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An integer linear programming model to support customer-driven material planning in synchronised, multi-tier supply chains

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This paper describes an integer linear programming model conceived as an alternative to a traditional material requirements planning (MRP) system for extending the concept of supply chain synchronisation upstream in a multi-tier supply chain. In this model, we assume there is an incumbent application for transmitting original equipment manufacturer (OEM) requirements to first-, second- and third-tier suppliers. The proposed model is regarded as being embedded within a web-enabled, multi-tier, supply chain information system that provides the application for transmitting the production requirements. The principal motivation for having second- and third-tier suppliers that are synchronised with OEM and first-tier activity is the significant inventory, lead time and responsiveness gains that can potentially be achieved. Here, inventory is considered as a whole across a supply chain, and stock-outs are prohibited for the first-tier supplier. For illustration purposes, an example based on a real, automotive case study is provided. The model results proved better in terms of inventory and bullwhip reduction than those found in a previous simulation-based approach. Also, a comparison of the proposed case results with those of a conventional MRP application is provided.

Keywords: MRP; supply chain management; integer linear programming; sequenced and synchronised suppliers; automotive sector

1. Introduction

Synchronisation in a multi-tier supply chain relates to a material requirements planning (MRP) where a supplier produces exactly to an original equipment manufacturer (OEM) requirement offset by an appropriate lead time (Lyons et al. 2005). Synchronised supply can be achieved by making accessible overall demand information directly along a supply chain (Holweg and Bicheno 2002). Thus, the use of production requirements' data from OEMs for material planning can provide high levels of synchronisation for suppliers operating in multi-tier supply chains such as those found in the automotive, machinery and aerospace manufacturing sectors. The use of OEM requirements' data for the determination of second- and third-tier material requirements conforms to the notion of the demand-driven supply chain (Lyons et al. 2012).

The simulation-based analyses by Holweg (2003) and Holweg et al. (2005) demonstrate the potential benefits of the synchronisation of sequenced components for an OEM and its first-tier suppliers in the automotive industry primarily in terms of reduced inventories. Coronado and Lyons (2008) investigated the benefits associated with extending synchronised sequencing of component deliveries to a second-tier supplier in two automotive supply chains. The authors regarded geographic proximity, time windows for deliveries and offset times between tiers as key supply chain variables and demonstrated via a spreadsheet-based simulation, synchronisation improvements in terms of bullwhip reduction, overall synchronisation levels and reduced supply chain inventories and cycle times. According to Coronado and Lyons (2008), the geographic proximity between the OEM and its suppliers proved to be an essential ingredient for the justification of the implementation of synchronised sequenced deliveries. In another study, Anne, Chedjou, and Kyamakia (2009), using a nonlinear dynamics approach, proposed an adaptive controller to support synchronisation and mitigate the effects of uncertainties (demand uncertainty, supply uncertainty, delivery uncertainty and forecasting uncertainty) in a three-tier supply chain. The authors used a bifurcation analysis in order to understand the performance of the supply chain over a range of different parameter settings. Also, Coronado, Lalwani, and Coronado (2011) proposed a simple approach to the measurement of synchronisation by comparing different values for units received and for units shipped. Liu, Young, and Ding (2011) proposed an integrated decision support system to make more efficient and effective global

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coordination decisions. A combination of qualitative and quantitative analysis and assessment functions was provided through the system's four key components: a global context modeller, a multi-criteria scoring modeller, a configurator and a coordinator. The evaluation of the decision system was undertaken through a case study within the automotive industry, which demonstrated the applicability of the system to provide decision support for realistic global manufacturing coordination problems.

The first MRP formulations were not associated with any type of optimisation. However, the maximisation of benefits (or minimisation of costs) and constraints related to replenishment, demand and capacity of available resources was gradually incorporated with the aim of providing optimal solutions to MRP problems (Yenisey 2006). It is important to highlight the work by Karni (1981) as one of the initial contributions in this field, where the use of integer linear programming for solving MRP problems is justified. Other authors such as Billington, McClain, and Thomas (1983), Escudero and Kamesam (1993), Rota, Thierry, and Bel (1997), Clark (2003), Giglio and Minciardi (2003), Yenisey (2006), Mula, Poler, and Garcia (2006, 2007), Mula, Poler, and Garcia-Sabater (2008) and Noori et al. (2008) have addressed the optimisation of MRP systems through different approaches to mathematical programming.

