Natalie Belmonte

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# ThreeDoor Analysis

The Monty Hall problem is a probability puzzle based on a game show setup of a prize behind one of three doors. After the contestant selects their door, another is revealed to not contain the prize. The contestant is then given the opportunity to change their chosen door or remain with their first choice. If they chose to switch to the second closed door, their chance of choosing the prize greatly increases. This can be shown by the GameTester() class's output below:

```
Chance of success over 10,000 trials, without changing curtain: 33.4%
Chance of success over 10,000 trials, when changing curtian: 66.28%
```

## <u>Simulation</u>

This project models the three doors with two random variables with values from 1-3: the original door selected by the contestant and the door with the prize. Using an if/else statement, it then determines if the contestant will win while staying with their given door or while changing. Since this is a short simulation with only one data point returned, the default trial amount during testing was set for 10,000.

## <u>Textbook Reasoning</u>

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Textbook page 34, question 2.20:
a. If the contestant choses a curtain at random, they have around a 1/3, or 33% chance of picking the curtain with the
   prize.
b. The host reveals an empty curtian, and gives the contestant a chance to change their door choice.
    i. If the contestant stays with their orginial choice, they have the same probability of winning (33%).
    ii. If they switch curtains, after selecting the one with the prize, they will not win the prize.
    iii. However, if they initially picked an empty curtian, changing their choice allows them to win the prize.
    iv. In general, switching curtians gives the contestant a 2/3, or 67%, chance of winning.
    v. As shown by the above program, switching curtains greatly increases the probability of winning the prize.
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The exercise above illustrates the key variables in the Monty Hall problem. However, it leaves one question unanswered: why does switching doors so greatly affect the probability of success?

When the first door is selected, the contestant has a 1/3 chance of guessing correctly. When given the opportunity to switch, only two doors remain. Although the amount of doors changes, the probability of the previously selected door holding the prize remains consistent. Conversely, once there are only two doors remaining, one with a probability of 1/3, the untouched door must have a 2/3 probability of containing the prize.

This principle, which applies to all probability problems, is established in Definition 2.6, Axiom 2, shown below.

> Suppose $S$ is a sample space associated with an experiment. To every event $A$ in $S$ ($A$ is a subset of $S$), we assign a number, $P(A)$, called the *probability* of $A$, so that the following axioms hold:
>
> Axiom 1: $P(A) \geq 0$.
> Axiom 2: $P(S) = 1$.

All events A in sample space S must add up to 1 (i.e. 100%). In this way, when the sample space decreases, the only remaining event not yet assigned a probability value is given the remainder of this total amount.

Although this reasoning seems hypothetical, it is mirrored in a concrete mathematical fashion through programs such as ThreeDoor, which simulate guesses through random variables and explore changing doors as a predetermined choice. While this exercise can be hand-solved, it is very effective to see an organic simulation generate the results that are expected by established statistical axioms.