Problem Set 1

Natasha Watkins

Problem 1

1. The state variables are B_t (barrels of oil left at time t) and p_t
2. The control variable is q_t, quantity of oil sold at time t
3. The transition equation is given as $B_{t+1} = B_t - q_t$
4. The sequence equation is given as $V(B_t) = \max_{q_1, q_2, \cdots} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t p_t q_t$

   The Bellman equation is given as $V(B_t) = \max_{q_t} p_t q_t + \left(\frac{1}{1+r}\right) V(B_{t+1})$
5. We can also write the Bellman equation as

$$V(B_t) = \max_{B_{t+1}} p_t(B_t - B_{t-1}) + \left(\frac{1}{1+r}\right) V(B_{t+1})$$

   Taking the derivative with respect to B_{t+1}, we find

$$p_t = \left(\frac{1}{1+r}\right) V'(B_{t+1})$$

$$V'(B_t) = p_t + \Big[-p_t + \left(\frac{1}{1+r}\right) V'(B_{t+1}) \Big] \tfrac{dB_{t+1}}{dB_t} = p_t$$

   So the Euler equation is

$$p_t = \left(\frac{1}{1+r}\right) p_{t+1}$$

6. If $p_{t+1} = p_t$ for all t, given $r > 0$, it is optimal to sell all of the oil in the first period.

   If $p_{t+1} > (1+r)p_t$ for all t, then it is optimal to sell oil in the last period - in the infinite case, the owner will never sell oil.

   Given this, an interior solution must satisfy $p_t \leq p_{t+1} < (1+r)p_t$