This paper proposes an integer linear programming formulation for extending synchronised sequencing in multi-tier supply chains as an alternative MRP model. The practical application of this model manifests itself in the form of a web-enabled, multi-tier supply chain information system in order to delegate to each member of the supply chain full responsibilities for the components they manufacture (Lyons et al. 2005). In our proposal, the use of this approach with suitable components allows second- and third-tier component requirements to be determined from the final vehicle demand. The validation of this model has been undertaken using the same data as in Coronado and Lyons (2008). The optimisation model was found to produce improved results in terms of inventory levels, bullwhip and policy-related parameters such as target safety stocks. Furthermore, the model is flexible enough to also consider additional decision variables such as safety stock, capacity constraints, backorders or independent requirements at each tier. The key contribution of this work concerns the application of integer linear programming to address material planning across multi-tier supply chains and the fresh insight that can be provided by such an approach. Previous efforts to address this challenge have primarily been addressed using simulation-based approaches. Furthermore, a comparison of the proposed case results with those of a conventional MRP application is carried out in order to examine the main differences, assumptions, additional data and information flows affecting the MRP results between the two approaches.

With respect to alternative approaches to supply chain production planning based on MRP-type environments, some authors have used mathematical programming as their means of investigation. These include McDonald and Karimi (1997), von Lanzenauer and Pil-Glombik (2002), Kreipl and Pinedo (2004), Chen and Lee (2004), Park (2005), Peidro et al. ("Fuzzy optimization" 2009), Peidro et al. (2010, 2012), Alemany et al. (2010) and Mula, Peidro, and Poler (2010). For an extensive review, we refer readers to Peidro et al. ("Quantitative Models" 2009) and Mula, Peidro, and Poler (2010).

The remainder of this article is arranged as follows. Section 2 describes the fundamental research problem. Section 3 describes the formulation of an integer linear programming model for extending synchronised material planning in two-tier supply chains. Section 4 proposes several extensions to the model. Section 5 validates and evaluates the model proposed. Section 6 offers conclusions and further research.

2. Description of the problem

The intention of the research was to address a customer-driven supply chain problem based on material planning synchronisation through demand and inventory information sharing across multiple supply chain tiers and to do so within an empirical context. In order to achieve this ambition, we were inspired by automotive supply chain arrangements where the first-tier (and occasionally second- and third-tier) suppliers are located on supplier parks in close proximity to the vehicle assembly plant. These proximate supply arrangements are characterised by the assembly of key modules and components by first-tier supply partners, which assemble and deliver the modules and components in the same sequence as the final assembly of the vehicles at the assembly plant (Lyons et al. 2012). Here, the typical flow of information along a key module or component value stream is instigated by the OEM and made available only to its immediate first-tier supplier, which makes new material requirements' information available for its second-tier supplier and so on. Thus, the requirements for first-tier components are transmitted by the OEM to the first-tier supplier. This communication is metronomic, triggered for every vehicle (typically within the pre-final assembly paint buffer) at a frequency close to the final assembly takt time. The first-tier supplier uses the requirements' file as input to its own internal MRP system. The suppliers' schedules are produced for each of the first-tier's component suppliers. This implies that the information generated by the OEM is visible to first but not second- or third-tier suppliers. It logically follows that the demand signal is amplified upstream in the chain, resulting in inefficiency and higher inventory levels along the supply chain (Coronado and Lyons 2008). This motivated the development of an integer linear programming model based on a simple proposition: making OEM production requirements' information available to second- and third-tier component suppliers would provide an opportunity for the material planning process under consideration to be more customer-driven and aligned with the OEM demand signal.

3. Integer linear programming for extending synchronised material planning in supply chains

3.1 *Assumptions and nomenclature*

This section formulates the alternative MRP model for synchronised suppliers for a multi-tier supply chain. This integer linear programming model makes the following assumptions:

- Second- and third-tier component demand can be determined from the OEM demand using a simple bill-of-materials parts explosion. For instance, for an automotive supply chain comprised of the vehicle assembler (OEM), the seat assembler, the rear centre headrest supplier and the headrest material supplier, it is possible to determine the second- and third-tier requirements from the available fixed OEM raw demand of first-tier components, that is, by using the OEM seat requirements, first and second tier can deduce their requirements for headrests and headrest covers.
- The implementation of this model assumes first- and second-tier suppliers to be in close proximity to the OEM assembly facility. Second- and third-tier component requirements are determined from the final requirements transmitted to the first-tier supplier, which is based on a frozen sequence. An external web application transforms the OEM raw demand into first-tier requirements. The creation of second-tier requirements requires the use of an offset time between the first- and second-tier suppliers to allow for production and transportation times between the tiers.
- Each raw material or part is assigned to just one second- and/or third-tier supplier.
- We consider initial stock levels for all components.
- We consider pre-defined safety stock target levels for all components.
- We consider predetermined delivery batch sizes for all components.
- Backorders are prohibited.
- Final vehicle demand, on-hand pipeline inventory and delivery batch sizes are used to determine second- and third-tier requirements for each supplier.

