

Exercise 6

1. $c_t = (1-\tau)[w_t(r_t - \delta)k_t] + k_t + T_t - k_{t+1}$
2. $c_t^{-\gamma} = \beta E_t \{ c_{t+1}^{-\gamma} [(r_{t+1} - \delta)(1-\tau) + 1] \}$
3. $a(1-l_t)^{-\xi} = c^{-\gamma} w_t(1-\tau)$
4. $r_t = \alpha K_t^{\alpha-1} (L_t e^{z_t})^{(1-\alpha)}$
5. $w_t = (1-\alpha) K_t^\alpha (L_t e^{z_t})^{-\alpha}$
6. $\tau[w_t l_t + (r_t - \delta)k_t] = T_t$
7. $z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z; \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$

Steady-state versions of these equations are given by

1. $\bar{c} = (1-\tau)[\bar{w}(\bar{r} - \delta)\bar{k}] + \bar{T}$
2. $\bar{c}^{-\gamma} = \beta E_t \{ \bar{c}^{-\gamma} [(\bar{r} - \delta)(1-\tau) + 1] \}$
3. $a(1-\bar{l})^{-\xi} = c^{-\gamma} \bar{w}(1-\tau)$
4. $\bar{r} = \alpha \bar{k}^{\alpha-1} (\bar{l} - e^{\bar{z}})^{(1-\alpha)}$
5. $\bar{w} = (1-\alpha) \bar{k}^\alpha (\bar{l} e^{\bar{z}})^{-\alpha}$
6. $\tau[\bar{w}\bar{l} + (\bar{r} - \delta)\bar{k}] = \bar{T}$
7. $0 = \epsilon_t^z; \epsilon_t^z \sim i.i.d.(0, \sigma_z^2)$