Q6.1 $m(a+bX) = a + b \times m(X)$

wtf

transformed to $Y_i = a + bx_i$

$$m(x) = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \text{so} \quad m(a+bx) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

because a & b are constants you can split up the mean

$$\frac{1}{N}\left(\sum_{i=1}^{n} a + \sum_{i=1}^{n} b x_i\right) = \frac{1}{N}\left(na + b\sum_{i=1}^{n} x_i\right)$$

constant so

$$= a + b\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

$$= a + b \cdot m(X)$$

Q 6.2 $\mathrm{cov}(X,X) = s^2$

$$\mathrm{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(X_i - m(X))(X_i - m(X))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(X_i - m(X))^2$$

$= s^2$ by the thing you showed before

Q 6.3 $\mathrm{cov}(X, a+bY) = b \times \mathrm{cov}(X,Y)$

transforms from $y_i = a + bx_i$

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\left[(a+by_i) - m(a+bY)\right]$$

from 6.1 $m(a+bX) = a + b \cdot m(X)$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\left[(a+by_i) - (a + b\cdot m(Y))\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\left[b(y_i - m(Y))\right]$$

$$= b\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))\right]$$

$$= b \cdot \mathrm{cov}(X,Y)$$

Q 6.4 $\text{cov}(a+bX, a+bY) = b^2\text{cov}(X,Y)$, notice $\text{cov}(bX, bX) = b^2X^2$

$$\text{cov}(a+bX, a+bY) = \frac{1}{N}\sum_{i=1}^{n}[(a+bX_i) - m(a+bX)][(a+by_i) - m(a+bY)]$$
$$= \frac{1}{N}\sum_{i=1}^{n}[(a+bX_i) - (a+bm(X))][(a+by_i) - (a+bm(Y))]$$
$$= \frac{1}{N}\sum_{i=1}^{n}[bX_i - bm(X)][by_i - bm(Y)]$$
$$= \frac{1}{N}\sum_{i=1}^{n}[b[X_i - m(X)]][b(y_i - m(Y))]$$
$$= b^2 \underbrace{\frac{1}{N}\sum_{i=1}^{n}(X_i - m(X))(y_i - m(Y))}_{\text{cov}}$$
$$= b^2\text{cov}(X,Y)$$

Q.6.5 True, because med(X) doesn't transform where the median is so its just adding the middle with constants so $\text{med}(a+bX) = a+b\,\text{med}(X)$ for $b>0$, similar to $m(a+bX) = a+bm(X)$

$IQR(a+bX) = a+bIQR(X)$ is False for $b>0$ because

$$IQR(a+bX) = Q_3(a+bX) - Q_1(a+bX)$$
$$= (a+bQ_3(X)) - (a+bQ_1(X))$$
$$= b[Q_3(X)) - Q_1(X)]$$
$$= bIQR(X) \quad \therefore \text{False} \neq a+bIQR(X)$$

for $b>0$

Q.6.6 $X^2, \sqrt{X} \neq (m(X))^2, \sqrt{m(X)}$

Ex. $X = \{1,2,3\}$

$X^2 = \{1,4,9\}$ $\quad (m(X))^2 = (2)^2 = 4$ $\quad 4 \neq \frac{14}{3}$

$\sqrt{X} = \{1, \sqrt{2}, \sqrt{3}\}$ $\quad m(X^2) = \frac{1+4+9}{3} = \frac{14}{3}$ $\quad \therefore$ mean $X^2 \neq (m(X))^2$

$m(X^2)$