Figure 1 depicts the information flows between the web-based system and the suppliers, which, in turn, upload available inventory, target safety stocks and batch size data for each of the selected components. Also, an intermediate source of demand information is used in order to offset the OEM requirements by an appropriate lead time to determine the first-tier requirements. Then, the optimisation model, integrated into the information system, determines the requirements for second- and third-tier suppliers. We refer readers to the research studies undertaken by Lin and Lin (2005), Jaksic and Rusjan (2008) and Chatfield (2009) for the establishment of lead times and safety stocks.

The nomenclature defines the indices, parameters and decision variables of the models (refer to Table 1).

The formulation of the model is as follows.

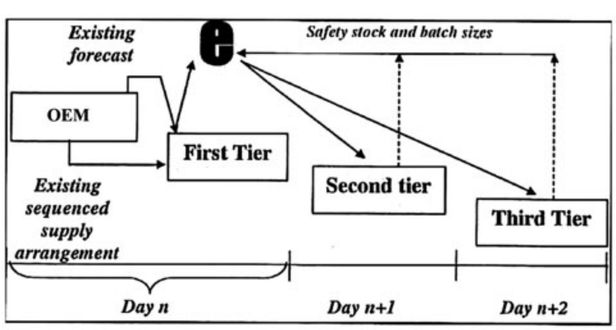

Figure 1. Information flows.

Table 1. Nomenclature.

Indices			
i	First-tier components ($i = 1 \ldots I$)		
T	Periods of time ($t = 1 \ldots T$)		
Model parameters			
$t1$	Offset time between first- and second-tier deliveries		
R_{it}	First-tier requirements of i in period t	$R0_i$	First-tier requirements of i in the initial period
$INC0_i$	Initial first-tier raw material inventory of i	$IND0_i$	Initial second-tier raw material inventory of i
aa_i	Predefined first-tier raw material inventory level of i	ab_i	Predefined second-tier raw material inventory level of i
ssC_i	Second-tier safety stock multiplication factor for i	ssD_i	Third-tier safety stock multiplication factor for raw material of i
ba_i	Second-tier batch size deliveries (units) of i	bb_i	Third-tier batch size deliveries (units) of raw material of i
Decision variables			
S_{it}	Second-tier deliveries of i in period t	F_{it}	Third-tier raw material deliveries of i in period t
INC_{it}	First-tier raw material inventory of i at the end of period t	IND_{it}	Second-tier raw material inventory of i at the end of period t
$IsslC_{it}$	First-tier raw material inventory lower to the inventory target level of i in period t	$IsslD_{it}$	Second-tier raw material inventory lower to the inventory target level of i in time period t
$IsshC_{it}$	First-tier raw material inventory higher to the inventory target level of i in period t	$IsshD_{it}$	Second-tier raw material inventory higher to the inventory target level of i in period t
ks_{it}	Number of lots to deliver of second-tier raw material of i at the period t	kt_{it}	Number of lots to deliver of third-tier raw material of i at the period t

3.2 *Objective function*

The objective of the model is to maintain the available inventory level close to the safety stock target level and, at the same time, to generate deliveries adjusted to requirement quantities for each component in a given period. Thus, the objective function minimises the first- and second-tier inventory levels lower and higher than the safety stock target levels and the positive deviation between second- and third-tier deliveries and demand quantities per product and period.

$$\text{Minimize } z = \sum_{i=1}^{I}\sum_{t=1}^{T}\left[IsslC_{it} + IsshC_{it} + IsslD_{it} + IsshD_{it} + (F_{it} - R_{it})\right] + \sum_{i=1}^{I}(S_{i,t=t1} - R0_i) + \sum_{i=1}^{I}\sum_{t>t1}^{T}(S_{it} - R_{i,t-t1}) \tag{1}$$

3.3 *Constraints*

The model uses the first-tier demand requirements from the OEM frozen production sequence files (with an offset time of $t1$) to determine the deliveries or raw demand for the second and third tiers for a given time period. The model compares the available inventory with a pre-defined safety stock target level and adds or subtracts from the raw demand to return available inventories to the desired level. The deliveries are adjusted to demand quantities and rounded to predetermined delivery batch sizes. In this section, we describe the constraints that are necessary for our proposal.

