Definition 3.11

Poisson probability distribution of random variable Y:

$$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$$

Theorem 3.11

Expected value of Poisson distribution:

$$E(Y) = \mu = \lambda$$

Variance of Poisson distribution:

$$V(Y) = \sigma^2 = \lambda$$

Theorem 3.14

Tchebysheff's Theorem for random variable Y, mean μ, and variance σ^2. For any constant $k > 0$:

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \quad \text{OR} \quad P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Definition 4.1

Distribution function of Y (any random variable):

$$F(y) = P(Y \leq y) \text{ for all } -\infty < y < \infty$$

Theorem 4.1

Properties of a distribution function:

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$
3. $F(y)$ is a nondecreasing function of y.

Definition 4.2

If $F(y)$ is continuous on $-\infty < y < \infty$, Y is a continuous random variable.

Definition 4.3

Probability density function of Y:

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

Theorem 4.2

Properties of a density function $f(y)$:

1. $f(y) \geq 0$ for all $-\infty < \mathrm{y} < \infty$
2. $\int_{-\infty}^{\infty} f(y)dy = 1$

Theorem 4.3

If Y has a density function $f(y)$ and bounds $a < b$, the probability that Y is on interval $[a, b]$:

$$P(a \leq Y \leq b) = \int_a^{\mathrm{b}} f(y)dy$$

Definition 4.5

Expected value of continuous random variable Y:

$$E(Y) = \int_{-\infty}^{\infty} yf(y)dy$$

Theorem 4.4

Expected value of continuous random variable Y:

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y)f(y)dy$$

Theorem 4.5

Let c be a constant, $g(Y), g_1(Y)$ ... $g_k(Y)$ be functions of Y.

1. $E(c) = c$
2. $E(cg(Y) = cE[g(Y)]$
3. $E(g_1(Y) + g_2(Y) + \cdots + g_k(Y) = E[g_1(Y)] + E[g_2(Y)] + \cdots + E[g_k(Y)]$

Definition 4.6

Y has a continuous uniform probability distribution if its density function is:

$$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 \leq y \leq \theta_2 \\ 0, & elsewhere \end{cases}$$

Theorem 4.6

If $\theta_1 < \theta_2$ and Y is uniformly distributed on $(\theta_{1,}\theta_2)$, then:

1. $E(Y) = \mu = \frac{\theta_1 + \theta_2}{2}$
2. $V(Y) = \sigma^2 = \frac{(\theta_2 - \theta_1)^2}{12}$

Definition 4.8

Y has normal probability distribution if its density function is:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)} \text{ when } \sigma > 0,\ -\infty < \mu < \infty, \text{ and } -\infty < y < \infty$$

Theorem 4.7

Expected value of normally distributed random variable Y:

$$E(Y) = \mu$$

Variance of normally distributed random variable Y:

$$V(Y) = \sigma^2$$

Definition 4.9

Y has a gamma distribution with parameters $\alpha > 0$ and $\beta > 0$ if the density function is:

$$f(y) = \begin{cases} \frac{y^{\alpha-1}e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, & 0 \leq y < \infty \\ 0, & elsewhere \end{cases} \text{, where } \Gamma(\alpha) \int_0^\infty y^{\alpha-1}e^{-y}dy$$

Theorem 4.8

Expected value of normally distributed random variable Y:

$$E(Y) = \mu = \alpha\beta$$

Variance of normally distributed random variable Y:

$$V(Y) = \sigma^2 = \alpha\beta^2$$

Definition 4.11

Y has an exponential distribution with parameter $\beta > 0$ if the density function is:

$$f(y) = \begin{cases} \frac{1}{\beta} e^{-y/\beta}, & 0 \leq y < \infty \\ 0, & elsewhere \end{cases}$$

Theorem 4.10

Expected value of exponentially distributed random variable Y:

$$E(Y) = \mu = \beta$$

Variance of exponentially distributed random variable Y:

$$V(Y) = \sigma^2 = \beta^2$$

Definition 5.1

Joint probability function of Y_1 and Y_2:

$p(y_1, y_2) = p(Y_1 = y_1, Y_2 = y_2)$ when $-\infty < y_1, y_2 < \infty$

Theorem 5.1

If Y_1 and Y_2 have joint probability function, then:

1. $p(y_1, y_2) \geq 0$
2. $\sum_{y_1, y_2} p(y_1, y_2) = 1$

Definition 5.2

Joint distribution function of Y_1 and Y_2:

$$p(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2) \text{ when } -\infty < y_1, y_2 < \infty$$

Definition 5.2

Jointly continuous random variables Y_1 and Y_2:

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f\left(t_{1,} t_2\right) dt_{2,} dt_1$$

Theorem 5.2

If Y_1 and Y_2 have joint distribution function, then:

1. $F(-\infty, -\infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$
2. $F(\infty, \infty) = 1$
3. $f(y1, y2) \geq 0$
4. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f\left(y_{1,} y_2\right) dy_{1,} dy_2$

Definition 5.4

Marginal probability function of Y_1 and Y_2:

$$p_1(y_1) = \sum_{all\ y_2} p(y_1, y_2) \qquad \text{AND} \qquad p_2(y_2) = \sum_{all\ y_1} p(y_1, y_2)$$

Marginal density function of Y_1 and Y_2:

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_2 \qquad \text{AND} \qquad f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1$$

Definition 5.5

Conditional discrete probability function of Y_1 and Y_2:

$$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2) = \frac{P(Y_1 = y_1|Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(y_1, y_2)}{p_2(y_2)} \text{ when } p2(y2) > 0$$

Definition 5.6

Conditional distribution function of Y_1 and Y_2:

$$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2)$$

Definition 5.7

Conditional density of Y_1, given $Y_2 = y_2$

$$f(y_1|y_2) = \frac{f(y_1,y_2)}{f_2(y_2)}$$

Conditional density of Y_2, given $Y_1 = y_1$

$$f(y_2|y_1) = \frac{f(y_1,y_2)}{f_1(y_1)}$$

Definition 5.8

Joint distribution variables Y_1 and Y_2 are independent if and only if

$$F(y_1, y_2) = F_1(y_1)F_2(y_2) \text{ for every } (y_1, y_2).$$

Otherwise, Y_1 and Y_2 are dependent.

Theorem 5.4

Marginal probability variables Y_1 and Y_2 are independent if and only if

$$p(y_1, y_2) = p_1(y_1)p_2(y_2) \text{ for every } (y_1, y_2).$$

Otherwise, Y_1 and Y_2 are dependent.

Marginal density variables Y_1 and Y_2 are independent if and only if

$$f(y_1, y_2) = f_1(y_1)f_2(y_2) \text{ for every } (y_1, y_2).$$

Otherwise, Y_1 and Y_2 are dependent.