

## Problem Set 1

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### Problem 1

1. The state variables are  $B_t$  (barrels of oil left at time  $t$ ) and  $p_t$
2. The control variable is  $q_t$ , quantity of oil sold at time  $t$
3. The transition equation is given as  $B_{t+1} = B_t - q_t$
4. The sequence equation is given as  $V(B_t) = \max_{q_1, q_2, \dots} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t p_t q_t$   
The Bellman equation is given as  $V(B_t) = \max_{q_t} p_t q_t + \left(\frac{1}{1+r}\right) V(B_{t+1})$
5. We can also write the Bellman equation as

$$V(B_t) = \max_{B_{t+1}} p_t (B_t - B_{t+1}) + \left(\frac{1}{1+r}\right) V(B_{t+1})$$

Taking the derivative with respect to  $B_{t+1}$ , we find

$$p_t = \left(\frac{1}{1+r}\right) V'(B_{t+1})$$

$$V'(B_t) = p_t + \left[-p_t + \left(\frac{1}{1+r}\right) V'(B_{t+1})\right] \frac{dB_{t+1}}{dB_t} = p_t$$

So the Euler equation is

$$p_t = \left(\frac{1}{1+r}\right) p_{t+1}$$

6. If  $p_{t+1} = p_t$  for all  $t$ , given  $r > 0$ , it is optimal to sell all of the oil in the first period.

If  $p_{t+1} > (1+r)p_t$  for all  $t$ , then it is optimal to sell oil in the last period - in the infinite case, the owner will never sell oil.

Given this, an interior solution must satisfy  $p_t \leq p_{t+1} < (1+r)p_t$