In the first place, we consider the set of constraints for second-tier material deliveries:

Constraint (2) is the first-tier inventory balance for the component i in the initial period.

$$INC_{it} = INC0_i + S_{it} - R0_i \quad \forall i, t = t1 \tag{2}$$

Constraint (3) is the first-tier inventory balance for the component i in the remaining periods.

$$INC_{it} = INC_{it-1} + S_{it} - R_{it-t1} \quad \forall i, t > t1 \tag{3}$$

Constraint (4) and constraint (5) control the first-tier inventories below pre-defined target inventory levels.

$$IsslC_{it} \geqslant (ssC_i \cdot aa_i) - INC_{it} \quad \forall i, t \tag{4}$$

$$IsslC_{it} \leqslant (ssC_i \cdot aa_i) \quad \forall i, t \tag{5}$$

Constraint (6) controls the first-tier inventories' upper pre-defined target inventory levels.

$$IsshC_{it} \geqslant INC_{it} - (ssC_i \cdot aa_i) \quad \forall i,t \tag{6}$$

Constraint (7) establishes the second-tier number of delivery lot sizes of the component i in the period t where $ba_i > 1$.

$$S_{it} = ks_{it} \cdot ba_i \quad \forall i,t \quad \text{where} \quad ba_i > 1 \tag{7}$$

Now, we consider the set of constraints for third-tier material deliveries:

Constraint (8) is the second-tier inventory balance for the component i in the initial period.

$$IND_{it} = IND0_i + F_{it} - R_{it} \quad \forall i, t = \text{first}(t) \tag{8}$$

Constraint (9) is the second-tier inventory balance for the component i in the remaining periods.

$$IND_{it} = IND_{it-1} + F_{it} - R_{it} \quad \forall i, t > \text{first}(t) \tag{9}$$

Constraint (10) and constraint (11) control the second-tier inventories below pre-defined target inventory levels.

$$IsslD_{it} \geqslant (ssD_i \cdot ab_i) - IND_{it} \quad \forall i,t \tag{10}$$

$$IsslD_{it} \leqslant (ssD_i \cdot ab_i) \quad \forall i,t \tag{11}$$

Constraint (12) controls the second-tier inventories' upper pre-defined target inventory levels.

$$IsshD_{it} \geqslant IND_{it} - (ssD_i \cdot ab_i) \quad \forall i,t \tag{12}$$

Constraint (13) establishes the third-tier number of delivery lot sizes of the component i in the period t where $bb_i > 1$.

$$F_{it} = kt_{it} \cdot bb_i \quad \forall i,t \quad \text{where} \quad bb_i > 1 \tag{13}$$

The model also contemplates non-negativity constraints and the definition of integer variables (14).

$$ks_{it}, kt_{it}, S_{it}, F_{it}, INC_{it}, IND_{it}, IsslC_{it}, IsshC_{it}, IsslD_{it}, IsshD_{it} \geq 0 \text{ and } ks_{it}, kt_{it} \in Z \tag{14}$$

$\forall i \in I, \forall t \in T$

Finally, some other decision variables can be defined as integers, for instance, if products are measured by units, but could change depending on the real-world problem where the model is applied.

4. Extensions

In this section, we describe some possible extensions to the previous general model that feasibly could be applied to different manufacturing environments.

4.1 *Safety stock multiplication factor as decision variable*

In cases where the safety stock multiplication factor has previously not been determined by external analytical or empirical formulations, it can be regarded as a decision variable. In order to facilitate this, the parameters ssC_i and ssD_i should be defined as decision variables, SS_i, instead of constant parameters and included in the non-negativity constraint (14). Also, the integrality condition should be defined if required. The rest of the model, namely the objective function and constraints (1)–(13), would remain unaltered.

4.2 *Capacity constraints*

The previous model presented pursues the extension of first-tier sequenced deliveries to the second- and third-tier suppliers based on OEM frozen demand with the aim of minimising inventories along the supply chain. In this model, we have not included capacity constraints because we consider they would be used for the supplier production planning model as another item of input data apart from the predicted demand sequence of our proposal, or even be considered in a complementary supply chain production planning model (see the Introduction section). Nevertheless, in cases where supply capacity constraints are considered primarily to extend sequenced deliveries, the model requires some modification in order to consider relevant capacity constraints. We refer to Billington, McClain, and Thomas (1983) and Mula, Poler, and Garcia (2006, 2007), Mula, Poler, and Garcia-Sabater (2008) for capacity-constrained MRP models; and McDonald and Karimi (1997), Peidro et al. ("Fuzzy Optimization" 2009), Peidro et al. (2010, 2012), Alemany et al. (2010), and Mula, Peidro, and Poler (2010) for supply chain production planning models with capacity constraints.

For example, the following capacity constraint could be incorporated:

$$R_{it} + Tid_{rt} - Tov_{rt} - Tsub_{rt} = CAP_{rt} \quad \forall i, r, t \tag{15}$$

Constraint (15) establishes the available capacity for normal, overtime and subcontracted production for deliveries where r is the set of considered productive resources, Tid_{rt} is the undertime of the resource r in period t, Tov_{rt} is the overtime considered and $Tsub_{rt}$ is the subcontracted time of the resource r in period t. Also, overtime (or subcontracted time) limits could be included as follows:

$$Tov_{rt} \leqslant ov_{rt} \cdot R_{it} \quad \forall i, r, t \tag{16}$$

Constraint (16) provides the overtime limits in terms of a percentage of the normal production working time, ov_{rt}.

Furthermore, the objective function should be modified in order to include the aspirations related to the new defined variables, Tid_{rt}, Tov_{rt} and $Tsub_{rt}$, in terms of cost or penalty coefficients. At this point, it would be useful to analyse the convenience of using linear programming or multi-objective linear programming models according to the problem to be modelled (Mula, Peidro, and Poler 2010; Peidro et al. 2012).

4.3 *Backorders*

With respect to backorders, they have been prohibited in our model in order to satisfy the efficacy of sequenced deliveries. Nevertheless, if capacity constraints are included, the consideration of backorders would be convenient in order to generate feasible plans if extra or subcontracted times are also limited. Thus, a new decision variable could be created, B_{it}, which represents the backorders of the component i in the period t. The constraints (2), (3), (8) and (9) would be replaced by (17), (18), (19) and (20), respectively. The data parameters $BC0_i$ and $BD0_i$ represent the initial backorders for the planning time horizon.

$$INC_{it} = INC0_i + S_{it} - R0_i - BC0_i + B_{it-1} \quad \forall i, t = t1 \tag{17}$$

$$INC_{it} = INC_{it-1} + S_{it} - R_{it-t1} - B_{it-t1-1} + B_{it-t1} \quad \forall i, t > t1 \tag{18}$$

$$IND_{it} = IND0_i + F_{it} - R_{it} - BD0_i + B_{it} \quad \forall i, t = \text{first}(t) \tag{19}$$

$$IND_{it} = IND_{it-1} + F_{it} - R_{it} - B_{it-1} + B_{it} \quad \forall i, t = \text{first}(t) \tag{20}$$

In addition to the previous constraints, the objective function should be modified in order to include the aspirations related to the newly defined variable, B_{it}, in terms of cost or penalty coefficient. At this point, it is apposite to analyse the appropriateness of using linear programming or multi-objective linear programming models to address the problem to be modelled (Mula, Peidro, and Poler 2010; Peidro et al. 2012).

4.4 *Complex bill of materials or alternative suppliers*

In the case of material requirements with more complex bills of material, when alternative suppliers exist or simply to reach a higher level of computational efficiency with the integer linear programming models, the same model formulation is run for each supplier with the same original OEM raw demand (with an offset time) and different supplier input data (inventory levels, target safety stocks and delivery batch sizes) instead of running a unique model for second- and third-tier deliveries. Thus, the external web application would transform the OEM raw demand into first- and second-tier requirements. The creation of third-tier requirements would require the use of an offset time ($t2$) between the second- and third-tier suppliers.

Therefore, the following model would be executed for each second-tier supplier separately.

$$\text{Minimize } z = \sum_{i=1}^{I}\sum_{t=1}^{T}[IsslC_{it} + IsshC_{it}] + \sum_{i=1}^{I} S_{it=t1} - R0_i + \sum_{i=1}^{I}\sum_{t>t1}^{T} S_{it} - R_{it-t1} \tag{21}$$

Subject to the constraints (2) to (7) and the corresponding non-negativity and integrality condition constraints.

The same model would be used for third-tier suppliers but with the corresponding third-tier demand provided by the demand web application (offset by $t2$ with respect to first-tier requirements) and by considering the related decision variables and data parameters for third-tier deliveries given in Table 1.

5. Application to an automotive supply chain

This application has considered real data from a section of the value stream of an automotive supply chain comprised of an automotive assembly plant (OEM), a seat assembler (first tier), a headrest manufacturer (second tier) and a supplier of headrest covers (third tier). These supply chain data were previously simulated using a spreadsheet approach by Coronado and Lyons (2008).

The analysis covers a total of 5 weeks, that is, 30 working days (T). The OEM sends production schedules to the first-tier supplier containing daily requirements for the following week. The alternative MRP is run weekly to plan the optimal second- and third-tier deliveries (Figure 2). The demand specified in the OEM production files was offset by 1 day ($t1$) to obtain the first-tier requirements (R_{it}). A sample of 10 headrests (i) was analysed. The batch sizes of second- and third-tier deliveries are equal to 28 units (ba_i, bb_i). A second-tier safety stock multiplication factor of 1.5 is applied (ssC_i, ssD_i) according to Coronado and Lyons (2008). Also, the following are considered: first- and second-tier initial inventory ($INC0_i$, $IND0_i$), first-tier requirements of i in period 0 ($R0_i$), and a buffer or pre-defined, first-tier and second-tier raw material inventory levels of i (aa_i, ab_i). In this case, each of the decision variables of the model is regarded as integers. More details on these data used can be found in Section 5.2.

5.1 *Implementation and resolution*

The proposed model has been developed using the MPL language, V4.2. The resolution has been carried out with the optimisation solver CPLEX 12.1.0. Finally, the input data and the model solution values were processed with the Microsoft SQL SERVER. The experiment was run on a server with two processors of 2.93 GHz (each one) and 48 GB of RAM.

5.2 *Evaluation of results*

For illustration and comparative purposes with the spreadsheet simulation undertaken by Coronado and Lyons (2008), we show the input data and results corresponding to the first week ($t = 1,\ldots,5$) and to the second tier from the OEM.

Table 2 depicts the details of the input data relating to the second-tier batch sizes (ba_i), first-tier initial inventory ($INC0_i$), first-tier requirements in the initial period ($R0_i$) and the pre-defined buffer for first-tier requirements (aa_i).

Table 3 shows the first-tier requirements' input data (R_{it}) and the results generated by spreadsheet simulation (Coronado and Lyons 2008), optimisation and conventional MRP in terms of deliveries (S_{it}) and first-tier inventory levels (INC_{it}). Table 3 indicates that the optimisation models yield improvements in component inventory held at the first tier. Also, the delivery quantities are more closely aligned at the second tier with the OEM demand requirements.

According to the model extension described in Section 4.1, Table 4 presents the results for the safety stock multiplication factor, where this parameter is defined as a decision variable, SS_i, instead of the established value of 1.5 for ssC_i and ssD_i.

Table 5 shows the bullwhip effect measure (BEM) from the OEM to the second-tier supplier for the t period, BEM_t, by aggregating the data for the 10 headrests considered in the three models presented in Table 3. According to Parra-Peña, Mula, and Campuzano-Bolarin (2012), the calculation of the BEM_t has been determined as a cumulative expression of the effect of individual period distortions, by using a conditional expression presented in Equation (22).

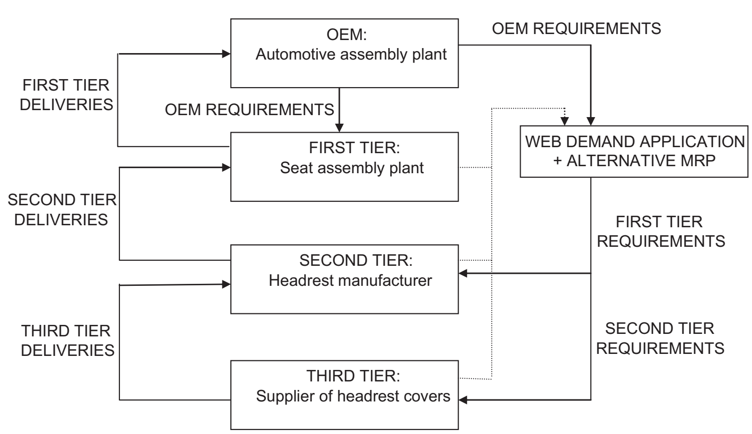

Figure 2. Application in an automotive supply chain.

Table 2. Input data.

i	ba_i	$INC0_i$	$R0_i$	aa_i
1	28	66	20	56
2	28	85	45	56
3	28	37	20	28
4	28	41	40	56
5	28	91	2	28
6	28	46	2	28
7	28	80	5	28
8	28	23	3	28
9	28	62	35	56
10	28	133	85	56

$$\mathrm{BEM}_t = \begin{cases} \mathrm{BEM}_{t-1} & \text{if} \quad S_{it} - R_{it} = 0 \\ \mathrm{BEM}_{t-1} + \dfrac{\mathrm{Var}(S_{it})}{\mathrm{Var}(R_{it})} & \text{if} \quad S_{it} - R_{it} \neq 0 \end{cases} \tag{22}$$

where the calculation of the variance of the OEM demand (R_{it}) is done with the variance formula because it must be calculated by using all dates available, that is, the dates associated with each period, from zero to the t period, while the formula of the variance of deliveries (S_{it}) is as indicated by Equation (23). This formula considers $n-1$ elements, because it is a sample variance (see Parra-Peña, Mula, and Campuzano-Bolarin 2012).

$$\mathrm{Var}(x) = \frac{(n-1)\left(\frac{x_n}{n}\right)^2 + \left(x_n - \frac{x_n}{n}\right)^2}{n-1} \tag{23}$$

The bullwhip measure reduced from 17.91 to 8.64 with the optimisation model (see Table 5). It is important to highlight that the optimisation model with SS_i as decision variables generates the lowest inventory levels (see Table 3) at the first tier. The BEM from the OEM to the second-tier supplier is higher than in the optimisation model with a calculated value of 1.5 for ssC_i and $ssD_{i,}$ This can be explained by the inclusion of a more smoothed plan of delivery quantities.

Table 6 illustrates the computational efficiency related to the first week ($t = 1,\ldots,5$) for first- and second-tier requirements. Also, we have included the computational efficiency when a unique model is run for each tier. In this case, second-tier deliveries and in both cases SS_i have been regarded as decision variables. The computational efficiency measures the computational effort required to solve models. The indicators are as follows: the number of iterations needed by the solver and used to reach the optimum solution, the number of model variables, the number of integers in the model, the number of constraints in the model, the number of non-zero elements in the constraints matrix that the model contains, the density of the constraints matrix and the CPU time required to obtain the model solution.

We compared our model with the previous spreadsheet simulations and found that we obtained better results in terms of average inventory levels and bullwhip effect in the first and second tier of the studied supply chain and did so without utilising high CPU times. The use of unique models for second-tier and third-tier suppliers would decrease considerably the complexity and CPU times because all the variables of the model are defined as integers. Also, we could optimise the second-tier deliveries by considering the safety stock multiplication factor as a continuous decision variable. Moreover, we have considered a weekly planning horizon and a daily planning period but daily runs are also desirable in some manufacturing environments.

Additionally, we have compared our proposal with the results obtained by a conventional MRP application. In such case, the same input data have been used and specific assumptions are as follows: safety stock is not flexible but mandatory for all periods (as assumed in a conventional MRP application) which means that all INC_{it} are maintained equal or over the safety stock values in all periods except, in some cases, the first period due to the initial stocks and initial requirements; and lot sizes have been met, launching multiples of such amounts when requirement plus safety stock overcome current stock. The optimisation proposals and spreadsheet simulation by Coronado and Lyons (2008) have achieved better results than conventional MRP in terms of average inventory levels and BEMs. Although the optimisation proposals consider flexible safety stocks or target stocks, smoothed inventory levels are provided along all considered time periods. The improvement achieved by the optimisation models could be extended to a third-tier supplier, which would receive the demand from the second-tier supplier MRP explosion.

Table 3. Input data and results.

			Spreadsheet simulation		Optimisation		Optimisation with SS_i as decision variables		Conventional MRP	
i	t	R_{it}	S_{it}	INC_{it}	S_{it}	INC_{it}	S_{it}	INC_{it}	S_{it}	INC_{it}
1	1	23	28	74	28	74	0	46	84	46
1	2	37	56	107	28	79	0	23	28	107
1	3	8	0	70	28	70	28	14	0	98
1	4	22	28	90	28	90	0	6	28	90
1	5	7	0	68	0	68	28	12	0	96
2	1	29	28	68	56	96	0	40	84	40
2	2	43	56	95	28	95	0	11	56	95
2	3	30	28	80	28	80	56	24	28	108
2	4	56	56	106	28	78	28	22	56	106
2	5	43	28	78	0	22	56	22	28	106
3	1	16	28	45	28	45	0	17	84	17
3	2	19	28	57	0	29	28	29	28	85
3	3	12	0	38	28	38	0	10	28	94
3	4	6	0	26	0	26	28	26	0	110
3	5	6	28	48	0	20	0	20	0	104
4	1	22	56	57	84	85	28	29	112	1
4	2	34	56	91	28	91	28	35	28	91
4	3	37	28	85	28	85	28	29	56	85
4	4	24	28	76	28	76	28	20	28	104
4	5	37	56	108	0	52	28	24	28	108
5	1	1	0	89	0	89	0	89	0	89
5	2	14	0	88	0	88	0	88	28	88
5	3	5	0	74	0	74	0	74	0	102
5	4	1	0	69	0	69	0	69	0	97
5	5	1	0	68	0	68	0	68	0	96
6	1	1	0	44	0	44	0	44	56	44
6	2	2	0	43	0	43	0	43	0	99
6	3	1	0	41	0	41	0	41	0	97
6	4	2	0	40	0	40	0	40	0	96
6	5	6	0	38	0	38	0	38	0	94
7	1	2	0	75	0	75	0	75	28	75
7	2	7	0	73	0	73	0	73	0	101
7	3	2	0	66	0	66	0	66	0	94
7	4	3	0	64	0	64	0	64	0	92
7	5	6	0	61	0	61	0	61	28	89
8	1	1	28	48	28	48	0	20	84	20
8	2	6	0	47	0	47	0	19	0	103
8	3	3	0	41	0	41	0	13	0	97
8	4	10	0	38	0	38	0	10	0	94
8	5	3	0	28	0	28	0	0	28	84
9	1	30	56	83	56	83	0	27	112	27
9	2	59	56	109	28	81	28	25	56	109
9	3	39	28	78	56	78	56	22	28	106
9	4	13	28	67	56	95	56	39	28	95
9	5	11	28	82	0	82	0	26	0	110
10	1	55	0	48	28	76	0	48	112	48
10	2	64	112	105	56	77	28	21	56	105
10	3	48	28	69	84	97	56	13	56	97
10	4	75	84	105	28	77	56	21	56	105
10	5	103	84	114	0	2	56	2	112	86
	Total	**1085**	**1148**	**3462**	**896**	**3182**	**728**	**1698**	**1652**	**4330**
	Aver.	**22**	**23**	**69**	**18**	**64**	**15**	**34**	**33**	**87**

Table 4. SS_i results.

i	1	2	3	4	5	6	7	8	9	10	Aver.
SS_i	0.2500	0.3929	0.7143	0.5179	2.6429	1.4643	2.3571	0.4643	0.4643	0.375	0.9643

Table 5. Bullwhip effect measures.

		Spreadsheet simulation		Optimisation		OPTIMISATION with SS_i as decision variables		Conventional MRP	
t	R_{it}	S_{it}	BEM_t (OEM – 2nd tier)	S_{it}	BEM_t (OEM – 2nd tier)	S_{it}	BEM_t (OEM – 2nd tier)	S_{it}	BEM_t (OEM – 2nd tier)
1	180	224		308		28		756	
2	285	364	8.01	168	1.71	112	0.76	280	4.74
3	185	112	8.91	252	6.23	224	4.33	196	7.48
4	212	224	13.19	168	8.64	196	7.61	196	10.76
5	223	224	17.91	0	8.64	168	10.27	224	15.48

Table 6. Computational efficiency.

Model	Iterations	Variables	Integers	Constraints	Non-zero	Density (%)	Time
2nd- and 3rd-tier deliveries	14,098	500	500	500	980	0.4	12.40 s
2nd- and 3rd-tier deliveries with SS_i optimisation	16,327	520	500	500	1280	0.5	12.84 s
2nd-tier deliveries	4463	250	250	500	490	0.8	3.92 s
2nd-tier deliveries with SS_i optimisation	8811	260	250	250	640	1	7.24 s

6. Conclusions

This paper continues the work by Coronado and Lyons (2008) and proposes an integer linear programming model for extending synchronised material planning across the second and third tiers of an automotive supply chain. Previously, the authors demonstrated via spreadsheet simulation the benefits of including second-tier synchronised planning in terms of bullwhip reduction and reduction of inventory levels and cycle times. In this paper, we have presented a proposal for applying the same synchronisation concept to a supply chain using a mathematical programming model. We have compared our model with the previous simulation models and also with a conventional MRP approach. The optimised results prove better in terms of inventory levels and bullwhip reduction. Also, our solution of the alternative MRP model can address this particular supply chain challenge with modest computational effort. Furthermore, the ease with which mathematical programming languages can be connected to company ERP systems and the generality of mathematical programming models provides the potential to be able to extend this approach to different manufacturing environments.

The use of metaheuristics for improving the efficiency of the solution model for more complex problems is a forthcoming piece of work. Also, there is significant research potential related to new organisational regimes in the supply chain associated with the consequences of the loss of production management responsibility of the first-tier suppliers and the subsequent implications for second- and third-tier suppliers. Finally, an imminent work is considering the analysis of the performance of the optimisation proposals when unexpected demand arrives and safety stocks must be used for accomplishing the requirements through a rolling horizon simulation.

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